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ON HYPERNUCLEAR π -MESONIC DECAY

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The theoretical treatment of the hypernuclear pi-mesonic decay is reformulated so that the pion distortion and the π -decay vertex renormalization are incorporated in a consistent manner, thus eliminating the dependence on the off-shell extensions of the π - N amplitude. Calculations are performed for some typical hypernuclei and compared to available experimental data.

The pi-mesonic decay of hypernuclei is quite an interesting problem, which has been extensively investigated [1-6]. In the free space the momentum q_0 released in the decay process $\Lambda \rightarrow N + \pi$ is only 100 MeV/c, which is much smaller than the nuclear Fermi momentum $p_F \sim 280$ MeV/c. Thus the π -decay of Λ hyperon in the nuclear medium tends to be strongly suppressed by the Pauli exclusion effect. It has been shown, however, that the distortion of the pion wave tends to diminish the Pauli suppression, yielding a significant enhancement of the π -decay rate [2,4-6]. Now the distorted pion waves and hence the decay rates depend sensitively on the particular form of the optical potential, which itself depends on the way how the π -nucleon scattering amplitude is extrapolated off-shell in order to build the potential. The π -decay rate is thus found to depend on the extrapolation.

It is the aim of this letter to show that this dependence is not quite physical. Our argument is inspired by the Beg-Agassi-Gal theorem [7], which concerns the scattering amplitude in a correlated medium. It states that when short range correlations prevent the overlap of two nucleons, creating a hole around each nucleon, the nuclear scattering amplitude should not depend on the off-shell behaviour of the elementary amplitude on the nucleon. This concept has been extended by Delorme, Ericson and Faldt [8] to an inelastic process, namely, the pion photoproduction one. These authors have shown that the independence on the off-shell form of the πN amplitude can be achieved by a proper renormalization of the elementary photoproduction amplitude, which depends on the particular form of the π -nucleus optical potential. They also gave the renormalization factors associated with the Kissinger type potential [9]. Here we recapitulate their argument and make use of their results.

We first give the physical argument why the independence of the decay upon the off-shell extrapolation should be achieved. We believe that the Λ -hyperon experiences in the nucleus a strong short-range repulsion with the neighbouring nucleons [10], which creates around it a correlation hole free from the presence of nucleons. This implies that the decay in fact occurs in a region of space which is void, i.e. where the emitted pion is on-shell. The π -decay rate cannot therefore depend on

the off-shell extrapolation of the π - N amplitude. For this argument to hold it is necessary, as shown in ref.[8], that the elementary process, here the transformation of the Λ -hyperon into a nucleon and a pion, be short ranged, the range should be smaller than the size of the correlation hole. This is the case if the pion is emitted in the weak vertex, together with the transformation of a Λ into nucleon. In the pole dominance model description [11] where pion emission occurs as a usual strong process, following the weak baryonic decay, the argument has to be transferred to the nucleon. The relevant correlations are then the NN ones and the range is that of the nucleonic pion source. In this case the renormalizations of the amplitudes are those of the πN vertex.

The π -decay (π^- - and π^0 -) Hamiltonian in free space is given by

$$H_\pi = \left[s_\pi + ip_\pi \frac{(\sigma q_0)}{q_0} \right] e^{iq_0 r} \quad (1)$$

with

$$s_{\pi^-} = 0.96 \times 10^{-7}, \quad p_{\pi^-} = -0.35 \times 10^{-7}, \quad (2)$$

$$s_{\pi^0} = -s_{\pi^-}/\sqrt{2}, \quad p_{\pi^0} = -p_{\pi^-}/\sqrt{2}, \quad (3)$$

where σ denotes the spin operator for the baryon. Eq.(3) reflects the $\Delta I = \frac{1}{2}$ rule. The spin-nonflip parity-violating interaction s_π is much stronger than the spin-flip parity-conserving interaction p_π :

$$p_\pi^2/s_\pi^2 \sim 1/9. \quad (4)$$

In the nuclear medium, the above H_π is usually transformed into

$$H_\pi = \left[s_\pi + ip_\pi \frac{(\sigma \nabla)}{q_0} \right] \psi_\pi^{(-)*}(q; r), \quad (5)$$

where $\psi_\pi^{(-)*}(q; r)$ is the pion distorted wave as a solution of the Klein-Gordon equation. However, this procedure is not the correct one and ψ_π should be replaced by

$\phi_\pi(r; r_i)$, where ϕ_π is the effective field which incorporates, as it should, the information that there is the Λ -hyperon (or the emitting nucleon if the pion is emitted at the strong vertex) at the point r_i and that the nuclear medium in a surrounding region is not homogeneous. Here ϕ_π should be taken at $r = r_i$, since we are dealing with a very short-range process.

