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EXTENDED CHAOTIC INFLATION
AND SPATIAL VARIATIONS OF THE GRAVITATIONAL CONSTANT

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ABSTRACT

An extended chaotic inflation scenario is proposed. In this scenario the values of the effective gravitational constant in different parts of the universe may differ from each other. Depending on the choice of a particular model, the value of the gravitational constant in our part of the universe either can be expressed through other coupling constants in the theory or can be determined with the help of anthropic considerations. In some models the weakness of the gravitational interactions may be related to the duration of inflation.

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In order to get a realistic theory of the evolution of the universe one needs building blocks of two types: different versions of the elementary particle theory and different versions of cosmological models. Then one may try to solve the puzzle by looking which building blocks fit together into a unique pattern.

With the invention of the inflationary universe scenario we got a new interesting class of models which may provide us with enough material to fulfil the cosmological part of the project. However, it would be naive to expect that we will get a complete theory very soon since the road to the realistic theory of all fundamental interactions takes a turn each few years. Moreover, the variety of different inflationary models is also very big, and it would be better to understand all kinds of qualitatively new possibilities they can offer before starting a tremendous work of constructing a complete and realistic cosmological theory.

The first versions of the inflationary universe scenario were based on the models where the process of inflation was determined by the evolution of just one scalar field ϕ . Even this simplest class of models leads to many unusual and unexpected properties of the universe, such as a non-trivial thermal history of the universe, the existence of a mechanism of formation of a scale-free (flat) spectrum of adiabatic density perturbations, the absence of the global end of the universe evolution etc. (For a review see e.g. ref. [1].)

However, in most of the realistic models of elementary particles there are many different scalar fields. This may lead to existence of two consequent stages of inflation (so-called combined scenario [2], double or multiple inflation [3, 4]), to generation of non-Gaussian perturbations with a non-flat spectrum [5 - 7], to appearance of exponentially big bubbles or domains with different energy density [4, 6], or with almost the same energy density but with a different density of baryons, or with density perturbations or different type inside different domains [1, 6, 8]. Domains formed at early stages of inflation may contain matter described by different laws of the low-energy physics [1]. In some Kaluza-Klein models even the effective dimensionality of space in different exponentially big domains of the universe may be different [9].

An investigation of such more complicated models is justified if by the study of such models one can learn something qualitatively new about the inflationary cosmology. Many interesting possibilities which may arise in theories with many scalar fields still remain unexplored. One of the most interesting models of that type suggested this year is the extended inflation scenario [10], which is essentially a realization of the old inflationary universe scenario by Guth [11] in the context of the Brans-Dicke theory [12]. In what follows the model of ref. [10] will be called an extended old inflation scenario. In this paper we are going to suggest another scenario, which will be called an extended chaotic inflation scenario, and to discuss some rather unexpected features of the new class of inflationary models.

Let us consider a theory with the action

$$A = \int d^4x \sqrt{-g} \left[\frac{\phi^2 R}{8b} - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) + L(\sigma) \right]. \quad (1)$$

Here $\phi^2 = \frac{b}{2\pi}$, where Φ is the Brans-Dicke field; σ is the inflaton scalar field. The case $V(\phi) = 0$ corresponds to the original version of the BD theory. In the first papers on extended old inflation it was assumed that $V(\phi) = 0$ and that the effective potential of the field σ has a deep minimum at $\sigma = 0$, as in the old inflationary universe scenario [10]. Then it was understood that a successful realization of this scenario is possible only for not very big values of b , which is incompatible with the experimental constraints on this parameter in the BD theory, $b > 500$ [13, 14], and to cure this problem it was suggested to add to the theory a nonvanishing effective potential of the field ϕ [13-15].

In what follows we will consider this theory with the potentials $V(\sigma)$ of the type used in the chaotic inflationary universe scenario. In such a case the value of the parameter b can be arbitrarily big and there is no need to assume the existence of a nonvanishing potential $V(\phi)$ of the BD field ϕ . We will return to a discussion of theories with a nonvanishing potential $V(\phi)$ in the end of the paper.

