CERN-TH. 5581/89

PHYSICS WITH A FUNDAMENTAL LENGTH^(*)

G. Veneziano

CERN, Geneva

(*) To appear in a memorial volume dedicated to DIMA KNIZHNIK

CERN-TH. 5581/89
November 1989

1. INTRODUCTION

Since the time physics was part of philosophy, physicists (philosophers) have been wondering about the infinite divisibility of space and time [1]. Is there a limit to the process of looking into smaller and smaller scales? Quantum field theory (QFT), as developed in the second part of this century, appears to answer this question in the negative. However, so far, QFT has failed to yield a consistent synthesis of the principles of general relativity and quantum mechanics: it is affected by infinities which, normally, but with important exceptions, can be swept under the rug in the so-called process of renormalization.

String theories supposedly avoid the above mentioned difficulties. How? A hint is that string theory, unlike point particle theory, contains a fundamental scale or length parameter. Although this fact had been stressed since the early days of dual string theory [2], the precise origin and meaning of this fundamental scale have long been clouded with mystery.

With the QCD reinterpretation of the old hadronic string the mystery is neatly resolved: a fundamental length, which is classically absent, emerges from quantization, through the so-called phenomenon of dimensional transmutation [3]. QCD loops diverge and the process of renormalization introduces a scale, the subtraction point. Renormalization group arguments guarantee that all physical quantities depend only on a particular combination of the subtraction point and of the coupling constant, which defines the fundamental scale Λ of QCD (and thus of its effective string theory).

The Scherk-Schwarz revolution of 1974 [4] takes again (super)string theory as a fundamental theory, rather than as the effective description of some non-perturbative dynamics of a local field theory. Thus, in the Theory of Everything (TOE) scenario for strings, the question of what the fundamental length stands for reappears in its full glory.

In this article I shall try to convey our present understanding of:

- i) the origin of a fundamental scale in string theory
- ii) the ways this scale may change our attitude towards a variety of problems in quantum field theories without affecting significantly their low energy predictions (when they exist!).
- iii) the way the new scale entails large departures from field theory at distances of its order or less.

I shall add immediately that many of our considerations will not depend on whether or not String Theory is what provides a fundamental length. Similar results will hold for any quantum-relativistic theory endowed with a (nicely behaved) physical short-distance cut-off.

2. ORIGIN OF THE STRING SCALE PARAMETER

By far the most powerful, conceptually and technically simple, formulation of string theory available today is the one based on the first quantized approach of Polyakov [5]. We shall thus explain how the fundamental length emerges naturally in that approach. The material of the next two sections is largely borrowed from some recent lectures of mine [6].

The starting point of [5] is the classical string action, generalized to the case of a string is moving in a set of arbitrary background fields (for simplicity we describe the closed bosonic string here):

$$S = -1/2 \int d^2 \xi \, g^{1/2} \{ g^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu G_{\mu\nu}(X) + (2) R(\xi) \phi(X)/4\pi \} + \dots \quad (1)$$

Here, as usual, X^μ is the string coordinate, $g_{\alpha\beta}$ is the metric on the world sheet, $G_{\mu\nu}$ and ϕ are the target-space metric and dilaton

background, and the dots indicate other possible backgrounds.

As everyone knows, the absolute normalization of the action is irrelevant at the classical level. There exists one conventional normalization but, since (1) is supposed to describe a TOE, we do not have to normalize our action with respect to anything else and we shall not.

At the quantum level, the action has to be transformed into a pure number whose absolute magnitude and phase are very relevant. Usually this is done by dividing the conventionally normalized action by $\hbar$, but here, again because of the TOE nature of string theory, we do not want to introduce any constant until we are really forced to do so. Since the background fields are arbitrary, we may just define their (a priori arbitrary) normalization so that the quantum theory is defined a la Feynman by the path integral:

$$Z (G_{\mu\nu}, \phi, \dots) = \int Dg^{\alpha\beta} DX^\mu \exp(-S_E(G_{\mu\nu}(X), \phi(X), \dots)) \quad (2)$$

We have introduced no fundamental constant (and, in particular, no length) yet. The only new feature of (2), as opposed to (1) is that the background fields must be assigned dimensions such that S_E (E denotes that we went over to Euclidean metric) is dimensionless. If we call the dimensions of X length, G has dimension $(\text{length})^{-2}$, ϕ has no dimension, etc. Obviously, Z will depend on the background fields but not on any arbitrary fundamental constant (except for gauge-fixing parameters which will not affect physical predictions).

