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UNIFIED-GAUGE FORMALISM AT TWO LOOPS FOR A CLASS OF PHYSICAL GAUGES

G. Leibbrandt^{*)}

CERN - Geneva

A B S T R A C T

A unifying prescription for axial-type gauges (light-cone, axial, temporal) is tested in Yang-Mills theory to two-loop order by evaluating a self-energy diagram and analyzing its corresponding pole structure. The calculation is free of ambiguities, but features a large number of overlapping divergences.

^{*)} Permanent address: Department of Mathematics and Statistics, University of Guelph, Guelph, Ontario, Canada N1G 2W1.

In massless Yang-Mills theory, with Lagrangian density

$$L_{YM} = -\frac{1}{4}(F_{\mu\nu}^a)^2 - \frac{1}{2\alpha}(n \cdot A^a)^2, \quad \alpha \rightarrow 0, \quad (1.1)$$

the axial-type gauges (pure axial, light-cone, temporal) are defined by

$$n_\mu^a = 0, \quad \mu = 0, 1, 2, 3, \quad n^2 = n_0^2 - \vec{n}^2, \quad (1.2)$$

where $n_\mu = (n_0, \vec{n})$ is a fixed vector and A_μ^a a massless Yang-Mills field,

$a = 1, 2, \dots, N^2-1$ for $SU(N)$; α is the gauge parameter. We use dimensional regularization and a $(+, -, -, -)$ metric. The propagator reads (for $\alpha = 0$)

$$G_{\mu\nu}^{ab} = -\frac{i\delta^{ab}}{(q^2+i\epsilon)} \left[\delta_{\mu\nu} - \frac{(q_\mu n_\nu + q_\nu n_\mu)}{q \cdot n} + \frac{n^2 q_\mu q_\nu}{(q \cdot n)^2} \right], \quad \epsilon > 0, \quad (1.3)$$

and the four-gluon vertex of zero-loop order

$$\begin{aligned} v_{\mu\lambda\rho\sigma}^{abcd} = & -ig^2 \left[f^{abec} f^{dcde} (\delta_{\mu\sigma} \delta_{\lambda\rho} - \delta_{\mu\rho} \delta_{\lambda\sigma}) + f^{ace} f^{bde} (\delta_{\mu\lambda} \delta_{\sigma\rho} - \delta_{\mu\rho} \delta_{\lambda\sigma}) \right. \\ & \left. + f^{ade} f^{cbe} (\delta_{\mu\sigma} \delta_{\lambda\rho} - \delta_{\mu\lambda} \delta_{\sigma\rho}) \right]. \end{aligned} \quad (1.4)$$

The poles of $(q \cdot n)^{-\beta}$, $\beta = 1, 2$, were handled originally by the principal-value (PV) prescription [3, 4, 6-9],

$$PV \frac{1}{q \cdot n} = \frac{1}{2} \lim_{\mu \rightarrow 0} \left[\frac{1}{q \cdot n + i\mu} + \frac{1}{q \cdot n - i\mu} \right], \quad \mu > 0,$$

but more recently by the unifying-gauge prescription for axial-type gauges, namely [10-13]

$$\frac{1}{q \cdot n} = \lim_{\epsilon \rightarrow 0} \frac{q \cdot n^*}{q \cdot n q \cdot n^* + i\epsilon}, \quad \epsilon > 0. \quad (1.5)$$

Formula (1.5) is a generalization of the prescription for the light-cone gauge developed independently by Mandelstam and the author. The noncovariant vector n_μ and its dual, n_μ^* , have the following structure:

$$n_\mu = \begin{cases} n_\mu^{lc} = (|\vec{n}|, \vec{n}), & n_0^2 = \vec{n}^2, \\ n_\mu^{ax} = (|\vec{n}_\perp|, \vec{n}), & n_0^2 < \vec{n}^2, \\ n_\mu^{temp} = (n_0, \vec{n}_\perp, -i|\vec{n}_\perp|), & n_0^2 > \vec{n}^2; \end{cases} \quad (1.6a)$$

1. Introduction

Noncovariant gauges of the axial type [1-4], such as the light-cone gauge, the temporal gauge, the pure axial gauge and the planar gauge, continue to gain in popularity [5], even though the associated Feynman integrals are trickier to handle than covariant-gauge integrals. The chief culprits, of course, are the unphysical poles of $(q \cdot n)^{-\beta}$, $\beta = 1, 2, 3, \dots$, which have been handled [6-9] in recent years with a unifying-gauge prescription [10-13]. As the name suggests, the latter incorporates all four gauges of the axial type. The unifying prescription generalizes the technique for the light-cone gauge that was developed independently by Mandelstam [14] and the present author [15] in the early 1980's.

During the past few years, the unified-gauge formalism has been applied to theories of varying complexity and found to give consistent results at the one-loop level [16-27]. A partial test at the two-loop level was carried out by Hüffel, Landshoff and Taylor who showed that the time-dependence of the Wilson-loop exponentiates to order g^4 [28]. In view of the advantages of a unified-gauge prescription for perturbation theory, I decided to test the formalism in detail at the two-loop level. Specifically, I shall work with the Yang-Mills self-energy diagram in Fig. 1, often called the sunset graph.

$$\left\{ \begin{aligned} (n_\mu^*)^* &= (|\vec{n}|, -\vec{n}), \\ n_\mu^* &= \begin{cases} (n_\mu^*)^* = (|\vec{n}_\perp|, -\vec{n}), \\ (n_\mu^*)^* = (n_0, \vec{n}_\perp, i|\vec{n}_\perp|). \end{cases} \end{aligned} \right. \quad (1.6b)$$

As already indicated, the unifying prescription (1.5) is valid for the pure axial, light-cone, temporal and planar gauge and yields satisfactory results to one loop [16-27].

The purpose of the present calculation is to test the unifying

prescription (1.5) in detail for the two-loop self-energy diagram, shown in

Fig. 1. A partial test had already been carried out in 1989 by Huffer, Landshoff and Taylor [28] on the Wilson loop.

