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THE FRACTAL STRUCTURE OF MULTIHADRON PRODUCTION AT HIGH ENERGIES*

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Abstract

We first give a short general introduction to intermittency and fractal structure, and discuss how self-similar patterns can arise in particle production. Then we show that at high energies, self-similar jet production leads to a universal power law behaviour for the multiplicity moments as a function of the relative rapidity $Y/\delta y$, once the rapidity intervals δy are large enough to be outside the usual resonance region ($\delta y \geq 1 - 2$). Data from the UA5 experiment at CERN (for $\sqrt{s} = 200, 546$ and 900 GeV) are found to agree very well with the predicted universal behaviour.

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1. Introduction

The study of multihadron production up to ISR energies ($\sqrt{s} \leq 60$ GeV) has revealed a number of general features, such as Feynman scaling or KNO scaling. With the advent of very high energy accelerators, these phenomenological laws have reached their limits of validity, and new general features of very high energy multihadron production have to be established. At sufficiently high energies one would moreover hope to reach a range in which such features can be related to predictable consequences of QCD.

In this talk, I want to discuss a new pattern of multiparticle production which sets in clearly only for $\sqrt{s} > 50$ GeV; the results I will present are based on joint work with I. Sarcevic¹. We find at these very high energies a universal power law behaviour of the multiplicity moments as a function of the rapidity interval. Such a behaviour follows from a self-similar, or fractal, pattern of multihadron production.

At very high energies, the multiplicity distribution dN/dy can be measured over a large range $0 \leq y \leq Y$. We can therefore subdivide this range into different size intervals δy which are all larger than that characteristic for the usual resonance correlations. If the invariant mass $M_{\pi\pi}$ of a two-pion system is of order $M_{\pi\pi} \approx 0.5 - 1.0$ GeV, the separation in rapidity of the two pions is about $\delta y_0 \approx 1 - 2$. The intervals we want to consider are to be larger than this value δy_0 .

Thus our considerations address a range of rapidity intervals complementary to that studied in recent "intermittency" investigations.² There one looks for a power-law dependence of multiplicity moments in rapidity intervals $\delta y \rightarrow 0$, i.e., for $\delta y < \delta y_0$. It has recently been asserted that the data in that δy range can be accounted for in terms of conventional short-range correlations^{3,4}, without any necessity for intermittent behaviour. We shall not deal with this question here, however, and instead concentrate on the possibility of self-similar behaviour for large rapidity separation, i.e., in the jet region.

The idea of self-similar multihadron production is not new. Hagedorn⁵ proposed more than 20 years ago that hadronic collisions would produce "fireballs", consisting of fireballs, which in turn consist of fireballs, and so on, with always the same structure down to the observed hadrons as the end of the cascade. Once we are away from this final step, which has an intrinsic scale fixed by the hadron masses, all steps are self-similar in their decay pattern. Such a fireball picture did not account for the jet structure observed in multihadron production and therefore is cited here only as an illustration. Within the framework of QCD, however, we expect that a jet will decay into jets, which in turn lead to jets, and so on, down to a final hadronisation stage - giving again rise to a self-similar pattern.

Since the study of self-similar structures in particle physics was started quite recently², we will spend the next two sections with a simple introduction to intermittency and fractal structure. Following that, we shall show how a decay cascade can

be related to these concepts. Finally, in section 5, we shall apply our formalism to high energy multihadron production and show that CERN collider data indeed show the behaviour expected in self-similar jet production.

2. Intermittency

We first want to illustrate how moments provide a measure of fluctuations, and then define what is meant by intermittency. Consider N objects (balls, particles, events, etc.), to be distributed over a d -dimensional unit volume $V \equiv R^d$ according to some given pattern. Subdivide this volume into n equal cells (bins, blocks) of size L^d , so that $n = (R/L)^d \equiv r^d$; r is our basic linear scale measure. In the m -th cell, there are k_m objects, and $\sum_{m=1}^n k_m = N$. We now define the normalized multiplicity moments of order l ,

$$f_l(r) \equiv \left[\frac{1}{n} \sum_{m=1}^n k_m^l \right] / \left[\left(\frac{1}{n} \sum_{m=1}^n k_m \right)^l \right]. \quad (1)$$

The probability to find an object in the m -th cell is $P_m(r) = k_m/N$, so that we can rewrite eq.(1) as

$$f_l(r) = r^{d(l-1)} \sum_{m=1}^n P_m^l. \quad (2)$$

Since $\sum P_m^l \leq \sum P_m = 1$, we have

$$1 \leq f_l(r) \leq r^{d(l-1)}, \quad (3)$$

in particular, $f_l(r=1) = 1$. A suitable quantity for the measure of fluctuations is, as we shall see shortly, the ratio

$$\lambda_l(r) \equiv [\ln f_l(r)/(l-1) \ln r], \quad (4)$$

which, using eq.(3), fulfills

$$0 \leq \lambda_l(r) \leq d. \quad (5)$$

Let us look at $\lambda_l(r)$ for two extreme distributions.

