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Wormholes Violate Quantum Mechanics in SQUIDS

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 $|\psi(\mathbf{p})|^2 = \delta(\mathbf{p})$

ABSTRACT

We have previously shown that wormholes may cause an observable violation of conventional quantum mechanics and the collapse of the wave function for a macroscopic object containing Avogadro's number $N_A \sim m_P/m_e$ of elementary particles. In this paper we apply this idea to SQUIDS, assuming a Hartle-Hawking wave function. We find a deviation of the coherent oscillations of the SQUID from conventional quantum-mechanical expectations, which can be distinguished from conventional dissipative effects.

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It has been suggested for some time that quantum-gravitational effects might destroy quantum-mechanical coherence [1]. This effect is established at the macroscopic level of black holes, but it is not yet clear whether quantum topological fluctuations cause a breakdown of coherence at the microscopic level. This lack of clarity is due to the absence of a reliable calculational scheme for quantum gravity. Such a framework should in principle be provided by string theory, but in practice...? The Euclidean approach to semiclassical calculations of at least some quantum gravitational effects leads to the wormhole calculus [2], but even within this approach the possible breakdown of coherence is still controversial [3,4].

In spite, or maybe because, of this theoretical confusion, we think it is interesting to investigate the circumstances under which such a microscopically-induced loss of coherence might become observable. Some time ago, it was shown [5] how such an effect would suppress off-diagonal matrix elements in a simple two-state system, and it was argued that $K - K$ mixing and neutron interferometry both gave comparable stringent upper bounds on non-Hamiltonian terms in the evolution equation for the density matrix. Recently, using in particular the wormhole calculus [2], we have argued [6] that quantum gravitational effects could cause a collapse of the wave function in position space which would be undetectably slow for microscopic particles, but could become important for macroscopic objects such as Schrödinger's cat.

Realizations of Schrödinger's cat by means of superconducting devices that exhibit quantum-mechanical behaviour on a macroscopic scale have been extensively discussed [7]. Based on superconducting rings with Josephson junctions (SQUIDs), these contain large numbers of microscopic particles, and one would therefore expect them to be good places to look for the possible microscopically-induced breakdown of quantum coherence. Indeed, we find that quantum gravity may well cause an observable loss of quantum coherence on a time scale of about 10^3 s.

Following Leggett in Ref. [7], we consider an RF SQUID with inductance $L \sim 1$ nH, ohmic resistance $R \sim 10^6 \Omega$ and capacitance $C \sim 0.1$ pF. The effective potential for the scalar phase parameter ϕ can be set up as a double well, so there are two distinct macroscopic states $\phi_{\pm}$, and the tunnelling frequency between them $\sim 10^6$ Hz. The time constant for the suppression of off-diagonal terms in the two-state density matrix by environmental coupling and thermal

fluctuations is $\sim 10^{-4}$ sec. We assume a Hawking-Hartle [8] ground state for the Universe, which is an eigenstate of the wormhole number, and a minimal gauge-invariant wormhole-matter coupling. After integration over the wormhole states, we find that the gauge coupling is not stochastic, but there is a new gauge-invariant non-local interaction. This induces a suppression of off-diagonal terms in the SQUID density matrix by a factor $e^{-\beta^2 l^2 N^2}$ where $N \sim 10^{19}$ is the number of electrons in the SQUID,

$$\beta \simeq \left(\frac{e^2}{2m_p} \right) \omega_0^2 \quad (1)$$

and $\omega_0 \sim 10^{11}$ Hz is the frequency of small oscillations, so that $\beta N \sim 10^{-3} s^{-1}$. The diagonal elements of the density matrix do not deviate from conventional quantum mechanical predictions. The quantum gravitational loss of coherence may be distinguished from environmental effects [9] by its different time dependence.

