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On the Treatment of E.W.R.C. in M.C. Event Generators for LEP *

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Abstract

The aim of this work is twofold: first the discussion of which is the status of the theoretical steps towards the precise-enough M.C. event generator, and second some prospects on how to deal with the event generators available at present. Therefore a simple analysis of the size and effect of the relevant corrections to be taken into account to fulfill the foreseen accuracy needed at LEP energies is presented. Additionally, a review of the different ways of implementing them developed so far and available at present in the form of M.C. event generators is presented. The intrinsic differences are commented and the real range of validity of each implementation is discussed.

1 Introduction

The foreseen accuracy of experimental measurements at LEP has motivated in the last years a vast theoretical effort in order to provide experimenters the precise predictions of the Standard Model necessary to extract clear information out of the data.

The necessity of including higher order effects in the calculations when predictions providing a systematic uncertainty of few percent are required has become clear not just to theorists but also to experimentalist who, mostly involved in the role of preparing the whole experimental set-up, in general do not have the necessary background to understand the technical aspects of the problem.

In the mean time, different groups of theorists have been working in trying to clarify the proper way of solving the problems. Different approaches have been developed and released to the experimentalists.

Some time ago, the increasing multiplicity of M.C. event generators (which actually should have been contemplated just as intermediate steps of the work, in spite that they were often released as "the final version"), together with the use of them without a clear knowledge of their contents and natural limitations originated an alarming number of results of simulations

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accuracy of each generator. Fortunately, this situation has been changing in the last months and at this moment, encouraging agreement between different calculations is being reported for most of the interesting processes for LEP energies.

The purpose of this paper is twofold: first the clarification of which is the status of the theoretical steps towards the precise-enough M.C. event generator, and second some prospects on how to deal with the use the M.C. event generators available at present.

The outline of the paper is as follows. In the next section we discuss an operative way of getting close to the problem: analyzing the evolution of M.C. calculations in the previous generation of accelerators (PETRA-PEP). Section 3 analyzes the obvious new needs for LEP energies. In section 4 the treatment the so-called oblique corrections is considered. Section 5 is devoted to the discussion of the different ways of including the infrared QED corrections. Section 6 deals with the way of implementing a reasonable calculation of the Z_0 width. In section 7 the common features and differences as well as the obvious limitations of each one of the methods reviewed are discussed. Finally an overview of the status of the questions and some prospects for experimentalists involved in the use of the existing M.C. event generators are presented.

2 The PETRA-PEP Monte Carlo event generators

The previous step from which the evolution of M.C. event generators towards the LEP requirements has started to develop is the set of programs which have been extensively used for the data analysis in PETRA and PEP. These calculations have been carefully confronted with data by all the experiments and therefore can be considered as accurate-enough for the energies in which they were working. The evolution of M.C. calculations to adapt their characteristics to the PETRA-PEP necessities can be schematized in the following steps:

A) The starting point is the use of programs consisting in just tree level calculations including just QED diagrams for two body final state processes. For some processes intended to be used to measure E.W. effects (μ pairs and τ pairs) some additional diagrams are taken into account: basically only the interference of QED diagrams with Z_0 ones at the tree level (customarily included just as a correction factor to the QED matrix element squared). At this level no hard photons are taken into account. For the calculation of the total cross section this is a reasonable approach already for processes such as Bhabha scattering (low Q^2) but can give just around a 10% accuracy for processes such as μ pair production (high Q^2) since:

i.- Weak corrections (even the ones due to the missing tree level diagrams) are very small in any case.

ii.- QED corrections (if no cut on additional photons or particle acollinearity are applied) are rather small for Bhabha but sizeable for μ pairs. As this point will be of paramount importance, let us discuss it in more detail: as it is well known, the cross section for the process $e^+e^- \rightarrow \mu^+\mu^-$ (which we will use as an example for simplicity) corrected with one loop initial state radiation and assuming that the dependence of σ_0 on s is smooth, can be written in the following way:

$$\sigma_1(s) = \sigma_0(s)(1 + \delta_1 + \beta \ln x_M) \quad (1)$$

where

$$\beta \equiv \frac{2\alpha}{\pi} \left(\ln \frac{s}{m_e^2} - 1 \right) \quad (2)$$

and

$$\delta_1 \equiv \frac{3}{4}\beta + \frac{\alpha}{\pi} \left(\frac{\pi^2}{3} - \frac{1}{2} \right) \quad (3)$$

z_M represents the maximum photon energy allowed by the detection cuts. If no photon cut or acollinearity cut is applied, then $z_M \rightarrow 1$ and the correction is:

$$\delta_1 \sim 8\% \quad (\sqrt{s} \sim 40 \text{ GeV}) \quad (4)$$

In this expression, the real photon contribution has been integrated over the photon energy and its infrared divergence has been cancelled with the one due to the virtual photon correction, leading to the term $\beta \ln z_M$ being δ_1 the remaining correction. Let's realize that since the Q^2 is very small for Bhabha events, the total correction is also small:

$$\delta_1 \sim 0.06\% \quad (5)$$

Nevertheless, since the presence of real bremsstrahlung photons is clearly detected in all channels affecting therefore all the differential cross sections, it becomes obvious that radiative corrections (which at the beginning of PETRA-PEP running where understood by many experimentalists just as the number of detectable real photons) should be taken into account somehow.

B) The following step was the development of computational methods to handle radiative processes at the tree level:

- i.- The use of helicity amplitudes to compute processes involving many Feynman diagrams (such as the radiative ones) [1].
- ii.- The application of more elaborated Monte Carlo techniques to handle the collinear peaks of the multidifferential bremsstrahlung cross sections [2]. Each real photon of energy E_k emitted by a charged particle p introduces in the matrix element M a factor

$$\frac{1}{E_k(1 - \beta_e \cos \theta_{pk})} \quad \beta_e = \sqrt{1 - \frac{m^2}{p_0^2}} \quad (6)$$

in such a way that if the photon direction is nearly parallel to p (collinear) then the cross section peaks. The dependence in E_k leads to the infrared divergence which can be regularized for instance just by taking a minimum photon energy ("hard photon").

These two tools allow the construction of M.C. event generators for studying specifically final states such as $e^+e^- \gamma$, $\mu^+\mu^-\gamma$, and so on. As experimentally the detection of a photon requires that it has an energy larger than a certain amount determined by the detector resolution, a natural definition of "hard photon" comes up.

C) The next step attacked is the construction of a M.C. event generator combining the radiative and the non-radiative cross section for a given process. This is obtained by calculating all the $O(\alpha)$ QED corrections to the Born QED diagrams ("reduced QED corrections"). When computing the real photon part, it turns out to have an infrared divergence which cancels out exactly with the one showing up in the interference of the tree-level diagrams with the virtual photon ones. To handle the problem, the solution customarily adopted is splitting the real photon integral into two parts:

- a part in which the photon energy is smaller than a certain limit x_0 . The photon energy integral is computed analytically in the limit $x \rightarrow 0$ (soft photon) and the result is added to the one of the virtual part cancelling out the infrared divergences.
- the part in which the photon energy is larger than the soft limit (hard photon). The cross section is treated as a radiative cross section at the tree level.

For instance, the $O(\alpha)$ QED corrections due to initial state radiation can be written as

$$\sigma_1(s) = \sigma_0(s)(1 + \delta_1 + \beta \ln x_0) + \int_{x_0}^{z_M} \sigma_0(s(1-x))\beta \left(\frac{1}{x} - 1 + \frac{x}{2} \right) dx \quad (7)$$

As actually the splitting of the real photon integral into soft and hard parts is arbitrary, the total result has to be independent on the value of the soft-hard limit (provided the s dependence of σ_0 is smooth enough).

D) For the processes allowing more accurate E.W. measurements ($\mu^+\mu^-$ and $\tau^+\tau^-$) further developments required for the pushing experimental accuracy were: including in the calculations all the Z_0 diagrams at the tree level and furthermore affected as well by the photonic corrections ("full QED corrections").