If the N objects are equidistributed over all the n cells, then $P_m = 1/n$, giving us $f_l(r) = 1 \forall l$. Hence we have

$$\lambda_l(r) = 0 \quad \forall l \quad (6)$$

for an equidistribution. On the other hand, if all objects are placed in one single cell, then $\sum P_m^l = 1$ and $\lambda_l(r)$ attains its upper limit,

$$\lambda_l(r) = d \quad \forall l. \quad (7)$$

Thus the larger $\lambda_l(r)$ is, the more does the distribution fluctuate. If $\lambda_l(r)$ remains non-zero even in the limit of arbitrarily large $r = R/L$, (i.e., either small cell size or large volume)

$$\lambda_l \equiv \lim_{r \rightarrow \infty} \lambda_l(r) > 0 \quad \forall l, \quad (8)$$

then we speak of intermittency² and call λ_l the (normalized) intermittency index. Intermittency thus implies a power-law dependence of the moments f_l on the scale measure r ,

$$f_l \sim r^{\lambda_l}, \quad (9)$$

with λ_l as the power.

Let us note that these considerations remain non-trivial only if $N \gg n$. If we keep N fixed as $n \rightarrow \infty$, then eventually all cells except some fixed number (independent of n) are empty, giving us $\lambda_l = d \quad \forall l$. This is a trivial form of intermittency: for fixed N and small enough cell size, all distributions fluctuate. Our extreme case of all objects in one cell is also of this type. What we are really looking for are situations in which the probability moments

$$g_l(r) \equiv \sum_{m=1}^n P_m^l \quad (10)$$

do not simply become constant for large r , but instead retain an r -dependence. In terms of $g_l(r)$, the intermittency indices (8) become

$$\lambda_l = d - \lim_{r \rightarrow \infty} [\ln g_l(r) / (l-1) \ln r], \quad (11)$$

so that for non-trivial intermittency, the second term on the r.h.s. of eq.(12) should not vanish. In the following section, we shall see that this implies distributions which have a fractal structure.

3. Fractal Structure

We begin again with a distribution function of some objects in a given d -dimensional space $V = R^d$, such as the distribution of probabilities in the previous section. For simplicity, let us look at the one-dimensional case. We subdivide the line R into $n = r = R/L$ intervals $m = 1, 2, \dots, r$; as before, r fixes our linear scale. We now want to count the number $n_F(r)$ of cells for which $P_m(r) \neq 0$. For a smooth distribution, $n_F(r) \sim r$. If this remains true even for $r \rightarrow \infty$, then the set of r segments of the line R becomes the one-dimensional continuum. On the other hand, if the objects are found in a finite number N of cells, no matter how large r is, or if we only have a fixed number N of objects, then the set of segments for which $P_m(r) \neq 0$ becomes for $r \rightarrow \infty$ a set of measure or dimension zero. The extreme cases thus give $d = 1$ and $d = 0$ for the dimension of the set of segments for which $P_m \neq 0$

in the limit of large r . There are, however, distributions which fall between these extremes⁶. To see this, we define as generalized dimension d_F the quantity

$$d_F \equiv \lim_{r \rightarrow \infty} [\ln n_F(r) / \ln r] \quad (12)$$

For the dimension of any smooth distribution in a one-dimensional space, we therefore recover $d_F = 1$, and for that of a distribution built up of spikes associated to a finite number of space points, we have again $d_F = 0$. The situation is changed, however, for self-similar distributions such as the Cantor set (fig. 1): the irregularities of this distribution are the same for all scales. After r steps, the Cantor set needs a scale of size $L = (1/3)^r$ to resolve its structure, and we need $n_F = 2^r$ such scale elements.

As a result, we now find

$$d_F = [\ln 2 / \ln 3] < 1 \quad (13)$$

for the generalized dimension of the Cantor set. The dimension (13) thus "geometrizes" self-similar fluctuating behaviour by associating it to a uniform distribution in a space of fractal dimension d_F . We shall therefore call fractal⁷ those systems, for which $d_F \neq 1$.

