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A covariant and gauge invariant string fragmentation model

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ABSTRACT

We present a fragmentation model, designed in particular to treat the decay of strings formed in soft hadronic interactions. The fragmentation model is based on classical string theory: the simplest possible local, covariant, gauge invariant fragmentation law is assumed. A very elaborate resonance table for low mass cluster decay is used. Resonances have a continuous mass spectrum, only stable hadrons are on mass shell. We treat quark-antiquark strings as well as quark-diquark strings and also more complicated structures (like $\bar{q}-qqqq$). A large amount of e^+e^- and lepton-nucleon data are used to fix parameters and to test the model.

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1. Introduction

Once in a string model of hadron-hadron interactions one assumes that each string fragments independent of all the others and also independent of the formation, one does not have to bother too much about the fragmentation procedure. A fragmentation model can simply be considered as a parametrization of data, where e^+e^- provides mainly information about quark-antiquark strings, whereas in lepton-nucleon scattering diquark-quark string fragmentation may be studied. So in a (phenomenological) sense, two models are equivalent when they describe "string fragmentation" data equally well.

From a theoretical point of view, the last statement is of course not true. We are always interested in having the "right" model, possibly derived from basic principles. Apart from esthetics, there are also reasons for phenomenologists to use a proper fragmentation model: in a hadronic collision, in particular in heavy ion collisions, there are plenty of strings, which certainly do interact. These string-string interactions, as well as interactions of strings with spectator nucleons in the case of nuclear collisions, depend very well on details of the fragmentation procedure, not just on the outcome. The space-time evolution of the decaying string has to be understood.

For these reasons it was decided to completely rewrite the fragmentation model used in the string model VENUS for hadronic collisions (now VENUS 3). The model will also include string-string interactions and string spectator interactions, however these aspects are not the subject of this paper.

Before going in medias res, here are some thoughts about strings: We desire to have a fragmentation model derived from basic principles. Nowadays the basic principle is QCD. Strings were introduced before QCD [1] as an interpretation of the successful Dual Resonance Model [2]. Despite the fact that neither a strict derivation of string theory from QCD exists nor a consistent quantum formulation, strings are still quite attractive, at least as a phenomenological description of hadron physics. It should also be noted, that QCD definitely shows stringlike behaviour (linear potential etc.). In this sense we consider classical string theory as fundamental, keeping however its limitations in mind, especially towards the end of a fragmentation cascade.

The structure of this paper is as follows: in chapter 2 we discuss string dynamics, chapter 3 describes the fragmentation model, and chapter 4 compares with data. Chapter 2 essentially provides the theoretical background, the reader not interested in, or familiar with, can easily skip this chapter.

2. Elements of string dynamics

In this chapter we discuss classical string theory (see refs. [3]), as the basic tool for later applications. A classical string is a two dimensional surface in the four-dimensional Minkowski space

$$x = x(\tau, \sigma) \quad (2.1)$$

with a spacelike parameter σ and a timelike one τ . Of course this is only one of infinitely

many parametrizations of this surface. A transformation from one parameter space to another

$$\begin{pmatrix} \tau \\ \sigma \end{pmatrix} \rightarrow \begin{pmatrix} \tilde{\tau}(\tau, \sigma) \\ \tilde{\sigma}(\tau, \sigma) \end{pmatrix} \quad (2.2)$$

is called gauge transformation, the group of such transformations is called gauge group. Of course the string action should not depend on this particular parametrization, so gauge invariance is a necessary requirement. Further restrictions should be locality and covariance. Concerning the question of gauge invariance it is useful to relate a metric g to a certain string parametrization via (using $\partial_1 \equiv \frac{\partial}{\partial \tau}$ and $\partial_2 \equiv \frac{\partial}{\partial \sigma}$):

$$g_{\alpha\beta} = \partial_\alpha x^\mu \partial_\beta x_\mu \quad (2.3)$$

where α and β assume the values 1 and 2. By using "dot" and "prime" as abbreviations for $\frac{\partial}{\partial \tau}$ and $\frac{\partial}{\partial \sigma}$, the metric can be written as

$$g = \begin{pmatrix} \dot{x}\dot{x} & \dot{x}x' \\ x'\dot{x} & x'x' \end{pmatrix}. \quad (2.4)$$

With M being the Jacobi matrix of a gauge transformation eq. (2.2), we get for the transformed metric $\tilde{g}$:

$$g = M^T \tilde{g} M \quad (2.5)$$

and therefore the action

$$S = \int L d\tau d\sigma = -\kappa \int \sqrt{-\det(g)} d\tau d\sigma \quad (2.6)$$

is clearly gauge invariant. Writing S explicitly as

$$S = \int L d\tau d\sigma = -\kappa \int \sqrt{(x'\dot{x})^2 - x'^2 \dot{x}^2} d\tau d\sigma \quad (2.7)$$

shows that S is also local and covariant. In fact S is the simplest local, covariant and gauge invariant expression, so a very attractive candidate for a string action. This action in fact measures the area of the string surface

$$S = -\kappa \int d^2A \quad (2.8)$$

which becomes obvious when we choose τ to be the time t and σ to be the length of the string: by defining

$$\gamma \equiv (1 - v_\perp^2)^{-1/2}; \quad \tilde{v}_\perp \equiv \frac{\partial \tilde{x}}{\partial t} - \frac{\partial \tilde{x}}{\partial l} \frac{\partial \tilde{x}}{\partial t}$$

and using $|\frac{\partial \tilde{x}}{\partial l}| = 1$ we find

$$S = -\kappa \int \frac{1}{\gamma} dl dt. \quad (2.9)$$

From the variational principle $\delta S = 0$ for a string path being varied by $\delta x(\tau, \sigma)$ we obtain the equations of motion