We work in the framework of the phenomenological wormhole calculus [2], according to which the effective action at length scales $\gg$ the wormhole size is

$$S = S_0[\phi] + \lambda(a + a^\dagger) S_I[\phi] \quad (2)$$

where $S_0(S_I)$ is the action without (induced by) wormholes, $a^\dagger(a)$ is the wormhole creation (annihilation) operator, and the λ are the wormhole couplings. For small couplings $\lambda \ll 1$, the Euclidean partition function is

$$Z_E = \langle \Omega | \langle \omega | \int d[\phi] e^{-(S_0[\phi] + \lambda(a + a^\dagger) S_I[\phi])} | \omega \rangle | \Omega \rangle \quad (3)$$

where $|\Omega\rangle$ ($|\omega\rangle$) is the ground state of the field theory (wormholes). If we denote the eigenstates of $\hat{A} \equiv (a + a^\dagger)$ by $|\alpha\rangle$ ($|\alpha\rangle \equiv |\alpha\rangle$), then

$$Z_E = \langle \Omega | \int d[\phi] \int d\alpha P(\alpha) e^{-(S_0 + \lambda\alpha S_I)} | \Omega \rangle = \int d\alpha P(\alpha) Z(\alpha) \quad (4)$$

where $P(\alpha) \equiv \langle \alpha | \omega \rangle^2$ and $Z_E(\alpha)$ is the partition function with shifted coupling constants $\{g\} \rightarrow \{g + \lambda\alpha\}$.

There are two operators in the wormhole calculus which can be identified with physical observables, namely $\hat{A} = (a + a^\dagger)$ and the number operator $\hat{N} \equiv a^\dagger a$. There are two corresponding natural candidates for the wormhole vacuum state $|\omega\rangle$:

(I) $|\omega\rangle \equiv |0\rangle$ is the no-wormhole Hartle-Hawking [8] ground state, one of the eigenstates $|n\rangle$ of the number operator $\hat{N}$. This possibility seems to be favoured by string theory [10].

In this case, using $\langle\omega|0\rangle = \frac{1}{\sqrt{\pi}} \exp(-\frac{\alpha^2}{2})$, we would have

$$\begin{aligned} Z_E &= \langle\omega| \int d[\phi] \frac{1}{\pi} \int d\alpha e^{-\frac{\alpha^2}{2}} e^{-iS_0 - \frac{1}{2}\lambda^2 S^2} |\Omega\rangle \\ &= \langle\omega| \int d[\phi] e^{-S_0 - \frac{1}{2}\lambda^2 S^2} |\Omega\rangle \end{aligned} \quad (5)$$

which becomes

$$Z_E = \langle\omega| \int d[\phi] e^{-iS_0 - \frac{1}{2}\lambda^2 S^2} |\Omega\rangle \quad (6)$$

in the Lorentz metric. The non-local term in the effective action (5), (6) resembles the “influence functional” of Feynman and Vernon [FV] [11]. FV studied a system (with co-ordinates q) coupled to an infinite set of harmonic oscillators (with co-ordinates x_i) via $H_I = \lambda \sum_i x_i q$. The influence functional was then obtained by integrating out the oscillator co-ordinates. The couplings to the wormhole operators in (2) are very analogous, and the physics is the same: the $q(\phi)$ system loses coherence as it evolves from a pure state to a mixture. Note, however, one difference: in the FV theory the oscillator co-ordinates were time-dependent, and the time-dependent couplings led to energy dissipation in the system, whereas in the wormhole case a and $a^\dagger$ are independent of space and time, so energy momentum is always conserved. The only signature of coherence loss is the associated increase in entropy.

(II) $|\omega\rangle \equiv |\alpha\rangle$ is an $\hat{A}$ eigenstate, so that $P(\alpha) = \delta(\alpha - \alpha_0)$ and the partition function reduces to $Z(\alpha_0)$, the only effect of wormholes is the shift $\{g\} \rightarrow \{g + \lambda\alpha_0\}$ in the couplings, and there is no loss of quantum coherence [3].