Of special importance and concern are the following points:

- (i) Can all two-loop integrals be calculated explicitly? In particular, can we obtain the coefficients of the single and double poles? What about the finite parts?
- (ii) Are any of the integrations ambiguous?
- (iii) Are there any unexpected technical problems?
- (iv) Do new integrals arise at the two-loop level which are absent at the one-loop level?

Before describing the highlights of this calculation, I should like to caution that the computation for the sunset diagram Fig. 1 is not quite complete. Furthermore, I decided to calculate everything by hand, because I wanted to be "in control of the situation", ready to develop new formulae if and when the need arose. It turned out to be a wise decision.

2. Structure of sunset graph

The self-energy graph in Fig. 1, sometimes called the sunset diagram, is

the most complicated of the two-loop diagrams. Its amplitude reads

$$\Pi_{\mu\nu}^{af}(p) = \frac{1}{3!} \iint \frac{dq dk}{(2\pi)^4 w} v_{\mu\lambda\sigma\rho}^{abcd} G_{\rho\alpha}^{de}(q) G_{\sigma\beta}^{ch}(p-k-q) G_{\lambda\xi}^{be}(k) v_{\alpha\theta\xi\nu}^{ghcf}, \quad (2.1)$$

$$dq = d^2 w_q, \quad dk = d^2 w_k;$$

$G_{\mu\nu}^{ab}$ and $v_{\alpha\beta\mu\nu}^{abcd}$ are given in eqs. (1.3) and (1.4), respectively. Let us write

$$\Pi_{\mu\nu}^{af}(p) = \delta^{af} [\Pi_{\mu\nu}(n^2=0) + \Pi_{\mu\nu}(n^2 \neq 0)], \quad (2.2)$$

where the "light-cone" part reads [29]

$$\Pi_{\mu\nu}(n^2=0) = 3\lambda(N)(2w-3)[A(p)\delta_{\mu\nu} - B(p)(k_\mu n_\nu + k_\nu n_\mu) - C(p)n_\mu n_\nu],$$

$$\lambda(N) = \frac{-i}{2(2\pi)^4 w} g^4 N^2, \quad (2.3)$$

while

$$\begin{aligned} \Pi_{\mu\nu}(n^2 \neq 0) &= \lambda(N)n^2 [\Pi(1)\delta_{\mu\nu} + \Pi(2)k_\mu k_\nu + \Pi(3)n_\mu n_\nu \\ &\quad + \Pi(4)(k_\mu n_\nu + k_\nu n_\mu) + \Pi(5)(q_\mu k_\nu + q_\nu k_\mu)], \end{aligned} \quad (2.4)$$

with $N = 3$ for $SU(3)$. The coefficients A, B, C , are the same as in ref. [29]:

$$A(p) = \iint dq dk [q^2 k^2 Q^2]^{-1} = \iint dR, dR = \frac{dq dk}{q^2 k^2 Q^2}, \quad Q_\mu = (p-k-q)_\mu, \quad (2.5a)$$

$$B(p) = \iint dR (k \cdot n)^{-1}, \quad (2.5b)$$

$$C(p) = \iint dR k \cdot q [q \cdot nk \cdot n]^{-1}. \quad (2.5c)$$

It is convenient to re-write the five terms in eq. (2.4) as follows:

$$\Pi(1)\delta^{\mu\nu} = 4\Pi_1^{\mu\nu} - 8\Pi_2^{\mu\nu} + 2n^2\Pi_3^{\mu\nu},$$

$$\begin{aligned} \Pi(2)k^\mu k^\nu &= -2\Pi_4^{\mu\nu} + 4n^2\Pi_5^{\mu\nu} - 4n^2\Pi_6^{\mu\nu} + 3n^4\Pi_7^{\mu\nu}, \\ \Pi(3)n^\mu n^\nu &= -\Pi_8^{\mu\nu}, \\ \Pi(4)(k^\mu n^\nu + k^\nu n^\mu) &= 4\Pi_9^{\mu\nu} + 4\Pi_{10}^{\mu\nu} + 2n^2\Pi_{11}^{\mu\nu} - 4\Pi_{12}^{\mu\nu} - 2n^2\Pi_{13}^{\mu\nu} \\ &\quad + \mu \leftrightarrow \nu, \\ \Pi(5)(q^\mu k^\nu + q^\nu k^\mu) &= 3\Pi_{14}^{\mu\nu} - 3n^2\Pi_{15}^{\mu\nu} + 2n^2\Pi_{16}^{\mu\nu} - \frac{3}{2}n^4\Pi_{17}^{\mu\nu} \\ &\quad + \mu \leftrightarrow \nu. \end{aligned} \quad (2.6)$$

The double integrals $\Pi_j^{\mu\nu}$, $j = 1, \dots, 17$, are listed in Appendix A. For example,

$$\Pi_1^{\mu\nu} = \delta^{\mu\nu} \iint dR \, k^2 (k \cdot n)^{-2}, \quad (2.7)$$

$$\Pi_2^{\mu\nu} = \delta^{\mu\nu} \iint dR \, q \cdot k (q \cdot nk \cdot n)^{-1}, \quad (2.8)$$

$$\Pi_3^{\mu\nu} = \delta^{\mu\nu} \iint dR (q \cdot k)^2 [(q \cdot n)^2 (k \cdot n)^2]^{-1}. \quad (2.9)$$

All integrals contain overlapping divergences, the more difficult ones being $\Pi_3^{\mu\nu}$, $\Pi_7^{\mu\nu}$, $\Pi_{11}^{\mu\nu}$, $\Pi_{13}^{\mu\nu}$, $\Pi_{16}^{\mu\nu}$, and $\Pi_{17}^{\mu\nu}$.

3. Initial steps in computation

(a) The first step is to apply the decomposition formula [30]

$$\frac{1}{q \cdot n (p \cdot q) \cdot n} = \frac{1}{p \cdot n} \left[\frac{1}{q \cdot n} + \frac{1}{(p \cdot q) \cdot n} \right], \quad p \cdot n \neq 0, \quad (3.1)$$

to all integrals containing multiple, noncovariant factors in the denominator.