These considerations are readily generalized from one to d dimensions. In terms of $r = R/L = n^{1/d}$, we define

$$d_F \equiv \lim_{r \rightarrow \infty} [\ln n_F(r) / \ln r] = d \lim_{n \rightarrow \infty} [\ln n_F(n) / \ln n], \quad (14)$$

so that now a smooth distribution in a d -dimensional space gives us $d_F = d$, and fractal behaviour implies $d_F < d$. Note that this does not require d_F to be fractional, i.e., non-integer.

The probability $P_m(r)$ to find an object in the m -th of n cells in a topological space V , which we had considered in the previous section, is now replaced by the probability $P_m(n_F)$ to be in the m -th of n_F elements of a fractal space. Since the distribution in this space is uniform, we simply have

$$P_m(n_F) = 1/n_F. \quad (15)$$

In view of eq.(14), the number n_F is related to (sufficiently large) r or n by

$$n_F(n) = r^{d_F} = n^{d_F/d}. \quad (16)$$

While in the previous section we had an irregular distribution of objects in n cells of the d -dimensional space V , we now have a uniform distribution over $n_F \leq n$ elements in a d_F -dimensional fractal space. The multiplicity moments (10) thus become

$$g_l(r) = g_l(n_F) = \sum_{m=1}^{n_F} P_m^l = n_F^{l(1-d_F/d)}, \quad (17)$$

and hence, using eqs. (16),

$$g_l(r) = r^{(l-1)d_F} \quad (18)$$

Inserting this into eq. (11), we obtain⁶

$$d_F = d - \lambda_l \quad (19)$$

for the fractal dimension of any system showing intermittent behaviour.

From eq. (19), we see immediately that in the absence of intermittency ($\lambda_l = 0$), $d_F = d$, at a sufficiently small scale, the original distribution in V is in this case smooth. Further, we note that the "trivial" intermittency obtained by distributing a fixed number of objects into an ever increasing number of cells, or all objects into one, leads with $\lambda_l = d \forall l$ to $d_F = 0$: it records the fluctuations due to a set of measure or "dimension" zero.

Eq. (19) also shows that the fractal structure for non-trivial intermittency is of the same type as that of the Cantor set, with $d_F < d$. Let us elaborate this point a little. When we go from a uniform distribution to one which has all objects in one cell even for $n \rightarrow \infty$, i.e., from the least to the most fluctuating distribution, the intermittency indices $0 \leq \lambda_l \leq d$ vary from their smallest to their largest value. To obtain a system with a fractal dimension $d_F < d$, we need objects in infinitely many cells, but such that the number of cells with $P_m \neq 0$ does not fill the entire d -dimensional volume. The "fractal dust" created in the Cantor set after r steps, with $r \rightarrow \infty$, is just of this type.

Let us now see how multiparticle production, in particular cascade production, fits into this picture.

4. A Simple Cascade Pattern*

Fig. 2 gives a specific decay pattern for a heavy "particle" (excited state, fireball or jet) of "mass" m_0 . After r decay steps, we have 2^r particles. The two particles, into which m_0 decays, each have a mass $m_1 = (1/\alpha)m_0$, with $\alpha \geq 2$. Energy conservation gives us

$$m_0^2 = 4[m_1^2 + \mathbf{p}_1^2] \quad (20)$$

where $\mathbf{p}_1$ is the momentum of each of the two particles of mass m_1 in the c.m. of their "parent" m_0 . Thus $\alpha = 2$ corresponds to break-up without kinetic energy; if the two particles have non-zero momenta in the c.m. of their parent, then $\alpha > 2$. Continuing the decay cascade, we get by self-similarity $m_r = (1/\alpha)m_{r-1} = (1/\alpha)^r m_0$ as the mass of each of $n(r) = 2^r$ particles after r steps.