As long as there is a non-trivial summation over α in the partition function (4), there will be some loss of coherence: this is absent only in the extreme case (II). The rate of loss of coherence depends, however, on the form of $P(\alpha)$: in Ref. [6], we assumed $P(\alpha) = \text{constant}$

and found that off-diagonal density matrix elements had a suppression $\propto e^{-\alpha^2}$. If we take the Hartle-Hawking [8] case (I): $P(\alpha) = \frac{1}{\sqrt{\pi}} e^{-\frac{\alpha^2}{2}}$, we find a suppression $\propto e^{-\beta^2 \ell^2 N^2}$, as will be shown later in this paper. Hartle and Hawking [8] have motivated their choice $|0\rangle$ of wave function by showing that it is the lowest energy eigenstate satisfying the Wheeler-DeWitt equation. Coleman, on the other hand, has argued that the wormhole state is forced into one particular $\hat{A}$ eigenstate $|\alpha\rangle$ for which the effective coupling constant $\Lambda_{eff} \rightarrow 0$ [3]. It is not clear whether this idea is technically correct, still less whether there is a unique state $|\alpha\rangle$: $\Lambda_{eff} = 0$. If not, coherence will still be lost. For definiteness, in what follows we use the Hartle-Hawking wave function $|0\rangle$.

We will now discuss the superconducting device [7] for which we expect a measurable loss of coherence. The reason we look at superconductivity is because the BCS ground state $|\Omega\rangle$ contains a macroscopic number ($N \sim 10^{19}$) in the same quantum state. The Green function $G(x, x')$ exhibits a long-range correlation:

$$G(x, x') \equiv \langle\Omega| T[\psi^\dagger(x)\psi(x')] |\Omega\rangle \rightarrow n_0 \quad (7)$$

for large $(x - x')$. Therefore the effect of the non-local term in Eq. (6), whose coupling $\lambda \sim 0(\frac{1}{m^2})$, may be enhanced by factors of the condensate density n_0 (7). Although the property (7) also holds for other spontaneously broken symmetries, superconductivity is the only laboratory phenomenon involving the macroscopic number of particles needed to make the loss of coherence observable*

All the macroscopic phenomena associated with superconductivity, such as infinite conductance, the Meissner effect, flux quantization and the Josephson effect can be simply explained from the fact that there is a spontaneous breaking of the $U(1)$ electromagnetic gauge symmetry to Z_2 [13]. In the BCS theory this is caused by Cooper pairing:

$$\langle\Omega| \psi_\alpha^{(+)}(x)\psi_\beta^{(+)}(x') |\Omega\rangle \equiv F^{(+)}(x, x') \epsilon_{\alpha\beta} \quad (8)$$

in two-component spinor notation, and the original $U(1)$ symmetry: $\psi(x) \rightarrow e^{i\Lambda(x)}\psi(x)$ is

* For example, laboratory QCD experiments only involve a microscopic number of Goldstone particles (pions). However, this might not be true in astrophysical situations such as the interiors of neutron stars [12].

broken to Z_2 : $\Lambda(x) = 0, \frac{\pi}{2}$. Associated with this spontaneous symmetry breaking is a Goldstone boson $\phi(x)$ which transforms according to $U(1)/Z_2$, i.e., $\phi(x) \rightarrow \phi(x) + \Lambda(x)$: $\phi(x) = \phi(x) + \frac{\pi\Phi}{2}$. Using $\phi(x)$, one can define gauge-invariant fermion fields $\tilde{\psi}(x) \equiv e^{i\phi(x)}\psi(x)$. The macroscopic phenomena listed above can be derived [13] from the classical equations of motion of the Goldstone field, the electromagnetic field $A_\mu(x)$ and the matter fields $\tilde{\psi}(x), \tilde{\psi}^\dagger(x)$, assuming a general Lagrangian of the form

$$\mathcal{L}_{sc}(\Lambda^\mu - \partial_\mu \phi, \tilde{\psi}, \tilde{\psi}^\dagger) \quad (9)$$

which is explicitly gauge-invariant. We see from Eq. (8) that the phase $\theta(x)$ of the local Cooper pair wave function $\Psi(x) \equiv e^{i\theta(x)}|\Psi(x)|$ is simply given by the Goldstone field: $\theta(x) = 2e\phi(x)$.

