CERN-TH.5590/89

ELECTROWEAK INTERACTIONS AND LEP PHYSICS

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Based on Lectures given at the
Cargèse Summer Institute on Particle Physics
18 July - 4 August, 1989

CERN-TH.5590/89
November 1989

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1. INTRODUCTION

These lectures on electroweak interactions start with a summary of the Glashow-Weinberg-Salam theory [1] and then cover more advanced subjects of present interest in phenomenology: the Higgs sector and the open problem of the experimental investigation on the origin of the Fermi scale of mass $G_F^{1/2}$; the experimental tests of the theory; and finally, an outlook on LEP physics.

The modern electroweak theory inherits the phenomenological successes of the $(V-A) \otimes (V-A)$ four-fermion low-energy description of weak interactions [2], and provides a well-defined and consistent theoretical framework including weak interactions and quantum electrodynamics in a unified picture.

As an introduction, in the following we recall some salient physical features of the weak interactions. The weak interactions derive their name from their intensity. At low energy the strength of the effective four-fermion interaction of charged currents is determined by the Fermi coupling constant G_F . For example, the effective interaction for muon decay is given by

$$\mathcal{L}_{eff} = (G_F/\sqrt{2}) [\bar{\nu}_\mu \gamma_\alpha (1 - \gamma_5) \mu] [\bar{e} \gamma^\alpha (1 - \gamma_5) \nu_e], \quad (1.1)$$

with [3]

$$G_F = 1.16637(2) \times 10^{-5} \text{ GeV}^{-2}. \quad (1.2)$$

In natural units $\hbar = c = 1$, G_F has dimensions of $(\text{mass})^{-2}$. As a result, the intensity of weak interactions at low energy is characterized by $G_F E^2$, where E is the energy scale for a given process ($E \approx m_\mu$ for muon decay). Since

$$G_F E^2 = G_F m_p^2 (E/m_p)^2 \approx 10^{-5} (E/m_p)^2, \quad (1.3)$$

where m_p is the proton mass, the weak interactions are indeed weak at low energies (of order m_p). The quadratic increase with energy cannot continue for ever, because it would lead to a violation of unitarity. In fact, at large energies the propagator effects can no longer be neglected, and the current-current interaction is resolved into current-W gauge boson vertices connected by a W propagator. The strength of the weak interactions at high energies is then measured by g_w , the

W - μ - ν_μ coupling, or, even better, by $\alpha_W = g_w^2/4\pi$ analogous to the fine-structure constant α of QED. In the standard electroweak theory, we have

$$\alpha_W = \sqrt{2} G_F m_W^2/\pi \approx \alpha/\sin^2 \theta_w \approx 1/30. \quad (1.4)$$

That is, at high energies the weak interactions are no longer so weak.

The range r_w of weak interactions is very short: it is only with the experimental discovery of the W and Z gauge bosons that it could be demonstrated that r_w is non-vanishing. Now we know that

$$r_w = \hbar/m_W c \approx 2 \times 10^{-16} \text{ cm}, \quad (1.5)$$

corresponding to $m_W \approx 81 \text{ GeV}$. This very large value for the W (or the Z) mass makes a drastic difference, compared with the massless photon and the infinite range of the QED force. The experimental limits on the photon mass [3] are listed in the following. From a laboratory experiment, one obtains $m_\gamma < 10^{-14} \text{ eV}$ by a method based on the vanishing of the electric field inside a cavity with conducting walls, predicted by Gauss' law. In fact, the exact r^{-2} behaviour of the electric field corresponds to $m_\gamma = 0$. From the observed distribution of planetary magnetic fields (the field should be damped by an extra factor $e^{-m_\gamma r}$ if $m_\gamma \neq 0$) the Pioneer probe to Jupiter obtained $m_\gamma < 6 \times 10^{-16} \text{ eV}$. Finally, indirect evidence from galactic magnetic fields indicates that $m_\gamma < 3 \times 10^{-27} \text{ eV}$. Thus, on the one hand, there is very good evidence that the photon is massless. On the other hand, the weak bosons are very heavy. A unified theory of electroweak interactions has to face this striking difference.

Another apparent obstacle in the way of electroweak unification is the chiral structure of weak interactions: in the massless limit for fermions, only left-handed quarks and leptons (and right-handed antiquarks and antileptons) are coupled to W's. This clearly implies parity and charge-conjugation violation in weak interactions.

The universality of weak interactions and the algebraic properties of the electromagnetic and weak currents [the conservation of vector currents (CVC), the partial conservation of axial currents (PCAC), the algebra of currents, etc.] have been crucial in pointing to a symmetric role of electromagnetism and weak interactions at a more fundamental level. The old Cabibbo universality for the weak charged current [4]:

$$J_\alpha^{\text{weak}} = \bar{\nu}_\mu \gamma_\alpha (1 - \gamma_5) \mu + \bar{\nu}_e \gamma_\alpha (1 - \gamma_5) e + \cos \theta_C \bar{u} \gamma_\alpha (1 - \gamma_5) d + \sin \theta_C \bar{u} \gamma_\alpha (1 - \gamma_5) s + \dots, \quad (1.6)$$

suitably extended, is naturally implied by the standard electroweak theory. In this theory the weak gauge bosons couple to all particles with couplings that are proportional to their weak charges, in the same way as the photon couples to all particles in proportion to their electric charges ($d' = \cos \theta_C d + \sin \theta_C s$ is the weak-isospin partner of u in a doublet).