This was essentially the state of the art in M.C. event generators used at PETRA-PEP before the requirements for LEP-SLC energies were taken into account. In spite that the complete one loop E.W. corrections for processes such as $\mu^+\mu^-$ existed already [3], they were not available in the form of M.C. event generators.

3 Requirements for a precise-enough M.C. for LEP-SLC

The first obvious realization about the requirements to obtain reliable M.C. event generators for LEP energies was that there was no strong reason to expect all the non-photonic one loop corrections to be negligible and therefore, for completeness they all were included in the M.C. event generators (though most of the corrections are really small in a given renormalization scheme and can be safely neglected). To do so, it was necessary to study in detail the one loop renormalization of the Standard Model choosing a clear and handy renormalization scheme such as the "on shell" one [4]. The calculations have been generally done in the Feynman gauge mainly because of the trivial form of the boson propagators it allows, in spite of the existence of unphysical fields which have to be taken into account in the diagrammatics. Nevertheless, some people have been using other gauges (unitary, ξ -gauge) allowing thus stringent comparisons.

This lead to a set of M.C. event generators which in principle were thought to be the Standard for LEP-SLC energies. They were orthodox theoretically speaking since they contained a full one-loop expansion of the Glashow-Weinberg-Salam theory for a given process, and thus they were by definition gauge-independent and divergency-free. In one hand, their contents were clear and reproducible (in the sense that since they were strictly full one-loop E.W. M.C. event generators, one knew exactly which Feynman diagrams were included and which not and one could compare exactly between different calculations using the same renormalization scheme) and in other hand nobody believed that a calculation of the full two-loop E.W. correction (which would have preserved the orthodoxy) was feasible.

At that moment the development of resummation of higher order terms for only "isolated" parts of the corrections was seen by many people with strong caution since it represented a loss of theoretical orthodoxy and clarity in favour of phenomenological requirements producing potentially just effective approaches.

The corrections to the Born terms can be classified as "weak" (non-photonic) and QED (photonic) ones. This division is sensible since the second corrections form a gauge invariant subset and helps to clarify the origin and effect of the corrections.

3.1 Weak corrections

The weak ones, thought were not the dominant ones at PETRA energies and can be estimated not to be the dominant ones either at LEP energies, are the most interesting ones since they contain the gauge theory character of the Standard Model, the dependence on unknown parameters as the Top and the Higgs masses and are also sensitive to "new physics".

In the "on shell" scheme and using the Feynman gauge, the most important ones for all the relevant observables turn out to be the oblique ones, essentially the Z_0 self energy which amounts up to 10% few GeV under and above M_Z . The oblique corrections at the one loop level do not depend on detection cuts. In any case it is obvious that if accuracies of about 1% are to be reached, higher order terms of these corrections have to be taken into account.

3.2 QED Corrections

The second ones, namely the QED corrections, do not have any special theoretical interest, but one shall recall that in the case of PETRA generators were the dominant ones and their size was strongly dependent on the detection cuts applied. Anyway, if no photon cut was applied, they were expected to amount less than 10%. This fact will not hold for LEP energies because of the following: near the Z_0 peak the QED corrections become large because the emission of initial state photons decreases the effective Q^2 in the Z_0 propagator. We can estimate numerically the effect of the presence of the resonance assuming that the Z_0 shape makes an effective cut in the maximum photonic energy of

$$x_M \sim \frac{E_Z}{E_b} = \frac{E_Z}{M_Z} \quad (8)$$

in such a way that the infrared term of the corrections becomes

$$\beta \ln \frac{E_Z}{M_Z} \sim -40\% \quad (9)$$

That is, at LEP energies, independently of the detection cuts (which may still enhance the size of the infrared correction) we will have always extremely important QED corrections

due to the neighbourhood to the Z_0 resonance. This enormous size of the QED one loop corrections implies that:

1. We cannot expect in any case higher order QED corrections to be small and therefore we have to take them into account to obtain reasonable accuracies.
2. As the bulk of these QED corrections comes from the infrared parts of the initial state radiation correction, and in the soft limit this part factorizes from the rest of the cross section, we shall keep in mind that the interplay among photonic and non-photonic corrections is important (if we do not take it into account we can be overestimating seriously the effect of oblique corrections).

Additionally some technical problems arise in the M.C. implementation of the soft-hard separation technique making the "pure one loop" calculation impractical:

- a) z_0 cannot be taken large because in this technique for the soft part we are neglecting the photon energy, and this can be a quite bad approximation since if we take $z_0 = 0.01$ for instance (what will match an experimental energy resolution of .5 GeV) then the actual dependence of σ_0 would give

$$\sigma_0(M_Z^2) - \sigma_0(M_Z^2(1 - 0.01)) \sim 0.1 \sigma_0(M_Z^2) \quad (10)$$

in such a way that to assure a variation of around 1 per mil we should actually take z_0 of about 0.001. In practice, this energy dependence of the soft cross section near the resonance can be taken into account approximately to correct it for the value of z_0 used and many M.C. do include this correction already.

- b) z_0 cannot be taken small because as we have seen, the soft cross section may become negative and therefore it is not possible to use it as a P.D.F. in a M.C. procedure.

3.3 Additional remarks

A last important remark in order is the following: in spite that the calculations discussed so far are intended to give us the cross section up to $O(\alpha^3)$ they are not exactly $O(\alpha^3)$ calculations. The reason is that actually the Z_0 width we are putting into the denominator of the Z_0 propagator is just an $O(\alpha)$ calculation.

Therefore, while we are off-resonance, $((s - M_Z)^2 \gg M_Z \Gamma_Z)$ the real part of the propagator dominates and then we have effectively an $O(\alpha)$ corrected calculation but when we are sitting on top of the resonance $((s - M_Z)^2 \rightarrow 0)$ this is no more true, and we end up with an $O(1)$ corrected cross section. This means that if we want our M.C. to be accurate enough on top of the Z_0 resonance, one of the things we need is including into the Z_0 propagator a more accurate calculation of the Z_0 width than the simple tree level one.

Therefore, the study of the complete E.W. one-loop calculation at LEP energies shows that there are essentially four aspects which have to be studied carefully to build M.C. event generators to fulfill the accuracy requirements of LEP experiments, namely:

- a) Higher order OBLIQUE corrections

- b) Higher order QED corrections
- c) Interplay of photonic and non-photonic corrections
- d) A more accurate evaluation of the Z_0 width

4 Oblique corrections

The oblique corrections are the ones to the boson propagators. As they are universal (in the sense that they don't depend on the specific process studied), they can be computed once for ever and included as basic building blocks of the theory as we will see now.