In a simply-connected, time-independent superconductor the field $\phi(x)$ can be gauged away by making the gauge transformation: $A_\mu(x) \rightarrow A_\mu(x) + \partial_\mu \Lambda(x)$ where $\Lambda(x) = -\phi(x)$. However, in multiply-connected regions, such as a superconducting ring, $\phi(x)$ can jump by a multiple of $\frac{\pi}{e}$ whereas $\Lambda(x)$ must be continuous. This leads to flux quantization: the magnetic flux $\Phi = \Delta\phi = \frac{\pi\pi}{e}$, and the Goldstone field $\phi(x)$ is a real dynamical variable. If we consider two homogeneous superconductors with a gap, the effective Lagrangian for the change $\Delta\phi(t)$ in ϕ across the junction must take the general form $L_J = f[\Delta\phi(t)]$, where the function f must be invariant under $\Delta\phi \rightarrow \Delta\phi + \frac{\pi\pi}{e}$, and hence can be written as

$$L_J = I_0 \cos(2e\Delta\phi(t)) + \text{higher harmonics} \quad (10)$$

A SQUID consists of such a superconducting ring with an insulating gap of thickness $\sim 20\text{\AA}$ and may be modelled by a shunt resistance R , capacitance C and inductance L as shown in Fig. 1. If an external magnetic flux Φ_{ex} is applied, then the total flux is $\Phi = \Phi_{ex} + LJ$ and the effective Hamiltonian for the system is [7]

$$H = \frac{1}{2}C\dot{\Phi}^2 + \frac{1}{2L}(\Phi - \Phi_{ex})^2 - \frac{I_0\Phi_0}{2\pi} \cos\left(\frac{2\pi\Phi}{\Phi_0}\right) \quad (11)$$

where the first term represents the energy of the electric field in the gap, the second is the energy of the magnetic field through the ring, I_0 is the critical current that is characteristic

of the junction, and $\Phi_0 = \frac{1}{2e} = 2 \times 10^{-15}$ Webers. If we set $\Psi_{ex} = \frac{\Phi_0}{2}$ and choose the circuit parameters: $1 < \frac{2\pi LJ}{\Phi_0} < \frac{5}{2}\pi$, then the effective potential has the double-well form shown in Fig. 2. The two classical solutions $\Phi_\pm = \Phi_{ex} \pm \Phi$: $|\dot{\Phi}| < \frac{\Phi_0}{2}$ corresponds to clockwise (anticlockwise) currents $I = \pm \frac{I_0}{2}$.

When the Goldstone field $\phi(x)$ is quantized, this system can exist in a linear combination $|\psi\rangle = a|\Phi_+\rangle + b|\Phi_-\rangle$, and there is a finite probability for tunnelling between the two classical states:

$$|\langle\Phi_+|\Phi_-\rangle|^2 = \left(\frac{\Delta U}{\omega_0}\right)^{\frac{1}{2}} \exp\left(-\frac{\Delta U}{\omega_0}\right) \quad (12)$$

where $\Delta U = I_0\Phi_0$ is the height of the barrier between the two wells, and $\omega_0 = \frac{2\pi}{\sqrt{LC}}$ is the frequency of small oscillations about the bottom of either well. For typical circuit parameters $L = 1\text{ nH}$, $C = 0.1\text{ pF}$, $I_0 = 10\mu\text{A}$, $\omega_0 = 10^{11}\text{ Hz}$. If the system is prepared in the state $|\Phi_+\rangle$ at time $t = 0$, then according to conventional quantum mechanics at a time t it will be in the pure state $|\psi\rangle = \cos\delta_0 t|\Phi_+\rangle + \sin\delta_0 t|\Phi_-\rangle$ and the density matrix will be

$$\rho(t) = \begin{pmatrix} \cos^2\delta_0 t & \cos\delta_0 t \sin\delta_0 t \\ \cos\delta_0 t \sin\delta_0 t & \sin^2\delta_0 t \end{pmatrix} \quad (13)$$

where

$$\delta_0 = \omega_0 \left(\frac{\Delta U}{\omega_0}\right)^{\frac{1}{2}} \exp\left(-\frac{\Delta U}{\omega_0}\right) \sim 10^6\text{ Hz} \quad (14)$$