$$\frac{\partial}{\partial \tau} \frac{\partial L}{\partial \dot{x}_\mu} + \frac{\partial}{\partial \sigma} \frac{\partial L}{\partial \dot{x}'_\mu} = 0 \quad (2.10)$$

with the boundary conditions (using the convention $\sigma \in [0, \pi]$),

$$\frac{\partial L}{\partial \dot{x}'_\mu} = 0 \quad \text{at } \sigma = 0, \pi \quad (2.11)$$

with the Lagrange density $L = -\kappa \sqrt{(\dot{x}^i \dot{x}^i)^2 - x'^2 \dot{x}^2}$. Again applying the variational principle, now however requiring the equations of motion, we obtain conservation laws. Invariance under translations $\delta x^\mu = \epsilon^\mu$ provides conserved currents (Noether's theorem):

$$\oint d\sigma \frac{\partial L}{\partial \dot{x}_\mu} + d\tau \frac{\partial L}{\partial \dot{x}'_\mu} = 0 \quad (2.12)$$

leading to the following definitions of the currents P_τ and P_σ :

$$P_\tau^\mu \equiv -\frac{\partial L}{\partial \dot{x}_\mu}; \quad P_\sigma^\mu \equiv -\frac{\partial L}{\partial \dot{x}'_\mu}. \quad (2.13)$$

Eq. (2.12) allows the definition of the string momentum as

$$P^\mu = \int_C d\sigma P_\tau^\mu + d\tau P_\sigma^\mu = \int_0^\pi d\sigma P_\tau^\mu \quad (2.14)$$

where C is an arbitrary curve on the string surface $x(\tau, \sigma)$ from $\sigma = 0$ to $\sigma = \pi$. The momentum P is conserved ($\partial P^\mu / \partial \tau = 0$).

To solve the equations of motion one should choose a gauge which simplifies the equations of motion. Orthonormal gauge

$$\dot{x}^i \dot{x}^i = 0, \quad \dot{x}^2 + x'^2 = 0 \quad (2.15)$$

does so. If we completely fix the gauge by setting

$$x_0 = \tau = t, \quad (2.16)$$

eq. (2.15) reads:

$$\dot{x}^i \dot{x}^i = 0, \quad (\dot{x}^2)^2 + (x')^2 = 1. \quad (2.17)$$

The currents eqs. (2.13) are now

$$P_\tau = \kappa \dot{x}, \quad P_\sigma = -\kappa x' \quad (2.18)$$

and the equations of motion eq. (2.10) are simply wave equations

$$\ddot{x}_\mu - x''_\mu = 0, \quad (2.19)$$

the boundary conditions eq. (2.11) are

$$x'(\tau, 0) = x'(\tau, \pi) = 0 \quad (2.20)$$

The solution of eqs. (2.19,20) is

$$\vec{x}(\tau, \sigma) = \frac{1}{2} [\vec{y}(\tau + \sigma) + \vec{y}(\tau - \sigma)] \quad (2.21)$$

where $\vec{y}$ is obviously the trajectory of one endpoint $\vec{y}(\tau) = \vec{x}(\tau, 0)$, called directrix. The directrix has to be periodic

$$\vec{y}(\tau + 2\pi) - \vec{y}(\tau) = \frac{2\vec{P}}{\kappa}. \quad (2.22)$$

Eq. (2.21) means: each point on the string may be obtained by a simple geometrical construction once the directrix $\vec{y}(\tau)$ is known. From eqs. (2.14,18,21) we also see that the momentum of a piece of string is generated by the momenta of the two corresponding directrix pieces:

$$dP(\tau, \sigma) = \frac{\kappa}{2} [\dot{y}'(\tau + \sigma)d(\tau + \sigma) + \dot{y}'(\tau - \sigma)d(\tau - \sigma)] \quad (2.23)$$

so the momentum for the string piece corresponding to $[0, \sigma]$ (from A to B in fig. 1) is

$$P[0 \rightarrow \sigma] = \int_0^\sigma dP = \frac{\kappa}{2} \int_{\tau-\sigma}^{\tau+\sigma} d\tau' \dot{y}'(\tau') = \frac{\kappa}{2} [\vec{y}(\tau + \sigma) - \vec{y}(\tau - \sigma)] \quad (2.24)$$

being proportional to the distance vector between two directrix points (DE in fig. 1). All this shows that the directrix piece from $D = \vec{y}(\tau - \sigma)$ to $E = \vec{y}(\tau + \sigma)$ determines the string piece from $A = \vec{x}(\tau, 0)$ to $B = \vec{x}(\tau, \sigma)$. We also find a section of the directrix related to the other string part from $B = \vec{x}(\tau, \sigma)$ to $C = \vec{x}(\tau, \pi)$. However equally well we can relate to this string piece the "antirectrix" (trajectory of the other end $\vec{x}(\tau, \pi)$) from F over C to G . It is obvious from fig. 1, that putting together the directrix piece D to E and the antirectrix piece F to G , corresponding to the two string pieces, we recover the full directrix (after a shift of the antirectrix by $\frac{1}{\kappa} \vec{P}$, which is the constant vector by which directrix and antirectrix differ).

All this discussion was aimed at writing down the laws for breaking and fusion. We do not know within this classical treatment where a string breaks or whether two strings fuse, but once we know the breakpoint, we know how to proceed. Let us just talk about string breaking, since fusion is just the inverse process, and rearrangement processes are a combination of breaking and fusion. As for the action we assume locality. If a breaking occurs, at (τ, σ) we have to make sure that for the future as well as the past we have periodic (anti-) directrices. And the directrices for future and past have to match properly in the

Fig. 1. A string with its directrix and antirectrix. The directrix segment DAE defines the string piece AB and the antirectrix FCG the string piece CB .

present. The only way to do so is to periodically continue (independently) the directrix corresponding to one string piece and the antirectrix corresponding to the other string piece into the future. This fully determines the time evolution of either string piece also for all the future (till the next break at least).