The corrected boson propagators can be related to the proper self energies through the Dyson equations (fig.1)

$$\begin{aligned} G_\gamma &= D_\gamma - D_\gamma \hat{\Sigma}^\gamma G_\gamma - D_\gamma \hat{\Sigma}^{\gamma Z} G_{\gamma Z} \\ G_Z &= D_Z - D_Z \hat{\Sigma}^Z G_Z - D_Z \hat{\Sigma}^{\gamma Z} G_{\gamma Z} \\ G_{\gamma Z} &= -D_Z \hat{\Sigma}^Z G_{\gamma Z} - D_Z \hat{\Sigma}^{\gamma Z} G_\gamma \\ G_W &= D_W - D_W \hat{\Sigma}^W G_W \end{aligned} \quad (11)$$

where $\hat{\Sigma}^\gamma$ is the proper photon self energy, $\hat{\Sigma}^{\gamma Z}$ is the proper $\gamma - Z_0$ mixing, $\hat{\Sigma}^Z$ is the proper Z_0 self energy and

$$D_\gamma = \frac{1}{s} \quad D_Z = \frac{1}{s - M_Z^2} \quad D_W = \frac{1}{s - M_W^2} \quad (12)$$

are the tree-level propagators. The solution of these equations is the following:

$$\begin{aligned} G_\gamma &= \frac{1}{s + \hat{\Sigma}^\gamma(s) - \frac{(\hat{\Sigma}^{\gamma Z}(s))^2}{s - M_Z^2 + \hat{\Sigma}^Z(s)}} \\ G_Z &= \frac{1}{s - M_Z^2 + \hat{\Sigma}^Z(s) - \frac{(\hat{\Sigma}^{\gamma Z}(s))^2}{s + \hat{\Sigma}^\gamma(s)}} \\ G_{\gamma Z} &= -\frac{\hat{\Sigma}^{\gamma Z}(s)}{(s + \hat{\Sigma}^\gamma(s))(s - M_Z^2 + \hat{\Sigma}^Z(s)) - (\hat{\Sigma}^{\gamma Z}(s))^2} \\ G_W &= \frac{1}{s - M_W^2 + \hat{\Sigma}^W(s)} \end{aligned} \quad (13)$$

in such a way that we can interpret the resulting propagators as a summation of all the 1PI contributions to the vector bosons self energies to all orders (see figure). If originally the self energies are calculated at the one-loop level, this implies adding up all the leading log contributions to the self energies:

$$\alpha^n \ln^n \frac{s}{m^2} \quad (14)$$

which, in fact, are just really important for the fermionic loops contribution.

The direct use of the result we have obtained from Dyson equations has a couple of drawbacks: the first is that the appearance of $(\hat{\Sigma}^{\gamma Z}(s))^2$ in the denominator introduces higher

order terms which affect the definition of the boson masses through the renormalization conditions. The second is that the resummation produced by Dyson equations leads to gauge dependent results which have to be treated with caution. As we will clarify later, the vector boson contributions to the boson self-energies can be pointed out as the responsible for this non-invariance. As in the Feynman gauge their size is essentially negligible, we can affirm that the numerical effect on the final results due to the gauge dependence of these equations would be very small, so that in principle we could just forget about the problem.

Different methods have been suggested so far to implement this resummation in practice, essentially they follow three different ideas, namely

- a) neglecting the effect of $(\hat{\Sigma}^{\gamma Z}(s))^2$ in the denominator and also forgetting about gauge non-invariance. This is done in the "propagator substitution method".
- b) trying to absorb the result of Dyson equations in the definition of a set of universal running coupling constants. In this absorption, the necessary terms to preserve gauge-invariance have to be resummed as well in the proper way. This is the basic idea of the STAR scheme.
- c) trying to absorb the result of Dyson equations in a set of forms factors preserving gauge-invariance. This has been recently claimed to be achieved by the Dubna-Zeuthen [5] collaboration.

4.1 Propagator substitution

If we just want to keep terms of $O(\alpha)$ in the denominator of the propagator with the leading-log terms resummed to all orders, we can neglect the terms

$$-(\hat{\Sigma}^{\gamma Z})^2 \quad (15)$$

and therefore we end up with the following rule

$$\begin{aligned} \frac{1}{s} &\rightarrow \frac{1}{s + \hat{\Sigma}^\gamma(s)} \\ \frac{1}{s - M_Z^2} &\rightarrow \frac{1}{s - M_Z^2 + \hat{\Sigma}^Z(s)} \\ \frac{1}{s - M_W^2} &\rightarrow \frac{1}{s - M_W^2 + \hat{\Sigma}^W(s)} \end{aligned} \quad (16)$$

which can be applied in any of the occurrences of the boson propagators (at the tree level and in the corrections themselves). This is the sense in which we can affirm that these corrections are universal. This simple substitution rule, introduces automatically the interplay of oblique corrections with the rest in the proper way.

The approximation made is sensible since the size of $\hat{\Sigma}^{\gamma Z}$ is rather small. Nevertheless, it has been shown that the situation changes sizeably if the Top mass is large (say > 150 GeV) and then, it becomes necessary the use of the full expression (eqs. 13) of the propagators to match the foreseen experimental needs.

4.2 The "improved Born" amplitude

A more serious attempt to take benefit of the universality of these oblique corrections can be obtained by studying specifically the four fermion interaction in the following way: The result of Dyson eq. can be seen also as a re-diagonalization of the neutral boson mass matrix after radiative corrections mixing Z_0 and photons have been included

$$(D_{\mu\nu})^{-1} = i g_{\mu\nu} \left(\frac{s + \hat{\Sigma}^\gamma(s)}{\hat{\Sigma}^\gamma Z(s)} \frac{\hat{\Sigma}^\gamma Z(s)}{s - M_Z^2 + \hat{\Sigma}^\gamma Z(s)} \right) \quad (17)$$

Neglecting in the self-energies all the imaginary parts (but, of course, that of the Z_0 one) which can be shown to be small and replacing their full s dependence by just a simple linear one like

$$\begin{aligned} \hat{\Sigma}^\gamma(s) &= s \hat{\Pi}^\gamma(M_Z^2) \\ \hat{\Sigma}^\gamma Z(s) &= s \hat{\Pi}^\gamma Z(M_Z^2) \end{aligned} \quad (18)$$

being

$$\begin{aligned} \hat{\Pi}^\gamma(M_Z^2) &\equiv \text{Re} \left(\frac{\hat{\Sigma}^\gamma(s)}{s} \right) \Big|_{s=M_Z^2} \\ \hat{\Pi}^\gamma Z(M_Z^2) &\equiv \text{Re} \left(\frac{\hat{\Sigma}^\gamma Z(s)}{s} \right) \Big|_{s=M_Z^2} \\ \hat{\Pi}^Z(M_Z^2) &\equiv \frac{\partial}{\partial s} \text{Re} [\hat{\Sigma}^Z(s)] - \frac{(\hat{\Sigma}^Z(s))^2}{s + \hat{\Sigma}^\gamma(s)} \Big|_{s=M_Z^2} \end{aligned} \quad (19)$$

which turns out to be a rather good approximation, the four-fermion amplitude can be written as

$$\begin{aligned} M_{NC} &= \frac{e^2}{1 + \hat{\Pi}^\gamma(M_Z^2)} Q_e Q_f \frac{1}{s} \gamma_\mu \otimes \gamma^\mu \\ &+ \frac{e^2}{4s_W^2 c_W^2} \frac{1}{1 + \hat{\Pi}^Z(M_Z^2)} \frac{1}{s - M_Z^2 + i \frac{\Gamma_Z}{M_Z}} \frac{1}{\Gamma_Z} \\ &\quad \gamma_\mu [I_3^e - 2s_W^2 Q_e - I_3^e \gamma_5] \otimes \gamma^\mu [I_3^f - 2s_W^2 Q_f - I_3^f \gamma_5] \end{aligned} \quad (20)$$

where Γ_Z denotes the physical width. This expression can be rewritten as

$$\begin{aligned} M_{NC} &= Q_e Q_f \frac{e^2(M_Z^2)}{s} \gamma_\mu \otimes \gamma^\mu \\ &+ \frac{G_{\mu K}}{\sqrt{2}} \frac{e^2(M_Z^2)}{s - M_Z^2 + i \frac{\Gamma_Z}{M_Z}} \frac{M_Z^2}{s} \gamma_\mu [I_3^e - 2s_W^2 Q_e - I_3^e \gamma_5] \otimes \gamma^\mu [I_3^f - 2s_W^2 Q_f - I_3^f \gamma_5] \end{aligned} \quad (21)$$

that is, by using a sort of "running" parameters $e(M_Z^2)$ and s_W^2 defined as

$$\begin{aligned} e^2(M_Z^2) &\equiv \frac{e^2}{1 + \hat{\Pi}^\gamma(M_Z^2)} \\ s_W^2 &\equiv \frac{s_W^2}{s_W^2 - s_W c_W} \frac{\hat{\Pi}^\gamma Z(M_Z^2)}{1 + \hat{\Pi}^\gamma(M_Z^2)} \end{aligned} \quad (22)$$

we are able to write an "improved" Born-like matrix element squared. The correction factor κ deviates from 1 only for a heavy top

$$\kappa \simeq 1 + \frac{3\alpha}{16\pi s_W^2} \frac{m_t^2}{M_W^2} \quad (23)$$

and also in practice

$$s_W^2 \approx s_W^2 + \frac{\alpha(M_Z^2)}{4\pi} \frac{3c_W^2}{4s_W^2} \frac{m_t^2}{M_W^2} \quad (24)$$

in such a way that the residual terms of the full expressions are of the same size as the vertex and box corrections to be included next.