Schrodinger's cat has always been a theoretical construct because in practice in any macroscopic quantum system thermal fluctuations mix the states in an unmeasurably short time. The SQUID system is unique in that, at the temperature $T = 10\text{ mK}$ at which the experiment can be performed, the system parameters may be chosen [9] such that $kT \ll \Delta U$ and $\omega_0 \ll \Delta U$, so that the two macroscopic states are not mixed by thermal fluctuations.

In the SQUID the dissipative coupling with the environment can be modelled by a finite shunt resistance R across the insulating gap [9]. This causes current losses and hence suppresses the tunnelling probability, so that the oscillation frequency $\delta_0 \rightarrow \frac{\delta_0}{R}(\Phi_+ - \Phi_-)^2$ and the off-diagonal terms in the density matrix (13) are modulated by a factor $\sim \exp(-\alpha\delta_0 t)$ with the

constant $\alpha = \left(\frac{A\Phi}{\Phi_0}\right)^2 \left(\frac{A}{4\pi}\right) \frac{1}{\hbar} \simeq 10^{-2}$. We will discuss later how this uninteresting effect can be distinguished from the novel wormhole effect that we now estimate.

We model the wormhole-matter coupling by a gauge-invariant term

$$S_I = \int d^4x \mathcal{L}_I(x) : \mathcal{L}_I(x) = \lambda \psi \not{D} \psi = \lambda \psi (\not{\partial} - e \not{A}) \psi \quad (15)$$

with

$$\lambda \sim e^{-S_{\text{wh}}} \left(\frac{m_e}{m_P}\right) \quad (16)$$

where $S_{\text{wh}} \simeq l_{\text{wh}}^2 m_P^2$ is the wormhole action. We shall assume that the wormhole size $l_{\text{wh}} \sim m_P^{-1}$ so that $S_{\text{wh}} \sim 1$ and $\lambda \sim 10^{-22}$. We shall estimate the rate of coherence loss in Eq. (6) by calculating the magnitude of the non-local term

$$W = \langle \Omega | \int d^4x \int d^4y \lambda^2 (\psi(x) \not{D} \psi(x) \psi(y) \not{D} \psi(y) | \Omega \rangle \quad (17)$$

in the BCS ground state. Taking the non-relativistic limit of $\mathcal{L}_I(x)$ (15), we obtain

$$\mathcal{L}_I(x) = \lambda \sum_{\alpha} \left[\frac{1}{2} \nabla \psi_{\alpha}^{\dagger} \cdot \nabla \psi_{\alpha} - \frac{e}{2m_e} (\psi_{\alpha}^{\dagger} (\nabla \psi_{\alpha}) - (\nabla \psi_{\alpha}^{\dagger}) \psi_{\alpha}) \cdot \mathbf{A} - \frac{e^2}{2m_e} \mathbf{A}^2 \psi_{\alpha}^{\dagger} \psi_{\alpha} + 0\left(\frac{v^2}{c^2}\right) \right] \quad (18)$$

The terms $\alpha \nabla \psi_{\alpha}$ do not contribute in the BCS ground state, so we are left with the London

$$\text{term} \quad \mathcal{L}_I(x) \simeq \frac{e^2}{2m_e} \mathbf{A}^2 \psi_{\alpha}^{\dagger} \psi_{\alpha} \quad (19)$$

and hence

$$W \simeq -\lambda^2 \left(\frac{e^2}{2m_e}\right)^2 \int d^4x \int d^4y \sum_{\alpha, \beta} \langle \Omega | \psi_{\alpha}^{\dagger}(x) \psi_{\alpha}(x) \psi_{\beta}^{\dagger}(y) \psi_{\beta}(y) \mathbf{A}^*(x) \cdot \mathbf{A}(x) \mathbf{A}^*(y) \cdot \mathbf{A}(y) | \Omega \rangle \quad (20)$$