(b) It is convenient to defer any vector or tensor integrals, whenever possible, to the second integration. Take, for instance, the term

$$I^{\mu\nu} = \int dq \int \frac{dk}{D} k^\mu k^\nu (q \cdot k)^2, \quad D = D(p, q, k, n). \quad (3.2)$$

Rather than perform the k -integration in (3.2), we first let $q_\mu \leftrightarrow k_\mu$,

$$I^{\mu\nu} \rightarrow \int dq \, q^\mu q^\nu \int \frac{dk (q \cdot k)^2}{D} = I_0^{\mu\nu}, \quad (3.3)$$

and only then integrate over dk in $I_0^{\mu\nu}$, by applying the procedure in part (c) below.

(c) To evaluate

$$\int \frac{dk (q \cdot k)^2}{D} = q_\sigma q_\lambda \int \frac{dk}{D} k^\sigma k^\lambda, \quad (3.4)$$

or any integral having the general form

$$\int \frac{dk (k_\mu, k_\nu, k_\mu k_\nu, k_\mu k_\nu k_\sigma, \dots)}{D}, \quad (3.5)$$

we apply the tensor method which is the only viable technique for two-loop integrals, in my opinion. The tensor method is trivial for covariant gauges, but requires "additional work" for noncovariant-gauge integrals. Two examples follow.

(i) For the integral $I_\mu = \int dk \, k_\mu / D$ we make either the ansatz [11]

$$I_\mu = A p_\mu + B n_\mu + C F_\mu, \quad (3.6a)$$

where F_μ is related to the dual vector n_μ^* , or the ansatz

$$I_\mu = A' p_\mu + B' n_\mu + C' n_\mu^*. \quad (3.6b)$$

Since F_μ is a null vector, the structure (3.6a) is computationally more convenient than (3.6b). In the temporal gauge, for instance,

$$F_\mu^{\text{temp}} = (\vec{n}_1 / \sigma, n_4 / \sigma, 1), \quad \sigma^2 = \vec{n}_1^2 + n_4^2.$$

Multiplication of (3.6a) successively by p_μ , n_μ , F_μ yields three equations which can be solved uniquely for A,B,C. We note that integration of I_μ in component form is not a viable alternative, simply because it becomes too messy, especially by the time the second integration over dq is completed.

(ii) Next we demonstrate the tensor method for the integral

$$K_{\mu\nu} = \int \frac{dk k_\mu k_\nu}{k^2(p-k)^2(k-n)^2} \quad (3.7)$$

The proper ansatz turns out to be

$$\begin{aligned} K_{\mu\nu} = & \{A\delta_{\mu\nu} + B(n_\mu F_\nu + n_\nu F_\mu) + C F_\mu F_\nu\} \\ & + (Dn_\mu n_\nu + E(p_\mu n_\nu + p_\nu n_\mu) + G p_\mu p_\nu + H(p_\mu F_\nu + p_\nu F_\mu)), \\ = & K_{\mu\nu}^{\text{pole}} + \{\text{finite terms}\}, \end{aligned} \quad (3.8)$$

where $K_{\mu\nu}^{\text{pole}}$ and (finite terms) $= f_{\mu\nu}$ contain the pole part and finite portion of $K_{\mu\nu}$, respectively. The coefficients A,B,...,H, are found in the usual way.

Successful application of the tensor method to the "first" integration over dk yields about a dozen distinct types of divergent integrals. We shall show in Section 4 how to represent the poles of these integrals in terms of the two scalar integrals X and Y.

4. Reduction of one-loop integrals

The general aim is to express the pole parts of all necessary one-loop integrals in terms of two scalar integrals Y and X:

$$Y(p) = \int \frac{dq}{q^2(p-q)^2} = \frac{i\pi^2 \Gamma(2-\omega) [\Gamma(\omega-1)]^2 (p^2)^{\omega-2}}{\Gamma(2\omega-2)}, \quad (4.1)$$

$$X(p) = \int dq [(\bar{p}-q)^2(q \cdot n)^2]^{-1}. \quad (4.2)$$

We shall demonstrate the procedure for the integral $K_{\mu\nu}$ given in eqs.(3.7) - (3.8):

$$K_{\mu\nu} = \{A\delta_{\mu\nu} + B(n_\mu F_\nu + n_\nu F_\mu) + C F_\mu F_\nu\} + f_{\mu\nu}. \quad (4.3)$$

We know from explicit computation [11] that only the first three coefficients, in square brackets, are proportional to simple poles. In other words, to obtain the divergent part of $K_{\mu\nu}$ it suffices to determine A,B,C. Multiplication of (4.3) in turn by $\delta^{\mu\nu}$, $n^\mu n^\nu$, $p^\mu p^\nu$ yields four equations:

$$[2\omega A + 2Bn \cdot F + 0] + \delta^{\mu\nu} f_{\mu\nu} = \int \frac{dk}{(k-p)^2(k \cdot n)^2} = X \quad (4.4)$$

$$[n^2 A + 2n^2 n \cdot F B + C(n \cdot F)^2] + n^\mu n^\nu f_{\mu\nu} = \int \frac{dk}{k^2(p-k)^2} = Y \quad (4.5)$$

$$[p \cdot n A + (n^2 p \cdot F + p \cdot n n \cdot F)B + n \cdot F p \cdot FC] + n^\mu p^\nu f_{\mu\nu} = \int \frac{dk p \cdot k}{k^2(p-k)^2 k \cdot n} = Z \quad (4.6)$$

or

$$[p^2 A + 2p \cdot n p \cdot F B + (p \cdot F)^2 C] + p^\mu p^\nu f_{\mu\nu} = \int \frac{dk(p \cdot k)^2}{k^2(p-k)^2(k \cdot n)^2} = W, \quad (4.7)$$

$$[2\omega A + 2Bn \cdot F + 0] = X - \delta^{\mu\nu} f_{\mu\nu} = X' \quad (4.8)$$

$$[n^2 A + 2n^2 n \cdot F B + C(n \cdot F)^2] = Y - n^\mu n^\nu f_{\mu\nu} = Y' \quad (4.9)$$

$$[p \cdot n A + (n^2 p \cdot F + p \cdot n n \cdot F)B + n \cdot F p \cdot FC] = Z - n^\mu p^\nu f_{\mu\nu} = Z' \quad (4.10)$$

$$[p^2 A + 2p \cdot n \cdot p \cdot F \cdot B + (p \cdot F)^2 C] = W - p^\mu p^\nu f_{\mu\nu} = W'. \quad (4.11)$$

First we solve system (4.8) - (4.10) for A and B, constructing the ratio $\langle \frac{A}{B} \rangle$ (4.8)-(4.10); next we solve system (4.8), (4.10), (4.11) for A and B, constructing $\langle \frac{A}{B} \rangle$ (4.8), (4.10), (4.11), and then finally equate the two ratios.