The basic conclusion of this approach is that a good implementation of the oblique corrections resummation near the Z_0 pole can be obtained already just by using in the tree level expressions

- i.- The running electric charge $e^2(s)$ in the photon diagrams.
- ii.- G_μ plus an "energy-dependent" width in the Z_0 diagrams.

4.3 The STAR scheme

One out of the most elaborated ways of treatment of the "oblique" corrections is the so-called STAR scheme [6], which can be regarded actually as a new renormalization scheme with running parameters. It takes full benefit of the universality of oblique corrections by including them already as basic building blocks of the theory that can be used to write the Feynman diagram amplitudes. This allows a fast evaluation of the effect of oblique corrections in any observable by using the following procedure: write the observable at the tree level in terms of "bare" parameters and include the relevant QED corrections; then just substitute the "bare" parameters by "starred" ones.

Basically, the idea of the method is as follows: First one starts with a renormalized lagrangian writing down the four-fermion amplitude in terms of "bare" parameters:

$$\begin{aligned} M_{NC} &= e_0^2 G_{AA} Q_e Q_f + e_0^2 [Q_e (\frac{I_3^e - s_W^2 Q_e}{s_0 c_0}) + Q_f (\frac{I_3^f - s_W^2 Q_f}{s_0 c_0})] G_{ZZ} \\ &+ e_0^2 (\frac{I_3^e - s_W^2 Q_e}{s_0 c_0}) (\frac{I_3^f - s_W^2 Q_f}{s_0 c_0}) G_{ZZ} \end{aligned} \quad (25)$$

Then one uses Dyson eqs. to obtain the full boson propagators G_{AA} , G_{ZZ} and G_{WW} and Z_0 mixing as it was done at the beginning of the section. The main difference at this point

is that since we are still dealing with "bare" quantities, the expression cannot be used as it is because we do not know what ϵ_0, s_0 are.

As we mentioned before, there are some theoretical problems which arise as a result of using Dyson equations to obtain the full propagators in this way, namely

- a) It is easy to realize that since the vector boson contributions to the self energies are not gauge invariant (fig.2), actually the results for the full boson propagators are not gauge invariant either.
- b) When computing Π_{ZA} (the $\gamma - Z$ mixing) we find terms proportional to M_W^2 which do not vanish when $Q^2 = 0$. This implies that there is a $\gamma - Z$ mixing term at $Q^2 = 0$, which means that the neutral sector has been misdiagonalized since we want the photon to have no weak component at all at $Q^2 = 0$. (This didn't happen in the previous method because there renormalized quantities are used, and one of the renormalization conditions is precisely that Π_{ZA} has to vanish for $Q^2 = 0$).

These problems come clearly from the non-abelian nature of the theory and to solve them we will have to perform a re-diagonalization of the neutral sector. The solution of both problems can be found realizing that the gauge dependence of the boson self energies cancels out at the one loop level with part of the weak vertex corrections¹ (fig.3). We know that every vertex in the theory will pick up corrections which can be treated as modifications of the coupling constants:

$$g_0 \rightarrow g_0(1 + g_0^2 \Gamma) \quad (26)$$

We can split Γ into two parts: $\Gamma = \Gamma' + \bar{\Gamma}$, the former one being gauge-dependent and the second one gauge-independent. Since the first one cancels with a similar part appearing in the oblique corrections, which are universal, it must be universal as well. The other way around, any process-dependent part of Γ has to be gauge-independent, because there is nothing to cancel with in the oblique corrections. Therefore we can include this part of the vertex correction by defining a new universal coupling constant

$$\tilde{g} \equiv g_0(1 + g_0^2 \Gamma') \quad (27)$$

and re-diagonalize the mass matrix with this new coupling. The proper treatment of this fact before using Dyson eq. ensures that problem "a" disappears and moreover, requiring that the new π_{ZA} must be purely transverse (proportional to Q^2), we isolate the actual part Γ' of Γ needed to fulfill requirement "b", namely

$$\Gamma' = \frac{-\Pi_{ZA}^L}{\tilde{g}g_0 M_W^2} \quad (28)$$

After this has been done, we can write a four-fermion amplitude which has on it all the vacuum polarization resummed properly, preserving gauge invariance and decoupling properly the oblique part from the remaining one:

¹This has been claimed by Lynn and Kennedy in spite that many people affirms that also some parts of the W boxes are needed to cancel the gauge dependence. This doesn't really affect the numerical results as commented above and, apparently, even if this is true, some minor changes would restore the rigorous validity of the method we are discussing [7]

Figure 1: Graphic representation of Dyson equations. The empty bubble stands for the 1 particle irreducible (1PI) self energy.

$$\begin{aligned} \text{Photon} &= \text{Photon} + \text{Photon} \text{ bubble} + \text{Photon} \text{ bubble} + \text{Photon} \text{ bubble} \\ \text{Z} &= \text{Z} + \text{Z} \text{ bubble} + \text{Z} \text{ bubble} + \text{Z} \text{ bubble} \\ \text{A} &= \text{A} + \text{A} \text{ bubble} + \text{A} \text{ bubble} + \text{A} \text{ bubble} \\ \text{W} &= \text{W} + \text{W} \text{ bubble} + \text{W} \text{ bubble} + \text{W} \text{ bubble} \end{aligned}$$

Figure 2: Boson contributions to the boson self energies.

Figure 3: Vertex diagrams which are gauge dependent.

inclusion of these effects everywhere in our calculation by means of the proper use of "starred" quantities, makes automatic the correct factorisation of these corrections with the rest and, also the inclusion of the oblique corrections in the Z_0 width.

If we neglect the imaginary parts (but the ones of the Z_0 self energy)

$$\begin{aligned} \text{Im}\Pi''_{AA} &\rightarrow 0 \\ \text{Im}\Pi''_{ZA} &\rightarrow 0 \\ \lambda &\rightarrow 0 \end{aligned} \quad (34)$$

then we end up with

$$M_{NC} = \frac{e^2}{q^2} + \frac{e^2}{s^2 c^2 q^2} + \frac{[I_3 - Q s^2][I_3' - Q' s^2]}{4\sqrt{2}s^2 c^2 G_{\mu\nu} \rho^* - i\sqrt{s}\Gamma_Z} \quad (35)$$

being then

$$M_Z^2 = \frac{e^2}{4\sqrt{2}s^2 c^2 G_{\mu\nu} \rho^*} \quad (36)$$

the same relation we have at the tree level and

$$\sqrt{s}\Gamma_Z^* = \text{Im}\Pi''_{ZZ} \quad (37)$$

so that what we have is just a Born-like matrix element but in terms of "starred" parameters. As, in fact, near the peak we have

$$\begin{aligned} \rho^* &\simeq 1 \\ G_{\mu\nu} &\simeq G_{\mu} \end{aligned} \quad (38)$$

the result we obtain consists in modifying the Born amplitude by substituting

i.- the "starred" electric charge $e^2(s)$ in the photon diagrams.

ii.- G_{μ} plus an "energy-dependent" "starred" width in the Z_0 diagrams.

which is numerically equivalent to the substitutions obtained in the previous sections.

