FINITE TEMPERATURE CALCULATIONS ON ANISOTROPIC LATTICES *

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We present results from a program of calculations for lattice gauge and matter fields theories at finite temperatures, realized by using different cut offs in the space and time directions (anisotropic lattices). Specifically, the analyses pertain to the ϕ^4 theory, Higgs models and QCD.

1. INTRODUCTION

Anisotropic lattice models have an anisotropy parameter ξ , which gives the cut-offs ratio in the "time" (τ) and "space" (σ) directions of the Euclidean lattice: $\xi = a_\sigma/a_\tau$, with a_σ, a_τ : lattice spacings¹. This additional parameter allows one to continuously and separately vary the scales in the space and time directions. Thus any desired physical temperature can be obtained, independently on the lattice (dimensionless) size. The analyses involve usually two steps: tuning of the couplings to achieve the desired cut-off anisotropy ξ ("calibration") and calculation of physical interests (spectrum, renormalization, etc.). To fix the ideas consider the action of a free scalar field:

$$S_0 = \frac{1}{2} \sum_n \left(\sum_{i=1,2,3} (\phi_{n+i} - \phi_n)^2 + \gamma^2 (\phi_{n+4} - \phi_n)^2 + M^2 \phi_n^2 \right) \quad (1.1)$$

leading to the propagator

$$G_0 = \left[4 \sum_i \sin^2 \frac{q_i}{2} + 4\gamma^2 \sin^2 \frac{q_4}{2} + M^2 \right]^{-1} \quad (1.2)$$

The cut offs, and thereby the anisotropy ξ appear in the renormalization process. The calibration of the lattice is obtained by demanding isotropy of the physical (dimensional) quantities at zero temperature. Introducing physical variables $\vec{p} = a_\sigma^{-1} \vec{q}$, $p_4 =$

$a_\tau^{-1} q_4$, $m = a_\sigma^{-1} M$, $\psi = a_\sigma^{-1} \phi$, the isotropy condition (at small p_μ) for the propagator eq.(1.2) implies $\gamma = \xi$. In the general case, we start with a lattice model defined in terms of some couplings κ_i and coupling anisotropies γ_i and do a scaling analysis in terms of a cut off and cut off anisotropy,

$$a_\sigma = R(\kappa)/\Lambda(\kappa, \gamma), \quad \xi = X(\kappa, \gamma) \quad (1.3)$$

and a numbers of physical quantities (e.g. masses). Here $R(\kappa)$ represents the renormalization group scaling. The calibration then means determine the functions $X(\kappa, \gamma)$ and $\Lambda(\kappa, \gamma)$ such as to be able to tune γ (and κ) to reproduce a desired a_σ, ξ at zero temperature after fixing the physical quantities; i.e., inverting eq.(1.3):

$$\gamma_i = \xi \eta_i(\xi, a_\sigma), \quad i = 1, 2, \dots \quad (1.4)$$

A non-zero physical temperature is achieved by fixing the physical "time" length

$$T = \frac{1}{l_\tau} = \frac{1}{N_\tau a_\tau} = \frac{\xi}{N_\tau a_\sigma} = \frac{\xi}{N_\tau} \Lambda(\kappa, \gamma)/R(\kappa) \quad (1.5)$$

with ξ given by eq.(1.3) and taking the thermodynamical limit $N_\sigma \rightarrow \infty; a_\sigma \rightarrow 0; N_\tau \rightarrow \infty, a_\tau \rightarrow 0$ (fixed T). With ξ at our disposal we can obtain a high enough temperature even with $N_\tau \gg 1$, such as not to violate this limit. Thermodynamic quantities, defined as derivatives with respect to $1/T$,

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are easily obtained with help of the chain rule :

$$\frac{\partial}{\partial(1/T)} = -\frac{\xi^2}{N_\tau a_\sigma} \frac{\partial}{\partial \xi} = -\frac{\xi^2}{N_\tau a_\sigma} \left(\eta + \frac{\partial \eta}{\partial \xi} \right) \frac{\partial}{\partial \gamma} \quad (1.6)$$

2. ϕ^4 AND HIGGS MODELS

For the ϕ^4 theory in 4 and 3 dimensions, as given by the action (see eq(1.2))

$$S = S_0 + \sum_n \frac{\lambda}{24} \phi_n^4 \quad (2.1)$$

the isotropy condition (imposed at zero temperature for the propagators analyzed within an 1-pole ansatz at fixed correlation length $R=2$) leads to the calibration as shown in Table1. We see that the deviations from the free field relation $\gamma = \xi$ are small, especially in the 4-dim. case. Since we can realize arbitrary small compactification radius we can effectively perform a dimensional interpolation between 4 and 3 dimensions. We can then study the variation of the character of the model in this interpolation by analyzing at each temperature the scaling behaviour of the renormalized coupling. For this we chose the renormalization procedure of², see also³, by fixing $z \equiv \frac{l_\tau}{r} = \frac{N_\sigma}{R} = 4 \pm .1$ and calculating λ_{ren} as function of the correlation length $r = Ra_\sigma$ at various temperature⁴ (see Fig.1). At least for correlation lengths less than 3, the picture changes drastically when l_τ (the inverse temperature) becomes less than r : for high temperature the 4-dim. model seems to have a similar scaling behaviour with that of the 3-dim. model. These results confirm the observation obtained from the Ising model³, where however no calibration was attempted and $\gamma = \xi$ has been assumed to hold. For the next step - the study of the Higgs model - the calibration itself is much more difficult. Now we have the action