A crucial, yet poorly understood, feature of string theory comes in at this point. On one hand, quantum consistency imposes constraints on the possible backgrounds in which the string can consistently move: these constraints, related to conformal and Weyl invariance of the quantum theory, enforce the vanishing of certain β -functions [7]. On the other hand, one can construct, out of Z , an effective action (through a sort of Legendre transform [8]) and find that the same conditions also follow from requesting, as is customary in field theory, that the action be stationary:

$$\delta\Gamma/\delta G_{\mu\nu} = \delta\Gamma/\delta\phi = \dots = 0 \quad (3)$$

Although these equations look very much like those of a classical field theory, I should stress that they came from quantizing a string theory: there is nothing like a classical field theory representing the low energy limit of a classical string theory [9].

Γ is, of course, very complicated, but one knows some of its very general properties at least in the limit in which only the massless backgrounds $G_{\mu\nu}$, ϕ (and the "torsion" $B_{\mu\nu}$) are kept and are slowly varying.

In general, Γ has a double infinite expansion:

$$\Gamma = \sum_g \Gamma_g \quad (4)$$

where $g=0,1,2,\dots$ is the genus (number of handles) of the two-dimensional Riemann surface described by the metric $g_{\alpha\beta}$. Because of the well-known relation,

$$4\pi (2-2g) = \int d^2\xi g^{1/2} (2)R(\xi) \quad (5)$$

and of the way the dilaton field ϕ enters the action one finds,

$$\Gamma_g = \int dx G^{1/2} \exp(2\phi(1-g)) L (\partial\phi, \dots) \quad (6)$$

This first expansion is the so-called string loop expansion: it is the equivalent, in string theory, of the usual field-theoretic loop expansion. The second expansion, which can be made genus by genus, is an expansion in derivatives of the background fields. It is also called the σ -model loop expansion and has no analog in field theory.

In particular, at the tree level ($g=0$), it reads:

$$\Gamma_0 = \int dx G^{1/2} \exp(2\phi) [(D-26)/12 - 1/4 R + 1/2 G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \dots] \quad (7)$$

The possible ground states, or vacua, of string theory are the possible stationary points of Γ . One such ground state, at tree level, is flat D=26 space (D=10 for superstrings at any finite order) with a constant dilaton field. Since $G_{\mu\nu}$ has dimensions $(\text{length})^{-2}$, a constant $G_{\mu\nu}$ introduces a fundamental length:

$$G_{\mu\nu} = \lambda_s^{-2} \eta_{\mu\nu}, \quad \eta_{\mu\nu} = \text{diag} (-1, +1, +1, \dots, +1) \quad (8)$$

where the component X^0 is to be identified eventually with ct.

We have thus discovered that only at the quantum level do we need a fundamental scale λ_s and this only because we have chosen a flat space time metric as our ground state! Incidentally, the observation that the appearance of a scale in Quantum Gravity is related to a vacuum expectation value of the metric had been made already, outside of the string context, by Fubini and coworkers [10].

One of the many, important roles of λ_s can be immediately appreciated by inserting (8) back into (2). We find that λ_s^2 is nothing but the Planck constant of string theory, the quantity one needs to convert the Nambu-Goto area action (which is formulated directly in flat space-time) into a pure number, a phase. If (8) is inserted into the expression for Γ , the derivative expansion becomes an expansion in $\lambda_s d/dx$, presumably a reliable expansion if the background fields change little (percentage-wise) over distances $O(\lambda_s)$.