For the M.C. implementation, the complete set of "starred" functions is available and for processes in which the Q^2 in the boson propagator lines changes from event to event (for instance radiative processes) the fact that their behaviour is smooth can be used to spare CPU time by using just tabulations and interpolation.

5 Infrared corrections

The infrared corrections are the dominant QED ones as argued before and the fact that their size depends so strongly on experimental detection conditions, makes them a complicated problem to treat in practice making unavoidable the intensive use of M.C. techniques.

A systematic and rigorous study of these corrections was done already many years ago (Y.F.S.) [8] and actually practical implementations of exponentiation techniques have been used already extensively for the study of the QED radiative corrections to resonance widths [9]. Nevertheless the clarification of the proper way of applying these techniques to the problem we are discussing is rather recent and we will try to describe just the different approaches developed for this purpose.

$$\begin{aligned} M_{NC} &= G_{ZZ} \frac{e^2}{s^2 c^2} [I_3 - Q(\bar{s}^2 - \bar{s}c^2 \frac{[ZA]}{[A]})][I_3' - Q'(\bar{s}^2 - \bar{s}c^2 \frac{[ZA]}{[A]})] \\ &+ \bar{s}^2 \frac{QQ'}{[A]} \end{aligned} \quad (29)$$

We can rewrite this amplitude in terms of finite parameters in the following way

$$\begin{aligned} M_{NC} &= \frac{e^2}{q^2(1 - i\text{Im}\Pi''_{AA})} \\ &+ \frac{e^2}{s^2 c^2} \frac{[I_3 - Q(s^2 - is c_\mu \text{Im}\Pi''_{ZA})][I_3' - Q'(s^2 - is c_\mu \text{Im}\Pi''_{ZA})]}{s^2 c^2 q^2(1 + (\text{Im}\Pi''_{AA})^2)(1 + \frac{1}{s^2 c^2}) + 4\sqrt{2}s^2 c^2 G_{\mu\nu} \rho^* - i\sqrt{s}\Gamma_Z} \end{aligned} \quad (30)$$

where e^2 , s^2 , $G_{\mu\nu}$ and ρ^* are finite and renormalization-scheme independent (since they depend only on "bare" quantities) functions of Q^2 . Namely

$$\begin{aligned} \frac{1}{e^2} &\equiv \frac{1}{e_0^2} - \text{Re}(\Pi'_{QQ} + 2\Pi') \\ s^2 &\equiv \text{Re} \left(\frac{\frac{1}{s_0^2} - (\Pi'_{3Q} + 2\Pi')}{\frac{1}{s_0^2} - (\Pi'_{QQ} + 2\Pi')} \right) \\ \frac{1}{4\sqrt{2}G_{\mu\nu}} &\equiv \frac{M_{H_{\mu\nu}}^2}{s_0^2} - \text{Re}(\Pi_{11} - \Pi_{33}^T - 2\Pi_{3Q}^T) \\ \frac{1}{\rho^*} &\equiv \frac{1}{\rho_0} - 4\sqrt{2}G_{\mu\nu} \text{Re}(\Pi_{33} - \Pi_{11}) \end{aligned} \quad (31)$$

being the rest of "starred" quantities just calculable functions of the four basic "starred" parameters. If we assume that $\rho_0 = 1$, that is only Higgs doublets, then ρ^* becomes also just a function of the remaining three "starred" functions, which must be considered then as the only fundamental ones. If we find a well-defined procedure of fixing their value at a given Q^2 we will have a well defined quantity. To do so, we use equation (29) to compute the QED amplitude in the Thompson limit, and from here we can just define

$$e^2(0) = 4\pi\alpha \quad (32)$$

In the same manner, to fix a value for s^2 we can impose that the real part of the Z_0 propagator vanishes at $q^2 = -M_Z^2$ and then we have

$$s^2 c^2 |Z = \frac{e^2}{4\sqrt{2}M_Z^2 G_{\mu\nu} \rho^* |Z} \frac{1}{1 + (\text{Im}\Pi''_{AA})^2 |Z} - \lambda |Z \quad (33)$$

And finally, the value of $G_{\mu\nu}$ is fixed by comparing the muon lifetime predicted using the equations we have obtained with the one deduced from the experiment (after subtracting and adding the proper corrections).

Therefore, we end up with a rigorous and well defined procedure of including all the oblique corrections in the calculation of 4-fermion amplitudes. This procedure, through allowing the

5.1 The "inductive" approach

The way higher order QED corrections have to be included can be studied in an inductive way analyzing which new terms show up when two loop corrections are calculated. This has been done for the initial state QED corrections for μ pair production [10] and the result can be written in the following way:

$$\sigma_x(s) = \sigma_0(s) \left\{ 1 + \delta_1 + \delta_2 + \beta \ln x_0 + \delta_1 \beta \ln x_0 + \frac{1}{2} \beta^2 \ln^2 x_0 \right\} + \int_{x_0}^1 \sigma_0(s') \left[\beta \left\{ \frac{1}{2} (1 + \delta_1 + \beta \ln x) - 1 + \frac{x}{2} \right\} + \delta_2^H \right] dx \quad (39)$$

where

$$\begin{aligned} \delta_2 &\equiv O(\alpha^2) \text{ correction independent of } x_0 \\ \delta_2^H &\equiv O(\alpha^2) \text{ non-leading hard correction} \\ s' &\equiv s(1-x) \end{aligned} \quad (40)$$

We can estimate the effect of the new corrections as we did for the one loop one near the resonance:

$$\begin{aligned} \sigma_2(s) &\sim \sigma_0(s) \left\{ 1 + \delta_1 + \delta_2 + \beta \ln \left(\frac{\Gamma_Z}{M_Z} \right) + \delta_1 \beta \ln \left(\frac{\Gamma_Z}{M_Z} \right) + \frac{1}{2} \beta^2 \ln^2 \left(\frac{\Gamma_Z}{M_Z} \right) \right\} \\ \delta_1 &= 9\% \\ \delta_2 &= -0.5\% \\ \beta \ln \left(\frac{\Gamma_Z}{M_Z} \right) &= -40\% \\ \delta_1 \beta \ln \left(\frac{\Gamma_Z}{M_Z} \right) &= -3.6\% \\ \frac{1}{2} \beta^2 \ln^2 \left(\frac{\Gamma_Z}{M_Z} \right) &= 8\% \end{aligned} \quad (41) \quad (42)$$

and it is obvious that while the non-infrared corrections δ_1, δ_2 fall rapidly (allowing us to expect the next term to be of the order of a per mil), the infrared ones do it slowly (the next-order terms can be estimated to be of the order of a percent). Moreover, looking at the soft part, we can see that the infrared terms exhibit a clear structure of the kind:

$$\sigma_0(s) \{ 1 + \delta_1 + \delta_2 \} \{ 1 + \beta \ln x_0 + \frac{1}{2} \beta^2 \ln^2 x_0 + \dots \} = \sigma_0(s) (1 + \delta_1 + \delta_2) x_0^\beta \quad (43)$$

The same can be seen concerning the hard part, namely, we can induce that the effect of resumming all the Sudakov logs will give us an expression like

$$\int_{x_0}^1 \sigma_0(s') \left[\beta (x^{\beta-1} (1 + \delta_1 + \delta_2) - 1 + \frac{x}{2}) + \delta_2^H \right] dx \quad (44)$$

The sum of these two parts gives us an expression of the total exponentiated cross-section including up to $O(\alpha^2)$ corrections exactly (the $O(\alpha)$ one can be obtained by just dropping all the δ_2 terms).

Lets realize that this expression of the cross section fixes up all the problems of QED corrections we warned at the beginning:

- i.- in one hand, it has resummed all the large infrared corrections.
- ii.- additionally, the problem with the range of x_0 values has disappeared: the soft cross section is defined positive for any value of x_0 (even for $x_0 = 0$ which would be the most precise election)

The problem of this method is that only information about the total energy loss due to photons is available. The rest has been implicitly integrated. The variable x represents essentially the photon energy sum if no transverse component is produced. Therefore, all we can expect to reproduce properly is the behaviour of the total integrated cross section.