Another crucial feature is that the charged weak interactions are the only known interactions that can change flavour: charged leptons into neutrinos or up-type quarks into down-type quarks. On the contrary, there are no flavour-changing neutral currents at tree level. This is a remarkable property of the weak neutral current, which is explained by the introduction of the GIM mechanism [5] and has led to the successful prediction of charm.

The natural suppression of flavour-changing neutral currents, the separate conservation of e, μ , and τ leptonic flavours, the mechanism of CP violation [6] through the phase in the quark-mixing matrix, are all crucial features of the Standard Model. Many examples of new physics tend to break the selection rules of the standard theory. Thus the experimental study of rare flavour-changing transitions is an important window on possible new physics.

In the following sections we shall see how these properties of weak interactions fit into the standard electroweak theory.

2. GAUGE THEORIES

In this section we summarize the definition and the structure of a gauge Yang-Mills theory [7, 8]. We will list here the general rules for constructing such a theory. Then in the next section these results will be applied to the electroweak theory.

Consider a Lagrangian density $\mathcal{L}[\phi, \partial_\mu \phi]$ which is invariant under a D dimensional continuous group of transformations:

$$\phi' = U(\theta^A) \phi \quad (A = 1, 2, \dots, D). \quad (2.1)$$

For θ^A infinitesimal, $U(\theta^A) = 1 + ig \Sigma_A \theta^A T^A$, where T^A are the generators of the group Γ of transformations (2.1) in the (in general reducible) representation of the fields ϕ . Here we restrict ourselves to the case of internal symmetries, so that T^A are matrices that are independent of the space-time coordinates. The generators T^A are normalized in such a way that for the lowest dimensional non-trivial representation of the group Γ (we use t^A to denote the generators in this particular representation) we have

$$\text{tr}(t^A t^B) = 1/2 \delta^{AB}. \quad (2.2)$$

The generators satisfy the commutation relations

$$[T^A, T^B] = i C_{ABC} T^C. \quad (2.3)$$

In the following, for each quantity V^A we define

$$V = \sum_A T^A V^A. \quad (2.4)$$

If we now make the parameters θ^A depend on the space-time coordinates $\theta^A = \theta^A(x_\mu)$, $\mathcal{L}[\phi, \partial_\mu \phi]$ is in general no longer invariant under the gauge transformations $U[\theta^A(x_\mu)]$, because of the derivative terms. Gauge invariance is recovered if the ordinary derivative is replaced by the covariant derivative:

$$D_\mu = \partial_\mu + ig V_\mu, \quad (2.5)$$

where V_μ^A are a set of D gauge fields (in one-to-one correspondence with the group generators) with the transformation law

$$V'_\mu = UV_\mu U^{-1} - (1/ig)(\partial_\mu U)U^{-1}. \quad (2.6)$$

For constant θ^A , V reduces to a tensor of the adjoint (or regular) representation of the group:

$$V'_\mu = UV_\mu U^{-1} = V_\mu + ig[\theta, V_\mu], \quad (2.7)$$

which implies that

$$V_\mu^C = V_\mu^C - g C_{ABC} \theta^A V_\mu^B, \quad (2.8)$$

where repeated indices are summed up.

As a consequence of Eqs. (2.5) and (2.6), $D_\mu \phi$ has the same transformation properties as ϕ :

$$(D_\mu \phi)' = U(D_\mu \phi). \quad (2.9)$$

Thus $\mathcal{L}[\phi, D_\mu \phi]$ is indeed invariant under gauge transformations. In order to construct a gauge-invariant kinetic energy term for the gauge fields V^A , we consider

$$[D_\mu, D_\nu] \phi = ig[\partial_\mu V_\nu - \partial_\nu V_\mu + ig[V_\mu, V_\nu]] \phi \equiv ig F_{\mu\nu} \phi, \quad (2.10)$$

which is equivalent to

$$F_{\mu\nu}^A = \partial_\mu V_\nu^A - \partial_\nu V_\mu^A - g C_{ABC} V_\mu^B V_\nu^C. \quad (2.11)$$

From Eqs. (2.1), (2.9), and (2.10) it follows that the transformation properties of $F_{\mu\nu}^A$ are those of a tensor of the adjoint representation

$$F'_{\mu\nu} = U F_{\mu\nu} U^{-1}. \quad (2.12)$$

The complete Yang-Mills Lagrangian, which is invariant under gauge transformations, can be written in the form

$$\mathcal{L}_{YM} = -\frac{1}{4} \sum_A F_{\mu\nu}^A F^{\mu\nu A} + \mathcal{L}[\phi, D_\mu \phi]. \quad (2.13)$$

For an Abelian theory, as for example QED, the gauge transformation reduces to $U[\theta(x)] = \exp[ieQ\theta(x)]$, where Q is the charge generator. The associated gauge field (the photon), according to Eq. (2.6), transforms as

$$V'_\mu = V_\mu - \partial_\mu \theta(x). \quad (2.14)$$

In this case, the $F_{\mu\nu}$ tensor is linear in the gauge field V_μ , so that in the absence of matter fields the theory is free. On the other hand, in the non-Abelian case the $F_{\mu\nu}^A$ tensor contains both linear and quadratic terms in V_μ^A , so that the theory is non-trivial even in the absence of matter fields.

3. THE STANDARD MODEL OF THE ELECTROWEAK INTERACTIONS

In this section, we summarize the structure of the standard electroweak Lagrangian and specify the couplings of $W^\pm$ and Z , the intermediate vector bosons (IVBs).

