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SYMMETRY BREAKING SCHEMES AND WW SCATTERING

John F. Donoghue^{*)}
CERN - Geneva

and

Carlos Ramirez
Physics Department
University of Massachusetts
Amherst, MA 01003, USA

ABSTRACT

We discuss how one can differentiate between various theories of the symmetry-breaking sector of electroweak interactions using WW scattering at high energies. The tool is the study of deviations from the universal lowest-order non-linear sigma model predictions for these amplitudes. The Higgs scheme and technicolour are used as examples, and these yield distinctly different behaviours.

^{*)} Permanent address: Physics Department, University of Massachusetts, Amherst, MA 01003, USA.

The study of WW scattering provides an important window to the symmetry breaking sector of the electroweak interactions.¹⁻³ If Higgs bosons exist and are light enough, they can be found as direct s-channel resonances in this process. However, even more interesting are the situations where either Higgs bosons are heavy and strongly interacting, or where other, non-Higgs, interactions are responsible for the symmetry breaking. In such situations, the electroweak sector becomes strongly interacting in the energy region above M_W . The scattering of longitudinal W_L , Z_L can be identified with that of the Goldstone bosons from the symmetry breaking, w , z , by means of an equivalence theorem¹

$$M(W_L, Z_L \dots) = M(w, z, \dots) + O\left(\frac{M_{WZ}}{E}\right) \quad (1)$$

valid even if the theory is strongly interacting. The corresponding amplitudes are determined by a non-linear sigma model following from the symmetries of the theory. This leads to universal forms of the scattering amplitudes to lowest order in the energy expansion. This is a powerful result analogous to the low energy theorems of QCD. The purpose of this article is to discuss how deviations from the universal low energy amplitudes, which can arise at the next order in the energy expansion, can differentiate among different origins of symmetry breaking.

The scattering of Goldstone bosons can be described by effective Lagrangians which incorporate the constraints of the symmetry breaking on the scattering amplitudes.^{4,5} These Lagrangians are written in an energy expansion, where higher numbers of derivatives imply higher powers of the energy when matrix elements are taken. At low energies, compared to the energy scale of the expansion, only those with the fewest derivatives will be important. That is, Lagrangians are written in an expansion

$$\mathcal{L} = a \text{Tr}(\partial_\mu U \partial^\mu U^\dagger) + \frac{b}{\Lambda^2} \text{Tr}(\partial_\mu U \partial^\mu U^\dagger \partial_\nu U \partial^\nu U^\dagger) + \dots \quad (2)$$

where

$$U = \exp(i \tau \cdot \phi / F) \quad (3)$$

is a matrix representation incorporating the Goldstone bosons ϕ . Matrix elements have the form

$$M = aE^2 + b \frac{E^4}{\Lambda^2} + \dots \quad (4)$$

where E is related to the energies involved. At energies $E^2 \ll \Lambda^2$, only the first term needs to be kept. It is within this framework that the low energy theorems are derived. Generally the lowest order Lagrangian is unique, and hence the scattering matrix elements are universal to all possible theories with this pattern of symmetry breaking. Although such a far reaching theorem is impressive, in many ways the correction to these results is even more interesting. At the next order in the energy expansion there are more possible Lagrangians consistent with the symmetry, and the deviations of the scattering amplitudes will no longer be universal but will depend on the coefficients of these Lagrangians. Different underlying theories will now yield different predictions for these coefficients and if we are able to measure these deviations we can differentiate between the various theories. This is the motivation for the present work.

Early studies of effective chiral Lagrangians focused on the universal behavior at order E^2 . In the 1980's there has been a renewal of interest in these techniques, with an emphasis on working at order E^4 , i.e., the second order in the energy expansion.⁵⁻¹¹ We now have a detailed phenomenological characterization of chiral Lagrangians in QCD and have learned how they behave in the real world. We have a modest theoretical understanding of the physics underlying the Lagrangians. It is this experience which we would like to transfer to the subject of WW scattering.