We are now going to discuss a simple but important example: a one-dimensional directrix, one period of which consisting of two linear segments ("yo-yo string"). For a one-dimensional directrix, straight lines with a tilt of 45° against vertical (in space-time) are mandatory, since the string end (represented by $\vec{y}(\tau)$) moves with the velocity of light (because of eqs. (2.17,20)). From eq. (2.21) it is clear that the corresponding string is a simple straight line ($q\bar{q}$ in fig. 2) stretched between directrix and antirectrix.

It is very instructive to investigate energy and momentum distribution along a one-dimensional yo-yo string. For the following we use $E \equiv p_0$ and $P \equiv p_3$ for energy and longitudinal momentum (no transverse momentum), the space-time coordinates are t and $z \equiv z_3$. From eqs. (2.14,18) we obtain for an arbitrary string element

$$dE = \kappa d\sigma; \quad dP = \kappa \dot{z} d\sigma. \quad (2.25)$$

The gauge fixing condition $z' \dot{z} = 0$ (from eq. (2.17)) requires either z' or $\dot{z}$ to be zero, an ordinary yo-yo having exactly two points with $z' = 0$: the two endpoints of the string (because of the boundary condition eq. (2.20) every string has to fulfil $\vec{x}' = 0$ at the endpoints). So we obtain from the gauge fixing conditions $z' \dot{z} = 0$ and $\dot{z}^2 + z'^2 = 1$, inside the string

$$\dot{z} \equiv \frac{\partial z}{\partial t} = 0; \quad |z'| \equiv \left| \frac{\partial z}{\partial \sigma} \right| = 1; \quad \text{for } \sigma \neq 0, \pi \quad (2.26)$$

and at the endpoints

$$z' \equiv \frac{\partial z}{\partial \sigma} = 0; \quad |\dot{z}| \equiv \left| \frac{\partial z}{\partial t} \right| = 1; \quad \text{for } \sigma = 0, \pi \quad (2.27)$$

Since these two domains (characterized by $z' = 0$ and $z' \neq 0$) behave so differently, we are going to discuss their contributions to energy and momentum separately. We use an index g (like glue) for the interior and an index q (like quark) for the endpoints. Energy and momentum of an inner piece of string of length dl are (using eqs. (2.25,26)):

$$dE_g = \kappa dl; \quad dP_g = 0 \quad (2.28)$$

whereas we get from eqs. (2.25,27) for an endpoint, during a time step dt with a corresponding movement dz of the endpoint, the following change of energy and momentum:

$$dE_q = \kappa s dt; \quad dP_q = \kappa s dz \quad (2.29)$$

(for $dt > 0$) where $s = +1(-1)$ means the endpoint has absorbed (emitted) a piece of string (or a piece of parameter space, to be precise). Eqs. (2.28,29) demonstrate, among other things, energy conservation: the energy dE_g gained by an endpoint by absorbing a piece of string is equal to the energy loss $-dE_q$ of the string due to its contraction. It is

Fig. 2. Space-time picture of a "yo-yo" string with period $2P/\kappa$.

also easy to see from eq. (2.29) that the two endpoints change momentum in an opposite way: $dP_q = -dP_{\bar{q}}$, guaranteeing momentum conservation. We are now going to integrate eq. (2.29). Let us consider one "basic cell" $OACB$ in fig. 2. The polygon OBC is half a period of the directrix, instead of the other half we consider the corresponding half period of the anti-directrix OAC (which is equivalent). So $OACB$ defines the string completely. We use for the left end (OBC) the index $\bar{q}$, for the right end the index q . At the turning points the momenta vanish

$$P_q(A) = P_{\bar{q}}(B) = 0 \quad (2.30)$$

and since this implies that at these points the parameter space specifying the endpoints consists of just one point (0 and π respectively), eq. (2.30) also requires the energy to be zero

$$E_q(A) = E_{\bar{q}}(B) = 0. \quad (2.31)$$

Now we can easily integrate eq. (2.29) backwards to point O to obtain the initial energy and momentum (we use $t_O = z_O = 0$):

$$\begin{aligned} E_q(O) &= \kappa t_A; & P_q(O) &= \kappa z_A \\ E_{\bar{q}}(O) &= \kappa t_B; & P_{\bar{q}}(O) &= \kappa z_B \end{aligned} \quad (2.32)$$

Using light cone coordinates $x^\pm = t \pm z$ and $p^\pm = E \pm P$ we get

$$p_q^+(O) = \kappa x^+(A); \quad p_{\bar{q}}^-(O) = \kappa x^-(B) \quad (2.33)$$

together with $p_q^-(O) = 0$ and $p_{\bar{q}}^+(O) = 0$. So eqs. (2.32,33) provide a simple relation between initial momenta and the length of directrix pieces, or in other words, we have a mapping "momentum space" to "real space" via

$$\Delta p = \kappa \Delta x. \quad (2.34)$$

Let us now discuss the "cutting rules" for such a yo-yo string (see fig. 3). Without interaction the string stretches between directrix $(t, y(t))$ and anti-directrix $(t, \bar{y}(t)) = (t, x(t, \pi))$. The point $B = (t, x(t, \sigma))$ be a breakpoint on the string at time t , dividing the string into two segments AB and BC with $A = (t, x(t, 0))$ and $C = (t, x(t, \pi))$. Directrix and anti-directrix corresponding to these segments are DEF with

$$\begin{aligned} D &= (t - \sigma, y(t - \sigma)) \\ F &= (t + \sigma, y(t + \sigma)) \end{aligned} \quad (2.35)$$

and GHI with

$$\begin{aligned} G &= (t - (\pi - \sigma), \bar{y}(t - (\pi - \sigma))) \\ I &= (t + (\pi - \sigma), \bar{y}(t + (\pi - \sigma))). \end{aligned} \quad (2.36)$$