To see how this is related to fractal structure, imagine a line of length m_0 . For $\alpha = 2$, the $n(r)$ masses m_r cover this line completely: they add up to m_0 . If $\alpha > 2$, however,

$$n(r)m_r = (2/\alpha)^r m_0 < m_0; \quad (21)$$

* Similar arguments concerning cascade decay are given in ref. 9.

in fact, for $r \rightarrow \infty$, $n(r)m_r \rightarrow 0$, even though the number $n(r)$ of particles becomes infinite. So the larger r , the less does $n(r)m_r$ cover the line of length m_0 . The dimension of the line $n(r)m_r$, for $r \rightarrow \infty$ is therefore less than one; $d = 1$ would require that $n(r)m_r$ covers a finite segment of m_0 even for infinite r . It is also larger than $d = 0$, since we have an infinite number of masses. Following the theory of fractal structures⁷ as sketched above, we therefore define as generalized dimension⁸

$$d_F \equiv \lim_{r \rightarrow \infty} \left\{ \frac{\ln n(r)}{\ln(m_0/m_r)} \right\} = \frac{\ln 2}{\ln \alpha} \quad (22)$$

For $\alpha = 2$, we recover with $d_F = 1$ the one-dimensional continuum corresponding to $n(r)m_r = m_0$; for $\alpha > 2$, however, $0 < d_F < 1$. Let us note at this point that the structure of the process remains the same if we keep m_r fixed at the same value $m_r = \mu$, but instead let $m_0 = \alpha^r \mu$ go to infinity as $r \rightarrow \infty$. In both cases we arrive at an infinite self-similar cascade: Either we have a fixed mass m_0 decaying into smaller and smaller masses $m_r = (1/\alpha)^r m_0$, or we have asymptotic decay products of mass μ originating from a heavier and heavier starting mass $m_0 = \alpha^r \mu$. The latter case will turn out to be the relevant one for the high energy limit of multihadron production.

5. High Energy Multihadron Production

We now have to connect the cascade decay pattern to multiplicity moments. Consider the multiplicity distribution dN/dy in the rapidity range $0 \leq y \leq Y$; here $Y = Y(s)$ denotes the maximum (laboratory) rapidity possible at a given c.m.s. collision energy $\sqrt{s}$. Now divide this range into $\bar{n}$ intervals (bins) of size δy . The normalized standard moment of order l is then defined as

$$C_l(\delta y) = \left(\frac{1}{\bar{n}} \sum_{m=1}^{\bar{n}} k_m^l / \left[\frac{1}{\bar{n}} \sum_{m=1}^{\bar{n}} k_m \right]^l \right), \quad (23)$$

where

$$k_m \equiv \int_{y_m}^{y_{m+1}} dy (dN/dy) \quad (24)$$

is the number of particles in the m -th rapidity bin of a given event. Since

$$\sum_{m=1}^{\bar{n}} k_m = \int_0^Y dy (dN/dy) = N \quad (25)$$

is the total number of particles in the event under consideration, we can rewrite Eq. (23) in the form

$$C_l(\delta y) = (Y/\delta y)^{l-1} \left\langle \sum_{m=1}^{\bar{n}} P_m^l \right\rangle \quad (26)$$

Here $P_m \equiv k_m/N$ is the probability to find a particle in the m -th bin, and we have used $\bar{n} = Y/\delta y$. In both Eqs. (23) and (26), the average $\langle \dots \rangle$ is taken over all available events.

In a typical multihadron production event, the probability P_m will fluctuate considerably from bin to bin and thus is difficult to relate to any theoretical scheme. For the case of a self-similar cascade, however, we can map the P_m from the usual one-dimensional rapidity space, where they fluctuate strongly, onto a fractal space, in which the fluctuations are "smoothed out" at the expense of reducing the dimension from $d = 1$ to $d_F \leq 1$. In the fractal space, we have $n \leq \bar{n}$ intervals, with $P_m = 1/n$. For a jet, the overall multiplicity grows as $\ln s \sim Y$, so that we identify $m_0 = Y$ and $m = \delta y$. From Eq. (22), we then get $P_m = (\delta y/Y)^{d_F}$, and hence

$$\sum_{m=1}^n P_m^l = (\delta y/Y)^{(l-1)d_F} \quad (27)$$

Inserting this in Eq. (26), we have

$$C_l(\delta y) = (Y/\delta y)^{(l-1)(1-d_F)} \quad (28)$$

Thus we obtain the result that multihadron production through a self-similar cascade will lead to multiplicity moments having a power-law dependence on the ratio $(Y/\delta y)$. In other words, we expect the ratio of logarithms,

$$\ln C_l(\delta y)/\ln(Y/\delta y) = (l-1)(1-d_F), \quad (29)$$

to become a constant when $Y/\delta y \rightarrow \infty$. The simple fractal model we had considered with the cascade of Fig. 2 leads to the l -dependence of Eq. (29). The concept of fractal dimension has been extended, however, to also include situations in which d_F depends on l (multifractals).⁶ The essential point of Eq. (29) therefore is that for large $(Y/\delta y)$ (i.e., away from the end of the cascade), the ratio of the logarithms in Eq. (10) becomes a constant; this constant could be different for different l .