In the presence of correlations the effective wave ϕ_π and the average wave ψ_π do differ and the correct treatment has to use the effective wave rather than the average one. This is, however, not a convenient procedure as the quantity easily accessible is the average wave as a solution of the Klein-Gordon equation. Delorme et al. [8] have then proposed to relate the effective wave to the average wave and incorporate this relation in the Hamiltonian through renormalization factors in such a way that once this renormalization procedure is done, one can forget the effective wave and work only with the average one. One should stress that each piece of the Hamiltonian, here the s_π and p_π terms of eq.(5), undergoes a different renormalization, which in addition is different for each adopted π -nucleus potential. In this work we make use of the results of ref.[8] for the Kissinger type potential and incorporate them into our calculation done with the MSU potential [12], which is of such nature. It should be noted here that the Kissinger type potentials are originally microscopic ones built from πN amplitude [13].

The renormalized Hamiltonian now reads

$$H_\pi = \left[s_\pi \ell^{(s)}(r) + ip_\pi \ell^{(p)}(r) \frac{(\sigma \nabla)}{q_0} \right] \psi_\pi^{(-)*}(q; r) \quad (6)$$

with the renormalization factors $\ell^{(s,p)}(r)$ given by

$$\ell^{(s)}(r) = 1, \quad \ell^{(p)}(r) = \ell(r), \quad (7)$$

where $\ell(r)$ is the LLEE factor for the $\Lambda N \pi$ vertex

$$\ell(r) = \frac{1}{1 + \frac{4\pi}{3}[c(r) + C(r)]}, \quad (8)$$

which we will use in association with the pion optical potential of the Kissinger

type as

$$2\omega U = -4\pi[b(r) + B(r)] + 4\pi\nabla \cdot L(r)[c(r) + C(r)]\nabla - 4\pi\left[\frac{p_1-1}{2}\nabla^2 c(r) + \frac{p_2-1}{2}\nabla^2 C(r)\right]. \quad (9)$$

Various quantities in eq.(9) are standard and to be referred to e.g. ref.[12]. $L(r)$ is the LLEE factor for the πN scattering amplitude, which has a Landau-Migdal parameter $\lambda=1.4$ in the MSU potential, somewhat different from the value $\lambda=1$ of the Beg theorem.

We should note here that Oset and Salcedo introduced a Landau-Migdal parameter g' in their many-body theoretical treatment of Λ -decay in the nuclear medium [2], which would be the equivalent of our vertex renormalization when $g' \simeq 1/3$.

The partial π -mesonic decay rate for the transition from a hypernuclear state $|{}^A_Z; J_i T_i \tau_i\rangle$ to a nuclear state $|{}^A_Z; J_f T_f \tau_f\rangle$ is expressed as [6]

$$\Gamma_\pi(J_i T_i \tau_i \rightarrow J_f T_f \tau_f) = \frac{2q}{1 + (\omega_q/M_\Lambda)} \left[s_\pi^2 S_\pi^{(s)}(i, f; q) + p_\pi^2 \left(\frac{q}{q_0}\right)^2 S_\pi^{(p)}(i, f; q) \right], \quad (10)$$

where $S_\pi^{(s)}$ and $S_\pi^{(p)}$ are given by

$$S_\pi^{(s,p)}(i, f; q) = \frac{4\pi}{2J_i + 1} \sum_K \left| \sqrt{A} \langle {}^A_Z; J_f T_f \tau_f | F_K^{(s,p)} \theta_\pi | {}^A_Z; J_i T_i \tau_i \rangle \right|^2, \quad (11)$$

$$F_K^{(s)} = \tilde{j}_K(q; r) \mathcal{C}^{(s)}(r) Y_K(\hat{r}), \quad (12)$$

$$F_K^{(p)} = -i \sum_{k=K \pm 1} (10K0|k0) f_K^k(q; r) \mathcal{C}^{(p)}(r) [\sigma \times Y_k(\hat{r})]_K$$

with

$$f_K^k(q; r) = \frac{1}{q} \frac{\partial \tilde{j}_K(q; r)}{\partial r} + \left\{ 1 + \frac{k(k+1) - K(K+1)}{2} \right\} \frac{\tilde{j}_K(q; r)}{qr}. \quad (13)$$

