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A Study of Jet Production Rates and a Test of QCD on the Z^0 Resonance

The OPAL Collaboration

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Abstract

Relative production rates of multijet hadronic final states of Z^0 boson decays, observed in e^+e^- annihilation around 91 GeV centre of mass energy, are presented. The data can be well described by analytic $O(\alpha_s^2)$ QCD calculations and by QCD shower model calculations with parameters as determined at lower energies. A first adjustment of $\Lambda_{\overline{MS}}$ and of the renormalisation scale μ^2 in $O(\alpha_s^2)$ QCD results in values similar to those obtained in the continuum of e^+e^- annihilations. Significant scaling violations are observed when the 3-jet fractions are compared to the corresponding results from smaller centre of mass energies. They can be interpreted as being entirely due to the energy dependence of α_s , as proposed by the nonabelian nature of QCD. The possibility of an energy independent coupling constant can be excluded with a significance of 5.7 standard deviations.

With the successful operation of the e^+e^- colliders SLC and LEP, hadronic decays of the Z^0 boson can be studied for the first time. Besides the possibility of precisely determining fundamental parameters of the standard model of electroweak interactions, these decays are also of particular interest for testing the validity of Quantum Chromodynamics (QCD), the nonabelian gauge theory of the strong interactions [1]. Within the framework of this theory, hadronic decays of the Z^0 are associated with the production of quarks and gluons, which subsequently materialise into jets of hadrons. The relative production rates of multijet events are determined by the value of the strong coupling constant, α_s , which - according to the concept of asymptotic freedom - is expected to decrease with increasing energy. Experimental studies of jet production rates are therefore well suited to test the basic ideas of QCD and to determine the free parameters of the theory.

In this letter, the relative production rates of multijet hadronic final states as observed with the OPAL detector [2] at the CERN e^+e^- collider, LEP, are presented. The detector offers a good acceptance for hadronic events over a solid angle of nearly 4π . In this analysis, mainly the information of the central jet chamber, the barrel and end cap lead glass electromagnetic calorimeters and the barrel time-of-flight counters are used in order to identify hadronic events and to reconstruct the energy and direction of flight of charged and neutral particles. The data were recorded between 88 GeV and 94 GeV center of mass energy and correspond to an integrated luminosity of about 540 nb^{-1} . The data are analysed in terms of testing the expectations of perturbative QCD. In a first step, the results are compared to QCD model calculations and to analytic $O(\alpha_s^2)$ QCD calculations with parameters as determined in the e^+e^- annihilation continuum at lower centre of mass energies. The second step is devoted to an adjustment of the QCD parameter $\Lambda_{\overline{MS}}$ and the renormalisation scale μ^2 in $O(\alpha_s^2)$ perturbative QCD, based for the first time on hadronic decays of the Z^0 . Finally, the production rates of 3-jet events are compared with similar data reported from studies at lower centre of mass energies as an experimental test of the energy dependence of α_s .

In addition to the event selection criteria described in an earlier publication which was based on a smaller event sample [3], further requirements are applied to the data in order to prepare a set of well contained and completely reconstructed events. For each event, at least five charged tracks originating from the vertex region (with a closest distance to the vertex of at most 5 cm perpendicular to the beam axis and 40 cm in the beam direction) and a minimum particle momentum of 150 MeV transverse to the beam are required. Each charged track must be reconstructed by at least 20 from a maximum of 159 hits in the central jet chamber, and the angle between the particle direction and the beam line has to exceed 20 degrees. Electromagnetic energy clusters as reconstructed in the lead glass calorimeter are required to be detected by at least three neighbouring lead glass blocks with a measured total energy of more than 200 MeV. No correction for energy deposited by associated charged particles is applied¹. At least three particles thus reconstructed in the electromagnetic shower counters are required for each event, in addition to the five charged particles mentioned above. For the reconstruction of particle energies, neutral particles are assumed to be photons, and charged particles to be pions. The total visible energy of each event has to exceed 40 % of the centre of mass energy, the absolute value of the vector sum of the momenta of all particles must be less than 40 % of the visible energy, and the angle between the principal event axis (the thrust axis) and the beam direction has to be larger than 26 degrees. Events with invariant masses of less than 2 GeV in both hemispheres defined by the plane perpendicular to the event axis are rejected in order to further decrease possible background from τ pair production.

After these requirements, 10385 events with a mean centre of mass energy of 91 GeV remain in

¹The partial double counting of energy from charged particles does not affect the following analysis of jet production rates.

the analysis. The expected background from beam-gas interactions, two-photon processes or lepton pair production are less than 0.1 % for each case. The overall efficiency of the event selection is about 80 %.