Using

$$\bar{y}(t) = y(t - \pi) + P \quad (2.37)$$

Fig. 3. Breaking of a "yo-yo" string. The new directrices are constructed according to the rules of classical string dynamics: we first determine the segments of the (anti)directrix corresponding to the string pieces AB and $C'B$; these segments are then periodically continued into the future.

we verify easily that, after the appropriate shift, the segments DEF and GHI provide a full period of the unperturbed string. As discussed earlier, we obtain the directrices of the two segments after the break by continuation of DEF ($\rightarrow DEFS\dots$) and of GHI ($\rightarrow GHIN\dots$). The corresponding antidirectrices can be easily constructed from the relation between directrix y and antidirectrix $\bar{y}$:

$$\bar{y}(t) = \frac{1}{2}(y(t + \pi) + y(t - \pi)), \quad (2.38)$$

so we get $BKM\dots$ and $BQT\dots$. We realize the identities

$$\|BJ\| = \|JK\|; \quad \|HJ\| = \|JI\| \quad (2.39)$$

and

$$\|BP\| = \|PQ\|; \quad \|EP\| = \|PF\|, \quad (2.40)$$

which provide a very simple procedure to actually construct the new directrices in numerical applications.

3. String fragmentation

We discussed in the last chapter string dynamics, including the case of a breaking string. For a simple one-dimensional yo-yo string this is illustrated in fig. 3. We did not yet specify any law determining where the string breaks. This clearly goes beyond any classical treatment. However we can restrict the variety of possible breaking laws by requiring certain properties. In the same way as for the string action S (see eqs. (2.6-8)) it can be shown that the simplest local, covariant and gauge invariant breaking law can be written as

$$dP(\tau, \sigma) \sim \sqrt{-\det g} \, d\tau d\sigma \quad (3.1)$$

with dP being the probability for a break at (τ, σ) , and g being the metric (eq. (2.3)). This means that the breaking probability is proportional to the corresponding area on the string surface: $dP \sim d^2A$ or

$$dP = (1 - P) \propto d^2A \quad (3.2)$$

with the "break probability" α as a parameter. This is the fragmentation law first suggested by Artru and Mennessier [4] and later also used by other authors [5]. It is so appealing because it is not just a good guess but rather a strict consequence of requiring very plausible properties: locality, covariance and gauge invariance. Another nice feature is that there is only one parameter (α) which should be the same for so different processes as for example diquark fragmentation into baryons or heavy quark fragmentation into heavy mesons.

For the following we restrict ourselves to yo-yo strings, which form a closed group among all possible strings, in the sense that a yo-yo breaks into two yo-yo's again. In fig. 4 we show an "elementary cell" of a half period directrix OAC ("quark" q_0) and the corresponding antidirectrix $OB'C$ ("antiquark" $\bar{q}_0$). Because of eq. (2.33) the coordinates

$x_0^+ = t_A + z_A$ and $x_0^- = t_B - z_B$ are related to the initial momenta of quark (p_0^+) and antiquark ($\bar{p}_0^-$) via

$$\|OA\| = x_0^+ = \frac{p_0^+}{\kappa}; \quad \|OB\| = x_0^- = \frac{\bar{p}_0^-}{\kappa}. \quad (3.3)$$

Let us consider a break-up at D into a "quark" ($DIJ \dots$) and an "antiquark" ($DEF \dots$). We may define "break-up momenta" $b_0^\pm$ via

$$\begin{aligned} \|UD\| &= \frac{b_0^+}{\kappa}, \\ \|VD\| &= \frac{b_0^-}{\kappa}. \end{aligned} \quad (3.4)$$

With $p_1^\pm$ being the parton momenta of the right substrings at E we find

$$\begin{aligned} \|EF\| &= \frac{p_1^+}{\kappa} = \frac{p_0^+ - b_0^+}{\kappa}, \\ \|EG\| &= \frac{\bar{p}_1^-}{\kappa} = \frac{\bar{p}_0^- - b_0^-}{\kappa}. \end{aligned} \quad (3.5)$$

A corresponding formula holds for the other substrings. The area of absolute past with respect to the break-up point D is given as

$$A \equiv \frac{1}{\kappa^2} \mathcal{A} = \frac{1}{\kappa^2} b_0^+ b_0^-. \quad (3.6)$$

All points D having the same value of A lie on a hyperbola in space-time, given by

$$(t+z)(t-z) = A. \quad (3.7)$$

As the other variable to fix D completely we choose the space-time rapidity

$$\eta \equiv \frac{1}{2} \ln \frac{t+z}{t-z} = \frac{1}{2} \ln \frac{b_0^+}{b_0^-}. \quad (3.8)$$

Using these variables $\mathcal{A}$ and η eq. (3.2) becomes

$$dP = (1-P) \frac{\alpha}{\kappa^2} d\mathcal{A} d\eta$$

leading to

$$dP(\mathcal{A}) = \alpha_0 e^{-\alpha_0 \mathcal{A}} d\mathcal{A} \quad (3.9)$$

We are now in a position to exactly define, step by step, how we proceed to fragment a yo-yo string into two substrings. The proton content is completely arbitrary, we treat $q\bar{q} - q$ strings in the same way as $q - \bar{q}$ strings or even more complicated structures are

Fig. 4. Relation between (anti)directrix segments and parton momenta. In the string model the energy discontinuities (at the endpoints in the case of a yo-yo) can be identified with partons!