The equality

$$\langle \frac{A}{B} \rangle (4.8), (4.10) = \langle \frac{A}{B} \rangle (4.8), (4.10), (4.11) \quad (4.12)$$

leads to two identities:

$$p \cdot F \cdot Y' - n \cdot F \cdot Z' = 0, \quad (4.13)$$

$$2(\omega-1)n \cdot F(p \cdot F \cdot Z' - n \cdot F \cdot W') + D_0 X' = 0, \quad (4.14)$$

which, in turn, yield the following relations among the four scalar integrals X, Y, Z, W:

$$Z = (p \cdot F / n \cdot F) Y + Z^{\text{finite}}, \quad (4.15)$$

$$W = \frac{1}{(n \cdot F)^2} \left[(p \cdot F)^2 Y + \frac{D_0}{2(\omega-1)} X \right] + W^{\text{finite}}, \quad (4.16)$$

where

$$Z^{\text{finite}} = n_\mu \langle p_\nu \cdot (p \cdot F / n \cdot F) n_\nu \rangle f^{\mu\nu}, \quad (4.17)$$

$$W^{\text{finite}} = \left[p_\mu p_\nu \cdot (n \cdot F)^{-2} \left\{ (p \cdot F)^2 n_\mu n_\nu + \frac{D_0}{2(\omega-1)} \delta_{\mu\nu} \right\} \right] f^{\mu\nu}, \quad (4.18)$$

$$D_0 = p^2 (n \cdot F)^2 - 2p \cdot n \cdot F \cdot p \cdot F + n^2 (p \cdot F)^2. \quad (4.19)$$

Systematic application of this technique to the other integrals enables us to express the divergent part of all one-loop integrals in terms of X(p) and Y(p). (See Appendix B.) For example, $K_{\mu\nu}$ in eq.(3.7) becomes

$$K_{\mu\nu}(p) = \frac{Y(p)}{(n \cdot F)^2} F_{\mu\nu}$$

$$+ \frac{X(p)}{2(\omega-1)} \left[\delta_{\mu\nu} - \frac{(n_\mu F_\nu + n_\nu F_\mu)}{n \cdot F} + \frac{n^2}{(n \cdot F)^2} F_{\mu\nu} \right] + k_{\mu\nu}^{\text{(finite)}}, \quad (4.20)$$

where $k_{\mu\nu}^{\text{(finite)}}$ is known.

5. Final integration

We illustrate the second integration, i.e. the integration over dq, by computing the double integrals $\Pi_1^{\mu\nu}$ and $\Pi_3^{\mu\nu}$.

5.1 The integral $\Pi_1^{\mu\nu}$

$$\Pi_1^{\mu\nu} = \delta^{\mu\nu} \iint \frac{dq \, dk}{q^2 Q^2 (k \cdot n)^2} \quad q \leftrightarrow k \quad \delta^{\mu\nu} \int \frac{dq}{(q \cdot n)^2} \int \frac{dk}{k^2 Q^2}, \quad (5.1)$$

$$= \delta^{\mu\nu} \int \frac{dq}{(q \cdot n)^2} Y(p \cdot q), \quad Q_\mu = (p \cdot q \cdot k)_\mu. \quad (5.2)$$

Substituting for Y(p-q) from eq. (4.1), we get

$$\Pi_1^{\mu\nu} = \frac{i \delta^{\mu\nu} \pi^2 \Gamma(2-\omega) [\Gamma(\omega-1)]^2}{\Gamma(2\omega-2)} \int \frac{dq}{[(p \cdot q)^2]^{2-\omega} (q \cdot n)^2}. \quad (5.3)$$

The remaining one-loop integral

$$\int dq [(p \cdot q)^2]^{2-\omega} (q \cdot n)^2]^{-1} = X(2-\omega), \quad (5.4)$$

can be computed explicitly to yield

$$\Pi_1^{\mu\nu} = \delta^{\mu\nu} \frac{i \pi^2 \Gamma(2-\omega) [\Gamma(\omega-1)]^2}{\Gamma(2\omega-2)}.$$

$$\begin{aligned}
& \frac{i\pi^2}{\rho^4 \Gamma(2-w)} \left\{ \frac{n^2 \Gamma(3-2w)}{2} \int_0^1 \frac{d\xi \xi^{2w-2}}{(1-B\xi)^{3/2}} [g(P)]^{2w-3} \right. \\
& + \Gamma(4-2w) \left\{ (\vec{P} \cdot \vec{N})^2 \int_0^1 \frac{d\xi \xi^{2w-3}}{(1-B\xi)^{5/2}} + 2i\rho P_4 (\vec{P} \cdot \vec{N}) \int_0^1 \frac{d\xi \xi^{2w-3}}{(1-B\xi)^{3/2}} \right. \\
& \left. \left. - \rho^2 P_4^2 \int_0^1 \frac{d\xi \xi^{2w-3}}{(1-B\xi)^{1/2}} \right\} [g(P)]^{2w-4} \right\}, \quad (5.5)
\end{aligned}$$

where

$$g(P) = P_4^2 + \frac{(\vec{P} \cdot \vec{N})^2}{\rho^2 (1-B\xi)}, \quad B = 1 - \sigma^2/\rho^2, \quad (5.6)$$

$$\rho^{\text{temp}} = \rho^{\text{ax}} = |\vec{n}_1|, \quad \rho^{\text{fc}} = |\vec{n}|; \quad \vec{P}_1 = \vec{P}_1, \quad P_3 = P_4, \quad P_4 = P_3, \quad (5.7)$$

$$\sigma^2 = \vec{n}_1^2 + n_1^2, \quad \vec{N}_1 = \vec{n}_1, \quad N_3 = n_4, \quad N_4 = n_3.$$

Clearly the double integral $\Pi_1^{\mu\nu}$ possesses only a simple pole as $w \rightarrow 2^+$.