Jet multiplicities are analysed in terms of the jet finding algorithm introduced by JADE [4,5]. In each hadronic event, the squares of the scaled pair masses,

$$y_{kl} = \frac{M_{kl}^2}{E_{vis}^2} \quad (1)$$

are determined for all pairs of particles k and l , where E_{vis} is the visible energy of the event. The pair mass M_{kl} is calculated according to the expression

$$M_{kl}^2 = 2 \cdot E_k \cdot E_l \cdot (1 - \cos\Theta_{kl}), \quad (2)$$

where E_k and E_l are the energies of the two particles k and l and Θ_{kl} is the angle between them. Those particles i and j with the smallest pair mass are replaced by a pseudoparticle with four-momentum $(\mathbf{p}_i + \mathbf{p}_j)$. The procedure is repeated until the scaled pair masses of all particle or pseudoparticle pair-combinations exceed a threshold value y_{cut} :

$$y_{kl} \geq y_{cut}. \quad (3)$$

The remaining clusters of particles and pseudoparticles are then taken as the final "jets".

The close agreement between jet rates and parton rates for identical values of y_{cut} was previously demonstrated for the centre of mass energy range of 22 to 60 GeV [4,5,6,7,8]. The results of a similar study for energies around the Z^0 pole, performed with the Lund QCD shower Monte Carlo program [9], are presented in Fig. 1. The relative production rates $R_n = \sigma_{n-jet}/\sigma_{tot}$ of resolvable n -parton and n -jet events are plotted for model events before and after hadronisation as well as after having passed a simulation of the OPAL detector.

In Fig. 1, the discrete symbols demonstrate the y_{cut} dependence of resolvable n -parton event rates. They are calculated from the final state partons of the events using the jet algorithm described above, before the process of hadronisation started. The corresponding results calculated from the final particles after hadronisation, assuming an ideal detector with infinite resolution and full acceptance for all particle species, are plotted as full lines in the same figure. The hadronisation process causes only minor changes compared to the initial parton multiplicities. For $y_{cut} \geq 0.03$, the production rates of 3-jet events are reduced by about 2% to 6%. This result is in agreement with what has been observed at lower centre of mass energies [5,7,8]. The magnitude of hadronisation effects are, however, model dependent. The HERWIG QCD shower model [10], which is based on a cluster hadronisation process rather than assuming string fragmentation, and a model based on the $O(\alpha_s^2)$ QCD matrix element of Ellis, Ross and Terrano [11,12,13] predict about the same size of hadronisation corrections as the Lund shower model. The NLLJET QCD shower model [14], using the same string fragmentation as in the Lund model but treating gluon masses differently, arrives at negligibly small corrections of about 1%. Since the corrections for hadronisation effects seem to be small but model dependent, the data are not corrected for these effects. This procedure has also been followed in earlier analyses [4,5,6,7,8] and corresponds to the assumption that hadronisation statistically has no influence on the reconstruction of jet production rates. The consequences of this assumption on the physical results of this analysis are discussed later.

The third stage of the model calculations shown in Fig. 1 includes the effects of the final acceptance and resolution of the detector and of initial state photon radiation, the latter being only a minor effect

for data measured on the Z^0 pole. A relative reduction of multijet event rates of about 15% is observed, which is mainly caused by the limited detector acceptance around the e^+e^- beam pipe². Acceptance corrections as shown in this figure must be applied to the data, if results from different experiments are to be compared to each other.

The relative production rates of multijet events observed in this experiment, corrected for detector acceptance and initial state radiation but not explicitly corrected for hadronisation effects, are listed in Table 1 for different jet resolution criteria of $y_{cut} = 0.005$ to 0.20, corresponding to minimum jet pair masses of about 6.4 GeV to 41 GeV. The errors given contain the statistical errors of the measurements and uncertainties of the detector correction. It should be noted that the data at different values of y_{cut} are not statistically independent from each other, since the total event statistics contributes to the results at each value of y_{cut} .

Several studies have been performed in order to estimate the systematic uncertainties of the experimental jet rates. The data were analysed during several development phases of the calibration constants for the different detector parts, and the analysis was repeated for several subsamples of the data recorded under various performance conditions of the LEP collider. The hadronic selection cuts were varied in reasonable intervals. Furthermore, the analysis was repeated by accepting only charged particles, as measured in the central drift chamber, and was also repeated using the information of the lead glass shower counters only. In all these cases and within the statistical errors of the data and of the model calculations, the final results agree well with those from the main analysis as given in Table 1. It is therefore concluded that the overall systematic uncertainty of the results is smaller than the statistical errors listed in Table 1; an additional systematic error of that size will be added if physical parameters are to be calculated from the data.