possible ($\bar{q} - qq\bar{q}$, $\bar{q}\bar{q} - qq\bar{q}\bar{q}$ etc.). In order to fix the break point D we first determine $\mathcal{A} = \kappa^2 A$ via integrating and inverting eq. (3.9):

$$\mathcal{A} = -\frac{1}{\alpha_0} \ln r \quad (3.10)$$

with $r \in [0, 1]$ being a random number. Before fixing D completely by determining η we have to be more specific about the break-up. We create a $q\bar{q} - \bar{q}\bar{q}$ pair with probability P_{diq} (fit parameter) and a $q - \bar{q}$ with $(1 - P_{\text{diq}})$. Concerning flavour we create a strange quark with probability P_{str} (fit parameter) and u as well d quarks with $(1 - P_{\text{str}})/2$. We then look into a resonance table (see VENUS 3.00 writeup) to determine for each substring the minimum mass m_{min} for the corresponding parton content. So the minimum mass for a $u\bar{d}$ system is the π^+ mass and so on. Suggested by the uncertainty principle we generate transverse momenta $\vec{p}_t$ and $-\vec{p}_t$ for the two partons at D , according to an exponential distribution

$$f(p_t) \sim p_t \exp \left[-\frac{p_t}{2} < p_t > \right] \quad (3.11)$$

with a fit parameter $< p_t >$ to be chosen in the order of the inverse proton size. Taking this value of p_t together with the minimum mass m_{min} we get a minimum transverse mass μ_{min} for each substring:

$$\mu_{\text{min}} = \sqrt{m_{\text{min}}^2 + p_t^2}. \quad (3.12)$$

Using for the transverse masses $\mu_{\pm}$ of the two substrings

$$\begin{aligned} \mu_+^2 &= p_0^+ b_0^- - \mathcal{A} \\ \mu_-^2 &= p_0^- b_0^+ - \mathcal{A} \end{aligned} \quad (3.13)$$

and using

$$\begin{aligned} b_0^+ &= \sqrt{\mathcal{A}} e^\eta \\ b_0^- &= \sqrt{\mathcal{A}} e^{-\eta}, \end{aligned} \quad (3.14)$$

we see that the requirement of minimum transverse masses restricts the rapidity η to be

$$\eta_- < \eta < \eta_+ \quad (3.15)$$

with

$$\eta_{\pm} = \ln \frac{\sqrt{\mathcal{A}} p_0^{\pm}}{\mu_{\pm}^2 + \mathcal{A}}; \quad \eta_- = \ln \frac{\mu_-^2 + \mathcal{A}}{\sqrt{\mathcal{A}} p_0^-}. \quad (3.16)$$

For $\eta_+ < \eta_-$ there is no solution, the string cannot be broken. Otherwise we determine the rapidity according to a constant distribution between η_- and η_+ :

$$\eta = \eta_- + \tau(\eta_+ - \eta_-) \quad (3.17)$$

with a random number $\tau \in [0, 1]$. From eq. (3.14) we see that the breakpoint is now fully determined.

This classical picture should be appropriate as long as the two substrings have a large mass. Whenever a small string mass occurs (say below 2 GeV) clearly quantum effects become important, most easily seen by the fact that a string with low mass is a hadron, and hadrons have discrete masses. Since we cannot deal with quantum strings properly, we try to correct for the quantum effect, in our classical model. First of all we introduce a cutoff parameter $m_{\text{clu}} = m_{\text{min}} + m_0$ which prevents strings with mass below m_{clu} to further split via the string fragmentation procedure. Such clusters are treated differently as we will see. We are going to discuss “exotic” quark configurations (like $\bar{q}qqq$) later, we first only consider clusters which are – concerning quark and antiquark content – hadrons. We circumvent for all but the lowest hadron states the problem of discrete mass: we allow the resonances to be off-mass-shell. For each quark configuration ($u\bar{d}, u\bar{s} \dots$) we have a table of numbers $m_1 < m_2 < \dots$ where the interval $[m_i, m_{i+1}]$ specifies the mass range for a certain resonance. This feature of resonances with continuous mass simplifies the fragmentation procedure, there is no correction needed to the breaking procedure as described above. However stable hadrons ($\Gamma \ll 1 \text{ GeV}$) have to have discrete masses, so whenever a string breaks in two pieces with at least one of these being a stable hadron (its mass falling into the lowest mass interval) we apply a correction procedure: we construct a new breakpoint D . We recall that D is specified by two parameters: the area $\mathcal{A} = b_0^+ b_0^-$ and the rapidity $\eta = \frac{1}{2} \ln(b_0^+/b_0^-)$. Since we do not want to modify the area law eq. (3.2), we are going to modify η and leave $\mathcal{A}$ fixed, if one stable hadron is involved (two hadrons will be treated later). So when the right substring is a stable hadron, with a required mass m we determine η to be (see eq. (3.16)):

$$\eta = \ln \frac{\sqrt{\mathcal{A}} p_0^+}{m^2 + p_t^2 + \mathcal{A}}; \quad (3.18)$$

if the hadron is left, we use

$$\eta = \ln \frac{m^2 + p_t^2 + \mathcal{A}}{\sqrt{\mathcal{A}} p_0^-}. \quad (3.19)$$

This guarantees the correct mass. If both substrings are stable hadrons, with masses m_+ and m_- , we have to redetermine both parameters, $\mathcal{A}$ and η , by solving the two equations (using $\mu_{\pm} = \sqrt{m_{\pm}^2 + p_t^2}$ and eqs. (3.13,14)):

$$\begin{aligned} \mu_+^2 &= p_0^+ \sqrt{\mathcal{A}} e^{-\eta} - \mathcal{A} \\ \mu_-^2 &= p_0^- \sqrt{\mathcal{A}} e^{\eta} - \mathcal{A} \end{aligned} \quad (3.20)$$

which leads to

$$\mathcal{A} = \frac{1}{2} (p_0^+ p_0^- - \mu_-^2 - \mu_+^2) - \sqrt{\frac{1}{4} (p_0^+ p_0^- - \mu_-^2 - \mu_+^2)^2 - \mu_-^2 \mu_+^2}. \quad (3.21)$$

The other parameter η is determined from eqs. (3.18) or (3.19). Because the mass of the hadron before correction is close to the real mass (= mass after correction), the new breakpoint D is in the vicinity of the old breakpoint, so it is really a correction in the sense of a small modification.

