The Vanishing of the Cosmological Constant from Four-Forms*

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Wormholes are not the only objects which can be used by the Baum-Hawking mechanism to neutralize the cosmological constant. A theory containing a four-form can also do the trick. There has been some considerable confusion about the dynamics of four-forms and in this report I provide a resolution to the problems.

1. INTRODUCTION.

A few years ago Baum¹ and Hawking² suggested a very simple and elegant mechanism for the vanishing of the macroscopic cosmological constant. The idea is rather straightforward: the cosmological constant defies its name by varying over a range of values. The BH mechanism provides a measure of probability for these values and selects the most probable value as zero. This would suggest that the universe we inhabit has vanishing cosmological constant. It is also believed that this result should hold to all orders.

*Invited talk at HEP'89, Madrid, to be published in *Nucl. Phys. B - Proceedings of the Europhysics Conference on High-Energy Physics, Madrid, Spain (1989)*. eds. F.Barriero and C.Lopez.

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Ever since physicists began taking the cosmological constant problem seriously, the proposed solutions have tended to be model dependent and have run into difficulties when more closely examined³. Owing to its generality the BH mechanism has not yet suffered this fate. All that is required by the mechanism is that our field theory contains a set of parameters whose values are undetermined by the dynamics. The recent interest in wormholes was prompted⁴ by the fact that in a theory with topological fluctuations the coupling constants and masses of the theory become undetermined. The BH mechanism would then select the values for these parameters in such a way as to neutralize the cosmological constant. I must emphasize that *even if wormholes do not exist*, as has been argued by some authors⁵, the BH mechanism is still generally valid. Wormhole theo-

ries are not the only ones which can take advantage of the mechanism.

An example of a theory which also allows application of the BH mechanism is one containing a four-form. Such an example was given in Hawking's original paper². Duff revived the discussion of this system⁶ by pointing out an omission in the original analysis, and which seemed to contradict the BH mechanism. However, there was also a flaw in this analysis as was recently shown by Lars Jensen and myself⁷. It turns out that the dynamics and statics of a four-form are more subtle than previously appreciated. Nevertheless, careful consideration of their properties shows that the BH mechanism does still work. In this note I would like to communicate the results of our analysis and refer the reader to the literature for details of the debate.

2. CHOOSING THE CORRECT EUCLIDEAN ACTION.

In discussing the BH mechanism one relies on the precepts of Euclidean space quantum field theory. In itself this is suspect for gravitational systems as the Einstein-Hilbert action is unbounded from below, corresponding to the fact that gravity is always an attractive force. Of course, our universe has a Lorentzian signature and we may question the validity of any results obtained from an *a priori* Euclidean analysis. However, for the moment let us concentrate on the problem as it was originally posed. The Lorentzian case will be briefly mentioned in closing.

To make matters simple, let us consider the fol-

lowing actions for a four-form on an open Euclidean manifold $\mathcal{M}$,

$$S_1 = \frac{1}{2} \int_{\mathcal{M}} \mathbf{F} \wedge \star \mathbf{F} \quad (1)$$

$$S_2 = \frac{1}{2} \int_{\mathcal{M}} \mathbf{F} \wedge \star \mathbf{F} - \int_{\mathcal{M}} \mathbf{d}(\mathbf{A} \wedge \star \mathbf{F}) \quad (2)$$

We are assuming that $\mathbf{F}$ is the curl of a three-form potential $\mathbf{A}$ i.e. $\mathbf{F} = \mathbf{dA}$. We have adopted the notation of forms in order to avoid a proliferation of indices. It also allows us to see clearly the geometry at work.

The first action is stationary under variations of $\mathbf{A}$, with the potential held fixed on the boundary $\partial\mathcal{M}$, when $\mathbf{d}\star\mathbf{F} = 0$. The solution to this equation of motion is that $\mathbf{F}$ is the dual of a constant,

$$\mathbf{F} = \star\phi \quad (3)$$

In the case of action (1) the constant ϕ is *determined* by the initial and final boundary values of the potential,

$$\int_{\mathcal{M}} \mathbf{F} = \int_{\partial\mathcal{M}_f} \mathbf{A}_f - \int_{\partial\mathcal{M}_i} \mathbf{A}_i = \phi \int_{\mathcal{M}} \eta \quad (4)$$

where $\eta = \star 1$ is the volume element. With knowledge of the potential on the boundaries and the intervening volume we have determined ϕ . The stationary value of the first action is

$$\langle S_1 \rangle = \frac{1}{2} \phi^2 \int_{\mathcal{M}} \langle \sqrt{g} \rangle d^4x \quad (5)$$

where ϕ is given by (4).

Action (2) is slightly different due to the second (boundary) term. This action is stationary under variations of $\mathbf{A}$, but this time holding the field strength $\mathbf{dA}$

fixed on the boundary. The solution in this case is that $\phi = \star dA_f = \star dA_i$ and it is up to us to specify its value. Thus ϕ is *undetermined* by the equations of motion, just what is required by the BH mechanism. The stationary value of this action is

$$\langle S_2 \rangle = -\frac{1}{2}\phi^2 \int_{\mathcal{M}} \langle \sqrt{g} \rangle d^4x \quad (6)$$

with a flip in sign due to the boundary contribution. This sign is all-important as we shall show.