The transition operators $F_K^{(s)}$ and $F_K^{(p)}$ come from the multipole expansion of the s_π and p_π parts, respectively, of H_π , with $\tilde{j}_K(q; r)$ being the k -th partial wave of

$\psi^{(-)*}$. In the no-distortion limit, $\tilde{j}_k(q; r)$ and $f_k^K(q; r)$ tend, respectively, to $j_k(qr)$ and $\pm j_k(qr)$ for $K = k \pm 1$, and $\mathcal{C}^{(s,p)}(r)$ to 1. The operator θ_π transforms a Λ hyperon into a nucleon. The pion momentum q and energy ω_q are determined by the relation

$$M_\Lambda + E({}^A_Z; J_i T_i) = M_N + E({}^A_Z; J_f T_f) + \frac{q^2}{2M_\Lambda} + \omega_q, \quad (14)$$

The total π -mesonic decay rate of a hypernucleus is given by summing the partial rates over all final nuclear states as

$$\Gamma_\pi(J_i T_i \tau_i) = \sum_f \Gamma_\pi(J_i T_i \tau_i \rightarrow J_f T_f \tau_f), \quad \pi = \pi^0 \text{ or } \pi^\pm. \quad (15)$$

For the decay of free Λ , eq.(15) is reduced to

$$\Gamma_\pi^{\text{free}} = \frac{2q_0}{1 + (\omega_{q_0}/M_N)} (s_\pi^2 + p_\pi^2) \quad (16)$$

and the free Λ decay rate Γ_Λ is expressed by

$$\Gamma_\Lambda = \Gamma_{\pi^0}^{\text{free}} + \Gamma_{\pi^\pm}^{\text{free}} = \hbar/(2.63 \times 10^{-10} \text{sec}) \quad (17)$$

Hereafter decay rates are always given in units of the free Λ decay rate Γ_Λ .

For illustration purpose, we present the calculated results for some typical hypernuclei. We use the Cohen-Kurath type wave functions [6] for p -shell (hyper)nuclei and the single-particle models with appropriate Woods-Saxon potentials for heavier (hyper)nuclei.

Table 1 compares the π -decay rates calculated with no pion wave distortion (FREE), taken from the previous work [6] and calculated within the present framework. (Notice that the absorption of the pion, incorporated through the imaginary part of the optical potential, should include only the true absorption, in which the

pion disappears. Processes such as quasi-elastic scattering, which degrade the pion energy, are not to be incorporated since they do not decrease the number of pions [14]. However, this distinction can be ignored at the small pion energy where true absorption dominates.) In ref.[6], the MSU effective-form potential was used, which includes only $\rho(r)$, $\nabla\rho(r)\nabla$ and $\nabla^2\rho(r)$ terms and not the LLEE-effect, and the vertex renormalization was not considered. Although this effective-form potential can reproduce low-energy π -nucleus scattering data, the associated pion distortion becomes extremely strong and gives very large enhancement of π -decay rates. In the present, more consistent, treatment the resulting enhancement is milder than that of ref.[6], although quite significant.

= Table 1 =

Table 2 illustrates the effect of the vertex renormalization, when the MSU full-form potential eq.(9) is used. The actual effect of the vertex renormalization tends to increase in heavier hypernuclei. This is because, as the pion distortion becomes more pronounced, the self-control effect also gets more significant. However, the vertex renormalization effect turns out not so drastic for the optical potential chosen here. This is due to the fact that the interaction strength p_π is much weaker than s_π as seen in eq.(4).