We want to stress that the break-up of a string into two substrings occurs completely arbitrary in the sense that each of the substrings may be a stable hadron, a resonance or a high mass string. This is the major difference to the Lund model, where one fragment has to be a hadron with a discrete mass. So in our model we have a “tree structure”: a string decays into two substrings, each substring may then decay into two substrings and so on. The Lund model has a “salami structure”: a hadron is chopped off at the end, then another one from the remaining string and so on.

The last step of the fragmentation procedure is resonance decay: all the primary hadrons (from string break-up) decay (if they are unstable) according to standard branching ratios. The off-shellness of resonances poses no difficulties. One has only to consider that some of the partial decays cannot occur because the energy is not available; we simply discard such decay modes. For details of the decay procedure see Ref. [6].

Exotic clusters are treated in a procedure called “cluster decay”, to be distinguished from ordinary “resonance decay” as well as from “string splitting”. Consider a cluster C of n quarks and m antiquarks with $n - m$ being an integer multiple of 3

$$C = (q_1 q_2 \dots q_n \bar{q}_1 \bar{q}_2 \dots \bar{q}_m) \quad (3.22)$$

where $q_i, \bar{q}_i$ are quark flavours ($u, d, s, \dots$). The probabilities to randomly choose a baryon (B), an antibaryon ($\bar{B}$), or a meson (M) are

$$P(B) = \frac{1}{N} \binom{n}{3}; \quad P(\bar{B}) = \frac{1}{N} \binom{m}{3}; \quad P(M) = \frac{1}{N} nm \quad (3.23)$$

with

$$N = \binom{n}{3} + \binom{m}{3} + nm. \quad (3.24)$$

According to these probabilities we are randomly selecting a hadron H , and then performing the decay

$$C \rightarrow H + C'. \quad (3.25)$$

With m_C being the cluster mass and m_H and $m_{C'}$ being the masses of the two decay products, we find the available centre-of-mass momentum to be

$$p_{cm} = \frac{1}{2m_C} \sqrt{(m_C^2 - m_H^2 - m_{C'}^2)^2 - (2 m_H m_{C'})^2}. \quad (3.26)$$

The momenta of the two decay products in the cm system of the cluster C are then $\pm p_{cm} \vec{u}$, with a random unit vector $\vec{u} \in S_2$. The mass m_H is a discrete hadron mass, $m_{C'}$ is either a hadron mass or - if C' is still an exotic state - a mass chosen randomly in the possible mass range. In the latter case, the cluster decay procedure is repeated for C' :

$$C' \rightarrow H' + C'' \quad (3.27)$$

and so on, till we are left just with ordinary hadrons.

Before coming to applications, we want to state some general remarks about our fragmentation model. A major motivation for keeping resonances off-shell is the fact that this fragmentation model was mainly constructed to be used in a model for hadronic interactions (VENUS 3). In particular for such reactions, interactions of produced resonances of the type

$$R_1 + R_2 \rightarrow R \rightarrow R_3 + R_4 + \dots \quad (3.28)$$

may occur, i.e. the two resonances fuse into a high excited resonance R before decaying again. Even if R_1 and R_2 were on-mass-shell, the fused object R will not in general, so one has to deal with off-shell resonances anyhow. If only discrete masses are considered, one has to correct again and again, which makes the whole approach questionable. Interactions of the type (3.28) are included in the model, however completely negligible for all examples to be discussed in the next chapter, therefore we do not discuss rescattering here [7].

Also with view to applications for hadronic collisions we included “exotic clusters”. Such objects are already needed for deep inelastic lepton nucleon scattering: in the simplest case the vector boson couples to one of the valence quarks of the nucleon to produce a diquark-quark string (see fig. 9). However it also happens that the boson kicks off a sea quark, leaving back in the remainder nucleon the corresponding antiquark, and thus we get an $q - q\bar{q}q\bar{q}$ string. After fragmentation we might be left with an exotic $\bar{q}q\bar{q}q$ system. Considering rescattering of resonances, we have to deal with exotics as well: by fusing for example an uuu and an $u\bar{s}$ we get the five quark state $uuu\bar{u}s$.

We finally would like to comment on the relation of our model to other fragmentation models. Such models may be classified according to symmetry properties: our model, as well as some others [5] based on the Artru-Mennessier model, takes the string picture seriously and provide a covariant, gauge invariant (= reparametrization invariant), energy and momentum conserving string breaking procedure. On the other extreme there is the Field-Feynman model. Instead of strings here one considers two independent partons moving into opposite directions, inspired by the experimentally observed jet-structure of produced particles. The two partons have to be considered in a certain frame, so the model is not covariant and of course not gauge invariant. And since the two jets are independent, energy and momentum are not conserved. The Lund model is somewhat in between. It is very similar to the Field-Feynman model in the sense that again two partons are considered, and the fragmentation law is formulated in terms of the momentum loss of a parton. However the two partons are linked by a colour field, which makes it possible to achieve energy and momentum conservation as well as covariance. However the string picture is not explicitly used, so it is not possible to apply the principle of gauge invariance, which leads to the derivation (!) of the simple breaking law eq. (3.2) in our model.

Unlike the other fragmentation models, this model does not bother about perturbative parton showers, since we are not interested in the fragmentation involving very high mass partons created in hard scattering processes. We are mainly interested in strings formed in soft hadronic collisions which might be considered to be unexcited yo-yo strings, without any gluon kinks. Nevertheless we are testing the model with e^+e^- and lepton-nucleon data up to quite high energies, however, expecting deviations above some energy.