Nevertheless, a reasonable approximation of the effect of photons in the differential cross section can be obtained by modifying a one-loop M.C. event generator with the following transformation:

- i.- in the soft part, eliminate the infrared terms $\beta \ln x_0$ from the corrections and multiply the cross section by x_0^β .
- ii.- in the hard photon part: substitute the piece $\beta \frac{1}{2}$ by $\beta x^{\beta-1} (1 + \delta_1)$.

Doing so, we are neglecting the effect of all the photons carrying an energy smaller than x_0 (but this is not a problem since we can take x_0 as small as we want) and additionally we are assuming that when the energy radiated is above this cut-off, then it is carried by just one photon. This is clearly an approximation since eq.(44) is adding the contribution from the radiation of an infinite number of photons but, however, when the radiated energy is large enough it tends to be carried by just one or at most two energetic photons. So the approximation is very good in this case.

5.2 The Structure Functions approach

A different way of attacking the problem is by using the Structure Functions approach [11] like in QCD. The effect of initial state radiation can be explained with the following picture: real electrons are particles "dressed" by photon radiation in such a way that the interacting "partons" in e^+e^- collisions in practice carry just a fraction of the real particle energy, being the rest lost in form of radiation when a collision takes place. The probability distribution of finding an electron with fractional energy

$$z = 1 - \sum_i \frac{E_{\gamma_i}}{\sqrt{s}/2} \quad (45)$$

at a momentum transfer squared $Q^2 = s$ is given by the Structure Function

$$D_e = (z, s) \quad (46)$$

in such a way that in analogy with the Drell-Yang processes in QCD we can write the total cross section as

$$\sigma(s) = \int dx_1 dx_2 D_e(z_1, s) D_e(z_2, s) \sigma_0(s x_1 x_2) \quad (47)$$

In this formula all the radiative corrections due to initial state radiation (virtual and real) are accounted for in the Structure Function expression, and all the other effects, like oblique corrections, have to be included explicitly in σ_0 .

The calculation of the Structure Function is obviously process independent and can be done by using Altarelli-Parisi evolution equations:

$$D_e(z, s) = \delta(1-z) + \int_{m_0^2}^s \frac{ds'}{s'} \frac{1}{2\pi} \int_0^1 \frac{da}{a} P(a) D_e\left(\frac{z}{a}, s'\right) \quad (48)$$

being

$$\begin{aligned} \alpha(s') &= \frac{\alpha}{1 - \frac{\alpha}{3\pi} \ln(\frac{s'}{m_e^2})} \quad \text{QED running coupling constant} \\ P(z) &= \frac{1+z^2}{1-z} - \delta(1-z) \int_0^1 dx \frac{1+x^2}{1-x} \quad \text{Kernel} \end{aligned} \quad (49)$$

This last expression is the so-called regularized vertex function. Its first term takes care of real photon emission (is just the well-known Weizsacker-Williams photon spectrum) whereas the second one reflects the contribution coming from virtual photons.

This equation can be solved analytically for all orders in the soft limit (the interesting one since we know that the bulk of photonic corrections comes from soft photonic radiation) leading to the Gribov-Lipatov [12] solution:

$$\begin{aligned} D_e(z, s) &= \frac{\exp\{\frac{3}{2}(2-2\gamma_E)\}}{\Gamma(\frac{3}{2})} (1-z)^{\frac{3}{2}-1} \\ \gamma &\equiv -6 \ln(1 - \frac{\alpha}{3\pi} \ln(\frac{s}{m_e^2})) \quad \gamma_E \equiv \text{Euler constant} \end{aligned} \quad (50)$$

In the soft limit (that is, neglecting the s dependence of σ_0) this leads to

$$\sigma(s) = \sigma_0(s) \frac{x_0^2}{\Gamma(\gamma+1)} \exp\left\{\frac{\gamma}{4}\left(\frac{3}{2} - 2\gamma_E\right)\right\} \quad (51)$$

which expanded in powers of α up to $O(\alpha^2)$ gives:

$$\begin{aligned} \sigma(s) &= \sigma_0(s) \left[1 + \frac{\alpha}{\pi} L(2l + \frac{3}{2}) + \left(\frac{\alpha}{\pi}\right)^2 L^2(2l^2 + \frac{10}{3}l + \frac{11}{8} - \frac{\pi^2}{3})\right] \\ L &\equiv \ln(\frac{s}{m_e^2}) \quad l = \ln x_0 \end{aligned} \quad (52)$$

This result is to be compared with the one coming from the complete $O(\alpha^2)$ calculation, which with the same notation can be written as

$$\begin{aligned} \sigma(s) &= \sigma_0(s) \left[1 + \frac{\alpha}{\pi} \left\{L(2l + \frac{3}{2}) - 2l + \frac{\pi^2}{3} - 2\right\}\right] \\ &+ \left(\frac{\alpha}{\pi}\right)^2 \left\{L^2(2l^2 + 3l + \frac{9}{8} - 2\zeta(2)) + L \quad (\dots) + \quad (\dots)\right\} \end{aligned} \quad (53)$$

revealing that the Structure Functions expansion reproduces exactly the $O(\alpha^2)$ calculation in the leading-log approximation. Moreover it accounts exactly for the $O(\frac{\alpha}{\pi}L)$ terms and very approximately for terms with $(\frac{\alpha}{\pi}L)^2$ with 1 powers smaller than 2. It can be seen that eq.(51) reproduces also the soft photon exponentiated equation at the leading-log level:

$$\sigma(s) \simeq \sigma_0(s) x_0^{\frac{2\alpha L}{\pi}} \left(1 + \frac{\alpha}{\pi} \frac{3}{2} L\right) \quad (54)$$

The Gribov-Lipatov equation includes all the leading-logs in the soft photon limit but, of course not for the hard photon case. Additionally the two equations above show differences due to the next-to-leading terms some of which can be included with the simple substitution rule $L \rightarrow L-1$. Actually, the Gribov-Lipatov solution can be manipulated following a simple procedure to include all the relevant terms we would like to have [11]. The basic idea is:

i.- Take the evolution equation (48) and substitute in the right part the lowest order expression of

$$D_e(z, s) = \delta(1-z) \quad (55)$$

to obtain the $O(\alpha)$ solution (which now will show a soft photon part as well as a hard photon one).

ii.- by comparing with the $O(\alpha)$ Gribov-Lipatov solution, the missing leading-log terms due to hard photons can be detected and included directly in the Gribov-Lipatov solution.

iii.- moreover, non-leading terms can be included by just substituting in the expression of the cross section (47) the $D_e(z, s)$ we want to complete and comparing the result with the exact one-loop calculation to look for the missing terms we want to pick up.

Of course, this can be done for $O(\alpha^2)$ as well and in any case, the infrared terms are obtained from the Structure Functions evolution equation whereas the rest of terms are obtained by comparison with the result of a complete Feynman diagram calculation.

The Structure Functions method, demonstrates that the induction made in the previous method about the behaviour of infrared corrections is correct because the leading-log terms show up naturally in the same way. In other hand, to reach the same accuracy attainable with the previous method, it needs a further elaboration of the calculational results obtained there and, as it has been described here, it gives no real additional information on the photon energy. Nevertheless, for M.C. purposes it provides a clear and easy way of including the effect of initial state radiation since eq. (47) is directly implementable in a M.C. event generator.