The path integrals for the amplitudes between configurations of fixed potential and fixed field strength are respectively,

$$\langle A_f | A_i \rangle = \int_i^f \mathcal{D}A \exp(-S_1) \sim \exp(-\langle S_1 \rangle) \quad (7)$$

$$\begin{aligned} \langle \star dA_f | \star dA_i \rangle &= \int_i^f \mathcal{D}A \exp(-S_2) \\ &\sim \delta(dA_f - dA_i) \exp(-\langle S_2 \rangle) \end{aligned} \quad (8)$$

In the former case ϕ is given by (4). The latter may be rewritten as

$$\langle \phi | \phi \rangle \sim \exp(-\langle S_2 \rangle) \quad (9)$$

and the BH mechanism will select which value of ϕ labels the vacuum.

3. NEUTRALIZING THE COSMOLOGICAL CONSTANT.

The complete set of equations of motion, gravity plus matter, are

$$\begin{aligned} \langle G_{\mu\nu} \rangle &= 8\pi G_N \langle T_{\mu\nu} \rangle - \Lambda_0 \langle g_{\mu\nu} \rangle \\ \langle d \star F \rangle &= 0 \end{aligned} \quad (10)$$

where $\langle T_{\mu\nu} \rangle$ is the background Euclidean stress tensor for the four-form,

$$T_{\mu\nu} = \frac{1}{4!} \left(4F_{\mu\lambda\rho\sigma} F_{\nu}{}^{\lambda\rho\sigma} - \frac{1}{2} g_{\mu\nu} F_{\lambda\rho\sigma\tau} F^{\lambda\rho\sigma\tau} \right) \quad (11)$$

This is the same for both actions, since boundary terms do not contribute to the equations of motion. Λ_0 is any bare cosmological constant in the original gravitational action. With the solution $\langle F \rangle = \langle \star \phi \rangle$ the stress tensor background is

$$\langle T_{\mu\nu} \rangle = \frac{1}{2}\phi^2 \langle g_{\mu\nu} \rangle \quad (12)$$

Therefore the cosmological constant from (10) is $\Lambda_{\text{cos}} = \Lambda_0 - 4\pi G_N \phi^2$ and the background curvature scalar is $\langle R \rangle = 4\Lambda_{\text{cos}}$.

The gravitational action is the familiar

$$S_G = -\frac{1}{16\pi G_N} \int_{\mathcal{M}} R \sqrt{g} d^4x + \frac{\Lambda_0}{8\pi G_N} \int_{\mathcal{M}} \sqrt{g} d^4x \quad (13)$$

Inserting the background solution for R and adding the background actions (5) and (6) we find the total actions are

$$\langle S_1^{\text{tot}} \rangle = \left(-\frac{\Lambda_{\text{cos}}}{8\pi G_N} + \phi^2 \right) V \quad (14)$$

$$\langle S_2^{\text{tot}} \rangle = -\frac{\Lambda_{\text{cos}}}{8\pi G_N} V \quad (15)$$

where V is the volume of the space. Closing the space to a four-sphere gives $V = 24\pi^2/\Lambda_{\text{cos}}^2$.

The semi-classical approximation to the action is $\exp(-\langle S^{\text{tot}} \rangle)$ and is a field theoretic measure of the probability for such a configuration. As $\Lambda_{\text{cos}} \rightarrow 0_+$ we find that $\exp(-\langle S_1^{\text{tot}} \rangle) \rightarrow 0$, $\exp(-\langle S_2^{\text{tot}} \rangle) \rightarrow \infty$. Duff began from action (1) and obtained his counterexample to the BH mechanism. However, if we use action (2) we find that the most probable configuration for $|\phi\rangle$ is such that the cosmological constant vanishes.

We can now see the flaws in the previous arguments by Hawking and Duff. Hawking² began with action (1) and solved the matter equations of motion. He then substituted the solution back into the action and varied the metric to find the gravitational equations of motion. However, as Duff was to emphasize⁶, this procedure is wrong: we should solve the gravitational and matter equations of motion *simultaneously* and then evaluate the stationary value of the action. This final value is different for the two procedures.

I have shown here that Duff's analysis is also incomplete - he began from the wrong action. We really ought to begin from the action (2). This choice is justified by the fact that it makes the vacuum state undetermined by the dynamics of the system, precisely what is required by the BH mechanism. I might add that if we had started from an *a priori* closed Euclidean manifold then proper consideration of the topology⁷ would have forced the total action into the form (15).

We have to keep in mind that the universe is actually Lorentzian. The Euclidean analysis is just a calculational aid to estimate amplitudes. We have to respect as many properties of the original Lorentzian theory before making our rotation. It turns out⁷, that there is a conserved charge in the theory which labels the vacuum. If we Wick rotate to the pseudo-Euclidean space, while still respecting this charge conservation, we find that the dynamics is again equivalent to that of action (2).

4. SUMMARY.

The cosmological constant is probably zero and the proof is probably right, but for the wrong reasons.

REFERENCES.

1. E.Baum, Phys. Lett. **133B** (1983) 185.
2. S.W.Hawking, Phys. Lett. **134B** (1984) 403.
3. S.Weinberg, Rev. Mod. Phys. **61** (1989) 1.
4. S.Coleman, Nucl. Phys. **B310** (1988) 643;
I.Klebanov *et. al.*, Nucl. Phys. **B317** (1989) 665.
5. J.Polchinski, Phys. Lett. **219B** (1989) 251;
W.G.Unruh, Phys. Rev. **D40** (1989) 1053.
6. M.J.Duff, Phys. Lett. **226B** (1989) 36.
7. M.J.Duncan and L.G.Jensen, CERN-TH.5520/89.