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An anomalous transformation law in supergravity models

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Abstract

We demonstrate the existence of an anomalous supersymmetric law of a composite operator in models of supergravity coupled to at least one chiral supermultiplet. After giving two different methods to prove this result, we briefly comment on its possible applications.

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In supersymmetric gauge theories, like SQCD, there exists an anomalous transformation law of the composite operator $\bar{\chi}z[1]$

$$\bar{\delta}(\bar{\chi}z) = (g^2/32\pi^2)\lambda\lambda, \quad (1)$$

where λ is the gaugino field, χ is the fermionic and z is the scalar part of the matter superfield Φ belonging to the N representation of $SU(N)$. Equation (1) is the $\theta=0$ component of the following equation

$$1/16 \bar{D}^2 \bar{\Phi} e^\nu \Phi = m\Phi\Psi + g^2/(32\pi^2)W^\nu W_\nu, \quad (2)$$

where W is a chiral superfield containing in its lowest component the gaugino field and Ψ is the chiral superfield transforming as the $\bar{N}$ representation of $SU(N)$. This transformation law provides very useful and exact relations between various condensates and play a role in determining whether supersymmetry can be spontaneously broken [2].

An analogous transformation law exists also in $N=1$ supergravity models coupled to at least one chiral multiplet and reads [3]

$$\bar{\delta}(\bar{\chi}z) = (1/384\pi^2)\psi_{ab}\psi_{ab}, \quad (3)$$

where $\psi_{ab}\psi_{ab} = e_\mu^\alpha e_\mu^\nu (D_\mu\psi_\nu - \mu \leftrightarrow \nu)$, e_μ^α is the vierbein field and ψ_μ is the gravitino field.

We recall that the model under consideration has a chiral $U(1)$ symmetry that transforms the matter fields as

$$(z, \chi) \rightarrow \exp(i\alpha)(z, \chi) \quad (4)$$

provided the superpotential is zero. The chiral current associated associated with this symmetry is anomalous and satisfies

$$D_\mu J^\mu = -(1/384\pi^2) * R_{abcd} R^{abcd}, \quad (5)$$

where $*R_{abcd} = 1/2 \epsilon^{abcd} R_{def}$. As a consequence of supersymmetry, equation (5) is related to equation (3) and to two other anomalous equations [3]. Of course the existence of an anomalous multiplet in supergravity is well known, but the relation of the first component of this multiplet to an anomalous transformation law and its relevance in the problem of supersymmetry breaking has been only recently examined.

In the following we will prove equation (3) using two different approaches, a diagrammatic one and the heat-kernel method in superspace, and we will comment on its applications for dynamical supersymmetry breaking in $N=1$ supergravity models.

The simplest way to show that equation (3) is correct is to observe that the term of lowest order in $\psi_{ab}\psi_{ab}$ is given by $(\partial_\mu\psi_\nu - \mu \leftrightarrow \nu)$ and we will show that this term can be obtained by calculating the relevant one-loop diagrams.

The origin of the anomalous behaviour is related to the definition of a composite operator. In fact we have to regularize it in a way compatible with the symmetries of the theory. We will perform this calculation by choosing the Pauli-Villars regularization. If we transform the composite field $\bar{\chi}z$, we obtain

$$\bar{\delta}(\bar{\chi}z) = -mz^2. \quad (6)$$

The Green functions, with the insertion of the transformed operator $-mz^2$, are not well defined. In order to define them we must also consider the insertion of $M_\tau z_\tau^2$ where z_τ is the regulator field with mass M_τ and take the limit $M_\tau \rightarrow \infty$ at the end of the calculation. In general this new contribution can be disregarded unless there is another factor in the Green function which behaves like $1/M_\tau$.