5.2 The integral $\Pi_3^{\mu\nu}$

A more challenging overlapping divergence occurs in the integral $\Pi_3^{\mu\nu}$,

$$\Pi_3^{\mu\nu} = \delta^{\mu\nu} \iint \frac{dq dk (q \cdot k)^2}{q^2 k^2 Q^2 (q \cdot n)^2 (k \cdot n)^2} = \delta^{\mu\nu} \int \frac{dq q^\sigma q^\lambda}{q^2 (q \cdot n)^2} \int \frac{dk k_\sigma k_\lambda}{k^2 Q^2 (k \cdot n)^2}. \quad (5.8)$$

Using formula (4.20) for the k -integral, we obtain

$$\begin{aligned}
\Pi_3^{\mu\nu} = \delta^{\mu\nu} \int \frac{dq q^\sigma q^\lambda}{q^2 (q \cdot n)^2} \left\{ \frac{Y(p-q)}{(n \cdot F)^2} F_\sigma F_\lambda \right. \\
\left. + \frac{X(p-q)}{2(w-1)} \left[\delta_{\sigma\lambda} - \frac{(n \cdot F_\lambda + n \cdot F_\sigma)}{n \cdot F} + \frac{n^2}{(n \cdot F)^2} F_\sigma F_\lambda \right] + k_{\sigma\lambda}(\text{finite}) \right\}, \quad (5.9)
\end{aligned}$$

where

$$\begin{aligned}
X &= X^{\text{pole}} + X^{\text{finite}}, \\
X^{\text{pole}}(p \cdot q) &= \frac{i\pi^2 n^2 \Gamma(2-w)}{2(\rho^2)^w} \int_0^1 \frac{d\xi \xi^{w-1}}{(1-B\xi)^{w-1/2}} \frac{1}{[(\vec{P} \cdot \vec{Q})^2 + \rho^2 (1-B\xi)(P_4 \cdot Q_4)^2]^{2-w}}, \\
X^{\text{finite}}(p \cdot q) &= \frac{i\pi^2 \Gamma(3-w)}{\rho^4} \left\{ (\vec{P} \cdot \vec{Q}) \cdot \vec{N} \int_0^1 \frac{d\xi \xi^{w-2}}{(1-B\xi)^{5/2}} \right. \\
& \quad \left. + 2i\rho(P_4 \cdot Q_4)(\vec{P} \cdot \vec{Q}) \cdot \vec{N} \int_0^1 \frac{d\xi \xi^{w-2}}{(1-B\xi)^{3/2}} - \rho^2 (P_4 \cdot Q_4)^2 \int_0^1 \frac{d\xi \xi^{w-2}}{(1-B\xi)^{1/2}} \right\}.
\end{aligned}$$

$$\left[\frac{(P_4 \cdot Q_4)^2 + \frac{(\vec{P} \cdot \vec{Q})^2}{\rho^2 (1-B\xi)}}{(\vec{P} \cdot \vec{Q})^2} \right]^{w-3}$$

To extract the coefficients of the double or single poles from (5.9) we need to compute the following two integrals:

$$(i) \quad K^{\sigma\lambda}(2-w) = \int \frac{dq q^\sigma q^\lambda}{q^2 [(p-q)^2]^{2-w} (q \cdot n)^2}; \quad (5.10)$$

$$\begin{aligned}
(ii) \quad X^{\sigma\lambda} &= \int \frac{dq q^\sigma q^\lambda}{q^2 (q \cdot n)^2} X^{\text{pole}}(p-q) = \frac{i\pi^2 n^2 \Gamma(2-w)}{2(\rho^2)^w} \int_0^1 \frac{d\xi \xi^{w-1}}{(1-B\xi)^{w-1/2}} \\
& \quad \int \frac{dq q^\sigma q^\lambda}{q^2 (q \cdot n)^2 [(\vec{P} \cdot \vec{Q}) \cdot \vec{N}]^2 + M(P_4 \cdot Q_4)^2]^{2-w}}, \quad (5.11)
\end{aligned}$$

with $M = \rho^2 (1-B\xi) - \rho^2 + n^2 \xi$; $\rho^{\text{temp}} = \rho^{\text{ax}} = |\vec{n}_1|$, $\rho^{\text{fc}} = |\vec{n}|$. $K^{\sigma\lambda}(2-w)$ and $X^{\sigma\lambda}$ may be evaluated with the help of the tensor method and the appropriate identities.

It is also worth noting, in the context of dimensional regularization, that [31]

$$\int \frac{dq}{q^2(q \cdot n)^2} \sim 0 \quad (5.12)$$

as expected, but that

$$\lim_{\omega \rightarrow 2^+} \int \frac{dq}{q^2[(p-q)^2]^{2-\omega}}(q \cdot n)^2 \quad (5.13)$$

is different from zero and finite.