From a comparison with the standard Einstein theory it follows that the effective gravitational constant $G(\phi)$ and the effective Planck mass $M_P(\phi)$ in the theory (1) depend on ϕ and are given by

$$G^{-1} = M_P^2 = \frac{2\pi\phi^2}{b}. \quad (2)$$

We will try to find inflationary solutions in the theories with $b \gg 1$. During inflation one can neglect $\frac{\dot{\phi}}{\phi}$ as compared with $H = \frac{\dot{a}}{a}$ (where $a(t)$ is the scale factor of the universe), $\ddot{\phi}$ as compared with $3H\dot{\phi}$ and $\dot{\phi}^2, \sigma^2$ as compared with the effective potential $V(\sigma)$. In such a case equations for a, ϕ and σ in the theory (1) with $V(\phi) = 0$ take a very simple form:

$$H = \frac{\dot{a}}{a} = \frac{2}{\phi} \left(\frac{b}{3} V(\sigma) \right)^{1/2}, \quad (3)$$

$$3H\dot{\phi} = \frac{4}{\phi} V(\sigma), \quad (4)$$

$$3H\dot{\sigma} = -V'(\sigma). \quad (5)$$

From eq. (3), (4) it follows that

$$\frac{\dot{a}}{a} = b \frac{\dot{\phi}}{\phi}. \quad (6)$$

This equation is very interesting. It says that independently of a choice of any particular theory of the type of (1)

$$\frac{a(t)}{a_o} = \left(\frac{\phi(t)}{\phi_o} \right)^b \quad (7)$$

during inflation. (The field ϕ practically does not change after inflation.) This result will be very important for us in what follows. From eq. (7) it follows in particular that for large b the value of the effective gravitational constant changes in time slowly as compared with $a(t)$. This implies that all theories $L(\sigma)$ which lead to inflation in the standard Einstein theory, lead to inflation in the theory (1) as well. To understand some novel features of inflation in the theory (1) let us consider e.g. the theory of the scalar field σ with the effective potential

$$V(\sigma) = \lambda \frac{\sigma^{2n}}{2n}. \quad (8)$$

It is known that in such theories the chaotic inflation scenario can be realized in the context of the Einstein theory [16]. This remains true in the theory (1), (8). Equations (4), (5) for this theory can be written in the following form:

$$\dot{\sigma} = -\phi \sigma^{n-1} \left(\frac{n\lambda}{6b} \right)^{1/2}, \quad (9)$$

$$\dot{\phi} = \sigma^n \left(\frac{2\lambda}{3nb} \right)^{1/2}. \quad (10)$$

From these equations it follows that

$$\frac{d}{dt} \left(\frac{2}{n} \sigma^2 + \phi^2 \right) = 0. \quad (11)$$

A solution of this equation can be most conveniently represented after the change of variables, $\varphi = \sigma \sqrt{\frac{2}{n}}$:

$$\varphi^2 + \phi^2 = c^2, \quad (12)$$

where c is an arbitrary constant (see Fig. 1), or, in a parametric form,

$$\phi = c \cdot \sin x(t), \quad (13)$$

$$\varphi = c \cdot \cos x(t), \quad (14)$$

where $x(t)$ obeys equation

$$\dot{x}(t) = \left(\frac{\lambda}{3b} \right)^{1/2} \left(\frac{nc^2}{2} \right)^{\frac{n-1}{2}} \cos^{n-1} x(t). \quad (15)$$

Let us consider e.g. the theory of a free massive scalar field σ with the effective potential $V(\sigma) = \frac{m^2 \sigma^2}{2}$ (i.e. $n = 1, \lambda = m^2$ in eq. (8)). In this case $\varphi = \sigma \sqrt{2}$ and eq. (15) reads

$$\dot{x}(t) = \frac{m}{\sqrt{3b}}, \quad (16)$$

which yields

$$\begin{aligned} \phi &= c \cdot \sin(x_o + \frac{m}{\sqrt{3b}} t), \\ \sigma &= \frac{\varphi}{\sqrt{2}} = \frac{c}{\sqrt{2}} \cdot \cos(x_o + \frac{m}{\sqrt{3b}} t), \\ a(t) &= a_o \cdot \left(\frac{\sin(x_o + \frac{m}{\sqrt{3b}} t)}{\sin x_o} \right)^b. \end{aligned} \quad (17)$$

For $\frac{m}{\sqrt{3b}} t > x_o$ the last equation describes the power-law inflation, $a(t) \sim t^b$, $b \gg 1$.