For this discussion we split the Lagrangian into two parts by separating the Higgs boson couplings:

$$\mathcal{L} = \mathcal{L}_{\text{symm}} + \mathcal{L}_{\text{Higgs}}. \quad (3.1)$$

We start by specifying $\mathcal{L}_{\text{symm}}$, which involves only gauge bosons and fermions:

$$\mathcal{L}_{\text{symm}} = -\frac{1}{4} \sum_{A=1}^3 F_{\mu\nu}^A F^{\mu\nu A} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \bar{\psi}_L i \gamma^\mu D_\mu \psi_L + \bar{\psi}_R i \gamma^\mu D_\mu \psi_R. \quad (3.2)$$

This is the Yang-Mills Lagrangian for the gauge group $SU(2) \otimes U(1)$ with fermion matter fields. Here

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \quad \text{and} \quad F_{\mu\nu}^A = \partial_\mu W_\nu^A - \partial_\nu W_\mu^A - g \epsilon_{ABC} W_\mu^B W_\nu^C \quad (3.3)$$

are the gauge antisymmetric tensors constructed out of the gauge field B_μ associated with $U(1)$, and W_μ^A corresponding to the three $SU(2)$ generators; ϵ_{ABC} are the group structure constants [see Eqs. (2.3)] which, for $SU(2)$, coincide with the totally antisymmetric Levi-Civita tensor (recall the

familiar angular momentum commutators). The normalization of the SU(2) gauge coupling g is therefore specified by Eq. (3.3).

The fermion fields are described through their left-hand and right-hand components:

$$\psi_{L,R} = [(1 \mp \gamma_5)/2]\psi, \quad \bar{\psi}_{L,R} = \bar{\psi}[(1 \pm \gamma_5)/2], \quad (3.4)$$

with γ_5 and other Dirac matrices defined as in the book by Bjorken–Drell [9]. In particular, $\gamma_3^2 = 1$, $\gamma_3^3 = \gamma_5$. Note that, as given in Eq. (3.4),

$$\bar{\psi}_L = \psi_L^\dagger \gamma_0 = \psi^\dagger [(1 - \gamma_5)/2] \gamma_0 = \bar{\psi} [\gamma_0 (1 - \gamma_5)/2] \gamma_0 = \bar{\psi} [(1 + \gamma_5)/2].$$

The matrices $P_\pm = (1 \pm \gamma_5)/2$ are projectors. They satisfy the relations $P_+ P_- = P_- P_+ = 0$, $P_+ + P_- = 1$.

The sixteen linearly independent Dirac matrices can be divided into γ_5 -even and γ_5 -odd according to whether they commute or anticommute with γ_5 . For the γ_5 -even, we have

$$\bar{\psi} \Gamma_E \psi = \bar{\psi}_L \Gamma_E \psi_R + \bar{\psi}_R \Gamma_E \psi_L \quad (\Gamma_E \equiv 1, i\gamma_5, \sigma_{\mu\nu}), \quad (3.5)$$

$$\bar{\psi} \Gamma_O \psi = \bar{\psi}_L \Gamma_O \psi_L + \bar{\psi}_R \Gamma_O \psi_R \quad (\Gamma_O = \gamma_\mu, \gamma_\mu \gamma_5). \quad (3.6)$$

whilst for the γ_5 -odd,

In the Standard Model the left and right fermions have different transformation properties under the gauge group. Thus, mass terms for fermions (of the form $\bar{\psi}_L \psi_R + \text{h.c.}$) are forbidden in the symmetric limit. In particular, all ψ_k are singlets in the minimal Standard Model. But for the moment, by $\psi_{L,R}$ we mean a column vector, including all fermions in the theory that span a generic reducible representation of $SU(2) \otimes U(1)$. The standard electroweak theory is a chiral theory, in the sense that ψ_L and ψ_R behave differently under the gauge group. In the absence of mass terms, there are only vector and axial vector interactions in the Lagrangian that have the property of not mixing ψ_L and ψ_R . Fermion masses will be introduced, together with $W^\pm$ and Z masses, by the mechanism of symmetry breaking.

The covariant derivatives $D_\mu \psi_{L,R}$ are explicitly given by

$$D_\mu \psi_{L,R} = \left[\partial_\mu + ig \sum_{A=1}^3 t_{L,R}^A W_\mu^A + ig' \frac{1}{2} Y_{L,R} B_\mu \right] \psi_{L,R}, \quad (3.7)$$

where $t_{L,R}^A$ and $\frac{1}{2} Y_{L,R}$ are the SU(2) and U(1) generators, respectively, in the reducible representations $\psi_{L,R}$. The commutation relations of the SU(2) generators are given by

$$[t_L^A, t_L^B] = i \epsilon_{ABC} t_L^C \quad \text{and} \quad [t_R^A, t_R^B] = i \epsilon_{ABC} t_R^C. \quad (3.8)$$

We use the normalization Eq. (2.2) [in the fundamental representation of SU(2)]. The electric charge generator Q (in units of e , the positron charge) is given by

$$Q = t_L^3 + \frac{1}{2} Y_L = t_R^3 + \frac{1}{2} Y_R. \quad (3.9)$$

Note that the normalization of the U(1) gauge coupling g' in Eq. (3.7) is now specified.