Before comparing our data with other measurements and with theoretical expectations, the basic ingredients of finite order QCD perturbation theory for jet production in e^+e^- annihilation will be briefly summarised. In second order perturbation theory, the relative production rates R_n of 2-, 3- and 4-parton events are quadratic functions of α_s :

$$\begin{aligned} R_2 &\equiv \frac{\sigma_2}{\sigma_{tot}} = 1 + C_{2,1} \cdot \alpha_s + C_{2,2} \cdot \alpha_s^2 \\ R_3 &\equiv \frac{\sigma_3}{\sigma_{tot}} = C_{3,1} \cdot \alpha_s + C_{3,2} \cdot \alpha_s^2 \\ R_4 &\equiv \frac{\sigma_4}{\sigma_{tot}} = C_{4,2} \cdot \alpha_s^2, \end{aligned} \tag{4}$$

where σ_{tot} is the total hadronic cross section and σ_n are the corresponding cross sections of n -parton event production. The k -th order QCD coefficients for n -jet production, $C_{n,k}$, are calculated by Kramer and Lampe [15,16]. Calculations in higher than second order are not available. The coefficients depend on the actual value of the resolution parameter y_{cut} chosen to define resolvable partons as well as on the prescription to recombine unresolvable partons, but do not exhibit an explicit dependence on the energy. Therefore, for a given value of y_{cut} , the energy dependence of the jet production rates R_n is only determined by the energy dependence of α_s . This situation allows one to analyse the energy evolution of α_s without the need to determine its actual size.

The coupling constant $\alpha_s(\mu)$ can be written as a function of $\ln(\frac{\mu^2}{\Lambda^2})$, where Λ is the QCD scale parameter which is to be determined by experiment and μ is the QCD renormalisation scale. In this article, the functional form of α_s given in [17] will be used when numerical values for Λ are quoted,

²At centre of mass energies off the Z^0 pole these corrections are partly cancelled by initial state radiation effects, which are larger in the e^+e^- continuum and of opposite size than the detector acceptance corrections.

and the number of active quark flavours in the α_s expression is taken to be 5. In e^+e^- annihilation, the renormalisation scale μ^2 is usually chosen to be the centre of mass energy of the hadronic system ($\mu^2 = E_{cm}^2$). Recent studies have shown, however, that a more general definition like

$$\mu^2 = f \cdot E_{cm}^2 \quad (5)$$

with $f = 0.001$ to 0.01 results in a better overall description of the observed production rates of 2-, 3- and 4-jet events [16,18,19,13,20] in $O(\alpha_s^2)$. The “optimisation” of μ , concurrently with the scale parameter Λ , is possible only for observables which are calculated in next-to-leading order, since the corresponding next-to-leading order coefficients like $C_{2,2}$ and $C_{3,2}$ in Eq. 4 exhibit an explicit dependence on the scale factor f [16]. Note that the energy scale μ has no direct physical interpretation because it is a parameter of which the *true* result of a calculation, if performed to *all* orders in perturbation theory, is independent. Finite order calculations, however, may depend on the detailed choice of μ .

In the following, the jet production rates observed at the highest e^+e^- centre of mass energies, as presented in Table 1 and Fig. 2, are to be compared with the predictions of perturbative QCD calculations and with the results obtained at lower energies. In Fig. 2a, the jet production rates observed at $E_{cm} = 91$ GeV are compared with complete QCD shower Monte Carlo calculations, which also include simulations of the hadronisation process. The Lund QCD shower model [9], with QCD and hadronisation parameters as optimised in a study by TASSO at $E_{cm} = 34.8$ GeV [6] ($\Lambda_{LLA} = 260$ MeV), provides a good description of the data at $E_{cm} = 91$ GeV. The HERWIG QCD shower model, with Λ_{LLA} adjusted to 120 MeV in this analysis, also describes the data well except for large values of y_{cut} , where a slight deficiency of 3-jet events indicates that this model underestimates the radiation of hard gluons.