We evaluate the integrand in the Hartree approximation by making pairwise contractions, either

via the Cooper pairing [13] of electrons of opposite spin, or via the conventional Green functions

$$G(x, x') \equiv \langle \Omega | \psi_{\alpha}^{\dagger}(x) \psi_{\beta}(x') | \Omega \rangle = \delta_{\alpha\beta} G(x - x') \quad (21)$$

(and its complex conjugate) for electrons of the same spin. The corresponding Green functions for the gauge-invariant spinors $\tilde{\psi}(x) = e^{-ie\phi(x)} \psi(x)$ are $\tilde{G}(+)$ and $\tilde{F}(+) = e^{\pm 2ie\phi} F(+)$, reflecting the gauge-invariance (-variance) of the $G(F)$ functions.

The F contraction of $W(20)$ is

$$\begin{aligned} W_I &\equiv -\lambda^2 \left(\frac{e^2}{2m_e}\right)^2 \int d^4x \int d^4y \sum_{\alpha, \beta} \underbrace{\psi_{\alpha}^{\dagger}(x) \psi_{\alpha}(x) \psi_{\beta}^{\dagger}(y) \psi_{\beta}(y)}_{\text{F contraction}} \underbrace{\mathbf{A}^*(x) \cdot \mathbf{A}(x) \mathbf{A}^*(y) \cdot \mathbf{A}(y)}_{\text{F contraction}} \\ &= \lambda^2 \left(\frac{e^2}{2m_e}\right)^2 \int d^4x \int d^4y \cdot 2 \sum_{\mathbf{k}, \mathbf{k}'} e^{-i2e\phi} \tilde{F}^{\dagger}(\mathbf{k}) e^{i2e\phi} \tilde{F}(\mathbf{k}') e^{i(\mathbf{k}-\mathbf{k}') \cdot (x-y)} \epsilon_{\gamma}^2 \end{aligned} \quad (22)$$

where we have expressed the F function in momentum space and ϵ_{γ}^2 is the number density (#/volume-time) of the photons. Terms involving $\mathbf{A}(x) \cdot \mathbf{A}(y)$ contractions are suppressed by the photon mass in the BCS ground state. The Cooper pair wave function is sharply peaked at $\mathbf{k} \simeq \mathbf{k}' \simeq \mathbf{k}_f$, the Fermi momentum, so that the $(x - y)$ -dependent exponential factor drops out. The number density of Cooper pairs is

$$\sum_{\mathbf{k}} F(\mathbf{k}) = \frac{N}{V} \quad (23)$$

and therefore the F term contribution to the wormhole interaction is

$$W_I(t) = -2\lambda^2 \left(\frac{e^2}{2m_e}\right)^2 t^2 \exp(-2ie\phi) N \exp(2ie\phi) N \quad (24)$$

Note that we have evaluated all the bilinear terms at equal times, so that the time-dependence

comes only from the time integration. There is also a G contraction term in W :

$$\begin{aligned} W_2 &= \lambda^2 \left(\frac{e^2}{2m_e} \right)^2 \int d^4x \int d^4y \sum_{\alpha, \beta} \underbrace{\psi_\alpha^\dagger(x) \psi_\alpha(x) \psi_\beta^\dagger(y) \psi_\beta(y)}_{\text{contraction}} \Lambda^*(x) \Lambda^*(y) \Lambda(y) \Lambda(y) \\ &= 2\lambda^2 \left(\frac{e^2}{2m_e} \right)^2 \int d^4x \int d^4y \sum_{k, k'} \tilde{G}(k) \tilde{G}(k') e_\gamma^2 \end{aligned} \quad (25)$$

Using the properties of the BCS ground state as before, we arrive at

$$W_2(t) = 2\lambda^2 \left(\frac{e^2}{2m_e} \right)^2 e_\gamma^2 N^2 \quad (26)$$

The total wormhole-induced interaction is the sum of (24) and (26):

$$W(t) = -\lambda^2 \left(\frac{e^2}{2m_e} \right)^2 e_\gamma^2 t^2 [N^2 - \exp(-2ic\phi) N \exp(2ic\phi) N] \quad (27)$$

whose consequences we now explore.