$$S_{GH} = S_G + S_H = -\frac{\beta}{N_c} \sum_P (ReTr P_{\sigma\sigma} + \gamma_G^2 ReTr P_{\sigma\tau}) - \sum_n \left(\kappa (Re(\phi^* u_\sigma \phi) + \gamma_H^2 Re(\phi^* u_\tau \phi)) - \lambda \phi^4 \right) \quad (2.2)$$

where $P_{\sigma\sigma}$, $P_{\sigma\tau}$ are the space-space, space-time plaquettes and u_σ , u_τ the space, time links, respectively. Here the relation to the bare mass is

$\kappa = 1/(M^2 + 6 + 2\gamma_H^2)$. In the previous case the procedure has been to give γ and find out ξ by measuring the correlation functions and see how they must be rescaled in order to achieve isotropy. Now, however, we have *two* coupling anisotropies to be absorbed into only *one* cut off anisotropy. This means that we cannot find a solution to the isotropy condition for each combination of values for γ_H , γ_G . Consider that we measure a number of observables, O_i . Then the isotropy condition implies a number of relations, say, of the form

$$\bar{O}_{i\sigma}(n; \gamma_H, \gamma_G) = \bar{O}_{i\tau}(\xi n; \gamma_H, \gamma_G), i = 1, 2, \dots \quad (2.3)$$

where the averages $\bar{O}_{i\sigma(\tau)}(n)$ are some correlation function over n sites in space (time) direction, respectively. The formal solution is then

$$\gamma_H = \xi \eta_H(\xi), \quad \gamma_G = \xi \eta_G(\xi) \quad (2.4)$$

which establishes a relation between γ_H and γ_G for each ξ . A procedure may be to give γ_G and tune γ_H to fulfil (2.3) and then read the rescaling ξ . This is very time consuming. Assume however that for a given γ_G we work at a value $\tilde{\gamma}_H$ near the correct one, γ_H . Then we can write

$$O_i(\gamma_H, \gamma_G) = \frac{\int O_i e^{-S(\gamma_H, \gamma_G)}}{\int e^{-S(\gamma_H, \gamma_G)}} = \frac{\int O_i e^{-\tilde{S} + \Delta\gamma_H^2(S_{H,\tau} - C)}}{\int e^{-\tilde{S} + \Delta\gamma_H^2(S_{H,\tau} - C)}}, \quad (2.5)$$

where $\tilde{S} = S(\tilde{\gamma}_H, \gamma_G)$, $\Delta\gamma_H^2 = \tilde{\gamma}_H^2 - \gamma_H^2$, $S_{H,\tau} = \kappa_\sigma \sum Re(\phi^* u_\tau \phi)$. Then, expanding the exponentials, we get the "isotropic" averages

$$\bar{O}_i(\gamma_H) \equiv \langle O_i \rangle_{\gamma_H} = \langle O_i \rangle_{\tilde{\gamma}_H}$$

$$+ \Delta\gamma_H^2 (\langle O_i S_{H,\tau} \rangle_{\tilde{\gamma}_H} - \langle O_i \rangle_{\tilde{\gamma}_H} \langle S_{H,\tau} \rangle_{\tilde{\gamma}_H})$$

expressed with help of averages calculated at the nonisotropic point $\tilde{\gamma}_H$, γ_G . Solving the eqs.(2.3) will then fix both ξ and $\Delta\gamma_H^2$ without further tuning. C is of course redundant, but choosing it to be $\langle S_{H,\tau} \rangle_{\tilde{\gamma}_H, \gamma_G}$, we see that it indicates that not $S_{H,\tau}$ (which is a global quantity) is decisive for the quality of the approximation, but its fluctuation $S_{H,\tau} - \langle S_{H,\tau} \rangle$ which mostly only grows as the square root of the volume.

3. QCD

The QCD action is (see also eq.(2.2))

$$S_{QCD} = S_G + S_F, \quad S_F = - \sum_{\langle nm \rangle} \bar{\psi}_n W_{nm} \psi_m, \\ W = 1 - \kappa \left(\sum_i \Gamma_i^\dagger u_i T_i + \gamma_F \Gamma_4^\dagger u_4 T_4 + \text{"h.c."} \right), \quad (3.1)$$

where $\Gamma_\mu^\pm = \tau \pm \gamma^\mu$, $r=1$ (0) gives Wilson (staggered) fermions, u_μ 's are link variables and T_μ translation operators. The relation to bare mass M is

