FLAVOUR DEMOCRACY AND THE LEPTON-QUARK HIERARCHY ^{*)}

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ABSTRACT

The mass hierarchy of the leptons and quarks is interpreted as a consequence of a coherent state phenomenon ("flavour democracy"). It is emphasized that particular forms of the mass matrices can arise from the coherent state basis. The violations of the "flavour democracy" turn out to be relatively large. Numerical examples are presented.

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In the standard electroweak model the masses of the leptons and quarks as well as the weak mixing angles enter as free parameters. Any further insight into the yet unknown dynamics which is responsible for the spectra of leptons and quarks and their mixing properties would be welcome as a step beyond the physics of the standard model.

A closer inspection of the mass spectra tells us that they are almost entirely dominated by the mass of the member of the third family. The spectra exhibit clearly a hierarchical pattern (see Fig. (1)): The masses of the members of the second family are small compared to those of the third one, and the members of the first family are light in comparison to the corresponding members of the second family.

Furthermore another hierarchical pattern emerges if we consider the weak interaction mixing parameters. Due to the observed smallness of the mixing angles the associated mixing matrix, if written in terms of mass eigenstates, is close to the unit matrix.

It has been emphasized some time ago^{1,2)} that the observed two hierarchies indicate the relevance of a specific symmetry. Nature seems to be somewhat close to the "rank one" limit, i.e. a limit in which all mixing angles vanish and the mass matrices of the $T_3 = +1/2$ sector and of the $T_3 = -1/2$ sector are proportional to the matrix

$$M_0 = \text{const.} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (1)$$

Thus the masses of the first two families vanish together with the mixing angles. It depends on yet unknown details of the mechanism which is responsible for the mass generation, whether such a limit can be achieved in a consistent way. We simply assume that this is the case, encouraged by the observed mass and mixing pattern.

We note that in this limit a specific basis in the three-dimensional unitary space of the three lepton-quark generations is singled out. Furthermore there exists a mass gap: The third generation is split in mass from the first two generations.

A mass matrix of the type given in eq. (1) can be obtained from another simple matrix $\tilde{M}$ by a unitary transformation:

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = U \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} U^{-1} = U \tilde{M} U^{-1} \quad (2)$$

Matrices of this type ("democratic mass matrices") have been considered in the past by a number of authors³⁻⁸⁾

More generally, any 3×3 matrix M of rank one can be written as a product

$$M = \text{const.} h_i h_k^* \quad (3)$$

where h_i ($i = 1, 2, 3$) is a 3-vector.

For simplicity we can assume that the components h_i are real, since in the absence of the masses of the first two generations the general matrix M can always be made real and symmetric by suitable unitary transformations of the fermion fields.

The two mass matrices given in eq. (2) are special forms of the general representation (3). If we choose $(h_i) = \text{const.} (0, 0, 1)$, we obtain the matrix (1). If $(h_i) = \text{const.} (1, 1, 1)$, the "democratic" mass matrix follows. We shall denote the latter basis as the "coherent state" basis.

In this paper we shall not discuss the dynamical origin of the rank one limit. It has been suggested that such a limit could be the result of a specific breaking of chiral symmetries⁹⁾, the consequence of a BCS like "pairing force"^{10,11)}, or the result of radiative corrections involving the mass terms of heavy fermions⁶⁾. Instead we shall discuss the pattern of mass generation for the first two generations in the coherent state basis. It will turn out that a particular "ansatz" for the quark and lepton mass matrices discussed by one of us long ago^{1,12)} is, in fact, suitable to describe the pattern of symmetry breaking in the coherent state basis.

In the coherent state basis the rank one mass matrix is a matrix with equal matrix elements. It has a symmetry $S(3)_L \times S(3)_R$, where $S(3)$ denotes the group of permutations of three elements. As emphasized above, the phenomenon of weak interaction mixing is supposed to be absent in this limit. Since the u-quark sector and the d-quark sector have the same mass structure, the symmetry of the mass term is

$$[S(3)_L \times S(3)_R]_U \times [S(3)_L \times S(3)_R]_D \times SU(2)_L \times SU(2)_R,$$

where $SU(2)_L \times SU(2)_R$ denotes the chiral group acting among the U-D flavor indices.