We also notice a very important dimensionless number which, according to eq. (7), appears to control the relative importance of higher genus contributions:

$$\Gamma_g \simeq (\alpha_{SL})^g; \quad \alpha_{SL} = \exp(-2\phi) \quad (9)$$

It is the string loop expansion parameter, related to the dilaton's expectation value [11], on which we shall come back later.

Incidentally, in the second quantized approach to string theory,

one would regard Γ_0 as a classical action to be quantized by standard field theoretic methods. Doing that looks somewhat artificial in string theory: trees and loops come naturally together because of the necessity (related to unitarity) to sum over Reimann surfaces of arbitrary genus (this is one of the issues on which Dima has so much contributed, see other authors in this book).

Furthermore, the quantum length λ_s appears already in the would-be classical action, underlining the fact that Γ_0 itself is the result of string quantization.

For all these reasons the only sensible attitude in string theory seems to be to stick to the first quantized approach, at least as far as perturbation theory is concerned: in principle, this approach leads to a quantum relativistic theory of all interactions in D dimensions, which can be solved by quantizing a two-dimensional field theory: the practical and conceptual consequences of this statement can hardly be overestimated.

Unfortunately the program of computing higher genus contributions to Γ is far from completed. This is hardly surprising: one has to do the analog of developing the Feynman rules for QFT, something that took some 20 years of hard work.

3. THE PLANCK CONSTANT AS ULTRAVIOLET CUT-OFF

We want to come now to the "raison d'être" of modern (super) string theory: the fact that it provides a consistent theory of quantum gravity. As we shall argue later on, this is due to the extremely nice ultraviolet behaviour of string loops which turn out to be cut-off, effectively, at momenta $O(\hbar/\lambda_s)$. In other words, the fundamental length, which we have already identified with Planck's constant, also plays the role of a short distance, ultraviolet cut-off.

This miracle is hardly unexpected: a string is a collection of harmonic oscillators and we know that, in the harmonic oscillator, Δx and Δp are both of order $(\hbar)^{1/2}$. Since, in string theory, the role of $\hbar$ is played by λ_s^2 , Δx has to come out $O(\lambda_s)$. The Planck constant of the

theory thus determines the spacial extent of the string and thus generates a form factor (as for any extended object) which damps large momenta.

If we wish to count at this point the number of fundamental, dimensionful constants in string theory, we find that there are only two: c , the speed of light and λ_s , the Planck constant. At first sight this looks like no gain with respect to having c , $\hbar$ and the string tension $1/\alpha'$, since, in order to come down from three to two, we had to choose a different system of units of mass ($\alpha'm$ replaces m and makes it a length). The gain, however, with respect to QFT, is that we have inherited, at no extra cost, a finite ultraviolet cut-off [12]!

As long as we are in a non-compact, Minkowski space, the actual value of λ_s is, of course, irrelevant. What the theory should determine are the ratios of λ_s (c) with some other physical length (speed) that we call cm . (or cm./sec.). In other words, the theory allows, in principle, to determine λ_s (c) in cm . (cm./sec.).

The situation changes drastically if some of the space-time dimensions are compact (as it has to be the case physically), say circles. In this case the ratio ρ of the compactification radius and λ_s is relevant i.e. it does matter how many Planck constants is the internal space large. Very amusing symmetries have been discovered [13] (for closed strings) relating large ρ to small ρ . They could lead [12] to a stabilization of the radius of the internal manifold to be (a precise multiple of) λ_s .

In the rest of this paper I shall use some string results and intuition, combined with the known field theoretic limits of string theory, to argue how the fundamental length λ_s can make itself felt, changing much of our present, field-theory-based (pre)conceptions on questions like infinities, regularizations, subtractions, quadratic and logarithmic divergences, running and renormalization group.

At the end, the usual low energy field theory result will emerge, even if somewhat reinterpreted and, in my opinion, finally free from

possible criticism on the way of handling infinities. At the same time (as it is the case in the standard model vis a vis Fermi's theory) the new length parameter (the W - Z mass in the SM) is bound to affect physical predictions at scales less or equal to λ_s . These more speculative aspects of string theory will be discussed at the end.