6. Summary

The Yang-Mills self-energy $\Pi_{\mu\nu}^{af}$ has been calculated in a unified-gauge formalism which includes the axial, temporal and light-cone gauge, and with an appropriate change of the gauge-fixing term - the planar gauge. The corresponding unified-gauge prescription

$$\frac{1}{q \cdot n} = \lim_{\epsilon \rightarrow 0} \frac{q \cdot n^*}{q \cdot n q \cdot n q \cdot n^* + i\epsilon}, \quad \epsilon > 0,$$

works to two loops for both simple and multiple poles, $(q \cdot n)^{-\beta}$, $\beta = 1, 2, 3, \dots$

The main results are:

$$(a) \quad \Pi_{\mu\nu}^{af}(p) = \delta^{af} \sum_{j=0}^2 \frac{1}{\epsilon^{2-j}} \left\{ A_j \delta_{\mu\nu} + B_j p_\mu p_\nu + C_j (p_\mu n_\nu + p_\nu n_\mu) + D_j n_\mu n_\nu \right. \\ \left. + E_j (p_\mu n_\nu^* + p_\nu n_\mu^*) + G_j (n_\mu n_\nu^* + n_\nu n_\mu^*) + H_j n_\mu n_\nu^* \right\}, \quad \epsilon = 2-\omega.$$

The coefficients $(A_0, \dots, H_0)$ can be obtained explicitly; the coefficients $(A_1, \dots, H_1)$ and $(A_2, \dots, H_2)$ are derivable but not necessarily in closed form.

(b) All single and double integrals are well defined. Ambiguities do not arise, and integrals of the type

$$\int \frac{dq (1, q_\mu, q_\mu q_\nu)}{q^2[(p-q)^2]^{2-\omega}} q \cdot n$$

can be evaluated explicitly.

(c) No additional problems are caused by overlapping divergences.

(d) The basic approach to the evaluation of two-loop noncovariant-gauge integrals is:

(i) to derive identities between certain one-loop integrals; for instance,

$$Z = (p \cdot F / n \cdot F) Y + Z_{\text{finite}}, \quad \text{i.e.}$$

$$\int \frac{dq p \cdot q}{q^2(p-q)^2 q \cdot n} = \frac{p \cdot F}{n \cdot F} \int \frac{dq}{q^2(p-q)^2} + \text{Finite Part};$$

(ii) to exploit these identities to express the divergent parts of all one-loop integrals in terms of the scalar integrals

$$Y = \int dq [q^2(p-q)^2]^{-1}, \quad \text{and } X = \int dq [(p-q)^2(q \cdot n)^2]^{-1};$$

(iii) to apply the tensor method to the first as well as the second integrals, i.e. to both the k- and q- integrations.

As for future work in this area, the most urgent task is to get the coefficients of all the poles. Knowledge of the coefficients $(A_0, \dots, H_0)$ of the double poles should give us further insight into the structure of the counterterms and hence into the renormalization program. An intriguing

question in this connection concerns the non-local terms: can they be handled consistently, for instance, in the framework of the Becchi-Rouet-Stora formalism? [32]

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Appendix A

The Double Integrals $\Pi_j^{\mu\nu}$

We list the seventeen double integrals $\Pi_j^{\mu\nu}$ referred to in eq. (2.6) in the main text. These integrals (see also ref. [33]) constitute the self-energy part $\Pi_{\mu\nu}(n^2 \neq 0)$ defined in eq. (2.4).

$$\Pi_1^{\mu\nu} = \delta^{\mu\nu} \iint dR \, k^2 (k \cdot n)^{-2},$$

$$\Pi_2^{\mu\nu} = \delta^{\mu\nu} \iint dR \, q \cdot k (q \cdot nk \cdot n)^{-1},$$

$$\Pi_3^{\mu\nu} = \delta^{\mu\nu} \iint dR (q \cdot k)^2 [(q \cdot n)^2 (k \cdot n)^2]^{-1},$$

$$\begin{aligned} \Pi_4^{\mu\nu} &= \iint dR \frac{1}{(k \cdot n)^2} k_\mu k_\nu, \\ \Pi_5^{\mu\nu} &= \iint dR \frac{Q^2}{(k \cdot n)^2 (Q \cdot n)^2} k_\mu k_\nu, \\ \Pi_6^{\mu\nu} &= \iint dR \frac{q \cdot Q}{q \cdot n (k \cdot n)^2 Q \cdot n} k_\mu k_\nu, \\ \Pi_7^{\mu\nu} &= \iint dR \frac{(q \cdot Q)^2}{(q \cdot n)^2 (k \cdot n)^2 (Q \cdot n)^2} k_\mu k_\nu, \\ \Pi_8^{\mu\nu} &= \iint dR \frac{q \cdot (p \cdot q) k \cdot Q}{q \cdot n (k \cdot n)^2 Q \cdot n} n_\mu n_\nu, \\ \Pi_9^{\mu\nu} &= \iint dR \frac{q \cdot k}{q \cdot n (k \cdot n)^2} k_\mu n_\nu, \\ \Pi_{10}^{\mu\nu} &= \iint dR \frac{q \cdot Q}{q \cdot nk \cdot n Q \cdot n} k_\mu n_\nu, \\ \Pi_{11}^{\mu\nu} &= \iint dR \frac{q \cdot Q k \cdot Q}{q \cdot n (k \cdot n)^2 (Q \cdot n)^2} k_\mu n_\nu, \\ \Pi_{12}^{\mu\nu} &= \iint dR \frac{q^2}{k \cdot n (q \cdot n)^2} k_\mu n_\nu, \\ \Pi_{13}^{\mu\nu} &= \iint dR \frac{(q \cdot Q)^2}{k \cdot n (q \cdot n)^2 (Q \cdot n)^2} k_\mu n_\nu, \\ \Pi_{14}^{\mu\nu} &= \iint dR \frac{1}{q \cdot nk \cdot n} q_\mu k_\nu, \\ \Pi_{15}^{\mu\nu} &= \iint dR \frac{q \cdot k}{(q \cdot n)^2 (k \cdot n)^2} q_\mu k_\nu, \\ \Pi_{16}^{\mu\nu} &= \iint dR \frac{k \cdot Q}{q \cdot n (k \cdot n)^2 Q \cdot n} q_\mu k_\nu, \end{aligned}$$

$$\Pi_{17}^{\mu\nu} = \iint dR \frac{q \cdot Q k \cdot Q}{(q \cdot n)^2 (k \cdot n)^2 (Q \cdot n)^2} q_\mu k_\nu.$$

Appendix B

Application of the technique developed in Section 4 leads to the following one-loop integrals as functions of the scalar integrals Y and X .