The coefficients in the chiral Lagrangian characterize the physics from higher energy which influences the interactions of Goldstone bosons. Different coefficients correspond to different processes and/or isospin channels. In general, the deviations from the lowest order results are dictated by the physics which occurs at the energy where the energy expansion breaks down.⁸⁻¹¹ For example, consider $\pi\pi$ scattering in the $J=1, I=1$

channel. The lowest order universal amplitude knows nothing about the ρ meson. However at next order the deviations are determined by the need to match on to the tail of the ρ . These are required in the presence of a resonance due to the unitarity relation. Similarly the q^2 variation of the pion form factor is determined by the presence of the ρ meson. These considerations are made firmer in several ways. One can consider the influence of general spin/isospin resonances which couple to two pions. Each of these predicts a unique pattern to the low energy constants, with the chiral coefficients proportional to $\Gamma_{\pi\pi}/m_\rho^5$. Low lying resonances will have a large influence on the chiral Lagrangians. This has been confirmed in the linear sigma model within the perturbative regime, treated to one loop order.⁵ The main determinant of the coefficients is the tail of the s channel sigma resonance. It is also confirmed phenomenologically in the excellent description of the observed chiral Lagrangian, which is characterized to a great extent as being due to the effects of the ρ meson.⁸⁻¹¹ In general, dispersion relations can be written for these coefficients. The dominance of dispersion relations by moderate energy physics is then the equivalent of the resonance dominance described above. The picture that has emerged then is that the coefficients are determined primarily by the low energy effects of the lightest resonances.⁸⁻¹¹ Note that one doesn't have to have enough energy to directly produce these resonances. The deviations in the scattering amplitude give a preview of the sign and magnitudes of resonant effects.

The categorization of the effective Lagrangian for the electroweak sector has been provided by Appelquist and Bernard¹² for the case of the custodial $SU(2)_L \times SU(2)_R$ of the Higgs sector, and by Langhittano¹³ assuming only the minimal $SU(2)_L$ invariance. The custodial symmetry is an extra global symmetry present in the case of Higgs doublets and with many extensions such as Technicolor. It guarantees that

$$\rho \equiv \left(\frac{M_w}{M_z \cos\theta_w} \right)^2 \quad (5)$$

is equal to unity, up to the usual small electroweak corrections. Because of the experimental constraints that $\rho \approx 1$, we will assume the custodial $SU(2)_L + R$ symmetry throughout this paper. In this limit, the Lagrangian also coincides with that written down for low energy QCD in the chiral $SU(2)$ limit by Gasser and Leutwyler.⁵ The result is

$$\begin{aligned} \mathcal{L}_{\text{eff}} &= \mathcal{L}_2 + \mathcal{L}_4 \\ \mathcal{L}_2 &= -\frac{v^2}{4} \text{Tr}(V_\mu V^\mu) \\ \mathcal{L}_4 &= \alpha_1 \text{Tr}(V_\mu V^\mu) \text{Tr}(V^\nu V_\nu) + \alpha_2 \text{Tr}(V_\mu V_\nu) \text{Tr}(V^\mu V^\nu) \\ &\quad + \alpha_3 \text{Tr}(F_{\mu\nu} (V^\mu, V^\nu)) \end{aligned} \quad (6)$$

where

$$\begin{aligned} F_{\mu\nu} &= \partial_\mu W_\nu - \partial_\nu W_\mu + ig[W_\mu, W_\nu] \\ V_\mu &= [D_\mu U] U^\dagger = (\partial_\mu + ig W_\mu) U U^\dagger \end{aligned} \quad (7)$$

with W_μ being the $SU(2)_L$ gauge field and U being given in Eq. 3. Here

$$F = v = 246 \text{ GeV} \quad (8)$$

A fourth operator in Ref. 12 can be eliminated by use of the equations of motion to this order. To this symmetry breaking sector Lagrangian is added the gauge boson kinetic energy terms

$$\begin{aligned} \mathcal{L}_{\text{tot}} &= \mathcal{L}_G + \mathcal{L}_{\text{eff}} \\ \mathcal{L}_G &= -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{2} \text{Tr}(F_{\mu\nu} F^{\mu\nu}) \end{aligned} \quad (9)$$

as well as fermions (which will not be needed for our work).