All fermion couplings to the gauge bosons can be derived directly from Eqs. (3.2) and (3.7). The charged-current (CC) couplings are the simplest. From

$$g(t^\dagger W_\mu^1 + t^2 W_\mu^2) = g \{ [(t^1 + it^2)/\sqrt{2}] [(W_\mu^1 - iW_\mu^2)/\sqrt{2}] + \text{h.c.} \} = g \{ [(t^\dagger W_\mu^-)/\sqrt{2}] + \text{h.c.} \}, \quad (3.10)$$

where $t^\pm = t^1 \pm it^2$ and $W^\pm = (W^1 \pm iW^2)/\sqrt{2}$, we obtain the vertex

$$V_{\psi\psi W} = g \bar{\psi} \gamma_\mu [(t_L^\dagger/\sqrt{2})(1 - \gamma_5)/2 + (t_R^\dagger/\sqrt{2})(1 + \gamma_5)/2] \psi W_\mu^\pm + \text{h.c.} \quad (3.11)$$

In the neutral current (NC) sector, the photon A_μ and the mediator Z_μ of the weak NC are orthogonal and normalized linear combinations of B_μ and W_μ^3 :

$$A_\mu = \cos\theta_w B_\mu + \sin\theta_w W_\mu^3 \quad \text{and} \quad Z_\mu = -\sin\theta_w B_\mu + \cos\theta_w W_\mu^3. \quad (3.12)$$

Equations (3.12) define the weak mixing angle θ_w . The photon is characterized by equal couplings to left and right fermions with a strength equal to the electric charge. Recalling Eq. (3.9) for the charge matrix Q , we immediately obtain

$$g \sin\theta_w = g' \cos\theta_w = e, \quad (3.13)$$

or equivalently,

$$\begin{aligned} \tan\theta_w &= g'/g, \\ e &= gg'/\sqrt{g^2 + g'^2}. \end{aligned} \quad (3.14)$$

Once θ_w has been fixed by the photon couplings, it is a simple matter of algebra to derive the Z couplings, with the result

$$\Gamma_{\psi\psi Z} = (g/2 \cos\theta_w) \bar{\psi} \gamma_\mu [t_L^3(1 - \gamma_5) + t_R^3(1 + \gamma_5) - 2Q \sin^2\theta_w] \psi Z^\mu, \quad (3.15)$$

where $\Gamma_{\psi\psi Z}$ is a notation for the vertex. In the minimal Standard Model, $t_R^3 = 0$ and $t_L^3 = \pm \frac{1}{2}$.

In order to derive the effective four-fermion interactions that are equivalent, at low energies, to the CC and NC couplings given in Eqs. (3.11) and (3.15), we anticipate that large masses, as experimentally observed, are provided for $W^\pm$ and Z by $\mathcal{L}_{Higgs}$. For left-left CC couplings, when the momentum transfer squared can be neglected with respect to m_W^2 in the propagator of Born diagrams with single W exchange, from Eq. (3.11) we can write

$$\mathcal{L}_{eff}^{CC} = (g^2/8m_W^2) [\bar{\psi} \gamma_\mu (1 - \gamma_5) t_L^\dagger \psi] [\bar{\psi}' \gamma^\mu (1 - \gamma_5) t_L^- \psi']. \quad (3.16)$$

By specializing further in the case of doublet fields such as $\nu_e e^-$ or $\nu_\mu \mu^-$, we obtain the tree-level relation of g with the Fermi coupling constant G_F measured from μ decay [see Eq. (1.2)]:

$$G_F/\sqrt{2} = g^2/8m_W^2. \quad (3.17)$$

By recalling that $g \sin\theta_w = e$, we can also cast this relation in the form

$$m_W = \mu_{Born}/\sin\theta_w, \quad (3.18)$$

with

$$\mu_{Born} = (\pi\alpha/\sqrt{2} G_F)^{1/2} = 37.2802 \text{ GeV}, \quad (3.19)$$

where α is the fine-structure constant of QED ($\alpha \equiv e^2/4\pi = 1/137.036$).

In the same way, for neutral currents we obtain in Born approximation from Eq. (3.15) the effective four-fermion interaction given by

$$\mathcal{L}_{\text{eff}}^{\text{NC}} = \sqrt{2} G_F q_0 \bar{\psi} \gamma_\mu [\dots] \psi \bar{\psi} \gamma^\mu [\dots] \psi, \quad (3.20)$$

where

$$[\dots] \equiv t_L^1 (1 - \gamma_5) + t_R^1 (1 + \gamma_5) - 2Q \sin^2 \theta_w \quad (3.21)$$

and

$$q_0 = m_W^2/m_Z^2 \cos^2 \theta_w. \quad (3.22)$$

All couplings given in this section are obtained at tree level and are modified in higher orders of perturbation theory. In particular, the relations between m_W and $\sin \theta_w$ [Eqs. (3.18) and (3.19)] and the observed values of q ($q = q_0$ at tree level) in different NC processes, are altered by computable electroweak radiative corrections, as discussed in Section 5.

The gauge-boson self-interactions can be derived from the $F_{\mu\nu}$ term in $\mathcal{L}_{\text{symm}}$, by using Eq. (3.12) and $W_\mu^\pm = (W^1 \pm iW^2)/\sqrt{2}$. Defining the three-gauge-boson vertex as in Fig. 1, we obtain ($V \equiv \gamma, Z$)

$$\Gamma_{W^-W^+V} = ig_{W^-W^+V} [g_{\mu\nu}(q-p)_\lambda + g_{\lambda\mu}(p-\tau)_\nu + g_{\lambda\nu}(\tau-q)_\mu], \quad (3.23)$$

with

$$g_{W^-W^+\gamma} = g \sin \theta_w = e \quad \text{and} \quad g_{W^-W^+Z} = g \cos \theta_w. \quad (3.24)$$