$$\int \frac{dk}{k^2(q-k)^2} = Y(q),$$

$$\int \frac{dk}{(q-k)^2(k \cdot n)^2} = X(q),$$

$$\int \frac{dk k_\mu}{k^2(q-k)^2 k \cdot n} = \frac{Y(q)}{n \cdot F} F_\mu + F.P.,$$

$$\int \frac{dk k_\mu k_\nu}{k^2(q-k)^2 (k \cdot n)^2} = \frac{Y(q)}{(n \cdot F)^2} F_\mu F_\nu$$

$$+ \frac{X(q)}{2(\omega-1)} \left[\delta_{\mu\nu} - \frac{(n_\mu F_\nu + n_\nu F_\mu)}{n \cdot F} + \frac{n^2}{(n \cdot F)^2} F_\mu F_\nu \right] + F.P.,$$

$$\int \frac{dk}{(q-k)^2 k \cdot n} = \frac{2q \cdot F}{n \cdot F} Y(q) + F.P.,$$

$$\int \frac{dk}{k^2(q-k)^2 (k \cdot n)^2} = -\frac{X(q)}{q^2} + F.P.,$$

$$\int \frac{dk k^2}{(q-k)^2 (k \cdot n)^2} = \frac{4(q \cdot F)^2}{(n \cdot F)^2} Y(q) + \left[-q^2 + \frac{2D_0}{(\omega-1)(n \cdot F)^2} \right] X(q) + F.P.,$$

$$\int \frac{dk k_\mu}{(q-k)^2 (k \cdot n)^2} = \left(q_\mu - \frac{q \cdot F}{n \cdot F} n_\mu \right) X(q)$$

$$+ \frac{F_\mu}{(n \cdot F)^2} \left[2q \cdot F Y(q) + (n^2 q \cdot F - q \cdot n n \cdot F) X(q) \right] + F.P.,$$

$$\int \frac{dk k^2}{(q-k)^2 k \cdot n} = \frac{2q \cdot F}{(\omega-1)(n \cdot F)^3} [(5-2\omega)(n \cdot F)^2 q^2$$

$$- (\omega+1)n \cdot F q \cdot F q \cdot n + n^2(q \cdot F)^2] Y(q) + F.P.,$$

$$\int \frac{dk k_\mu k_\nu}{k^2(q-k)^2 k \cdot n} = \frac{Y(q)}{2(n \cdot F)^2} \left\{ \frac{n \cdot F q \cdot F}{\omega-1} \delta_{\mu\nu} - \frac{q \cdot F}{\omega-1} (n_\mu F_\nu + n_\nu F_\mu) \right.$$

$$+ \frac{n^2 q \cdot F}{(\omega-1)n \cdot F} F_\mu F_\nu$$

$$+ n \cdot F(q_\mu F_\nu + q_\nu F_\mu) - q \cdot n F_\mu F_\nu \left. \right\} + F.P.,$$

where $D_0 = n^2(q \cdot F)^2 - 2q \cdot n n \cdot F q \cdot F + q^2(n \cdot F)^2$, and F.P. means "finite part".

Appendix C

Consider the double integral

$$\Pi^{\mu\nu} = \int dq \int dk R^{\mu\nu}(k, p, q, n),$$

and define

$$\Pi^{\mu\nu} = \int dk R^{\mu\nu}(k, p, q, n) = E^{\mu\nu, \text{pole}} + E^{\mu\nu, \text{finite}}$$

Then

$$\Pi^{\mu\nu} = \int dq (E^{\mu\nu, \text{pole}} + E^{\mu\nu, \text{finite}}) = \Pi^{\mu\nu}(\text{sub-div}) + \Pi^{\mu\nu}(\text{sub-finite}),$$

with

$$\Pi^{\mu\nu}(\text{sub-divergent}) = \int dq E^{\mu\nu, \text{pole}},$$

$$\Pi^{\mu\nu}(\text{sub-finite}) = \int dq E^{\mu\nu, \text{finite}}.$$

In this appendix we list all $\Pi_j^{\mu\nu}(\text{sub-div})$, $j = 1, 2, \dots, 16$, the expressions for $\Pi_j^{\mu\nu}(\text{sub-finite})$ being discussed elsewhere.

$$\Pi_1^{\mu\nu}(\text{sub-div}) = \delta^{\mu\nu} \int \frac{dq}{((p-q) \cdot n)^2} Y(q),$$

$$\Pi_2^{\mu\nu}(\text{sub-div}) = \delta^{\mu\nu} \int \frac{dq (p-q)_\sigma}{(p-q)^2 (p-q) \cdot n} \cdot \frac{Y(q)}{n \cdot F} F^\sigma,$$

$$\begin{aligned} \Pi_3^{\mu\nu}(\text{sub-div}) = & \delta^{\mu\nu} \int \frac{dq (p-q)^\sigma (p-q)^\lambda}{(p-q)^2 ((p-q) \cdot n)^2} \left[\frac{F_\sigma F_\lambda}{(n \cdot F)^2} Y(q) + \right. \\ & \left. + \frac{1}{2(\omega-1)} \left[\delta_{\sigma\lambda} - \frac{(n_\sigma F_\lambda + n_\lambda F_\sigma)}{n \cdot F} + \frac{n^2}{(n \cdot F)^2} F_\sigma F_\lambda \right] X(q) \right], \end{aligned}$$

$$\Pi_4^{\mu\nu}(\text{sub-div}) = \int \frac{dq (p-q)^\mu (p-q)^\nu}{(p-q)^2 ((p-q) \cdot n)^2} Y(q),$$

$$\Pi_5^{\mu\nu}(\text{sub-div}) = \int \frac{dq (p-q)^\mu (p-q)^\nu}{(p-q)^2 ((p-q) \cdot n)^2} X(q),$$

$$\Pi_6^{\mu\nu}(\text{sub-div}) = \int \frac{dq (p-q)^\mu (p-q)^\nu}{(p-q)^2 ((p-q) \cdot n)^2 q \cdot n} \left[-\frac{2q \cdot F}{n \cdot F} Y(q) \right],$$