The scattering amplitudes for WW scattering will involve only α_1 and α_2 . At energies where the equivalence theorem applies, we find tree level amplitudes

$$\begin{aligned} M(W_L^+ W_L^- \rightarrow Z_L Z_L) &= \frac{s}{v^2} + \frac{4}{v^4} [2\alpha_1 s^2 + \alpha_2 (t^2 + u^2)] \\ M(W_L^+ W_L^- \rightarrow W_L^+ W_L^-) &= -\frac{u}{v^2} + \frac{4}{v^4} [2\alpha_1 (s^2 + t^2) + \alpha_2 (2u^2 + s^2 + t^2)] \end{aligned}$$

$$M(Z_L Z_L \rightarrow Z_L Z_L) = \frac{8}{v^4} (s^2 + t^2 + u^2) (\alpha_1 + \alpha_2)$$

$$M(W_L^\pm Z_L \rightarrow W_L^\pm Z_L) = \frac{t}{v^2} + \frac{4}{v^4} [2\alpha_1 t^2 + \alpha_2 (s^2 + u^2)]$$

$$M(W_L^\pm W_L^\mp \rightarrow W_L^\pm W_L^\mp) = -\frac{s}{v^2} + \frac{4}{v^4} [2\alpha_1 (t^2 + u^2) + \alpha_2 (2s^2 + t^2 + u^2)] \quad (10)$$

where s, t, u are the Mandelstam variables ($s + t + u \approx 0$). Formulas to one loop order in partial wave projections are equivalent to the $\pi\pi$ scattering results quoted in Ref. 7 with $m_\pi = 0$. (Note however the factor of four difference between the normalization of α_i in Ref. 7 and the present work.). Specifically the partial wave amplitudes are

$$T_0^0 = \frac{s}{16\pi F^2} + \frac{s^2}{12\pi F^4} (11\alpha_1 + 7\alpha_2) + B_0^0(s)$$

$$T_1^1 = \frac{s}{96\pi F^2} + \frac{s^2}{24\pi F^4} (\alpha_2 - 2\alpha_1) + B_1^1(s)$$

$$T_0^2 = -\frac{s}{32\pi F^2} + \frac{s^2}{6\pi F^4} (\alpha_1 + 2\alpha_2) + B_0^2(s)$$

$$T_2^0 = \frac{s^2}{60\pi F^4} (2\alpha_2 + \alpha_1) + B_2^0(s)$$

$$T_2^2 = \frac{s^2}{120\pi F^4} (2\alpha_1 + \alpha_2) + B_2^2(s) \quad (11)$$

where the $B_J^I(s)$ are unitarity effects given in Fig. 2 and Eq. 23 of Ref. 7.

The authors of Refs. 12, 13 have given predictions for the coefficients α_i within the Higgs theory, in the logarithmic approximation; that is, one loop corrections are calculated and the $\ln(s/m_H^2)$ corrections are extracted. These are then used as estimates for the

coefficients α_i . This approximation does not work within the context of QCD, where the logarithmic corrections are much smaller than the tree level contributions.^{8,10} It also does not work in the perturbative region of the linear sigma model, where the exchange of the sigma particle is more important than pion loops.⁵ Therefore we do not expect these

logarithmic estimates to be dominant in the Higgs model, at least for most of the parameter space.

Let us contrast two theories: 1) The heavy Higgs limit of the Higgs sector of the standard model and 2) Technicolor theories. The former is equivalent to the linear sigma model with a large sigma mass. It is characterized by an s-channel scalar resonance. When exchanged in the t or u-channels, the scalar has the effect of decreasing other amplitudes as energy increases, such as the behavior in Fig. 1. There is no completely compelling model for Technicolor. For the purposes of the present work we follow most work on Technicolor in assuming that the theory mimics QCD, but with the scale being increased by a factor of several thousand. Thus we can use our experience with the phenomenology of QCD to give expectations for this theory. Here the prime effect is expected to be the presence of a techni-rho meson. This produces an $I = 1$ s channel process, as well as influences on t and u-channel processes, as presented pictorially in Fig. 2.