The partial widths for the decay of W and Z into a massless fermion-antifermion pair, including first-order strong and electroweak radiative corrections, are given by

$$\Gamma(W \rightarrow f\bar{f}) = N(G_F m_W^3/6\pi\sqrt{2})(1 + \delta_f^W) \quad (3.25)$$

$$\Gamma(Z \rightarrow f\bar{f}) = N(G_F q_0 m_Z^3/24\pi\sqrt{2})[1 + (1 - 4|Q_f| \sin^2 \theta_w)^2(1 + \delta_f^Z)], \quad (3.26)$$

where $q_0 = 1$ for doublet Higgses; δ_f^W and δ_f^Z are known electroweak radiative corrections, to be discussed in Sections 5 to 9 and are given explicitly, for example, in Ref. [10]; and

$$\begin{aligned} N &= 1 && \text{for leptons,} \\ N &= 3(1 + \alpha_s/\pi + \dots) && \text{for quarks.} \end{aligned} \quad (3.27)$$

Fig. 1 The three-gauge boson vertex

Here α_s is the QCD coupling. We can define α_s according to the $\overline{\text{MS}}$ prescription. In this case the second-order correction is also known [11]. From experimental determinations of A_{QCD} [12], we obtain a value of $\alpha_s(m_Z)$ in the range

$$\alpha_s(m_Z) = 0.11 \pm 0.01. \quad (3.28)$$

In the case of W decays, formula (3.25) refers to the sum of all down-quarks associated with a given up-quark. For a particular down-quark, q' , a factor $|V_{qq'}|^2$ would appear, where $V_{qq'}$ is the relevant term of the Cabibbo-Kobayashi-Maskawa matrix ($\sum_{q'} |V_{qq'}|^2 = 1$ by unitarity). Note that by writing $\Gamma_{\text{Born}} \approx G_F^2 M^3$, instead of $\Gamma_{\text{Born}} \approx g^4 M^3 \approx \alpha^2 M^3/\sin^2 \theta_w$, we make the Born approximation more precise (i.e. $\delta_f^{W,Z}$ smaller). In fact the radiative corrections are large on both α and $\sin^2 \theta_w$. On the other hand, there are no leading logs in the scale dependence of the Fermi coupling G_F [13]; $\delta_f^{W,Z}$ are very small ($\delta_f \approx 0.1\%$) if $m_i \approx m_Z$ and there are no additional heavy-fermion generations [provided the physical values of $m_{W,Z}$ and $\sin^2 \theta_w$ are inserted in Eqs. (3.25) and (3.26)]. The electroweak radiative corrections become considerably larger if m_i is large or if weak isospin multiplets with large mass splittings exist [10]. We shall come back to this case in Sections 5 to 9. With the exception of this case, for example, we obtain

$$\Gamma(W \rightarrow e\nu) = 224 \quad (m_W/80 \text{ GeV})^3 \text{ MeV}, \quad (3.29)$$

$$\Gamma(Z \rightarrow e^+e^-) = 83.5 \quad (m_Z/91 \text{ GeV})^3 \text{ MeV}, \quad (3.30)$$

$$\Gamma(Z \rightarrow \nu_e \bar{\nu}_e) = 166 \quad (m_Z/91 \text{ GeV})^3 \text{ MeV}. \quad (3.31)$$

Note that the dependence of $\Gamma(Z \rightarrow ee)$ on $\sin^2 \theta_w$ is very small for $\sin^2 \theta_w \approx 0.23$. If a careful determination of the partial widths is required, one should implement the dependence of $\sin^2 \theta_w$ on m_Z given by Eq. (5.1) and take δ_f^W and δ_f^Z into account, which are large if $m_i > m_Z$. This will be done in Sections 5 to 9.

For $W \rightarrow t\bar{b}$ and $Z \rightarrow t\bar{t}$ (and perhaps $Z \rightarrow b\bar{b}$) the quark-mass corrections cannot be neglected. The widths in Born approximation are then modified as follows [14]:

$$\begin{aligned} \Gamma_{\text{Born}}(W \rightarrow t\bar{b}) &= (G_F m_W^3/2\pi\sqrt{2}) \{ [(m_W^2 - m_t^2 - m_b^2)^2 - 4m_b^2 m_t^2]/m_W^4 \}^{1/2} \\ &\quad \times \{ 1 - (m_t^2 + m_b^2)/2m_W^2 - 1/2[(m_t^2 + m_b^2)/m_W^2]^2 \} \\ &\approx (G_F m_W^3/2\pi\sqrt{2}) (1 - \epsilon)^2 (1 + \epsilon/2), \end{aligned} \quad (3.32)$$

$$\Gamma_{\text{Born}}(Z \rightarrow Q\bar{Q}) = (G_F q_0 m_Z^3/8\pi\sqrt{2}) [\beta^3 + (1 - 4|Q_Q| \sin^2 \theta_w)^2 1/2 \beta(3 - \beta^2)], \quad (3.33)$$

with $\epsilon = m_t^2/m_W^2$ and $\beta = [1 - (4m_b^2/m_Z^2)]^{1/2}$. The QCD corrections of order α_s are also modified with respect to the massless case and are given in Ref. [15].

The leptonic branching ratios are independent of m_W and m_Z (at fixed $\sin^2 \theta_w$). We obtain

$$\text{BR}(W \rightarrow e\nu) = 0.089, \quad m_t = 40 \text{ GeV}, \quad (3.34)$$

$$\text{BR}(W \rightarrow e\nu) \approx 0.109, \quad m_t > m_W - m_b, \quad (3.35)$$

$$\text{BR}(Z \rightarrow e^+e^-) \approx 0.034, \quad (3.35)$$

$$\text{BR}(Z \rightarrow \sum_{i=e,\mu,\tau} \nu_i \bar{\nu}_i) \approx 0.20, \quad (3.36)$$

if $m_t > m_Z/2$ and $\sin^2 \theta_w \approx 0.23$.