4. Inclusive spectra

We applied the model (referred to as VENUS 3) to calculate particle production in e^+e^- , μp and $\bar{\nu} p$ reactions. An e^+e^- event $\{e^+e^-\}$ is related to quark-antiquark string fragmentation as a simple superposition (see fig. 5):

$$\{e^+e^-\} = \frac{1}{\sum_i e_i^2} \sum_i e_i^2 \{q_i - \bar{q}_i\} \quad (4.1)$$

where the weights are determined from the quark charges. The e^+e^- energy determines which flavours have to be considered. In the following figures we use the convention: data are dots, model results are histograms. In fig. 6 we consider inclusive spectra for e^+e^- at 14 GeV: data and model agree almost perfectly for the distributions of transverse momentum p_t of charged particles, rapidity y of charged particles and the energy fraction x of photons. The transverse momentum shows essentially the exponential behaviour of the input distribution eq. (3.11), however quite remarkable is the correct concave shape of the photon x -distribution. Photon spectra are strongly affected by η resonance decay, we may consider this as some support of our low mass cluster treatment (concerning the production of flavour neutral resonances, see [6]). The deviation from an ideal plateau in the rapidity distribution in fig. 6, namely the shallow peak at large $|y|$, is due to the decay of heavy mesons: there are plenty of $c - \bar{c}$ strings, forming $q\bar{c}$ and $\bar{c}q$ mesons on the outside of the string, therefore we get a contribution at large rapidities. In figs. 7,8 we show the same distributions, however for higher energies: 22 and 34 GeV. The photon energy fraction distributions are energy independent (scaling), data and model agree nicely. The transverse momentum distribution in the model is almost energy independent, whereas the data show an increase of the p_t tails with energy. This effect is well known to be due to a perturbative parton cascade, not included in our model. By looking at the rapidity distributions we see that these missing high p_t particles are located at central rapidities. The tails of the rapidity distributions are always reproduced properly.

We are now turning to antineutrino proton scattering: $\bar{\nu} + p \rightarrow \mu^+ + \text{"string"}$, see fig. 9. Considering only the contributions with $\Delta s = 0$, we have two possibilities: The W^- boson couples to a u quark which transforms into a d quark, forming a $d - ud$ string together with the remainder diquark $p - u = ud$ (see fig. 9); the other case occurs when the W^- couples to a d quark from the sea, to be transformed into a $\bar{u}$ quark, forming correspondingly a $\bar{u} - uudd$ string. For the experiments we are studying, the kinematic cuts are such that the latter contribution can be neglected, so $\bar{\nu} p$ scattering is simply $d - ud$ string fragmentation:

$$\{\bar{\nu} p\} = \{d - ud\} \quad (4.2)$$

In fig. 10 we show distributions of the longitudinal momentum fraction x for $\bar{\nu} p$ scattering at 6.2 GeV ($d - ud$ string fragmentation with a string mass of 6.2 GeV). Momentum fraction refers to the maximum possible momentum of a produced particle in the string cm system, along the longitudinal axis (experimentally the string momentum can be reconstructed since in the reaction $\bar{\nu} + p \rightarrow \mu^+ + \text{"string"}$ the momenta of $\bar{\nu}$ and μ^+ are known). The data can be at least equally well reproduced as with the former VENUS fragmentation routine

Fig. 5. String formation in e^+e^- annihilation.

Fig. 6. Inclusive spectra for e^+e^- annihilation at 14 GeV: transverse momentum (upper plot) and rapidity distributions (middle plot) of charged particles, and energy fraction distributions of photons (lower plot). The data (dots) are from ref. [10].

Fig. 7. Same as fig. 6, but 22 GeV.

Fig. 8. Same as fig. 6, but 34 GeV.

Fig. 9. String formation in $\bar{\nu}p$ scattering.

Fig. 10. Longitudinal momentum distribution for π^+ (upper plot) and π^- (lower plot) for $\bar{p}p$ scattering at 6.2 GeV (string mass). The dashed curve is a calculation including string-string interactions (no effect). The data (dots) are from ref. [11].

(Field-Feynman procedure, see [8]), although now much fewer parameters enter. The distributions in fig. 10 are only affected by one parameter: the string breaking probability α in eq. (3.2). Since on the diquark side (negative x) the first particle has to contain the diquark, being a baryon usually, the pions are at least second in the chain, and therefore most likely slower than the baryon. On the other side, where the quark fragments (positive x), a pion is usually first and fastest. For these reasons we get the forward-backward asymmetry for the π^- , seen in fig. 10. The dashed line contains final state interactions, not showing any effect.

We finally turn to muon proton scattering ($\mu + p \rightarrow \mu' + \text{"string"}$, see fig. 11). The intermediate boson couples to either a quark or an antiquark, forming a $q - (p - q)$ or $\bar{q} - (p - \bar{q})$ string. The probability of a certain quark (antiquark) flavour i being involved is [9]:

$$P_i = N \int dx_B dy \frac{2\pi\alpha^2}{(-q^2)^2} s x_B(1 + (1 - y)^2) e_i^2 q_i(x_B) \quad (4.3)$$

where the usual variables are used:

$$x_B \equiv \frac{-q^2}{2pq}, \quad y \equiv \frac{pq}{pk} \quad (4.4)$$

with k, q and p being the four momenta of the incoming muon, the photon and the proton; $q_i(x_B)$ is the parton momentum distribution function, e_i is the parton charge, and N is the normalization. The integration of (4.3) has to be performed over the appropriate acceptance area in the $x_B - y$ plane, in order to compare with a specific experiment. Since the quark flavour i in eq. (4.3) may refer to valence quarks, sea quarks or sea antiquarks, we get contributions from $q - qq$, $q - qqq$ and $\bar{q} - qqq$ strings. The weights P used for the following figures, are given in table 1.

string	P [%]
$u - ud$	67.66
$d - uu$	8.46
$u - \bar{u}ud$	8.80
$d - \bar{d}ud$	2.20
$s - \bar{s}ud$	0.94
$\bar{u} - uud$	8.80
$\bar{d} - dud$	2.20
$\bar{s} - sud$	0.94

Table 1: The weights of individual string contributions for the $\mu - p$ reactions discussed in this paper.