We write the terms in the Lagrangian which are relevant to the insertion of this operator

$$L = L_0 + L_1 \quad (7)$$

where

$$L_0 = -1/2(\partial_\mu z_\tau^* \partial^\mu z_\tau + M_\tau^2 |z_\tau|^2 + \bar{\chi}_\tau \gamma \cdot \partial \chi_\tau + \bar{\chi}_\tau \chi_\tau) \quad (8)$$

and

$$L_1 = \bar{\psi}_\mu (\partial^\mu \hat{z}_\tau^*) \chi_\tau - 1/2 M_\tau \bar{\psi}_\mu \psi^\mu \hat{z}_\tau^2, \quad (9)$$

where we take $z = A + iB$ (where A and B are real fields) and $\hat{z} = A + i\gamma_5 B$ [5]. In this expression we have put to zero the terms proportional to $\gamma \cdot \psi$ because the gravitino does not propagate in the internal lines and the terms proportional to $\partial \cdot \psi$ must be put to zero at the end of the calculation due to the equations of motion. The diagrams that contribute to this calculation are shown in fig.1. We observe that there are still divergent terms which

cancel each other and that the diagrams contain a term behaving like $1/M$, as can be easily seen by power counting.

The contribution from the first diagram is equal to

$$I_1 = (1/(192\pi^2))(p-p')^2 - M^2/(32\pi^2)f)\delta_{\nu\sigma}\psi_{\nu p'}\psi_{\sigma p'}, \quad (10)$$

whereas the contribution from the second is equal to

$$I_2 = 1/(32\pi^2)(\delta_{\nu\sigma}(fM^2 - 1/6(p-p')^2 - 1/6pp') + 1/(12p_\sigma p'_\sigma)\psi_{\nu p'}\psi_{\sigma p'}, \quad (11)$$

where f is a divergent term (the sum of these divergent terms is zero).

If we sum the two diagrams we will obtain

$$I = 1/(182\pi^2)(p_\sigma p'_\sigma - pp'\delta_{\nu\sigma})\psi_{\nu p'}\psi_{\sigma p'}. \quad (12)$$

This is exactly the lowest order term of the right handside of equation (3).

It is clear that the complete calculation along this line is practically impossible. In order to have the complete answer we will repeat the same calculation using the heat kernel method in superspace [5].

We recall that the action, written in superspace [6], is given by

$$I = 1/2 \int dz (E/R)g(S) + \int dz E\bar{S}S, \quad (13)$$

where S is a chiral superfield coupled to the supergravity multiplet, E is the determinant of the supervierbein and R is one of the three superfield with the Bianchi identities for the torsion and the curvature can be solved [7].

We now write this action in a chiral subspace, introducing a set of coordinates $y = (y^\mu, y^\nu, y^m)$, defined so that

$$E_\alpha^M = \rho\delta_\alpha^M, \quad (14)$$

where $M = (\mu, \nu, m)$, y^μ, y^ν are anticommuting variables and y^m are commuting variables. With respect to this set of coordinates the chirality condition can be written as

$$D^\alpha S = \rho\partial^\alpha S. \quad (15)$$

In this gauge a chiral superfield is independent of y^μ . It is then possible to write (g) in the chiral subspace, obtaining

$$I = \int d^4 y^m d^2 y^\mu \epsilon(g(S) + -1/4(\bar{D}^2 - 8R)\bar{S}S), \quad (16)$$

where ϵ is the restriction of E to the chiral subspace. If we now perform a chiral transformation $\delta S = i\Lambda S$, where Λ is an infinitesimal chiral superfield the variation of the action then becomes

$$\delta I = \int d^4 y^m d^2 y^\mu \epsilon(\delta g/\delta S - 1/4(\bar{D}^2 - 8R)\bar{S})i\Lambda S. \quad (17)$$

However we observe that also the functional integral measure transforms, yielding another term in the generating functional

$$\delta I_m = \int d^4 y^m d^2 y^\mu \epsilon \lim_{y \rightarrow y'} 1/\epsilon \delta^{(6)}(y - y') i\Lambda. \quad (18)$$