To get a complete picture one should know when inflation starts and when it finishes. One can easily check by the use of eqs. (8)-(10) for $V(\sigma) = \lambda \frac{\sigma^{2n}}{2n}$ that $V(\sigma) \gg \dot{\phi}^2, \dot{\sigma}^2$ and inflationary regime (15) is realized only if the field $\varphi = \sigma \sqrt{\frac{2}{n}}$ is bigger than its final value at the inflationary stage φ_f , where

$$\varphi_f \sim \phi_f \sqrt{\frac{n}{3b}} = M_P(\phi_f) \sqrt{\frac{n}{6\pi}}, \quad (18)$$

see line OA at Fig. 1. On the other hand, one can really speak about inflation of a classical space-time only if the energy density is smaller than the Planck density, which implies in particular that $V(\sigma) < M_P^4(\phi)$. Moreover, one can easily show that the gradient energy of inhomogeneities $\delta\phi$ and $\delta\sigma$ produced during inflation is smaller than the potential energy density $V(\sigma)$ (which is necessary for inflation to occur) only if the same condition $V(\sigma) < M_P^4(\phi)$ is satisfied [17, 14]. The corresponding quantum boundary for ϕ and φ is shown at Fig. 1. The curve OB corresponds to the theory $V(\sigma) = \frac{m^2 \sigma^2}{2}$, the curve OC - to the theory $\frac{\Delta\phi^4}{4}$, and the curve OD - to the theory $\frac{m^2 \sigma^2}{2} + \frac{\Delta\phi^4}{4} \ln \frac{\sigma}{m}$.

As it is argued in [1], the quantum boundary corresponds to the most natural initial conditions for chaotic inflation. Moreover, as it is obvious from eq. (7) and Fig. 1, the closer the initial values of the fields to the quantum boundary, the bigger is the volume of the corresponding domains after inflation. Therefore we will assume that typical initial values ϕ_i and φ_i (or σ_i) of the fields ϕ and φ (or σ) are close to the quantum boundary,

$$V(\sigma_i) \sim M_P^4(\phi_i) \sim \frac{4\pi^2 \phi_i^4}{b^2}. \quad (19)$$

Let us first try to understand possible implications of this condition in a different kind of theories where $V(\sigma)$ has a minimum near $\sigma = 0$, as required e.g. in the old inflationary

universe scenario [11] and in the first versions of the new inflationary universe scenario [18]. In such theories eq. (19) yields

$$\phi_i \sim \left(\frac{b^3 V(0)}{4\pi^2} \right)^{1/4}. \quad (20)$$

In the extended old scenario [10] the probability of the bubble formation is exponentially suppressed, whereas the field ϕ linearly grows with time. Since the most natural initial value of this field, as we have argued, is fixed by the condition (20), the typical value ϕ_f of the field ϕ at the end of inflation (when tunneling occurs) should be exponentially big and can be expressed through $V(0)$ and those parameters of the theory which give contribution to the Euclidean action corresponding to the probability of the bubble formation. One then could try to use it as a possible explanation of the big value of the Planck mass $M_p \sim \phi_f / \sqrt{2\pi}$, i.e. of the weakness of the gravitational interactions.

Unfortunately, as we already mentioned, the old extended scenario [10] should be extended even further by introducing an effective potential $V(\phi)$, which is absent in the original BD theory [14, 15]. In such a case the final value of the Planck mass is not of a dynamic origin but is just fixed by hand. However, it does seem possible to get an exponential suppression of the effective gravitational constant in more complicated models, e.g. in the theories describing two subsequent stages of inflation: old and chaotic one.

Now let us return to the extended chaotic inflation in the theory (8). In this case the effective Planck mass in different parts of the universe may take all possible values, corresponding to different values of ϕ_f at the line OA. Due to inflation the present distribution of the scalar field $\phi(x) \sim \phi_f(x)$ in the observable part of the universe is practically homogeneous, even if the original distribution of the fields $\phi_i(x)$ and $\sigma_i(x)$ was not exactly homogeneous. That is why to a good accuracy we can speak about the gravitational *constant*, even though at extremely large distances the values of this *constant* differ from each other.

However, inhomogeneities of the field ϕ , just as of any other scalar field, are produced due to quantum fluctuations during inflation [19]. This leads to large-scale inhomogeneities of the effective gravitational constant $G(\phi)$ which may serve as an additional source of perturbations necessary for galaxy formation.