In Fig. 2b, the data are compared with the analytic $O(\alpha_s^2)$ QCD calculations of Kramer and Lampe [15,16], which have been carried out for values of y_{cut} between 0.01 and 0.14. In order to verify the compatibility of our data with the theoretical expectations based on lower energy data, the values of the QCD parameters are taken from a fit [18] to the jet production rates observed by MARK-II at 29 GeV centre of mass energy [8]. The results of the calculations at $E_{cm} = 91$ GeV are shown in Fig. 2b for two different energy scales $\mu^2 = E_{cm}^2$ and $\mu^2 = 0.0017 \cdot E_{cm}^2$ with $\Lambda_{\overline{MS}} = 230$ MeV and 110 MeV, respectively ³. The data are reasonably well described by the calculations using the small energy scale, while the choice $\mu^2 = E_{cm}^2$ fails to describe the 2- and 3-jet production rates for $y_{cut} \leq 0.05$ and 4-jet rates in general. This behaviour has also been observed at lower centre of mass energies [18,19,13,20]. Note that the $O(\alpha_s^2)$ QCD calculations predict unphysical (i.e. negative) jet production rates for small values of $y_{cut} < 0.01$, depending on the actual values of $\Lambda_{\overline{MS}}$ and μ^2 . The calculations can therefore not be applied in this regime of y_{cut} , where the experimental definition of jets is still reliable and meaningful ($y_{cut} = 0.01$ corresponds to jet pair masses of 9 GeV/ c^2 at $E_{cm} = 91$ GeV). Further note that the production of 5-jet events is an $O(\alpha_s^3)$ process which is not described in $O(\alpha_s^2)$. A good description of 2-, 3- and 4-jet event rates in regions where a sizeable fraction of 5-jet events is observed ($y_{cut} < 0.015$) is therefore not expected. Apart from these theoretical restrictions, it is nevertheless remarkable to see that analytical calculations in $O(\alpha_s^2)$ perturbative QCD, with parameters as optimised at $E_{cm} = 29$ GeV, are able to describe the data at $E_{cm} = 91$ GeV so well, especially if small energy scales are used.

After comparing the jet ratios from hadronic Z^0 decays with calculations as optimised to lower energy data, the QCD parameters $\Lambda_{\overline{MS}}$ and μ^2 are to be determined from the current data sample.

³The values of $\Lambda_{\overline{MS}}$ given in [18] were determined using a different expansion of the α_s expression; they have therefore been converted to the expression as given in [17].

Since the optimisation of μ^2 is only meaningful for observables which are calculated in next-to-leading order perturbation theory, 4-jet rates, which are calculated only in leading order, should not be included in such a determination. In the following, we therefore present an adjustment of $\Lambda_{\overline{MS}}$ and μ^2 in the *differential* distribution of $D_2(y)$, where for each hadronic event y is the value of y_{cut} at which the jet multiplicity changes from 3 to 2. In contrast to the *integral* presentation of jet production rates as given in Fig. 2, the statistical errors in bins of $D_2(y)$ are independent from each other since each event enters the distribution only once. The distribution $D_2(y)$ can be calculated from the integrated 2-jet production rates R_2 , as presented in Table 1, by the following expression ($y \equiv y_{cut}$):

$$D_2(y) = \frac{R_2(y) - R_2(y - \Delta y)}{\Delta y}. \quad (6)$$

The distribution $D_2(y)$ as calculated from Table 1 is shown in Fig. 3. A fit to the $O(\alpha_s^2)$ QCD calculations of Kramer and Lampe, performed in the region of $y_{cut} \geq 0.010$ for the reasons mentioned above, results in $\Lambda_{\overline{MS}} = (93^{+12}_{-11})$ MeV and $\mu^2/E_{cm}^2 = 0.0012^{+0.0003}_{-0.0002}$ with a χ^2 of 12.9 for 11 degrees of freedom. The corresponding theoretical distribution is displayed in Fig. 3. Similar results are obtained using the $O(\alpha_s^2)$ QCD calculations of Ellis, Ross and Terrano [11] parametrised by Kunszt and Nason [21]. Including the corrections for hadronisation as predicted by the Lund QCD shower model [9] alters the results given above to $\Lambda_{\overline{MS}} = 129^{+13}_{-12}$ MeV, while μ^2/E_{cm}^2 remains unchanged. Also shown in Fig. 3 is the distribution for the fit result obtained if μ^2 is constrained to E_{cm}^2 ($\Lambda_{\overline{MS}} = 239^{+25}_{-24}$ MeV and $\chi^2 = 8.2$ for $y_{cut} \geq 0.05$ and 8 degrees of freedom). Taking into account the systematic uncertainties on the experimental jet rates, the uncertainties about the influence of hadronisation and variations of the results if determined in different intervals of y , the final results of this study, assuming five different flavours of quarks, are

$$\begin{aligned} \Lambda_{\overline{MS}} &= 80 \text{ MeV} - 180 \text{ MeV} \\ \frac{\mu^2}{E_{cm}^2} &= 0.001 - 0.003. \end{aligned}$$