4. PHYSICS MUCH ABOVE THE FUNDAMENTAL LENGTH

In this section we shall discuss the low energy, large distance limit of string theory arguing that the known results of Quantum Field Theory are recovered in a (perhaps) novel way. Quantum field theory thus emerges as an effective theory approximately valid at distances much larger than the string scale λ_s . The situation is thus reversed as compared to looking at string theory as being an effective theory for QCD.

We have already said that the tree level effective action of string theory closely resembles a classical field theory action, e.g. Einstein's theory of gravity coupled to a gauge theory. In order to show that there is a correspondence with QFT we have to argue that string loops reproduce, in some sense, at large distances, those of ordinary QFT.

I said "in some sense" since we know that, even at large distance, this cannot be, strictly speaking, the case. Indeed quantum gravity has no field theory limit in four dimensions and gauge theories have no field theory limit in more than four dimensions. All we can hope to recover is a regulated version of QFT loops which goes over to renormalized perturbation theory in the low energy limit for renormalizable interactions and to something new for non-renormalizable ones.

Let us consider, to be more specific, a string theory which has a perturbatively stable vacuum consisting of a number d of flat, infinite dimensions and a number d_c of compact dimensions. For the sake of simplicity, let us assume that the radii of the compact dimensions are all comparable and given by R_C . We shall also take $R_C > \lambda_s$.

There are two important features of string loops that have to be used in order to establish the connection with field theory.

- i) In string loops virtual momenta are effectively cut-off at the string scale $\hbar/\lambda_s$. Thus loops are UV finite.
- ii) The contribution of light particles circulating in the loop must reproduce, for internal momenta much smaller than $\hbar/R_C$, the field theoretic expression. Such expressions contain the effective coupling constant in d-dimensions given by:

$$\alpha_d = \alpha_{SL} / V_C \quad (10)$$

where $V_C \simeq (R_C)^d$ is the volume of the compactified space.

As discussed in more detail in ref.[6], these two features imply the following consequences:

- a) A typical string loop is "down" by a power of α_{SL} relative to a tree. Such a contribution comes from large (i.e. $O(\hbar/\lambda_s)$) internal momenta.
- b) If the external momenta are of order $\hbar/\lambda_s$, the above is the actual order of the loop correction. Assuming α_{SL} to be small, we conclude that the bare (i.e. tree level) parameters have physical meaning: they are to be identified with the short distance effective couplings, masses, etc.
- c) If the external momenta are much larger than $\hbar/\lambda_s$, $\hbar/R_C$, the problem becomes a string-theoretic one and cannot be tackled by field theory techniques (for some examples, see sect. 5).
- d) If the external momenta are much smaller than $\hbar/\lambda_s$, $\hbar/R_C$, which is the case we are interested in for the moment, the loop correction can be enhanced relative to its typical value because of infrared singularities. It is precisely through these IR singularities that one recovers well-known QFT effects. Let us list some of them:

- i) As a result of logarithmic enhancements one recovers:
 - the running of gauge couplings in $d=4$ [6];
 - the running of gravitational couplings in $d=4$ in the presence of a cosmological constant [6]. The latter phenomenon could be relevant in connection with the Cosmological Constant problem [14].
- ii) As a result of power-like infrared singularities one recovers large effects forbidden at tree level e.g.
 - light-light scattering, which, in string theory can occur at tree level, but is depressed by four powers of λ_s d/dx . At one loop, however, the graph contributing to $F_{\mu\nu}^4$ in the effective action is badly infrared singular with the electron mass playing the role of IR (mass) regulator. One thus recovers the field theoretic result $O(\alpha^2(p/m_e)^4)$.
- the famous Adler-Bell-Jackiw triangle anomaly in $d=4$ (and similar ones for other values of d). It is well known [15], indeed, that the ABJ anomaly can be seen either as a consequence of UV divergences or as due to a singular IR behaviour in the massless limit.
- iii) Non-renormalizable interactions are no problem whatsoever. To the contrary, their bad UV behaviour has a counterpart in their better IR limit. Consequently, there are no large quantum gravity corrections in $d=4$, if the cosmological constant is zero, or gauge coupling renormalization if $d>4$. In particular, the running of gauge couplings stops at the (de)compactification scale $\hbar/R_C$, which thus emerges as the true Grand Unification scale. This unique property of $d=4$ may have something to do [16] with why compactification to $d=4$ should be preferred over compactification, say, to $d=6$ or to no compactification at all.
- iv) Non-asymptotically free theories (such as QED or ϕ^4) have all the right to exist in string theory, while they are likely to be trivial in QFT [17]. As explained, the bare coupling constant basically