We proceed as follows. To estimate the corrections due to a scalar resonance we couple the scalar to the Goldstone modes of the theory

$$\mathcal{L}_H = g_H \text{Tr}(\partial_\mu U \partial^\mu U^\dagger) \quad (12)$$

in a manner consistent with the $SU(2)_L \times SU(2)_R$ custodial symmetry. At energies below the mass of the scalar, the effect of the Higgs may be integrated out to form an effective interaction. In a standard procedure, we truncate this at order E^4 to find at tree level⁸

$$\mathcal{L}_{\text{eff}} = \frac{g^2}{2m_H^2} \text{Tr}(\partial_\mu U \partial^\mu U^\dagger) \quad (13)$$

which translates into

$$\alpha_1 = \frac{64\pi}{3} \frac{F^4}{m_H^5} \quad (14)$$

$$\alpha_2 = 0$$

where Γ is the decay width for $H \rightarrow W_L^+ W_L^- Z_L Z_L$. This relation has not yet utilized the perturbative treatment of the Higgs potential in order calculate the coefficient g . In this regard it is more general than the perturbative treatment. However if the mass is light enough that a perturbative treatment is valid, then the result can be quoted to one loop order⁵

$$\alpha_1 = \frac{1}{2\lambda} + \frac{1}{24\pi^2} \left(\ln \frac{m_H^2}{\mu^2} - \frac{3.5}{6} \right) \quad (15)$$

$$\alpha_1 = \frac{1}{12\pi^2} \left(\ln \frac{m_H^2}{\mu^2} - \frac{1.1}{6} \right) \quad (15)$$

where μ is the scale used to specify the theory. The $1/\lambda$ term in Eq. 15 is exactly the same as the result of Eq. 14 when the perturbative expression for masses and couplings are used.

For the Technicolor theory, we couple the techi-rho up to the Goldstone bosons.

Using an antisymmetric tensor field this is of the form

$$\mathcal{L}_\rho = \bar{g} \rho_{\mu\nu}^i \text{tr}(\tau^i \xi^\dagger \partial_\mu U \partial_\nu U \xi) \quad (16)$$

where

$$U = \xi \xi^\dagger \quad (17)$$

When the techi-rho is integrated out,^{5,8,9} one obtains

$$- \alpha_1 = \alpha_2 = \frac{\bar{g}^2}{2m_\rho^2} = \frac{192\pi \Gamma v^4}{m_\rho^5} \quad (18)$$

where here Γ is the decay rate of the techi-rho to longitudinal gauge bosons. The pattern $\alpha_2 = -\alpha_1 > 0$ is characteristic of vector particles in much the same way that $\alpha_1 > 0$ and $\alpha_2 \approx 0$ are characteristic of scalars. These patterns are quite distinct.

The individual rates that seem the most sensitive to these patterns are $W^+W^- \rightarrow Z^0Z^0$, $W^+Z^0 \rightarrow W^+Z^0$ and $Z^0Z^0 \rightarrow Z^0Z^0$. Comparison with Eq. 10 shows that the E^4

behavior in $W^+W^- \rightarrow Z^0Z^0$ has opposite signs in the two models. Similarly $W^\pm Z^0 \rightarrow W^\pm Z^0$, which is just the crossed version of the same amplitude, has deviations from the lowest order result of a very different character in the two theories. The reaction $Z_L^0 Z_L^0 \rightarrow Z_L^0 Z_L^0$ is a theoretically clean laboratory, because the lowest order result vanishes. The

Higgs model would predict a positive amplitude, while Technicolor would keep this amplitude zero (or nearly so), because the techi-rho does not have couplings to neutral gauge bosons.

The expectations that we have described are under the assumption that the lightest particles in the theory are low enough in mass to have some effect on the effective Lagrangian. Where would this be valid? In Ref. 8, we have given a criterion for this, such that the tree level coefficient induced is smaller than the scale dependence of the coefficients when calculated at one loop order. When that criterion is applied, we would be most confident about this analysis for masses

$$\begin{aligned} m_H &< 1.8 \text{ TeV} \\ m_{\rho T} &< 2.7 \text{ TeV} \end{aligned} \quad (19)$$

where we have assumed widths less than 1 TeV for the higher values of masses.

However, the general methodology could still be useful outside this range. The two parameters α_1 and α_2 characterize the deviations from the lowest order result, and are sensitive to the underlying theory. The parameters are also calculable nonperturbatively using lattice methods for masses outside the range of Eq. 19. Hopefully by the time this result is phenomenologically needed, the lattice techniques will be adequate for such a task.