In classical superconductivity, the Goldstone field ϕ and the electron number N are c-numbers, so that $W(t) = 0$ classically and wormholes have no effect. However, when ϕ , or in this case the flux Φ which is equivalent to it, is quantized, then the operators ϕ and N do not commute and $W(t) \neq 0$ in general. To evaluate $W(t)$ (27), it is not correct to use as commutation relation $[\phi, N] = i$, since, being periodic, ϕ is not a Hermitian operator. In promoting φ to the operator $\hat{\phi}$ we face the usual operator ordering ambiguity. This could be serious since our whole result is based on studying the consequence of promoting a classical field φ to the quantum operator $\hat{\phi}$. We resolve this problem by choosing the unique operator ordering in which the conserved charges $\hat{Q}(\phi)$ of the original Hamiltonian $\hat{H}$ are also conserved in the wormhole extension: $\hat{H}_0 = \hat{H}_0 + \hat{\omega}$. Formally, the time evolution of the density matrix is determined by

$$\rho(t) = e^{-itH_0 + iW} \rho(0) e^{itH_0 + iW^\dagger} \quad (28)$$

where H_0 is the conventional quantum-mechanical Hamiltonian. Since wormholes conserve

energy, we have

$$\rho(t) = e^{i\hat{W}} [e^{-itH_0} \rho(0) e^{itH_0}] e^{i\hat{W}^\dagger} \quad (29)$$

where the quantity in square brackets is just the conventional density matrix (13). Taking the time derivative of (29) we obtain the non-linear Schrödinger equation obeyed by $\hat{\rho}(t)$:

$$\dot{\hat{\rho}}(t) = i[\hat{H}_0, \hat{\rho}] + i\hat{W} \dot{\hat{\rho}} + \dot{\hat{W}} \hat{\rho}^\dagger \quad (30)$$

Making our choice of operator ordering and keeping only the leading order term in (27), we obtain the modified Schrödinger equation

$$\dot{\hat{\rho}}(1) = i[\hat{H}_0, \hat{\rho}] - 4\beta^2 N^2 t [\hat{\Phi}, [\hat{\Phi}, \rho]] \quad (31)$$

where

$$\beta^2 = \lambda^2 \left(\frac{e^2}{2m_e} \right)^2 e_\gamma^2 \quad (32)$$

In the $|\Phi\rangle$ representation, Eq. (31) reads

$$\dot{\rho}(\Phi, \Phi', t) = i[H_0, \rho] - 4\beta^2 N^2 t (\Phi - \Phi')^2 \rho(\Phi, \Phi', t) \quad (33)$$

which has the solution

$$\rho(\Phi, \Phi', t) = \rho_0(\Phi, \Phi', t) e^{-2\beta^2 N^2 t^2 (\Phi - \Phi')^2} \quad (34)$$

Thus the wormhole interaction suppresses the off-diagonal mixing matrix elements, which are the tunnelling amplitudes between distinguishable macroscopic states. Such behaviour could have been anticipated on general intuitive grounds: the states $|\Phi\rangle$ and $|\Phi'\rangle$ are centred at different values of Φ , but have a spread and hence an overlap which defines the corresponding tunnelling amplitude. As the states become better localized with time, one expects the overlap to diminish.

The density matrix for our two-state system is now given by

$$\rho(t) = \begin{pmatrix} \cos^2 \delta_0 t & \cos \delta_0 t \sin \delta_0 t \exp(-2\beta^2 N^2 t^2) \\ \exp(-2\beta^2 N^2 t^2) \cos \delta_0 t \sin \delta_0 t & \sin^2 \delta_0 t \end{pmatrix} \quad (35)$$

in which we see explicitly the non-quantum-mechanical time evolution of the system. Probability is conserved since $\text{Tr} \rho(t) = 1$, but $\text{Tr} \rho^2(t) < 1$ and hence the system becomes mixed. Hence wormholes collapse the wave function of the system, and the rate of collapse $\sim \exp(-\beta^2 N^2 t)$ means that objects with large numbers of particles behave more like classical objects.