We should like to differentiate between the coherent state basis and the mass eigenstate basis of the rank 1 limit, which we denote as the h-basis (h referring to heavy, since in that basis the heavy lepton - quark states, i.e. τ^0 , b^0 , t^0 , are eigenstates, together with the other massless fermions). The connection between the two systems is given by

$$\begin{aligned} \begin{bmatrix} e^0 \\ \mu^0 \\ \tau^0 \end{bmatrix} &= U \begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix} \\ \begin{bmatrix} u^0 \\ c^0 \\ t^0 \end{bmatrix} &= U \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \\ \begin{bmatrix} d^0 \\ s^0 \\ b^0 \end{bmatrix} &= U \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} \end{aligned} \quad (4)$$

$$U = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix}$$

Here l_1 , l_2 and l_3 are the lepton-states in the coherent state basis, and e^0 , μ^0 , τ^0 the states in the h-basis. The superscript denotes that these states are not the physical states (with their physical masses), but the corresponding states in the rank 1 limit, in which both e^0 and μ^0 are massless. Due to this fact the $e^0 - \mu^0$ directions in the h-space are not uniquely defined. One could just as well choose

another basis, in which e^0 and μ^0 are rotated into each other by an arbitrary angle θ . This arbitrariness will disappear, however, once the symmetry breaking is introduced. Similar remarks are valid for the U- and D-quark systems.

In order to obtain a realistic mass and mixing pattern, the permutation symmetry must be broken. Thus breaking should be such that the observed hierarchy $m_u \ll m_c$, $m_d \ll m_s$ is reproduced in a natural way. Furthermore one should aim to involve as few extra parameters as possible and to obtain relations between the mixing angles and mass eigenvalues.

We shall proceed as follows. First the $S(3)_L \times S(3)_R$ symmetry is broken down to $S(2)_L \times S(2)_R$. This symmetry breaking keeps the first family massless. Finally the $S(2)_L \times S(2)_R$ symmetry is broken, and all three families are massive:

$$\begin{aligned} S(3)_L \times S(3)_R \\ \downarrow \\ S(2)_L \times S(2)_R \\ \downarrow \\ ./. \end{aligned}$$

It is important to note that the symmetry breaking is arranged such that the breaking term has the same structure for the U- and D-type quarks.

The first stage of the symmetry breaking is achieved by adding a breaking term which violates the $S(3)$ -symmetry, but keeps the $S(2)$ -symmetry intact. One way to describe the resulting (Hermitian) mass matrix is as follows:

$$M = \text{const.} \begin{bmatrix} 1 & 1 & 1+\alpha \\ 1 & 1 & 1+\alpha \\ 1+\alpha^* & 1+\alpha^* & 1+\beta \end{bmatrix} \quad (5)$$

(α complex parameter, β real parameter, the leptonic and the U- and D-Matrix are described by different parameters α , β).

In the h-basis the mass matrix can easily be worked out. One finds:

$$M = \text{const.} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{2}{3}(-\alpha - \alpha^* + \beta) & -\frac{\sqrt{2}}{3}(-\alpha + 2\alpha^* + \beta) \\ 0 & -\frac{\sqrt{2}}{3}(2\alpha - \alpha^* + \beta) & 3 + \frac{1}{3}(2\alpha + 2\alpha^* + \beta) \end{pmatrix} \quad (6)$$

As expected, this matrix describes one massless state, a light state and a heavy state. A further diagonalization which will not be carried out here would lead to the corresponding mass eigenstates.

We should like to emphasize that the mass matrix denoted above has the same structure both for the charged leptons and for the U- and D-quarks, due to the special nature of the symmetry breaking. This does not lead to a particular constraint for the leptons, but it implies for the quarks that in this limit (rank 2 limit) the massless u- and d-quarks remain unmixed with respect to the weak interactions, i.e. the Cabibbo angle vanishes.

A special case deserves to be discussed, namely the case $\beta = \alpha + \alpha^*$. In this case the (2,2) - matrix element in eq. (6) vanishes, and we are led to the particular structure of the mass matrix discussed by one of us in ref. (1). In this case a relation between the mass eigenvalues and the weak interaction mixing angle follows, which is not in disagreement with the experimental data and constraints on the $t - m$ mass.