If the symmetry breaking sector is strongly interacting, the lowest order predictions of WW scattering are model independent. We have shown how the deviation from these predictions, at next order in the energy expansion, can be used as a signature of the underlying theory, and have given expectations for this pattern based on the experience of chiral symmetry breaking studies.

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References

- 1) J.M. Cornwall, D.N. Levin and G.Tiktopoulos, Phys. Rev. **D10**, 1145 (1974).
M.S. Chanowitz and M.K. Gaillard, Nucl. Phys. **B261**, 379 (1985).
- G. Gounaris, R. Kogeler and H. Neufeld, Phys. Rev. **D34**, 3257 (1986).
- 2) M.S. Chanowitz, M. Golden and H. Georgi, Phys. Rev. **D36**, 1490 (1987); Phys. Rev. Lett. **57**, 2344 (1986).
- 3) M.S. Chanowitz, Ann. Rev. Nucl. Part. Sci. **38**, 323 (1988).
- 4) S. Weinberg, Physica **96A**, 327 (1979).
H. Georgi, Weak Interactions and Modern Particle Physics (Benjamin, Palo Alto, 1985).
- 5) J. Gasser and H. Leutwyler, Ann. Phys. (NY) **158**, 142 (1984).
- 6) J. Gasser and H. Leutwyler, Nucl. Phys. **B250**, 465, 520, 539 (1985).
J.F. Donoghue, E. Golowich and B.R. Holstein, Phys. Rev. **D30**, 587 (1984).
E. Golowich, Phys. Rev. **D36**, 3516 (1988).
- J.F. Donoghue, B.R. Holstein and Y.C. Lin, Phys. Rev. **D37**, 2423 (1988).
J.F. Donoghue, Nucl. Phys. **B7A**, 59 (1989).

- G. Ecker, A. Pich and E. de Rafael, Nucl. Phys. **B303**, 665 (1988).
F. D'Ambrosio and D. Espriu, Phys. Lett. **B175**, 237 (1986).
J.L. Goity, Z. Phys. **C34**, 341 (1987).
- J.F. Donoghue, B.R. Holstein and Y.C. Lin, Phys. Rev. Lett. **55**, 2766 (1988).
- J. Bijnens, A. Bramon and F. Cornet, Phys. Rev. Lett. **61**, 1453 (1988).
- J.F. Donoghue and D. Wyler, Nucl. Phys. **B316**, 289 (1989).
D. Issler, SLAC-Pub., (to appear).
- J. Bijnens, A. Bramon and F. Cornet, Munich Preprint, (to appear).
- J. Kambor, J. Missimer and D. Wyler, Zurich preprint, (to appear).
- 7) J.F. Donoghue, C. Ramirez and G. Valencia, Phys. Rev. **D38**, 2195 (1988).
- 8) J.F. Donoghue, C. Ramirez and G. Valencia, Phys. Rev. **D39**, 1947 (1989).
- 9) G. Ecker, J. Gasser, A. Pich and E. de Rafael, Nucl. Phys. **B321**, 311 (1989).
G. Ecker, J. Gasser, H. Leutwyler, A. Pich and E. de Rafael, Bern preprint BUTP/89/4 (to appear).
- 10) J.F. Donoghue and B.R. Holstein, preprint UMHEP-325 "Pion Transitions and Models of Chiral Symmetry", Phys. Rev. **D** (to appear).
- 11) M. Praszalowicz and G. Valencia, preprint BNL-43148 (to appear).
- 12) T. Appelquist and C. Bernard, Phys. Rev. **D22**, 200 (1980).
- 13) A.C. Longhitano, Nucl. Phys. **B188**, 118 (1981).

Figure Captions

Fig. 1. Schematic representation of the physics involving a scalar Higgs particle in WW scattering. The dashed line represents the lowest order universal prediction, while the solid line represents the full result with a scalar resonance. a) The $I = 0, J = 0$ channel with an s-channel resonance. b) The $I = 1, J = 1$ channel with t-channel exchange.

Fig. 2 Schematic representation of the physics involving a techni-rho meson in WW scattering. The dashed line represents the lowest order universal prediction, while the solid line represents the full result with a vector resonance.
a) The $I = 0, J = 0$ channel with t-channel exchange.
b) The $I = 1, J = 1$ channel with an s-channel resonance.

a)

b)

Fig. 1

a)

b)

Fig. 2