On an extremely large length scale the variations of the gravitational constant produced by quantum fluctuations may be very big even if originally the field ϕ was almost homogeneous in the whole universe (which is possible e.g. if the universe originally was closed and small). Indeed, as it was shown in [20, 17] in the context of the chaotic inflation scenario, generation of large long-wave perturbations of scalar fields may lead to the process of permanent self-regeneration of inflationary domains. This process makes the distribution

of the scalar field inhomogeneous on extremely large length scales (much bigger than the size of the observable part of the universe), so that at each given moment of time the classical scalar fields take *all* their possible values in some parts of the universe.

The same result remains true for the extended chaotic inflation as well. The necessary (and sufficient) condition for the appearance of the self-regeneration regime [20] is that the change of the scalar fields ϕ and σ during the typical time H^{-1} in accordance to the classical equations of motion (9), (10) is smaller than the amplitude of the long-wave inhomogeneities of these fields $|\delta\phi| \sim |\delta\sigma| \sim \frac{H}{2\pi}$ produced at the same time due to freezing of quantum fluctuations with the wavelength bigger than H^{-1} during inflation [19]. One can easily check that this condition is satisfied if

$$\frac{3\pi^2\phi^4}{b^3} \left(1 + \frac{\phi^2}{\varphi^2} \right) < V(\sigma) < \frac{4\pi^2}{b^3} \phi^4, \quad (21)$$

the last inequality corresponding to the quantum (Planck) boundary (19).

From eq. (21) it follows that the inflationary universe with most natural initial conditions (i.e. not far from the quantum boundary (19)) enters regime of self-regeneration. During this regime the universe becomes filled with all possible values of the fields ϕ and σ , independently of their initial values in the region (21). Consequently, the universe after inflation will contain many exponentially large domains in which the Planck mass $M_p(\phi)$ and the gravitational constant $G(\phi)$ take all possible values from 0 to ∞ .

But if different parts of the universe are filled by different values of the field ϕ , is there any way to explain why in our part of the universe this field takes a particular value corresponding to the extremely big Planck mass $M_p \sim 10^{19} \text{ GeV}$?

A simplest idea is that the parts of the universe filled by different fields inflate differently, so we should live in a part of the universe, filled now by the field $\phi \sim \phi_f$, which experienced bigger inflation and consequently occupies bigger volume. According to eq. (7), the total amount of inflation is given by $\left(\frac{\phi_f}{\phi_i} \right)^6$. This result has a simple graphic interpretation: The bigger is the angle between the ray OA at Fig. 1 and the ray showing from the origin O to any particular point (φ_i, ϕ_i) at the quantum boundary OB (or OC, or OD, depending on the choice of $V(\sigma)$), the bigger inflation is. The domains of the universe, containing fields (φ_i, ϕ_i) corresponding to the biggest angle, after inflation occupy the biggest fraction of the volume of the universe. Then with the help of eq. (2) and the fact that $\phi_f^2 = \phi_i^2 + \varphi_i^2$ at $b \gg 1$ one can easily calculate the present value of M_p :

$$M_p \sim \sqrt{\frac{2\pi}{b}(\phi_i^2 + \varphi_i^2)}. \quad (22)$$

From this point of view the theory with $V(\sigma) = \frac{m^2\sigma^2}{2}$, curve OB, may seem unfavourable since the maximal inflation corresponds to infinitely large σ , and, consequently, to infinitely large value of M_p . For the theory $\frac{\lambda\sigma^4}{4}$, the answer also does not look particularly good since the rate of inflation does not depend on a particular choice of (φ_i, ϕ_i) on the line OC. The most interesting case is the theory $\frac{m^2\sigma^2}{2} + \frac{\lambda\sigma^4}{4} \ln \frac{\sigma}{m}$, curve OD. Since $\ln \frac{\sigma}{m}$ is a slowly changing function, the solutions of the corresponding equations to a good accuracy also can be represented as circles (13), (14), and eq. (22) remains approximately true, with $\varphi \sim \sigma$. In this case it can be shown that for a sufficiently small λ the biggest inflation occurs for $\sigma_i \sim 2m/\sqrt{\lambda}$ (point M at the curve OD), which leads to the Planck mass $M_p \sim 5 \cdot 10^7 m/\sqrt{b}$ for $\lambda \sim 10^{-14}$. This means that the small value of the coupling constant $\lambda \sim 10^{-14}$, which is necessary in order to get density perturbations $\frac{\delta\rho}{\rho} \sim 10^{-5}$ on a galaxy scale, and the weakness of gravitational interactions (existence of the very large mass scale $M_p \sim 10^{19} \text{ GeV}$) in this model are somehow related to each other.