$$\Pi_7^{\mu\nu}(\text{sub-div}) = \frac{1}{2} \int \frac{dq (p-q)^\mu (p-q)^\nu}{(p-q)^2 ((p-q) \cdot n)^2 (q \cdot n)^2} P_7^{\text{div}}(q),$$

$$\Pi_8^{\mu\nu}(\text{sub-div}) = \frac{1}{2} n^\mu n^\nu \int \frac{dq q \cdot (p-q)}{(p-q)^2 q \cdot n (p-q) \cdot n} \left[-2X - \frac{4q \cdot F}{q \cdot n n \cdot F} Y(q) \right],$$

where

$$\begin{aligned} \Pi_9^{\mu\nu}(\text{sub-div}) = & n^\nu \int \frac{dq (p-q)^\mu (p-q)^\sigma}{(p-q)^2 ((p-q) \cdot n)^2} \cdot \frac{F^\sigma}{n \cdot F} Y(q), \\ \Pi_{10}^{\mu\nu}(\text{sub-div}) = & n^\nu \int \frac{dq (p-q)^\mu}{(p-q)^2 q \cdot n (p-q) \cdot n} \left[-\frac{2q \cdot F}{n \cdot F} Y(q) \right], \\ \Pi_{11}^{\mu\nu}(\text{sub-div}) = & \frac{1}{2} n^\nu \int \frac{dq (p-q)^\mu}{(p-q)^2 ((p-q) \cdot n)^2 q \cdot n} P_{11}^{\text{div}}(q), \\ \Pi_{12}^{\mu\nu}(\text{sub-div}) = & n^\nu \int \frac{dq (p-q)^\mu}{(p-q)^2 (p-q) \cdot n} X(q), \\ \Pi_{13}^{\mu\nu}(\text{sub-div}) = & \frac{1}{2} n^\nu \int \frac{dq (p-q)^\mu}{(p-q)^2 (q \cdot n)^2 (p-q) \cdot n} P_{13}^{\text{div}}(q), \\ \Pi_{14}^{\mu\nu}(\text{sub-div}) = & \int \frac{dq (p-q)^\mu}{(p-q)^2 (p-q) \cdot n} \cdot \frac{F^\nu}{n \cdot F} Y(q), \\ \Pi_{15}^{\mu\nu}(\text{sub-div}) = & \int \frac{dq (p-q)^\mu (p-q)^\lambda}{(p-q)^2 ((p-q) \cdot n)^2} \left[\frac{F^\nu F_\lambda}{(n \cdot F)^2} Y(q) + \right. \\ & \left. + \frac{1}{2(\omega-1)} \left[\delta_\lambda^\nu - \frac{(n^\nu F_\lambda + n_\lambda F^\nu)}{n \cdot F} + \frac{n^2}{(n \cdot F)^2} F^\nu F_\lambda \right] X(q) \right], \\ \Pi_{16}^{\mu\nu}(\text{sub-div}) = & - \int \frac{dq (p-q)^\mu}{(p-q)^2 ((p-q) \cdot n)^2 q \cdot n} P_{16}^{\text{div}}(q), \\ \Pi_{17}^{\mu\nu} = & \left\{ \int \frac{dq}{q^2 (q \cdot n)^2 ((p-q) \cdot n)^3} \left[\frac{dk((p-q)^2 \cdot k^2)}{k^2 Q^2 k \cdot n} \right. \right. \\ & \left. \left. + \frac{1}{2} \int \frac{dq}{q^2 (q \cdot n)^2 ((p-q) \cdot n)^3} \left[\frac{dk((p-q)^2 \cdot k^2)}{k^2 Q^2 (k \cdot n)^2} \right] \right\} \right. \\ & \cdot \left[q \cdot Q(k_\mu q_\nu + k_\nu q_\mu) + q \cdot k(Q_\mu q_\nu + Q_\nu q_\mu) \right], \quad Q_\mu = (p \cdot q \cdot k)_\mu, \end{aligned}$$

$$\begin{aligned}
P_7^{\text{div}}(q) &= \frac{4q \cdot F Y(q)}{(\omega-1)(n \cdot F)^3 q \cdot n} \left[n^2 (q \cdot F)^2 - 2q \cdot n n \cdot F q \cdot F - (4\omega-7) q^2 (n \cdot F)^2 \right] + \\
&+ 2 X(q) \left[-2q^2 + \frac{D_0(q)}{(\omega-1)(n \cdot F)^2} \right] - P_{13}^{\text{div}}(q), \\
P_{11}^{\text{div}}(q) &= -\frac{2q \cdot F}{n \cdot F} \left[\frac{q \cdot (p \cdot q)}{q \cdot n} + \frac{(p \cdot q) \cdot F}{n \cdot F} \right] Y(q) + \\
&+ \frac{X(q)}{(n \cdot F)^2} \left[(p \cdot q) \cdot n q \cdot F n \cdot F - q \cdot (p \cdot q) (n \cdot F)^2 \right. \\
&\left. + q \cdot n n \cdot F (p \cdot q) \cdot F - n^2 q \cdot F (p \cdot q) \cdot F \right], \\
P_{16}^{\text{div}}(q) &= \left[\frac{q \cdot (p \cdot q)}{n \cdot F} F^\nu + \frac{(p \cdot q) \cdot F}{n \cdot F} q^\nu \right] Y(q) + \\
&+ \frac{(p \cdot q)^\lambda}{(n \cdot F)^2} \left[\frac{q \cdot F n \cdot F}{\omega-1} \delta_\lambda^\nu - \frac{q \cdot F}{\omega-1} (n^\nu F_\lambda + n_\lambda F^\nu) + \right. \\
&\left. + n \cdot F (q_\lambda F^\nu + q^\nu F_\lambda) + \left[\frac{n^2 q \cdot F}{(\omega-1) n \cdot F} - q \cdot n \right] F^\nu F_\lambda \right] Y(q); \\
D_0(q) &= n^2 (q \cdot F)^2 - 2q \cdot n n \cdot F q \cdot F + q^2 (n \cdot F)^2.
\end{aligned}$$

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Figure Caption

Fig. 1 Two-loop Yang-Mills self-energy