coincides with the coupling at the string or compactification scale. Hence, by definition, if there is a Landau pole in the field theory limit, such a pole is above the string scale, i.e. in the region where string or higher dimensional corrections cannot be neglected. Vice versa, the low energy coupling is bounded from above by the fact that it is $< O(1)$ at the string scale and that it decreases towards larger distances. This will allow to recover bounds [18] on elementary Higgs masses and couplings.

In conclusion, all the good results of QFT are recovered by string theory. Furthermore, string theory makes sense of some "bad" field theories (including one we know for sure to belong to Nature, gravity) and of the Kaluza-Klein idea because of its beautiful UV cut-off, hence, in the end, because of its fundamental length parameter.

We shall now see how this parameter is likely to affect physics around and below the Planck scale in a much more drastic way.

5. PHYSICS BELOW THE FUNDAMENTAL LENGTH

In the last few years a few attempts have been made to understand string theory at short distances i.e. in the region where one expects it to differ drastically from field theory.

It is still not clear how to define off-shell correlation functions in string theory. For the time being one is obliged to proceed differently from QFT in order to explore short distances. So far two methods have been tried: i) High energies and ii) High temperatures.

i) High Energy

The situation in this approach has been recently reviewed by the author [19]. I shall recall here the main conclusions. Hereafter d will stand for the number of non-compact dimensions.

- a) The high energy collision of gravitationally interacting particles (e.g. two gravitons) is characterized by a classical

interaction range, the so-called Schwarzschild radius:

$$R_S = (GE)^{1/(d-3)} \quad (11)$$

where G is Newton's constant. String theory enters in two ways: i) It introduces a new scale, the already much-mentioned string length parameter λ_S . ii) It relates G and λ_S through the (closed) string unification formula [20]:

$$\lambda_P^{(d-2)} = \alpha_d \lambda_S^{(d-2)} \quad \lambda_P^{(d-2)} = G\lambda \quad (12)$$

It is easy to see that, even at energies much larger than $\hbar/\lambda_S$, R_S can be smaller than λ_S for a sufficiently small α_d , i.e. for a Planck length which is much smaller than the string length.

The two cases have to be distinguished carefully:

- i) $R_S > \lambda_S$. This case includes the infinite energy limit at fixed Newton constant. One finds that expected Classical General Relativity effects can be recovered from flat space-time calculations. These effects dominate in this regime while string-scale phenomena remain screened inside R_S , which plays the role of a horizon. In order to arrive at this result a non-trivial resummation of leading terms to all orders in the string loop expansion is necessary. If one works at fixed genus, as E goes to infinity, one obtains the result that the S-matrix is trivial [21], in striking disagreement with CGR expectations. The point is, of course, that the genus that dominates at a given energy depends on the energy and goes to infinity with it.

To be sure, in this case calculations can only be made order by order in the parameter R_S/b , assumed to be small. As b approaches R_S , strong classical relativity effects start to emerge. Although technical complications prevent so far obtaining definite results, it seems very likely [22] that one goes over to a situation in which the collision is accompanied by the formation of a (Kerr) black hole [23]. If this is the case Hawking radiation with a thermal spectrum should be seen in the final state.

ii) $R_S < \lambda_S$

This case corresponds to large, but not extreme, energies or, if we prefer, to strings which are large in Planck-length units. We expect (and find!) string-size effects to be very relevant in this regime. In particular:

a) If we work at fixed c.m. angle, we find [24] that the Fourier transform from impact parameter to momentum transfer is always dominated by scales larger than λ_S .