However, this may be not a final answer. Indeed, the volume of the universe after inflation depends on initial conditions only as some power of $\frac{\phi_i}{\phi_*}$. Meanwhile the probability to live and to ask questions about the value of the gravitational constant is most certainly exponentially suppressed for the values of G outside some narrow region. Indeed, the process of formation of stars, galaxies, of light elements and even the existence of baryonic matter in the universe crucially depend on the value of G . For example, with the gravitational constant G much smaller than its present value the speed of the universe expansion at a given temperature becomes more slow, the deviation from thermal equilibrium which is necessary for the baryosynthesis after inflation becomes too small, and the corresponding part of the universe becomes empty and devoid of observers [21]. In this sense, the use of anthropic considerations, which can be justified in the context of the chaotic inflation scenario [1], may save the theories with $V(\sigma) = \frac{m^2\sigma^2}{2}$ or $\frac{\lambda\sigma^4}{4}$ and may be sufficient for the determination of the order of magnitude of the gravitational constant in our part of the universe.

To get a final solution of the problem one needs a more detailed investigation of the stage of self-regeneration of the universe. Indeed, eq. (7) is valid even with an account taken of the process of self-regeneration, but only for a typical degree of inflation near a certain point [17]. If one is interested in a typical history and a typical degree of inflation of a domain of a hypersurface of a given density $\rho \sim 10^{-29} \text{ g} \cdot \text{cm}^{-3}$ (and this is what we are interested in), then the answer may be different. In particular, as it was shown in [17] in the context of the chaotic inflation scenario, the main part of the volume of such a hypersurface is formed during not very probable but volume-producing quantum jumps of the scalar field near the quantum boundary. In the extended chaotic inflation scenario a similar investigation probably will show that in many theories the main part of the physical volume of the universe is formed due to quantum jumps of the fields ϕ and σ near some

particular point at the quantum boundary (e.g. near the point M at the curve OD). However, due to a long and complicated process of diffusion of the fields ϕ and σ in the region (21), the distribution of the probability to come from this point to some particular point ϕ_f at the line OA may be very wide. This again would make anthropic considerations appropriate.

The effects discussed in the present paper appear not only in the theory (1). Similar effects may occur in any theory in which an effective potential of some scalar fields has flat directions.

Let us finally discuss the possibility that the effective potential $V(\phi)$ of the Brans-Dicke field does not vanish. If this potential is inflationary [15], then we just have a more complicated two-stage chaotic inflation scenario of the type studied in [2 - 4, 15]. The value of the gravitational constant finally will be fixed by the position of the minimum of $V(\phi)$. However, if the potential $V(\phi)$ is not of inflationary type (as assumed e.g. in [14]), we may have another game to play. For example, the amplitude of density perturbations generated during inflation in the theory of a massive scalar field is proportional to the ratio $\frac{m}{M_p}$, where M_p is the Planck mass during inflation and reheating. Efficiency of baryosynthesis also depends on the value of $M_p(\phi)$ at that time. Depending on the choice of a particular inflationary trajectory in the extended chaotic inflation scenario, this value may differ by many orders of magnitude from the present value of M_p . This may provide us with a wider class of theories according to which an efficient process of baryogenesis is possible and $\frac{\delta\rho}{\rho} \sim 10^{-5}$ in some exponentially large parts of the universe, in one of which we live now.

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Fig. 1

FIGURE CAPTIONS

Fig. 1 During inflation the Brans-Dicke scalar field ϕ and the inflaton field $\varphi = \sqrt{\frac{2}{n}}\sigma$ according to the classical equations of motion (3) - (5) move along a circle (13), (14) with the radius c depending on initial conditions. Classical evolution starts at some point (ϕ_i, φ_i) at the quantum boundary $V(\sigma) \sim M_p^4(\phi_i)$ (curves OB, OC, OD for three different theories). Inflation finishes at the point (ϕ_f, φ_f) where the circle crosses the line OA, eq. (18).