For qualitative arguments it is important to notice, that quark-diquark ($q - qq$) strings are dominant, and among those $u - ud$ strings are most likely. We first consider μp scattering

at 11.4 GeV. In fig. 12 we show longitudinal momentum distributions of pions and kaons. Again we see the strong forward-backward asymmetry for π^+ and K^+ , since for $u - ud$ strings only at the forward end of the string, π^+ and K^+ can be produced at the end. The pion distributions are steeper than the kaon distributions which is obvious after writing eq. (3.9) for leading particles as

$$dP(x, m^2) = \frac{\alpha_0}{\Delta\eta} \exp \left[-\alpha_0 m^2 \frac{1-x}{x} \right] \frac{dx}{x} dm^2 \quad (4.5)$$

where x is the momentum fraction and m the mass of the particle. We see from eq. (4.5) that heavier particles have flatter distributions. In fig. 13 momentum fraction distributions of baryons are shown: protons antiprotons, lambdas and antilambdas. We again observe forward-backward asymmetries, which may be summarized as follows: the more the quark content of a produced hadron and a string end differ, the faster the distribution approaches zero for $|x| \rightarrow 1$. The reason is that for large differences in the quark content, many other particles in the chain of produced hadrons are closer to the string end. Since the diquark on the minus side differs by just one quark from a proton or a lambda, these distributions are rather flat for $x < 0$. The quark on the other side differs by two quarks from p and Λ , so the distributions fall faster towards $x \rightarrow 1$. The difference between the forward quark and $\bar{p}$ or $\bar{\Lambda}$ is 4 quarks, so the distributions fall faster than for p and Λ for $x \rightarrow 1$. The largest difference in quark content is, however, observed for the diquark side ($x < 0$) and for $\bar{p}, \bar{\Lambda}$ production: the difference of 5 quarks results in a very fast drop for $x \rightarrow -1$. In the former VENUS fragmentation (Field-Feynman) these quark counting arguments were used as an input, by using different splitting functions for different particle species, depending on the difference in quark content. Now we get this behaviour for free, just from the relation between quark content and the number of steps from an endpoint to produce a particle. To avoid confusion: the fragmentation procedure in VENUS 3 breaks the string at arbitrary points, not successively from an endpoint. Nevertheless we can analyse results by considering a chain of produced particles from one string end to the other.

An alternative variable, stretching the central region and compressing the fragmentation region, is the rapidity

$$y = \frac{1}{2} \ln \frac{E + P_{\parallel}}{E - P_{\parallel}} \quad (5.6)$$

where E is the energy and $P_{\parallel}$ the longitudinal momentum of a produced particle. In figs. 14 and 15 we display rapidity distributions of π^+ and π^- in μp reactions at string energies of 7 GeV, 12.1 GeV, 15.7 GeV and 18.7 GeV. Whereas the height of the distributions ($y(0)$) remains almost invariant with energy, the width increases clearly. So we observe a rapidity plateau, expected from Lorentz invariance. There is more π^+ than π^- production, since there are more u than d quarks in the string.

Fig. 11. String formation in μp scattering.

Fig. 12. Longitudinal momentum distributions of pions and kaons for μp scattering at 11.4 GeV (string energy). The data (dots) are from ref. [12].

Fig. 13. Longitudinal momentum distributions of protons, antiprotons, lambdas and antilambdas for μp scattering at 11.4 GeV (string energy). The data (dots) are from ref. [12].

Fig. 14. Rapidity distributions of π^- for μp scattering at 7.0 GeV, 12.1 GeV, 15.7 GeV and 18.7 GeV (string energy). The data (dots) are from ref. [13].

Fig. 15. Rapidity distributions of π^+ for μp scattering at 7.0 GeV, 12.1 GeV, 15.7 GeV and 18.7 GeV (string energy). The data (dots) are from ref. [13].

5. Some remarks about the code

The code VENUS 3.00 (available as a single fortran file VENUS300 FORTRAN from WERNERK at CERNVM) is mainly constructed and documented to simulate hadron-hadron, hadron-nucleus or nucleus-nucleus collisions. However there is an option to simulate pure string fragmentation (used in this paper). For a superposition of different strings the weights and the flavour content of the endpoints has to be provided as input, this is not done automatically in the code. The code contains much more than described in this paper, like string-string interactions and string-spectator interactions, so one can treat for example lepton-nucleon scattering. This is discussed in a separate publication.

6. Conclusion

We presented a string fragmentation model where both string evolution and string breaking occurs consistently according to classical string theory. The main requirements are locality, covariance and gauge invariance (as well as minimality, since there are more complicated solutions of those three requirements). Gross features of the results are determined by just one parameter, the break probability α (see eq. (3.2)). In view of later applications for soft hadronic collisions, we not only treat $q - \bar{q}$ strings but also $q - qq$ strings and more exotic strings like $\bar{q} - qqq$. We consider resonances with continuous masses, only the stable hadrons are produced with mass shell constraint, requiring only a little correction procedure. We investigate e^+e^- data, and – most important with view to hadronic collisions – lepton-nucleon data, the latter ones probing quark-diquark strings. A publication describing string-string interactions is under preparation.

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