These results are in good agreement with previous determinations of $\Lambda_{\overline{MS}}$ and μ^2 in jet production rates observed in the continuum of e^+e^- annihilations [18,19,20]. For μ^2 fixed to E_{cm}^2 the final result is $\Lambda_{\overline{MS}} = 200 \text{ MeV} - 450 \text{ MeV}$, which is also compatible with previous results (see e.g. [22]).

In order to further study the experimental significance for the energy dependence of α_s , the 3-jet production rates observed at $E_{cm} = 91$ GeV are to be compared with the results obtained at lower centre of mass energies. In the region of $y_{cut} \geq 0.04$ the 3-jet rates observed at $E_{cm} = 91$ GeV are a factor of 0.77 ± 0.03 smaller than the corresponding rates observed at $E_{cm} = 29$ GeV [8], and a factor of 0.84 ± 0.04 and 0.89 ± 0.04 smaller than the data at $E_{cm} = 31.6$ GeV and 44 GeV [5], respectively. This comparison therefore establishes the observation of significant scaling violations. In the framework of QCD, these scaling violations can be consistently attributed to the energy dependence of α_s (c.f. Eq. 4).

This is further demonstrated in Fig. 4, where the 3-jet event production rates available so far [4,5,6,7,8,23] are plotted as a function of the centre of mass energy. The jet production rates recently reported by the MARK-II collaboration in a study of hadronic decays of the Z^0 boson [24] are compatible with our measurements, but do not contribute significantly to this study due to their much larger statistical errors. Those results are therefore not included in Fig. 4. In Figure 4a, the data are presented for different values of y_{cut} . Also shown are the predictions of the $O(\alpha_s^2)$ QCD calculations by Kramer and Lampe for two different choices of μ^2 , with $\Lambda_{\overline{MS}}$ optimised to describe the e^+e^- continuum data at $E_{cm} = 29$ GeV to 60 GeV [18]. The data at $E_{cm} = 91$ GeV agree well with the

theoretical expectations, and follow the trend of the energy dependence of R_3 observed at lower centre of mass energies. Note that, in contrast to the actual value of $\Lambda_{\overline{MS}}$, the energy dependence of R_3 , and therefore of α_s , is rather insensitive to variations of the energy scale μ^2 in the theoretical calculations.

In Figure 4b, the 3-jet production rates observed at $y_{cut} = 0.08$ are compared to the fit results of two $O(\alpha_s^2)$ QCD calculations as well as to the assumption that α_s does not depend on energy. Model calculations show that this hypothesis results in a constant rate of 3-jet events for data above $E_{cm} = 25$ GeV (see also [5]). Some of the models studied [9,10,11,12,13], however, indicate that the hadronisation corrections for jet rates at $E_{cm} = 91$ GeV may decrease by about 4% compared to the lower centre of mass energies. Therefore, in addition to the statistical error and a systematic error of the same size as discussed above, a further systematic uncertainty of $\pm 4\%$ is added to the data at 91 GeV⁴. Adding these errors in quadrature results in a total relative error of 6.7% for R_3 at $E_{cm} = 91$ GeV and $y_{cut} = 0.08$, such that $R_3 = 17.5\% \pm 1.2\%$ is finally used for the following calculations of χ^2 .

The assumption that α_s is constant results in a χ^2 of 45.8 for the data above $E_{cm} = 25$ GeV (8 degrees of freedom). The QCD fits provide a significantly better description of the data with $\chi^2 = 5.3$ and 6.2 for 8 degrees of freedom for $\mu^2 = 0.0017 \cdot E_{cm}^2$ ($\Lambda_{\overline{MS}} = 107 \pm 4$ MeV) and $\mu^2 = E_{cm}^2$ ($\Lambda_{\overline{MS}} = 250 \pm 11$ MeV), respectively. The differences in χ^2 rule out the possibility of an energy independent α_s with a significance of 5.7 standard deviations. Without the data at $E_{cm} = 91$ GeV, the significance is 4.6 standard deviations.