b) If we work at fixed impact parameter we find that, unlike in the situation for $R_S > \lambda_S$, nothing dramatic happens as we reach values of $b < R_S$. The string length parameter appears to limit [12] the possible size of black holes to be larger than λ_S . Equivalently, because of Hawking's relation:

$$k_B T_{\text{Haw.}} = \hbar / R_S \quad (13)$$

one finds that the black hole's temperature cannot exceed the limiting string (Hagedorn) temperature:

$$k_B T_{\text{Hag.}} = \hbar / \lambda_S \quad (14)$$

A background and λ_S -independent characterization of an irrelevant metric deformation ΔG can be written as:

$$G^{-2} \partial_\Delta^2 G \gg 1 \quad (15)$$

ii) High temperature.

Recent investigations [25] of string thermodynamics at temperatures above $T_{\text{Hag.}}$ suggest that there is a drastic reduction in the number of degrees of freedom as compared to the field-theoretic case. This is evidenced by the behaviour of the free energy F (say at

genus one) in a given volume V as a function of the temperature:

$$F_{\text{string}} (k_B T V)^{-1} \simeq (\alpha' k_B T) / \lambda_S^d \quad (16)$$

as opposed to:

$$F_{\text{field th.}} (k_B T V)^{-1} \simeq (k_B T / \hbar) d^{-1} \quad (17)$$

It is amusing that the factor involved in going from the point to the string case is very similar to the one encountered [22] in the small impact parameter behaviour of the tree-level phase shift in the Planckian energy collision case, as if there was a correspondence between high T and small b .

This reduction of the relevant short distance degrees of freedom in string theory fits very well, of course, with the high energy collision results. It is also in line with the results of ref. [26] and with duality [13].

6. ENLARGED UNCERTAINTY AND EQUIVALENCE PRINCIPLES?

One can try to make abstraction from these result and ask whether or not distances shorter than λ_S can ever be probed in string theory. A few results give strong hints that they cannot and, at the same time, suggest (at least) two more general formulations of such impossibility: a) An Enlarged Uncertainty Principle ; b) An Enlarged Equivalence Principle. Let us briefly discuss them in succession:

a) A simple way to summarize the findings of ref.[24] on the scale Δx probed in a string-string collision at energy E and scattering angle θ i.e. at momentum transfer $\Delta p = \theta E$, is given by the formula:

$$\Delta x = \hbar / \Delta p_S + \alpha' \Delta p_S \quad (18)$$

where:

$$\Delta p_S = \Delta p / \sqrt{1+g} \quad (19)$$

and $\langle g \rangle$ is the average genus (loop order) contributing to the process in that kinematical regime. The first term in (18) looks like the usual uncertainty principle apart from the fact that the momentum transfer per loop Δp_S replaces the overall momentum transfer. The second term is a typical string effect. Obviously, the first term dominates over the second for $\Delta p_S < (\hbar/\alpha')^{1/2}$, while the opposite is true for $\Delta p_S > (\hbar/\alpha')^{1/2}$. In any event, the $x+1/x$ structure of (18) yields a lower bound on Δx :

$$\Delta x > \lambda_S$$

This is the enlargement of the uncertainty principle caused by the string length parameter!

b) In a recent paper [27] one has tried to give an explicit argument in favour of the conjecture [9] that an enlarged equivalence principle lies behind string theory. The basic idea behind is simple: we know that string theory contains general relativity and, hence, that it obeys, at least at large scales, Einstein's equivalence principle. Mathematically this statement can be expressed as the invariance of the string partition function Z of eq. (2) under a change of the space-time metric $G_{\mu\nu}$ which corresponds to a General Coordinate Transformation (GCT).