This expression is not well defined and we regularize it with the insertion of the operator $\exp(i\Delta)$, where

$$\Delta = 1/16(D^2 - 8R)(\bar{D}^2 - 8R). \quad (19)$$

It is important to observe that Δ respects both supersymmetry and chiral invariance and it is a very natural generalization of the laplacian in superspace. This rather lengthy calculation for δI_m has already been performed and the result is the following [7]

$$\lim_{t \rightarrow 0} \lim_{y \rightarrow y'} \exp(t\Delta) \epsilon^{-1} \delta^{(6)}(y - y') = 1/(48\pi^2) W^{\alpha\beta\gamma} W_{\alpha\beta\gamma}, \quad (20)$$

where W is the Weyl chiral superfield and we have not written the terms which are proportional to the matter fields on shell. If we use equations (17), (18) and (20), observing that a change of variables does not change the value of the functional integral we obtain

$$\langle (\delta g/\delta S)S - 1/4\bar{D}^2\bar{S}S + 1/(48\pi^2)WW \rangle = 0. \quad (21)$$

Now if we take the lowest component of equation (21) we get

$$\bar{\delta}(\bar{\chi}z) = -(\delta g/\delta z)z + (1/384\pi^2)\psi_{ab}\psi_{ab}, \quad (22)$$

which is exactly equation (3) once we put the superpotential to zero.

Having this anomalous transformation law, we note that, once we take the vacuum expectation value of both sides in equation (3), local supersymmetry will be spontaneously broken if the gravitino condensate is different from zero [3]. The consequences of having a non zero gravitino condensate were investigated in [8] and [9].

In reference [11] it was shown that the Eguchi-Hanson instanton has the right number of spin 3/2 zero modes to give a gravitino condensate different from zero. In [3] this condensate was calculated in a semiclassical approximation and shown to be finite. We recall that the Eguchi-Hanson instanton is asymptotically locally flat and has zero action. The first of these properties means that the topology is not euclidean and in fact opposite points with respect to the origin must be identified. It is then difficult to give a physical interpretation to these classical solutions and to establish whether they contribute to the functional integral. In [12] it was argued that the exclusion of these solutions from the functional integral does not lead to any contradiction but, given our present understanding of the quantization of gravity, this problem does not yet have a definite answer. Recently 't Hooft [13] has suggested a physical interpretation of these instantons showing that they do not have unusual properties and in some respect are similar to black holes. Another argument that can support the importance of the contribution of the Eguchi-Hanson instanton to the functional integral is the following. Let us consider the supergravity model presented in [8], with a scalar superfield coupled with the Euler characteristic superfield. The model has a $U_n(1)$ symmetry which is broken by the chiral anomaly having a contribution from the gravitino field only. This model is invariant if we transform also the pseudoscalar field coupled to the $*RR$ term. In order to construct an effective lagrangian with the correct symmetry properties we must introduce as low energy fields the scalar superfield S and the Weyl multiplet WW , which contains in its lowest component the gravitino condensate.

Imposing that the action is invariant under the transformations

$$W \rightarrow e^{i\alpha} W \quad (23)$$

$$S \rightarrow S - i\alpha b \quad (24)$$

we obtain the following superpotential

$$g = SU + bU \text{Log}(U/M_s^2), \quad (25)$$

where $U=WW$ and M_s is a scale that determines the value of the gravitino condensate. If we choose the Kähler potential depending only on $|U|$ and $\text{Re}S$ then the action is invariant under this symmetry. We observe that this symmetry is not sufficient to determine completely the form of the Kähler potential. Apart from the interpretation of the different terms present in (23), the superpotential has the same form as the one proposed in [10], for the case of gaugino condensation. We think that it would be important to understand the differences between the various cases (having gravitino and/or gaugino condensation) and to see if, in the superstring-inspired models, there are other symmetries that can constrain the form of the Kähler potential.

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Fig. 1

FIGURE CAPTIONS

1. Diagrams contributing to the anomalous variation of the composite operator $\bar{\chi}\chi$.