In summary, the multijet event production rates of hadronic final states of the Z^0 boson are well described by perturbative QCD calculations and by QCD Monte Carlo models with parameters as optimised at lower centre of mass energies. Both $\Lambda_{\overline{MS}}$ and μ^2 are adjusted in $O(\alpha_s^2)$ by a fit to the differential distribution $D_2(y)$, giving $\mu^2/E_{cm}^2 = 0.001 - 0.003$ and $\Lambda_{\overline{MS}} = 80$ MeV - 180 MeV. If μ^2 is fixed to E_{cm}^2 , the result is $\Lambda_{\overline{MS}} = 200$ MeV - 450 MeV ($\alpha_s(91\text{GeV}) = 0.124 \pm 0.008$), providing a good description of the data only for large jet resolution parameters of $y_{cut} \geq 0.05$. The errors of these results are largely determined by systematic uncertainties in the hadronisation process. Significant scaling violating effects are observed when the jet rates are compared to similar results obtained at lower centre of mass energies. Within the framework of perturbative QCD calculations, these scaling violations are attributed to the specific energy dependence of α_s . The energy evolution of the relative 3-jet event production rates, observed in the center of mass energy range from 29 GeV to 91 GeV, excludes the possibility of an energy independent coupling constant with a significance of 5.7 standard deviations. The QCD predictions reproduce the energy dependent effects seen in the data. The evidence for the running α_s does not depend on the choice of the renormalisation scale μ^2 used in the theoretical calculations.

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⁴Since a smaller hadronisation correction would lead to a *decrease* of this data point relative to the other results and therefore *enhance* the evidence against a constant α_s , the inclusion of this uncertainty is regarded to be a conservative choice.

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y_{cut}	R_2	R_3	R_4	R_5
0.005	12.1 ± 0.4	40.7 ± 0.7	32.5 ± 0.9	14.7 ± 0.7
0.010	26.0 ± 0.6	50.1 ± 0.8	19.8 ± 0.7	4.1 ± 0.4
0.015	36.1 ± 0.7	50.5 ± 0.9	12.2 ± 0.6	1.14 ± 0.22
0.020	44.8 ± 0.7	47.1 ± 0.9	7.7 ± 0.5	0.39 ± 0.14
0.030	56.2 ± 0.7	40.0 ± 0.9	3.80 ± 0.35	---
0.040	64.1 ± 0.7	33.5 ± 0.8	2.37 ± 0.29	---
0.050	70.0 ± 0.7	29.0 ± 0.8	0.98 ± 0.17	---
0.060	74.4 ± 0.6	24.8 ± 0.8	0.84 ± 0.22	---
0.080	82.4 ± 0.6	17.5 ± 0.7	0.11 ± 0.06	---
0.100	87.2 ± 0.5	12.8 ± 0.6	---	---
0.120	89.88 ± 0.46	10.12 ± 0.53	---	---
0.140	92.85 ± 0.39	7.15 ± 0.44	---	---
0.170	95.60 ± 0.32	4.40 ± 0.35	---	---
0.200	97.51 ± 0.25	2.49 ± 0.26	---	---

Table 1. Experimental n-jet event production rates, R_n , in % of the total hadronic cross section. The data are corrected for the final acceptance and resolution of the detector. No explicit corrections for hadronisation effects are applied.

Fig. 1. The influence of hadronisation as well as of initial state radiation and detector acceptance on the reconstruction of n-jet rates, simulated in model calculations, as a function of the jet resolution parameter y_{cut} .

Fig. 2a. Two-, three-, four- and five-jet event rates observed at $E_{cm} = 91 \text{ GeV}$ as a function of the jet resolution parameter y_{cut} , compared to two different QCD model calculations.

Fig. 2b. The same data as shown in Fig. 2a, but compared with the $O(\alpha_s^2)$ QCD calculations by Kramer and Lampe with parameters as optimised at $E_{cm} = 29 \text{ GeV}$.

Fig. 3. The measured distribution $D_2(y)$ as defined in the text, compared with the $O(\alpha_s^2)$ QCD expectations for fitted values of $\Lambda_{\overline{\text{MS}}}$ and μ^2 (full line) and of $\Lambda_{\overline{\text{MS}}}$ for $\mu^2 = E_{\text{cm}}^2$ (dotted line).

Fig. 4a. Three-jet event production rates observed at different centre of mass energies, compared to $O(\alpha_s^2)$ QCD calculations with two different renormalisation scales μ^2 .

Fig. 4b. The energy dependence of the three-jet event production rates at $y_{cut} = 0.08$, compared with several assumptions about the energy dependence of α_s . The data points at 22 GeV are not included in the fits [5]. The errors shown are statistical only.