The total widths Γ_W and Γ_Z are given to a good approximation by summing up the rates for $V \rightarrow f\bar{f}$ discussed above, because the rare decays of W/Z in the Standard Model are really rare and can be neglected. For example, for $m_t > m_W - m_b$, we obtain

$$\Gamma_W \equiv \Gamma(W \rightarrow e\nu)/\text{BR}(W \rightarrow e\nu) = 2.06(m_W/80 \text{ GeV})^3 \text{ GeV}. \quad (3.37)$$

We now turn to the Higgs sector [16] of the electroweak Lagrangian. Here we simply review the formalism of the Higgs mechanism applied to the electroweak theory. In the next section we shall discuss in more detail the physics of the electroweak symmetry breaking. The Higgs Lagrangian is specified by the gauge principle and the requirement of renormalizability to be

$$\mathcal{L}_{\text{Higgs}} = (D_\mu \phi)^\dagger (D^\mu \phi) - V(\phi^\dagger \phi) - \bar{\psi}_L \Gamma^\dagger \psi_R \phi - \bar{\psi}_R \Gamma^\dagger \psi_L \phi^\dagger, \quad (3.38)$$

where ϕ is a column vector including all Higgs fields; it transforms as a reducible representation of the gauge group. The quantities Γ (which include all coupling constants) are matrices that make the Yukawa couplings invariant under the Lorentz and gauge groups. The potential $V(\phi^\dagger \phi)$, symmetric under $SU(2) \otimes U(1)$, contains, at most, quartic terms in ϕ so that the theory is renormalizable. Spontaneous symmetry breaking is induced if the minimum of V —which is the classical analogue of the quantum mechanical vacuum state (both are the states of minimum energy)—is obtained for non-vanishing ϕ values. Precisely, we denote the vacuum expectation value (VEV) of ϕ , i.e. the position of the minimum, by v :

$$\langle 0|\phi(x)|0\rangle = v \neq 0. \quad (3.39)$$

The fermion mass matrix is obtained from the Yukawa couplings by replacing $\phi(x)$ by v :

$$M = \bar{\psi}_L \mathfrak{M} \psi_R + \bar{\psi}_R \mathfrak{M}^\dagger \psi_L, \quad (3.40)$$

with

$$\mathfrak{M} = \Gamma \cdot v. \quad (3.41)$$

In the minimal Standard Model, where all left fermions ψ_L are doublets and all right fermions ψ_R are singlets, only Higgs doublets can contribute to fermion masses. There are enough free couplings in Γ , so that one single complex Higgs doublet is indeed sufficient to generate the most general fermion mass matrix. It is important to observe that by a suitable change of basis we can always make the matrix $\mathfrak{M}$ Hermitian, γ_5 -free, and diagonal. In fact, we can make separate unitary transformations on ψ_L and ψ_R according to

$$\psi_L = U \psi_L', \quad \psi_R = V \psi_R' \quad (3.42)$$

and consequently

$$\mathfrak{M} \rightarrow \mathfrak{M}' = U^\dagger \mathfrak{M} V. \quad (3.43)$$

This transformation does not alter the general structure of the fermion couplings in $\mathcal{L}_{\text{symm}}$. For quarks, the Cabibbo-Kobayashi-Maskawa unitary transformation relates the mass eigenstates d, s ,

and b to the CC eigenstates d', s', b' , i.e. the states coupled by W emission to u, c , and t , respectively. The NC is then automatically diagonal in flavour at tree level (GIM mechanism [5]). In the case of leptons, if the neutrinos are massless then clearly there is no mixing.

If only one Higgs doublet is present, the change of basis that makes M diagonal will at the same time diagonalize also the fermion-Higgs Yukawa couplings. Thus, in this case, no flavour-changing neutral Higgs exchanges are present. This is not true, in general, when there are several Higgs doublets. But one Higgs doublet for each electric charge sector—i.e. one doublet coupled only to u -type quarks, one doublet to d -type quarks, one doublet to charged leptons—would also be all right [17], because the mass matrices of fermions with different charges are diagonalized separately. For several Higgs doublets it is also possible to generate CP violation by complex phases in the Higgs couplings [18]. In the presence of six quark flavours, this CP violation mechanism is not necessary. In fact, at the moment, the simplest model with only one Higgs doublet seems adequate for describing all observed phenomena.

We recall that the Standard Model, with N fermion families with the observed quantum numbers, is automatically free of γ_5 anomalies [19] owing to cancellation of quarks with lepton loops.

We now consider the gauge-boson masses and their couplings to the Higgs. These effects are induced by the $(D_\mu \phi)^\dagger (D^\mu \phi)$ term in $\mathcal{L}_{\text{Higgs}}$ [Eq. (3.38)], where

$$D_\mu \phi = \left[\partial_\mu + ig \sum_{A=1}^3 t^A W_\mu^A + ig' (Y/2) B_\mu \right] \phi. \quad (3.44)$$