$$\kappa = 1/2(M + 3r + \gamma_F r). \quad (3.2)$$

At each finite β , going to κ_c we may find a massless pion - however, this does not mean that we can obtain a reasonable continuum theory, since all other physical masses would then be infinit (they are non-zero in lattice units). Starting at $\beta = \infty, \kappa = 1/8$, there will be lines extending to $\kappa < \kappa_c, \beta < \infty$ on which physics at energies lower than the cut off remains invariant. We obtain then the correct mass ratios since the two scales - as given e.g. by the gluon and pion mass - are correlated. This correlation is wrong in the quenched approximation hence we may not be able to obtain the correct spectrum here. Even more doubts concern the quenched approximation results obtained at high temperature, where the contribution of the fermions may completely change the structure of the configurations. For QCD with dynamical fermions on anisotropic lattices there are to date only results in perturbation theory. There has been a series of works ⁵, completed recently by the calculation of the fermionic selfenergy ⁶. One obtains for the propagator S_F (to lowest order in the external momenta p),

$$S_F^{-1} = S_{F,0}^{-1} + \Sigma = \sum_i \gamma_i \sin(p_i a_\sigma) + \gamma_F \gamma_4 \sin(p_\tau a_\tau) \\ + M + g^2 \frac{N_c^2 - 1}{2N_c} (M \Sigma_2 + \sum_i p_i a_\sigma \Sigma_{1,\sigma} + p_4 a_\sigma \frac{\gamma_F}{\xi} \Sigma_{1,\tau}) \quad (3.3)$$

The isotropy condition gives

$$\gamma_F = \xi \eta_F(\xi) = \xi(1 + g^2 c_F(\xi)), \\ c_F(\xi) = \frac{N_c^2 - 1}{2N_c} (\Sigma_{1,\sigma}(\xi) - \Sigma_{1,\tau}(\xi)), \quad (3.4)$$

(the expressions for the various selfenergy terms are given in ref.6). The nontrivial η_F of eq.(3.4) leads

via eq.(1.6) to corrections to the physical energy which are necessary to get agreement with continuum perturbation theory. Moreover, the previously found overshooting of the energy is substantially reduced ⁶. This stresses the importance of the dynamical fermions. Nonperturbative studies concern at present only quenched QCD. The calibration here is simply that of pure QCD ⁷. We then assume that the fermions just "feel" the anisotropy induced by the YM fields, i.e., we take

$$\gamma_F = \xi = X_G(\gamma_G). \quad (3.5)$$

In the following we work at $\gamma_G = 1$ ($\xi = 1$) and at $\gamma_G = 2$ where the nonperturbative YM calibration ⁷ gives $\xi = 2.6$. Preliminary results have been presented in ref.8. From the "naive" relation eq.(3.2) we have

$$1/\kappa(\xi) - 1/\kappa(1) = 2(\xi - 1). \quad (3.6)$$

This seems well satisfied by our data, in particular we get $\kappa_c(\xi = 2.6) \simeq .112$, $\kappa_c(\xi = 1.0) \simeq .171$, in rather good agreement with eq.(3.6). The meson mass is calculated from propagators along the τ direction. We also compute the "screening mass" from σ -direction propagators. Notice that $M = \xi \sqrt{2(\cosh \bar{M} - 1)}$ ($\bar{M}$: the actual slope measured from the propagator) ⁸. If the propagator would be dominated by a free particle pole then of course the mass would be identical with the "screening mass" ^{8,9}. However in the interaction case we expect a more complicated structure. In this case, since the temperature breaks the Euclidean invariance, we expect to see different behaviour in the τ and σ direction. As an example, from the weak coupling propagator eqs.(3.3,4) we can read the ratio of masses measured in the τ and σ directions, as function of the temperature ξ/N_τ : $m(\tau)/m(\sigma) \sim 1 + g^2(c_F(\xi, N_\tau) - c_F(\xi, \infty))$. The nonperturbative quenched results of Fig.2 show indeed a mass growing with the temperature at variance with a practically constant screening mass. We interpret the growing mass as a signal for the building up of collective effects, leading to a bump in the propagator which with increasing temperature moves to higher energies. However, we may have large finite size errors, therefore we are now

repeating this analysis with larger lattices and varying only ξ but not N_τ . This procedure has also been chosen to compare confined and deconfined (τ -direction) propagators (see Fig.3). We find here a striking difference in behaviour - above T_c the square of the pseudo scalar mass is no longer linear in $1/\kappa(\sim m_q)$ and remains much higher, without any indication of going toward zero. This is in qualitative agreement with results for the screening mass. We want however to repeat also these calculations at a higher N_τ . In spite of these observations and of the general criticism to the quenched approximation we think that the qualitative picture obtained so far is correct. In any case these are the first results for the mass (measured along the τ direction) at temperatures around the deconfining transition.

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d	lattice	γ	λ	ξ
4	$8^3 16$	2	750	2.03(1)
			12000	2.08(1)
4	$8^3 24$	3	750	3.00(1)
			12000	3.25(1)
3	$8^3 16$	2	750	2.06(1)
			12000	2.14(1)
3	$8^3 24$	3	750	3.10(1)
			12000	3.40(1)

TABLE 1 : Calibration of the ϕ_4^4 model.

FIG.1 : λ_{ren} for ϕ_4^4 (various T) and ϕ_3^4 .

FIG.2 : Temperature dependence below T_c .

FIG.3 : Confining and deconfining region.