= Table 2 =

For the π^- -decay of ${}^{12}_\Lambda\text{B}(J_\pi^\pi = 1^-)$, there is a special reason why the vertex renormalization is so ineffective. Its π^- -decay rate is dominated by the transition to the ground 0^+ state of ${}^{12}\text{C}$; only the spin-nonflip s_π interaction contributes to the 1^- to 0^+ transition. On the other hand, if the ground state of ${}^{12}_\Lambda\text{B}$ were 2^- , then the 2^- to 0^+ transition would be dominated by the spin-flip p_π interaction and hence this partial decay rate would be directly influenced (reduced by a factor 2) by the vertex renormalization.

We compare our results with the available experimental data on the π^- and π^0

decay rates of ${}^{12}\text{C}$. Table 3 displays the $\Gamma_{\pi^-}/\Gamma_\Lambda$ and $\Gamma_{\pi^0}/\Gamma_\Lambda$ values calculated by the present treatment and those measured by two groups [15,16]. We have obtained the π^0 decay rate in the last column by multiplying the measured π^0 branching ratio [16] by the total decay rate from ref.[15]. The two experimental data are barely overlapping within their uncertainties. The theoretical value is just between the two experimental values. It is desirable that the experimental data reach better precision in near future.

= Table 3 =

In order to illustrate the selective character of the π -decay, an example is shown in fig.1, where the calculated excitation spectra for the π^- decay of ${}^{12}_\Lambda\text{B}(J_\pi^\pi = 1^-, 2^-)$ going to various states of ${}^{12}\text{C}$ are compared. The patterns of the spectra are completely different in dependence on the spin of the parent hypernucleus, indicating the usefulness of the π -decay as a hypernuclear spectroscopic tool.

= Fig. 1 =

The pionic decay of hypernuclei has been reformulated by emphasizing the self-consistent treatment of the pion wave distortion and the π -decay vertex renormalization. For microscopic pion optical potentials with different extrapolations of the πN amplitude, the result does not depend on the optical potential, as a consequence of the generalized Beg-Adassi-Gal theorem. The results calculated in this framework show a significant enhancement of the π -decay rate due to the pion wave distortion, which is, however, rather mild and stabilized compared to the previously obtained ones. We have also explored the sensitivity of our results to the introduction of a Landau-Migdal ξ with $\xi \neq 1$ in

$$\ell(r) = \frac{1}{1 + \frac{1}{3}4\pi[c(r) + C(r)]}.$$

We have taken $\xi=1.4$ (the same value as appears in the MSU potential), and we have checked that this does not change our numerical results by more than 3%. Thus the

present line of theoretical calculations should be able to give sound predictions for various hypernuclear π -mesonic decay processes.

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Table 1. Comparison of the π -decay rates obtained by the present
 treatment with those calculated without pion distortion
 (FREE) and those taken from ref.[6]. See the text for details.

	FREE	ref.[6]	present
$^{12}\text{B}_\Lambda$	$\Gamma_{\pi^0}/\Gamma_\Lambda$	0.028	0.066
	$\Gamma_{\pi^-}/\Gamma_\Lambda$	0.175	0.356
$^{12}\text{C}_\Lambda$	$\Gamma_{\pi^0}/\Gamma_\Lambda$	0.078	0.169
	$\Gamma_{\pi^-}/\Gamma_\Lambda$	0.058	0.134

Table 2. The $\Gamma_{\pi^-}/\Gamma_\Lambda$ values for typical hypernuclei calculated
 without and with the vertex renormalization.

	$\Gamma_{\pi^-}/\Gamma_\Lambda$	
	No V.R.	V.R.
$^{12}\text{B}_\Lambda$	0.291	0.288
$^{29}\text{Si}_\Lambda$	0.033	0.028
$^{91}\text{Zr}_\Lambda$	0.0049	0.0041

Table 3. Comparison of the calculated and experimental π^- and π^0 decay rates of ^1_2C .

	Calc	Exp
		ref.[15] ref.[16]
$\Gamma_{\pi^0}/\Gamma_{\Lambda}$	0.130	0.06^{+08}_{-05} $0.21 \pm .07$
$\Gamma_{\pi^-}/\Gamma_{\Lambda}$	0.099	0.05^{+06}_{-03}

Figure caption

Fig. 1. The calculated excitation spectra for the π^- decay of ^1_2B with $J^\pi = 1^-$ ($J^\pi = 2^-$ (right) going to various states of ^1_2C).