In the discussion of the cascade of Fig. 2, we had seen that there are two ways to make m_0/m , large. Similarly, we can increase $Y/\delta y$ by letting $\delta y \rightarrow 0$ at fixed Y or by letting $Y \rightarrow \infty$ for $\delta y \geq \delta y_0$, where δy_0 denotes the lower bound needed to exclude the "resonance" regime. In a number of intermittency studies the first alternative was investigated.² As already mentioned, there are now claims^{3,4} that this will only bring us into the region of the usual short-range two-particle correlations, which signal the end of the cascade. We want to investigate here if multihadron production takes place in form of self-similar cascade as $\sqrt{s} \rightarrow \infty$. If we keep $\delta y > \delta y_0$, the moments of the multiplicity distribution must then vary with $Y/\delta y$ according to Eq. (29).

Let us now estimate the lower bound δy_0 for the rapidity bins to be used. For a two-pion system of invariant mass $M_{\pi\pi} = 0.5$ GeV, the rapidity separation between

the two pions is $\delta y \sim 1$; we have here assumed each pion to have transverse mass $m_T = (p_T^2 + m_\pi^2)^{1/2} \approx 0.5$ GeV. For $M_{\pi\pi} = 1$ GeV, we get $\delta y \sim 2.3$, so that rapidity windows $\delta y \geq \delta y_0 \sim 1-2$ ought to put us above the end region of the cascade.

In Fig. 3, we now illustrate schematically the prediction resulting from our consideration. To have a well defined case, we look at $C_l(Y/\delta y)$ for $\delta y_0 \sim 1.8$ (i.e. $M_{\pi\pi} \sim 0.7$ GeV) and some fixed l . As $\sqrt{s}$ varies from 10 GeV to 1800 GeV, the end of the power-law behaviour predicted by Eq. (10) shifts from $\ln(Y/\delta y) = 0.5$ to 1.5. Beyond this end point, the logarithmic moments will flatten out; how much, depends on the details of the short range correlations. If we increase our $M_{\pi\pi}$ cut-off to 1 GeV, i.e., take $\delta y_0 = 2.3$, then for $\sqrt{s} = 10$ GeV, we have power-law behaviour up to $\ln(Y/\delta y) \sim 0.25$ only. To have a reasonable range within which to test the power-law (Eq. (29)), we therefore need $\sqrt{s} \sim 50$ GeV or higher. The dashed line in Fig. 3 shows the predicted form as $\sqrt{s} \rightarrow \infty$.

Before we can apply our considerations to actual data, we have to address two somewhat technical points. Since even at the highest available energies, the multiplicities per bin become quite small when $\delta y \rightarrow 1-2$, we should, before we study the effect of self-similar cascading, try to eliminate purely statistical fluctuations (which lead to a behaviour similar to the "trivial" intermittency of section 2.). In Ref. 2, it is proposed to do this by considering the factorial moments $\bar{F}_l$, rather than the standard moments C_l . For $l = 2-5$, the two forms are related by

$$\begin{aligned} F_2 &= C_2[1 - (1/\bar{N}C_2)] \\ F_3 &= C_3[1 - (3C_2/\bar{N}C_3) + (2/\bar{N}^2C_3)] \\ F_4 &= C_4[1 - (6C_3/\bar{N}C_4) + (11C_2/\bar{N}^2C_4) - (6/\bar{N}^3C_4)] \\ F_5 &= C_5[1 - (10C_4/\bar{N}C_5) + (35C_3/\bar{N}^2C_5) - (50C_2/\bar{N}^3C_5) + (24/\bar{N}^4C_5)] \end{aligned} \quad (30)$$

Here $\bar{N}$ is the average multiplicity. We see that for $\sqrt{s} \rightarrow \infty$, and hence $\bar{N} \rightarrow \infty$, the two forms become equivalent, since C_l/C_{l+1} also vanishes in that limit. Following Ref. 2, we shall consider the data for the factorial moments; it turns out, however, that up to $\delta y \sim \delta y_0$, the two forms do not differ very much. Below δy_0 , the standard moments are strongly enhanced by statistical fluctuations.