It is well known that the algebraic structure of the mass matrix given in ref. (1) can be derived in particular schemes of the spontaneous symmetry breaking involving several representations of "Higgs" scalars^{1,12,13}. Here we find that this structure appears, if in the coherent state system the special case $\beta = \alpha + \alpha^*$ is adopted. It remains to be seen whether this case could be derived directly from a particular ansatz of the symmetry breaking.

One interesting possibility is to associate the breaking of the permutation symmetry with effects of radiative corrections. The β -term appears in the diagonal (3,3)-element, while the α -term shows up in the non-diagonal elements (i, 3) or (3, k) (i, k: 1,2). In a second order radiative correction one would expect the associated self-energies to factorize in lowest order, i.e. to the orders $O(\alpha)$, $O(\beta)$ ^{6,14}. No second non-zero eigenvalue of the mass matrix is obtained to this order, since a factorizable 3 x 3 matrix has still rank 1. Correspondingly one has $\beta = \alpha + \alpha^* + O(\alpha^2)$.

Thus in dynamical schemes based on radiative corrections, e.g. by involving heavy fermions as discussed in ref. (6) the constraint $\beta = \alpha + \alpha^*$ should be valid in a good approximation.

We proceed to the next stage of symmetry breaking which we discuss only for the special case $\beta = \alpha + \alpha^*$. The breaking of the $S(2)_L \times S(2)_R$ symmetry can be described by a complex parameter γ as follows:

$$\begin{pmatrix} 1-(\gamma+\gamma^*) & 1 & 1+\alpha+\gamma \\ 1 & 1+(\gamma+\gamma^*) & 1+\alpha-\gamma \\ 1+\alpha^*+\gamma^* & 1+\alpha^*-\gamma^* & 1+\alpha+\alpha^* \end{pmatrix} \quad (7)$$

Transforming this matrix to the h-basis, one finds

$$\begin{pmatrix} 0 & -\sqrt{3}(\gamma+\frac{1}{3}\gamma^*) & -\frac{\sqrt{6}}{3}\gamma^* \\ -\sqrt{3}(\frac{1}{3}\gamma+\gamma^*) & 0 & -\sqrt{2}\alpha^* \\ -\frac{\sqrt{6}}{3}\gamma & -\sqrt{2}\alpha & 3+\alpha+\alpha^* \end{pmatrix}, \quad (8)$$

i.e. one finds a structure of the mass matrix which is close to the one given in ref. (1).

It is well known¹³, that this structure is able to reproduce the observed mixing and mass pattern, which suggests that the "true" mass matrices of the leptons and quarks are probably not far from such a pattern. Thus a simple way to study the actual violations of the $S(3) \times S(3)$ symmetry would be to start from the pattern of ref. (1) and deduce the mass matrices in the coherent state basis. One finds

$$\begin{pmatrix} 0 & A & 0 \\ A & 0 & B \\ 0 & B & C \end{pmatrix} \rightarrow \frac{C}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{B}{\sqrt{18}} \begin{pmatrix} 2 & 2 & -1 \\ 2 & 2 & -1 \\ -1 & -1 & -4 \end{pmatrix} \quad (9)$$

$$+ \frac{A}{\sqrt{3}} \begin{pmatrix} 1 & 0 & -1 \\ 0 & -1 & 1 \\ -1 & 1 & 0 \end{pmatrix}$$

$$= \begin{bmatrix} \frac{C}{3} + \frac{2B}{\sqrt{18}} + \frac{A}{\sqrt{3}} & \frac{C}{3} + \frac{2B}{\sqrt{18}} & \frac{C}{3} - \frac{B}{\sqrt{18}} - \frac{A}{\sqrt{3}} \\ \frac{C}{3} + \frac{2B}{\sqrt{18}} & \frac{C}{3} + \frac{2B}{\sqrt{18}} - \frac{A}{\sqrt{3}} & \frac{C}{3} - \frac{B}{\sqrt{18}} + \frac{A}{\sqrt{3}} \\ \frac{C}{3} - \frac{B}{\sqrt{18}} - \frac{A}{\sqrt{3}} & \frac{C}{3} - \frac{B}{\sqrt{18}} + \frac{A}{\sqrt{3}} & \frac{C}{3} - \frac{4B}{\sqrt{18}} \end{bmatrix}$$

The parameters A, B, C are taken to be real. Possible phases have already been absorbed in the definition of the weak currents.