Eq. (47) can be used to compute the total cross section as well as to take into account distribution effects unsensitive to transverse components (let's realize that transverse information has been implicitly integrated in the Structure Functions that we are using). To account for transverse effects, two possible ways have been proposed:

(a) ONE-PHOTON: The total cross section is computed using eq.(47) but when the amount of energy radiated is larger than a certain x_0 , the one-hard-photon exponentiated cross section (the same one as in the previous method) is used instead of the soft one. In this way, the total cross section is correct and the main distributions show the approximation of assigning the total photonic cross section to just one photon.

(b) MULTI-PHOTON: The total cross section is computed also using eq.(47) but when the amount of energy is larger than a certain z_0 , a multiphoton production algorithm is applied [13]. The basic idea is the following: when the photonic energy is above a certain quantity, use is made of an iteration of the evolution equation (48) to obtain an expansion in α rather than an exponentiated solution like the Gribov-Lipatov one. For instance, iterated to third order this gives

$$D_e(z, s) = \delta(1-z) + \frac{\alpha}{2\pi} \int_{s'} P(z) + \left(\frac{\alpha}{2\pi}\right)^2 \int_{s''} \int_{s'} \frac{da}{a} P\left(\frac{z}{a}\right) P(a) + \left(\frac{\alpha}{2\pi}\right)^3 \int_{s'''} \int_{s''} \int_{s'} \frac{da}{a} \int_{b'} \frac{db}{b} P\left(\frac{z}{ba}\right) P(a) P(b) \quad (56)$$

This provides the following basic ingredients to treat multiphoton production:

i.- the number of produced photons is computed using a Poissonian distribution

$$\begin{aligned} B(0) &= 1 - \bar{n} + \frac{\bar{n}^2}{2} - \frac{\bar{n}^3}{6} + \dots = e^{-\bar{n}} \\ B(1) &= \bar{n} B(0) \\ B(2) &= \frac{\bar{n}^2}{2} B(0) \dots \\ \bar{n} &= \frac{\alpha}{2\pi} L(-1 - \frac{3}{4}) \end{aligned} \quad (57)$$

which in fact can be directly deduced from the iterated solution of the evolution equation integrating over z and dropping non-dominant terms.

ii.- the longitudinal energy for each photon is generated using the Weizsaker-Williams expression that shows up as the Kernel in the evolution equation.

iii.- to obtain the transverse energy of each photon, the Structure Function keeping information on the transverse momentum is used $D_e(z, p_T, s)$.

$$\sigma(s) = \int dz_1 dz_2 d^2 p_{T1} d^2 p_{T2} \sigma_0(s') D_e(z_1, p_{T1}, s) D_e(z_2, p_{T2}, s) \quad (58)$$

This function verifies the evolution equation

$$\frac{\partial D_e(z_1, p_{T1}, s')}{\partial s'} = \frac{\alpha}{2\pi} \frac{1}{s' - m_e^2} \int_z^1 \frac{da}{a} P(a) \int \frac{d^2 q_T}{\pi} \delta D_e\left(\frac{z}{a}, p_T - \frac{z}{a} q_T, s'\right) \quad (59)$$

where δ guarantees a correct kinematic splitting at the vertex which iterated up to $O(\alpha)$ gives

$$D_e(z_1, p_T, s) = B(0) \delta(1-z) \delta^2(p_T) + B(1) P(z) \frac{1}{p_T^2 + (1-z)^2 m_e^2} + O\left(\frac{p_T^2}{E_s^2}\right) \quad (60)$$

showing, therefore, the approximated photon p_T distribution.

5.3 Y.F.S.

The most elaborated and rigorous way of attacking the problem is based upon the early work of Yennie, Frautschi and Suura [14]. A careful study of the infrared terms leads to the following general master equation

$$\begin{aligned} \sigma &= \sum_{n=0}^{\infty} \frac{1}{n!} \int \frac{d^3 q_1 d^3 q_2}{q_1^2 q_2^2} \left(\prod_{i=1}^n \frac{d^3 k_i}{k_i^2} \tilde{S}(p_1, p_2, k_i) \theta\left(\frac{2k_i^0}{\sqrt{s}} - \epsilon\right) \right) \\ &\quad \delta^4(p_1 + p_2 - q_1 - q_2 - \sum_{i=1}^n k_i) \\ &\quad \exp\left\{ 2\alpha \operatorname{Re} B(p_1, p_2) + \int \frac{d^3 k}{k_0} \tilde{S}(p_1, p_2, k) \theta\left(\epsilon - \frac{2k^0}{\sqrt{s}}\right) \right\} \\ &\quad [\tilde{\beta}_0(p_1, p_2, q_1, q_2) + \sum_{i=1}^n \frac{\tilde{\beta}_1(p_1, p_2, q_1, q_2, k_i)}{\tilde{S}(k_i)} + \sum_{i,j=1}^n \frac{\tilde{\beta}_2(p_1, p_2, q_1, q_2, k_i, k_j)}{\tilde{S}(k_i) \tilde{S}(k_j)}] \end{aligned} \quad (61)$$

where

$$\begin{aligned} 2\alpha B(p_1, p_2) &= \frac{i\alpha}{4\pi^2} \int \frac{d^4 k}{k^2 - m_e^2} \left(\frac{2p_1 - k}{k^2 - 2p_1 k} + \frac{2p_2 + k}{k^2 + 2p_2 k} \right)^2 \Rightarrow \text{virtual photon} \\ \tilde{S}(p_1, p_2, k) &= -\frac{\alpha}{4\pi^2} \left(\frac{p_1}{p_1 k} - \frac{p_2}{p_2 k} \right)^2 \Rightarrow \text{real photon} \end{aligned} \quad (62)$$

which are the universal parts due to radiation, and the $\tilde{\beta}$ functions contain all the non-infrared parts and are, obviously, process-dependent. Their expression depends on the order of perturbation expansion one wants to keep in the non-infrared part and this is related directly with the number of hard photons one wants to reproduce correctly. If one hard photon is to be seen, one needs the use of a complete $O(\alpha)$ QED Feynman diagram calculation to compute the functions $\tilde{\beta}_0$ and $\tilde{\beta}_1$ which in this case will represent

- $\tilde{\beta}_0$: is the complete $O(\alpha)$ virtual+soft photon part of the cross section after eliminating the virtual infrared contribution.
 - $\tilde{\beta}_1$: is the complete $O(\alpha)$ real photon part of the calculation after subtracting explicitly the infrared part coming from the factorisation of the infrared factor $\tilde{S}(p_1, p_2, k)$ times $\tilde{\beta}_0$ at the Born level.
- This can be called "first order exponentiation" and, obviously, if two real photons have to be seen, a second order exponentiation is needed for which the master equation is the same but $\tilde{\beta}_0$, $\tilde{\beta}_1$ and $\tilde{\beta}_2$ have to be computed using an $O(\alpha^2)$ calculation.
- From the master equation we can learn two things:

included as the Z -gamma mixing and the oblique ones) and has been calculated already [16]. The difference between using the STAR scheme for the inclusion of oblique corrections or not in this calculation is of $O(\alpha^3)$ and can be neglected in front of the hadronic uncertainties.

7 Comparison

We are going to point out the obvious limitations and advantages of the different methods reviewed here for the treatment of the questions of OBLIQUE corrections resummation and infrared correction EXPONENTIATION and multiphoton production.

Concerning the first point it is obvious that the first method is just an approximation, which can be rather good if we are working in a region in which ΣZ is by far larger than $\Sigma^* Z$ and is very easy to implement in any already existing calculation. The "improved BORN amplitude" is an step ahead which contains actually all the relevant elements to ensure an accuracy in this part of the calculation well below the hadronic uncertainties. Finally, the STAR scheme is the most rigorous way of dealing with the problem, out of the three reviewed here, besides being the most complicated one, though the difference with the previous method can be smaller than the uncertainties coming from other parts of the calculations. The implementation of both methods in an existing calculation is not difficult provided you have the STAR functions already available. It is important to point out that since in all the methods the basic ingredients are the one loop self energies, they are seriously in trouble if the Top mass turns out to be large. In this case some correction factors should be included or even better, the result of a two loop self energy calculation.