Is our expected collapse (35) of the SQUID density matrix observable? The most uncertain factor in β (32) is the wormhole coupling constant λ , which we take (16) to be $\sim m_e/m_P \sim 10^{-22}$. The number density of the electromagnetic field $\epsilon^2 \sim \omega_0^4$, where, for our choice of parameters: $L = 10^{-9} \text{ m}$ and $C = 10^{-13} \text{ F}$ we have $\omega_0 \sim \frac{2\pi}{\sqrt{LC}} \sim 10^{11} \text{ Hz}$, and $\delta_0 \sim 10^6 \text{ Hz}$. With these parameter choices, $\beta \sim 10^{-23} \text{ s}^{-1}$, which means that the time constant for collapse of the system is $\frac{10^{23}}{N}$. Taking a SQUID ring of volume 1 mm^3 and taking $\frac{1}{N} \sim 10^{22} \text{ cm}^3$ for the density of superconductivity electrons, we have $N \sim 10^{19}$ and hence the collapse time

$$t_c \equiv \frac{1}{\beta N} \simeq 10^3 \text{ s} \quad (36)$$

This is clearly subject to very considerable uncertainties, but may be fast enough to be measurable.

The off-diagonal terms determine the interference between the two states of the system. The magnetic flux through the SQUID ring is a classical observable and unlike *any other* quantum system it is possible to measure the state of the system without destroying the interference pattern. For instance, as suggested by Leggett [14], one may consider measuring the magnetic flux states Φ_I by shooting a beam of polarized neutrons through the ring. The outgoing flux of neutrons is then correlated to the states of the SQUID. The interference pattern of the neutrons which can be measured will have a time dependence given by

$$P(t) = \sin \theta \exp(-\beta^2 N^2 t^2) \exp(-\alpha t) \quad (37)$$

where the angle between the successive minima $\theta = (\frac{2\pi\Phi}{m_n}) B \Delta t$, where $(\frac{2\pi\Phi}{m_n})$ is the neutron magnetic moment, B is the average magnetic field in the SQUID ring and Δt is the time

that neutrons spend in the magnetic field of the SQUID. For a good correlation we must have $\Delta t \ll \delta_0^{-1} (\sim 10^6 \text{ sec})$ which is easily satisfied. The hard part in the experiment would be to separate the Caldeira-Leggett suppression $\exp(-\alpha \delta_0 t)$ with $\alpha \delta_0 \sim 10^{+4} \text{ sec}$ from the wormhole suppression $\sim \exp(-\beta^2 N^2 t^2)$ with $\beta N \sim 10^{-3} \text{ sec}^{-1}$. The most favourable situation would be to devise a SQUID with a high shunt resistance ($R > 1 \text{ m}\Omega$) and a large number of Cooper pairs ($N > 10^{19}$), in which case we would have a small α and a large and hopefully detectable wormhole suppression.

The analysis of this paper (see also Ref. [6]) can be viewed as a derivation of Avogadro's number N_A . This counts the number of elementary particles in one gram of material, which was a unit of mass chosen by classical scientists as a convenient measure for objects that behave classically. The number N of electrons in our macroscopic SQUID is clearly $\propto N_A$, whilst $\beta \propto \frac{\Delta}{m_e} \propto \frac{1}{m_P}$. We see that the system behaves classically on a time-scale $t \sim \frac{1}{\beta N} \propto \frac{m_P}{N_A}$. Fixing the time-scale over which objects behave classically, we find $N_A \propto \frac{m_P}{m_e}$. In other words, large objects behave classically because m_P is large on a particle physics scale, and the fact that $m_P \sim 10^{19} \text{ GeV}$ leads us to expect that $N_A = 10^{23}$ as observed.

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FIGURE CAPTIONS

- 1) The lumped circuit model of the SQUID.
- 2) Sketch of the double-well potential (11).

- Fig. 1 -

- Fig. 2 -