The matrices multiplying A and B are, of course, traceless. The matrix multiplied by B respects the $S(2) \times S(2)$ symmetry.

Using eq. (9), we can study the violations of the $S(3)_L \times S(3)_R$ -symmetry.

As an example we use the quark mass values $m_u = 5.1$ MeV, $m_d = 8.9$ MeV, $m_s = 145$ MeV, $m_c = 1.36$ GeV, $m_b = 5.7$ GeV (these are the central values, given e.g. in ref. (15) — these masses are normalized at an energy scale of $\mu = 1$ GeV). For m_l we choose the two values:

- a) m_l (1 GeV) = 130 GeV (this corresponds to $m_l(m_l) \approx 80$ GeV)
- b) m_l (1 GeV) = 200 GeV (this corresponds to $m_l(m_l) \approx 120$ GeV).

The resulting mass matrices (in GeV units) in the coherent state basis are:

$$M_{\text{lepton}} = 0.595 \begin{bmatrix} 1.35 & 1.35 & 0.82 \\ \sim & 1.34 & 0.84 \\ \sim & \sim & 0.31 \end{bmatrix} \quad (10)$$

$$M_D = 1.9 \begin{bmatrix} 1.24 & 1.23 & 0.88 \\ \sim & 1.22 & 0.90 \\ \sim & \sim & 0.56 \end{bmatrix}$$

$$a) M_U = 43.3 \begin{bmatrix} 1.15 & 1.15 & 0.93 \\ \sim & 1.14 & 0.93 \\ & & 0.711 \end{bmatrix}$$

$$b) M_U = \frac{200}{3} \cdot \begin{bmatrix} 1.12 & 1.12 & 0.94 \\ \sim & 1.12 & 1.12 \\ \sim & \sim & 0.77 \end{bmatrix}$$

Considering the various mass matrices given above it becomes evident that there is a fairly large breaking of the $S(3)_L \times S(3)_R$ symmetry. The largest deviations from one occur in the main diagonal. In our approach the breaking terms are much larger than discussed in ref. (10). However, there no particular pattern is obtained in the h-basis, while we obtain the ansatz of ref. (1) in this basis, which would allow to derive relations and constraints for the mixing angles in terms of the mass eigenvalues.

The main purpose of this investigation has been to see whether one could find a way to understand the departure from the simple "flavor democratic" mass matrix in terms of particular corrections setting constraints for the mixing angles. This is possible; however, one must be prepared to allow for fairly large correction terms (of the order of 30 % and more). It remains to be seen whether such corrections can be obtained as radiative corrections in specific field theoretic models.

One is invited to speculate whether the symmetry "flavor democracy" (and the associated coherent state basis) as well as its breaking have a deeper physical meaning. In the past the appearance of a mass gap phenomenon (here the dominance of the lepton-quark mass spectrum by the member of the third family) has been successfully understood by the introduction of coherent states, which turned out to be the "simple" states in a specific dynamical sense. In the BCS theory of superconductivity the "coherent states" are given by correlated pairs of electrons, which are mixed by the long range BCS force. In QCD the pseudoscalar neutral mesons give a similar example. The mass eigenstates are highly mixed, while the coherent states are simply the "quark model eigenstates" $\bar{u}u, \bar{d}d$ or $\bar{s}s$.

Thus one could speculate that the coherent states of leptons and quarks, e.g. the state $l_3 = \frac{1}{\sqrt{3}}(e + \mu + \tau)$, is "simpler" in a dynamical sense than a mass eigenstate.

It seems to us that the best way to accommodate such a notion is in an approach, in which the leptons and quarks are considered as bound states. But there could be other dynamical schemes as well. In any case the coherent state basis of the leptons and quarks should be kept in mind as a starting point for exploring the yet unknown dynamics which is behind the phenomenon of mass generation and flavor mixing for the quarks and leptons.

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Fig. 1. The lepton - quark mass spectrum on a logarithmic scale. The generation index is used as the x-coordinat. In the figure the top-mass is assumed to be of the order of 100 GeV, the electron mass is not shown.