Concerning the second point, it is obvious that only the infrared terms in the soft photon limit do factorize rigorously and thus can be exponentiated. This result is taken into account in the same manner by all the three methods presented. A complete $O(\alpha^2)$ calculation was needed not just to corroborate in an inductive way this fact, but also to provide all the methods the source to obtain the missing relevant terms which cannot be deduced directly from the study of the infrared parts.

We can first classify all the methods in two groups: the ones producing only one hard photon, and these producing multiphotons. From the first group, the main differences between the "inductive" approach and the Structure Functions one provided the proper $D_e(z, s)$ is taken to include non-leading effects, is that the second method gives always the right answer for the longitudinal components (even in the soft photon case) whereas the first one will always show a gap in distributions such as acollinearities. For the hard part, the distributions will be the same by construction and the results you can expect from both methods for the total cross section are essentially the same [7].

The second group is obviously superior to the first one for the study of differential distributions affected by multiphotons though the results for the total cross section must be essentially the same. As a drawback, they tend to be somewhat slower because of the multiphoton production procedure in spite that this can be adjusted by means of the arbitrary soft-hard energy limit, which actually acts as a regulator on the number of photons produced. Both treatments of multiphoton events can be expected to give roughly the same answer being the only relevant differences the following:

- In one hand, the treatment of multiphoton event looks more rigorously done in the Y.F.S. method for two reasons: first, the complete photon four-momentum distribution is known taking, thus, care of correlations among photons, and second, the "master"

1. how the infrared terms do factorize and appear exponentiated in the soft photon approximation

2. how the differential cross section for multiphoton production giving the full photon four-momenta information shows up naturally in the soft limit and can be corrected to produce hard photons by computing properly the β functions.

It can be easily seen that the master equation predicts a Poissonian distribution of the number of photons if only infrared terms are taken into account (that is, only β_0 is kept and the soft limit is taken), which is in very good agreement with the one obtained in the previous method.

In practice, the M.C. implementation of this method is rather involved since the rigorous treatment of the multiphotonic phase space and four-momentum conservation requires a quite complicated procedure [15].

6 The Z_0 width

As we saw already, at least the next order corrections to the Z_0 width should be included in a calculation in order not to loose accuracy when sitting on the peak.

The Z_0 width is given at the tree level by the imaginary part of the one loop Z_0 self energy $Im\hat{\Sigma}_Z^{(1)}(s)$. If we use the full boson propagators, we can be sure that some higher order terms are already included, like

$$-\frac{(\hat{\Sigma}^{*Z}(s))^2}{s + \hat{\Sigma}^*Z(s)} \quad (63)$$

Besides of that, when using the STAR scheme, the fact that the "starred" width is a function of the "starred" basic parameters, guarantees that resummed oblique corrections to the Z_0 width have been automatically included.

In any case, what is still missing is the two loop irreducible part of the self energy $Im\hat{\Sigma}_Z^{(2)}(s)$. To include these corrections in a reasonable way, we can use the fact that at the Z_0 pole we know that

$$\begin{aligned} Im\hat{\Sigma}_Z^{(2)}(M_Z^2) &= M_Z \Gamma_Z^{(0)} \\ Im\hat{\Sigma}_Z^{(2)}(M_Z^2) &= M_Z (\Gamma_Z^{(1)} - \Gamma_Z^{(0)}) \end{aligned} \quad (64)$$

and additionally the fact that the imaginary part of the self energy behaves around the the peak as

$$\hat{\Sigma}^*Z(s) \propto s \quad s > 0 \quad (65)$$

Therefore a reasonable way of including the relevant corrections to the width is simply by taking

$$Im\hat{\Sigma}_Z^{(2)}(s) \simeq \hat{\Sigma}_Z^{(2)}(M_Z^2) + \frac{s}{M_Z^2} M_Z (\Gamma_Z^{(1)} - \Gamma_Z^{(0)}) \quad (66)$$

where $\Gamma_Z^{(1)}$ includes the Z_0 three body decay width as well as strong corrections (which are responsible for most of the uncertainty) and the electroweak ones (but the ones already

formula provides the correct way of including a correct calculation up to any order in perturbation theory, that is, in spite that just a Poissonian distribution of the number of photons or a simple photon four-momenta spectrum can be utilized to approximate the differential cross section in a M.C. implementation, we can use afterwards the "master" equation to correct for the approximation used and thus obtain the right result up to the desired perturbation order. This cannot be done rigorously in the Structure Functions multiphoton approach reviewed here because it uses a Poissonian distribution which cannot be corrected for the non-infrared effects and this affects the photon-by-photon longitudinal and transverse variables, which are the only ones that we can control. Additionally in the generation of the photon transverse momentum as reviewed here $O(\frac{p_T^2}{E^2})$ terms are neglected and this can be a rather bad approximation when hard transverse photons are studied.

- In other hand, the Structure Functions approach, as mentioned before, gives always the right longitudinal distributions regardless on the soft-hard limit taken to regulate the number of produced photons. In the Y.F.S. method, a precise acollinearity distribution can be obtained only by lowering the soft-hard limit, thus producing many undetectable photons which represent a waste of computing time.

8 Conclusions

The relevant modifications to the previous generation of M.C. event generators (PETRA - PEP) needed "a priori" for working at LEP energies have been discussed. The different methods developed so far to implement them have been briefly reviewed showing their main features and also pointing out their similarities and differences.

We cannot see the methods reviewed here as the final answer to the problem of perturbative expansions in the E.W. theory, but rather as an effective treatment of the relevant questions needed at present to reach the requested accuracy in LEP-SLC experiments. In any case, they all constitute an step ahead in the clarification of the real meaning of perturbation theory and its confrontation with the experiment.

The main conclusion is that if we want to get around 1% accurate theoretical predictions for processes dominated by the Z_0 peak influence we have to check that the M.C. event generator that we are using, is at least an $O(\alpha)$ corrected calculation and that the authors have foreseen some treatment of all the following fundamental aspects:

- a) Resummation of OBLIQUE corrections
- b) Exponentiation of infrared QED corrections
- c) Interplay of photonic and non-photonic corrections
- d) A more accurate evaluation of the Z_0 width than the simple tree level one

For most of the applications any combination of the methods reviewed here can produce already the required accuracy. A nice example on this fact is the excitingly good agreement between the $\nu\bar{\nu}\gamma$ M.C. event generator programs NNGG03 (Maza-Martinez-Miquel)[17] and KORALZ (adapted for Colas-Mirabito-Was [18] for neutrino production) which have been written independently and using completely different techniques:

NNGG03 is a program based upon a complete $O(\alpha)$ QED corrected $\nu\bar{\nu}\gamma$ calculation which uses the STAR scheme for the oblique corrections, the "inductive" method for the treatment of infrared QED corrections and includes the Z_0 width in the way explained here.

KORALZ is a program based upon a complete $O(\alpha)$ QED corrected $\nu\bar{\nu}$ calculation which uses the "propagator substitution" method for the oblique corrections, the Y.F.S. scheme to treat the infrared QED corrections (including also leading and next-to-leading logarithmic terms of the $O(\alpha^2)$ QED corrections) and to produce multiphoton events and includes also corrections to the Z_0 width.

For some applications, however, the use of one technique instead of another may be of paramount importance. You can realize that twofold:

- if possible IN ADVANCE, because it is easy to understand that some techniques presented here, by construction, are not designed to give accurate results in the special observable you might be interested in (this is just matter of information), or
- A POSTERIORI, because after debugging the programs and checking that they give the same answer in the observables they are supposed to do it, you get different results for the observable that you are calculating.

In this last case, a M.C. implementation combining the most accurate techniques for each of the problems discussed here should be the starting point to try to understand whether still further neglected corrections should be taken into account or not.

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