Here t^A and $1/2 Y$ are the $SU(2) \otimes U(1)$ generators in the reducible representation spanned by ϕ . Not only doublets but all non-singlet Higgs representations can contribute to gauge-boson masses. The condition that the photon remains massless is equivalent to the condition that the vacuum is electrically neutral:

$$Q|v\rangle = (t^3 + 1/2 Y)|v\rangle = 0. \quad (3.45)$$

The charged W mass is given by the quadratic terms in the W field arising from $\mathcal{L}_{\text{Higgs}}$, when $\phi(x)$ is replaced by v . We obtain

$$m_W^2 W_\mu^+ W^{-\mu} = g^2 [(t^+ v/\sqrt{2})]^2 W_\mu^+ W^{-\mu}, \quad (3.46)$$

whilst for the Z mass we get [recalling Eq. (3.12)]

$$1/2 m_Z^2 Z_\mu Z^\mu = [g \cos \theta_w t^3 - g' \sin \theta_w (Y/2)] v^2 Z_\mu Z^\mu, \quad (3.47)$$

where the factor of $1/2$ on the left-hand side is the correct normalization for the definition of the mass of a neutral field. By using Eq. (3.45), relating the action of t^3 and $1/2 Y$ on the vacuum v , and Eqs. (3.14), we obtain

$$1/2 m_Z^2 = (g \cos \theta_w + g' \sin \theta_w)^2 |t^3 v|^2 = (g^2/\cos^2 \theta_w) |t^3 v|^2. \quad (3.48)$$

For Higgs doublets

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad v = \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad (3.49)$$

we have

$$|t^3 v|^2 = v^2, \quad |t^3 v|^2 = 1/4 v^2, \quad (3.50)$$

so that

$$m_W^2 = \frac{1}{2}g^2 v^2, \quad m_Z^2 = \frac{1}{2}g^2 v^2 / \cos^2 \theta_w. \quad (3.51)$$

Note that by using Eq. (3.17) we obtain

$$v = 2^{-3/4} G_F^{-1/2} = 174.1 \text{ GeV}. \quad (3.52)$$

It is also evident that for Higgs doublets

$$\phi_0 = m_W^2 / m_Z^2 \cos^2 \theta_w = 1. \quad (3.53)$$

This relation is typical of one or more Higgs doublets and would be spoiled by the existence of Higgs triplets etc. In general,

$$\phi_0 = \sum_i [(t_i)^2 - (b_i)^2 + t_i] v_i^2 / \sum_i 2(t_i^2) v_i^2 \quad (3.54)$$

for several Higgses with VEVs v_i , weak isospin t_i , and z-component b_i . These results are valid at the tree level and are modified by calculable electroweak radiative corrections, as discussed in Section 5.

If only one Higgs doublet is present, then the fermion-Higgs couplings are in proportion to the fermion masses. In fact, from the Yukawa couplings $g_{\phi f i} (\bar{f}_L \phi f_R + \text{h.c.})$, the mass m_f is obtained by replacing ϕ by v , so that $g_{\phi f i} = m_f / v$.

With only one complex Higgs doublet, three out of the four Hermitian fields are removed from the physical spectrum by the Higgs mechanism and become the longitudinal modes of W^+ , W^- , and Z . The fourth neutral Higgs is physical and should be found. If more doublets are present, two more charged and two more neutral Higgs scalars should be around for each additional doublet.

Finally, the couplings of the physical Higgs H to the gauge bosons can be simply obtained from $\mathcal{L}_{\text{Higgs}}$, by the replacement

$$\phi(x) = \begin{pmatrix} \phi^+(x) \\ \phi^0(x) \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ v + (H/\sqrt{2}) \end{pmatrix}, \quad (3.55)$$

[so that $(D_\mu \phi)^\dagger (D^\mu \phi) = \frac{1}{2}(\partial_\mu H)^2 + \dots$], with the result

$$\begin{aligned} \mathcal{L}[H, W, Z] = & g^2 (v/\sqrt{2}) W_\mu^+ W^{-\mu} H + (g^2/4) W_\mu^+ W^{-\mu} H^2 \\ & + [(g^2 v Z_\mu Z^\mu)/(2\sqrt{2} \cos^2 \theta_w)] H + [g^2/(8 \cos^2 \theta_w)] Z_\mu Z^\mu H^2. \end{aligned} \quad (3.56)$$

We have thus completed our summary of the standard electroweak theory and of the W^+ , Z couplings.

4. THE HIGGS AND BEYOND: THE PROBLEM OF THE FERMI SCALE

The gauge symmetry of the Standard Model was difficult to discover because it is well hidden in nature. The only observed gauge boson that is massless is the photon. The graviton is still unobserved even at the classical level of gravitational waves; the gluons are presumed massless but unobservable because of confinement, and the W and Z weak bosons carry a heavy mass. Actually the main difficulty in unifying weak and electromagnetic interactions was the fact that e.m. interactions have infinite range ($m_\gamma = 0$), whilst the weak forces have a very short range, owing to $m_W, m_Z \neq 0$.

The solution of this problem is in the concept of spontaneous symmetry breaking, which was borrowed from statistical mechanics.

Consider a ferromagnet at zero magnetic field in the Landau-Ginzburg approximation. The free energy in terms of the temperature T and the magnetization $\mathbf{M}$ can be written as

$$F(\mathbf{M}, T) = F_0(T) + \frac{1}{2}\mu^2(T)\mathbf{M}^2 + \frac{1}{4}\lambda(T)(\mathbf{M}^2)^2 + \dots \quad (4.1)$$

This is an expansion which is valid at small magnetization, and which is the analogue in this context of the renormalizability criterion; $\lambda(T) > 0$ is assumed for stability; F is invariant under rotations, i.e. all directions of $\mathbf{M}$ in space are equivalent. The minimum condition for F reads

$$\partial F / \partial \mathbf{M} = 0, \quad [\mu^2(T) + \lambda(T)\mathbf{M}^2]\mathbf{M} = 0. \quad (4.2)$$