The question is whether or not Z is invariant under a larger class of transformations which affect the metric non-trivially but only on very short scales, i.e. on scales much shorter than λ_S . In order to see if this is the case we have considered [27] the scattering of point-like and of string-like particles by massless, gravitational shock waves (which are precisely those relevant for High Energy superstring collisions) before and after adding ripples in the metric that live on scales much shorter than λ_S . We have found that, for point particles, the ripples influence the scattering process, while strings appear to be "blind" to such modifications. We thus have a confirmation that short distance degrees of freedom are irrelevant in string theory.

7. A SCALE-FREE (TOPOLOGICAL) PHASE?

I should finally mention an idea on a possible phase transition in string theory in which even the fundamental length λ_S disappears. We have seen that λ_S has come from having chosen, for phenomenological reasons, a string ground state consisting of Minkowski space time.

At very high temperatures and energy densities of the type possibly realized in the early universe, it is certain that Minkowski space-time was far from describing the average background in which strings were propagating. What could have been a more appropriate background?

An amusing possibility would be a background that is more symmetric than Minkowski and which does not force us to introduce a fundamental scale as a vacuum parameter (see again ref[10] for similar ideas outside of the string context). Two possibilities come to mind:

i) A non-trivial metric which is homogeneous of degree -2 in the coordinates:

$$G_{\mu\nu} \sim X^{-2} \quad (21)$$

Since -2 is the physical dimension of $G_{\mu\nu}$ we do not need to introduce any dimensional parameter in this case. The metric is naturally of the de Sitter type (if we also insist that it is homogeneous and isotropic) and is characterized by a pure number γ :

$$G_{\mu\nu} = \gamma X_0^{-2} \eta_{\mu\nu} \quad (22)$$

Study of string propagation in these metrics [28] shows that nowhere a fundamental length comes into play. However, what goes on depends rather crucially on the parameter γ . As is clear from the criterion (15), small γ corresponds to the dangerous case in which string effects may show up. Indeed one finds that, in this case,

strings become highly unstable and blow up in size, a phenomenon reminiscent of Jean's instabilities and galaxy formation. Maybe this will lead to some sort of self-sustained inflation in string theory [29]. Another possibility is that, at $\gamma < 1$, the instability we have found signals the transition to an even more dramatic background:

$$\text{ii) } G_{\mu\nu} = 0 \quad (23)$$

In this case no metric-in-the-mean can be defined and the metric is entirely due to quantum fluctuations. This could correspond to a purely topological phase [30] for string theory. It also corresponds to a zero string-tension situation, which has been advocated to occur [31] above the limiting Hagedorn temperature. Maybe the early universe was in this topological phase and that the very concept of distance -and of a minimal scale- has only come out "later".

REFERENCES

1. Democritus, ca 400 B.C.; Zenon, ca. 300 B.C.
2. see e.g. G. Veneziano, Phys. Reports, 9C (1974) 199.
3. S. Coleman and E. Weinberg, Phys. Rev. D7 (1973) 1888; D. J. Gross and F. Wilczek, Phys. Rev. D8 (1973) 3633.
4. J. Scherk and J. H. Schwarz, Nucl. Phys. B81 (1974) 118.
5. A. M. Polyakov, Phys. Lett. 103B (1981) 207, 211; see also M. Ademollo et al., Nuovo Cim. 21A (1974) 77.
6. G. Veneziano, DST workshop on Superstring Theory (H.S. Mani and R. Ramachandran eds.) World Scientific Publ. Co. (1988) page 1.
7. see e.g. C. Lovelace, Phys. Lett. 135B (1984) 75; E.S. Fradkin and A.A. Tseytlin, Phys. Lett. 158B (1985) 316; *ibid* 160B (1985) 69; Nucl. Phys. B261 (1985)1; C. G. Callan D. Friedman, E.J. Martinec and M. J. Perry, Nucl. Phys. B262 (1985) 593;
8. T. Kubota and G. Veneziano, Phys. Lett. 207B (1988) 419 and references therein.
9. G. Veneziano, in "Strings and Gravitation", talk presented at the 5th Marcel Grossmann meeting (Perth, Aug.1988), Boston University preprint, BU-HEP-88-47 (1988).
10. V. de Alfaro, S. Fubini and G. Furlan, Nuovo Cim. A50 (1979) 523; *ibid*. B57 (1980) 227.
11. E. S. Fradkin and A. A. Tseytlin, ref. [7]; E. Witten, Phys. Lett. 149B (1984) 351.
12. G. Veneziano, Europhysics Lett. 2 (1986) 133.
13. K. Kikkawa and M. Yamasaki, Phys. Lett. 149B (1984) 357; N. Sakai