The second problem arises because the available data have not been analysed such as to give the moments C_l as we have defined them in Eq. (23), or the equivalent F_l . Instead, experimental data are presented in terms of the moments

$$\tilde{C}_l(\delta y) = \left(\frac{1}{\bar{n}} \sum_{m=1}^{\bar{n}} k_m^l \right) / \left(\frac{1}{\bar{n}} \sum_{m=1}^{\bar{n}} k_m \right)^l \quad (31)$$

and the corresponding $\tilde{F}_l$: numerators and denominators are separately averaged over all events. As a consequence, they do not become unity for $\delta y = Y$, and different incident energies lead to different starting points.

We strongly suggest a reanalysis of the data, normalizing event by event and then averaging. To use the existing data, we are forced to shift the curves by their value at $\delta y = Y$, so that we will now look at the behaviour of

$$\ln \tilde{f}_l(\delta y) - \ln \tilde{f}_l(\delta y = Y) \equiv \ln f_l(\delta y); \quad (32)$$

by definition, $\ln f_l(\delta y)$ will then vanish for $\delta y = Y$ at all energies and for all l .

A complete experimental determination of moments for $l = 2 - 5$, with the definition (32), is given by the UA5 Collaboration, for $\sqrt{s} = 546 \text{ GeV}$ and for $\sqrt{s} = 200$ and 900 GeV . According to our considerations, the results for $\ln f_l(\delta y)$ at different energies but for fixed l should fall on one universal curve, up to $\ln(Y/\delta y) \sim 1.2(200 \text{ GeV})$, $1.3(546 \text{ GeV})$ and $1.4(900 \text{ GeV})$, if we use $\delta y_0 = 1.8$ as smallest bin size. In Figs. 4-7, we see that the data agree very well with this prediction; there is no noticeable systematic deviation from straight-line behaviour, nor any indication for different slopes at different energies. We should emphasize here that the production pattern which we have observed is an intrinsic irregularity of the data – it does not involve any theoretical input. Such a behaviour does arise naturally, however, in a fractal production scheme.

Let us now define a slope measure by

$$\lambda_l \equiv \frac{1}{(l-1)} [\ln f_l(\delta y) / \ln(Y/\delta y)]. \quad (33)$$

A fractal cascade then gives us, from Eq. (29), the l -independent result

$$\lambda_l = (1 - d_F); \quad (34)$$

however, a multifractal picture still allows λ_l to depend on l . Straight-line fits to the data (Figs. 4-7) give the following values for the slope measures

$$\lambda_2 = 0.140 \pm 0.008 \quad (35a)$$

$$\lambda_3 = 0.172 \pm 0.009 \quad (35b)$$

$$\lambda_4 = 0.203 \pm 0.013 \quad (35c)$$

$$\lambda_5 = 0.228 \pm 0.016 \quad (35d)$$

Before drawing any conclusions about this noticeable but not very strong l -dependence, we would prefer to see if it persists when the experimental moments are re-determined according to Eq. (23) or the corresponding $\tilde{f}_l(\delta y)$, rather than the presently used averaging according to Eq. (31).

We note that our considerations predict forthcoming data from the Tevatron ($\sqrt{s} = 1800 \text{ GeV}$) to fall on the curves in Figs. 4-7. The region of power-law behaviour in this case is expected to persist to a somewhat higher value of $\ln(Y/\delta y)$. It will

be very interesting to see whether this holds true also for multihadron production at LEP, or whether $q\bar{q}$ initial state leads to a different hadronization pattern.

On a theoretical level, we now obviously face the challenging problem of deriving self-similar multihadron production in general, and the observed intermittency measures λ_l in particular, from the multi-jet structure of QCD. For first steps in this direction, see ref.12.

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Figure Captions

Fig. 1 The Cantor Set.

Fig. 2 Self-similar Cascade.

Fig. 3 Schematic illustration of the multiplicity moments $C_l(Y/\delta y)$ at fixed l and $\delta y_0 = 1.8$ for different incident energies $\sqrt{s}$. The dashed line shows the continuation for $\sqrt{s} \rightarrow \infty$.

Figs. 4-7 Experimental data on multiplicity moments $\ln f_l(\delta y)$ ($l = 2, 3, 4, 5$) as a function of $\ln(Y/\delta y)$ for the energies $\sqrt{s} = 200$ GeV (squares), $\sqrt{s} = 546$ GeV (diamonds) and $\sqrt{s} = 900$ GeV (crosses) and the straight line fit to the data at large δy .

Fig. 1

Fig. 2

Fig. 4

Fig. 3

Fig 6

Fig 5

Fig 7