and I. Senda, *Prog. Theor. Phys.* 75 (1986) 692; A. Giveon, E. Rabinovici and G. Veneziano, *Nucl. Phys.* B322 (1989) 167.

14. T.R. Taylor and G. Veneziano, *Phys. Lett.* 228B (1989) 311.

15. see e.g. T. Banks, Y. Frishman, A. Schwimmer and S. Yankielowicz, *Nucl. Phys.* B177 (1981) 157.

16. T.R. Taylor and G. Veneziano, *Phys. Lett.* 212B (1988) 147.

17. see e.g. L. Landau, "Niels Bohr and the Development of Physics", ed. W. Pauli (Pergamon Press London, 1955); K.G. Wilson, *Phys. Rev. D* 6 (1972) 419; M. Aizenmann, *Comm. Math. Phys.* 86 (1982) 1; J. Frohlich, *Nucl. Phys.* B200 (1982) 281.

18. R. Dashen and H. Neuberger, *Phys. Rev. Lett.* 50 (1983) 1987. For a recent review, see e.g. P. Hasenfratz, *Lattice '88 Conference*, (Batavia, Ill. Sept. 1988), *Nucl. Phys. (Proc. Suppl.)* 9 (1989) 3.

19. G. Veneziano, "An Enlarged Uncertainty Principle from Gedanken String Collisions", talk presented at the Superstring '89 workshop, Texas A&M University (March 1989), CERN-TH. 5366/89, and references therein.

20. see e.g. D. J. Gross, J. A. Harvey, E. Martinec and R. Rohm, *Nucl. Phys.* B256 (1985) 253.

21. D. J. Gross and P. F. Mende, *Phys. Lett.* 197B (1987) 129; *Nucl. Phys.* B303 (1988) 407.

22. D. Amati, M. Ciafaloni and G. Veneziano, *Int. Journ. Mod. Phys.* 3A (1988) 1615; and in preparation.

23. see e.g. S. W. Hawking, *Phys. Rev. D* 13 (1976) 191.

24. D. Amati, M. Ciafaloni and G. Veneziano, *Phys. Lett.* B216 (1989) 41.

25. J.J. Atick and E. Witten, *Nucl. Phys.* B310 (1988) 291; see also: Ya I. Kogan, *JETP Lett.* 45 (1987) 709; B. Sathiapalan, *Phys. Rev. D* 35

(1987) 3277.

26. M. Karliner, I. Klebanov and L. Susskind, *Int. Journ. Mod. Phys.* A3 (1988) 1981; T. Yoneya, "On the Interpretation of Minimal Length in String Theory", Univ. of Tokio preprint (1989); K. Konishi, G. Paffuti and P. Provero, "Minimum Physical Length and the Generalized Uncertainty Principle in String Theory", Univ. of Pisa preprint, IFUP-TH 46/89.

27. M. Fabbrichesi and G. Veneziano, "Thinning out of Relevant Degrees of Freedom in Scattering of Strings", preprint CERN-TH.5509/89.

28. N. Sanchez and G. Veneziano, "Jeans-like Instabilities for Strings in Cosmological Backgrounds", preprint CERN-TH. 5491/89.

29. Y. Aharonov, F. Englert and J. Orloff, *Phys. Lett.* B199 (1987) 366; N. Turok, *Phys. Rev. Lett.* 60 (1988) 549.

30. See, e.g., E. Witten, "The Search for Higher Symmetries in String Theory", talk presented at the Royal Society Meeting on String Theory, London (Dec.1988).

31. See, e.g., P. Salomonson and B. S. Skagerstam, *Physica A* 158 (1989) 499.