There are two cases. If $\mu^2 > 0$, then the only solution is $\mathbf{M} = 0$, there is no magnetization, and the rotation symmetry is respected. If $\mu^2 < 0$, then another solution appears, which is

$$|\mathbf{M}_0|^2 = -\mu^2/\lambda. \quad (4.3)$$

The direction chosen by the vector $\mathbf{M}_0$ is a breaking of the rotation symmetry. The critical temperature T_{crit} is where $\mu^2(T)$ changes sign:

$$\mu^2(T_{\text{crit}}) = 0. \quad (4.4)$$

It is simple to realize that the Goldstone theorem holds. It states that when spontaneous symmetry breaking takes place, there is always a zero-mass mode in the spectrum. In a classical context this can be proven as follows. Consider a Lagrangian

$$\mathcal{L} = |\partial_\mu \phi|^2 - V(\phi) \quad (4.5)$$

symmetric under the infinitesimal transformations

$$\phi \rightarrow \phi' = \phi + \delta\phi, \quad \delta\phi_i = i\delta\theta_{ij}\phi_j. \quad (4.6)$$

The minimum condition on V that identifies the equilibrium position (or the ground state in quantum language) is

$$(\partial V / \partial \phi_i)(\phi_i = \phi_i^0) = 0. \quad (4.7)$$

The symmetry of V implies that

$$\delta V = (\partial V / \partial \phi_i) \delta\phi_i = i\delta\theta_{ij}(\partial V / \partial \phi_i)\phi_j = 0. \quad (4.8)$$

By taking a seconde derivative at the minimum $\phi_i = \phi_i^0$ of the previous equation, we obtain

$$\partial^2 V / \partial \phi_k \partial \phi_i (\phi_i = \phi_i^0) \eta_{ij} \phi_j^0 + (\partial V / \partial \phi_i)(\phi_i = \phi_i^0) \eta_{ik} = 0. \quad (4.9)$$

The second term vanishes owing to the minimum condition, Eq. (4.7). We then find

$$\partial^2 V / \partial \phi_k \partial \phi_i (\phi_i = \phi_i^0) \eta_{ij} \phi_j^0 = 0. \quad (4.10)$$

The second derivatives $M_{\phi_i \phi_j}^2 = (\partial^2 V / \partial \phi_i \partial \phi_j)(\phi_i = \phi_j = \phi^0)$ define the squared mass matrix. Thus the above equation in matrix notation can be read as

$$M^2 \phi^0 = 0, \quad (4.11)$$

which shows that if the vector (ϕ^0) is non-vanishing, i.e. there is some generator that shifts the ground state into some other state with the same energy, then ϕ^0 is an eigenstate of the squared mass matrix with zero eigenvalue. Therefore, a massless mode is associated with each broken generator.

When spontaneous symmetry breaking takes place in a gauge theory, the massless Goldstone mode exists, but it is unphysical and disappears from the spectrum. It becomes, in fact, the third helicity state of a gauge boson that takes mass. This is the Higgs mechanism. Consider, for example, the simplest Higgs model described by the Lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \{(\partial_\mu - ieA_\mu)\phi\}^2 + \frac{1}{2}\mu^2\phi^*\phi - (\lambda/4)(\phi^*\phi)^2. \quad (4.12)$$

Note the 'wrong' sign in front of the mass term for the scalar field ϕ , which is necessary for the spontaneous symmetry breaking to take place. The above Lagrangian is invariant under the $U(1)$ gauge symmetry

$$A_\mu \rightarrow A'_\mu = A_\mu - (1/e)\partial_\mu\theta(x), \quad \phi \rightarrow \phi' = \phi \exp[i\theta(x)]. \quad (4.13)$$

Let $\phi^0 = v \neq 0$, with v real, be the ground state that minimizes the potential and induces the spontaneous symmetry breaking. Making use of gauge invariance, we can make the change of variables

$$\phi(x) \rightarrow (1/\sqrt{2})[e(x) + v] \exp[i\tilde{\chi}(x)/v], \quad A_\mu(x) \rightarrow A_\mu - (1/ev)(\partial_\mu\tilde{\chi}(x)). \quad (4.14)$$

Then $e = 0$ is the position of the minimum, and the Lagrangian becomes

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}e^2v^2A_\mu^2 + \frac{1}{2}e^2\tilde{\chi}^2A_\mu^2 + e^2vA_\mu + \mathcal{L}(\tilde{\chi}). \quad (4.15)$$

The field $\tilde{\chi}(x)$, which corresponds to the would-be Goldstone boson, disappears, whilst the mass term $\frac{1}{2}e^2v^2A_\mu^2$ for A_μ is now present; e is the massive Higgs particle.

The Higgs mechanism is realized in well-known physical situations. For a superconductor in the Landau-Ginzburg approximation the free energy can be written as

$$F = F_0 + \frac{1}{2}\mathbf{B}^2 + [(\nabla - 2ie\mathbf{A})\phi]^2/4m - \alpha|\phi|^2 + \beta|\phi|^4. \quad (4.16)$$

Here $\mathbf{B}$ is the magnetic field, $|\phi|^2$ is the Cooper pair (e^-e^-) density, $2e$ and $2m$ are the charge and mass of the Cooper pair. The 'wrong' sign of α leads to $\phi \neq 0$ at the minimum. This is precisely the non-relativistic analogue of the Higgs model of the previous example. The Higgs mechanism implies the absence of propagation of massless phonons (states with dispersion relation $\omega = kv$ with constant v). Also the mass term for $\mathbf{A}$ is manifested by the exponential decrease of $\mathbf{B}$ inside the superconductor (Meissner effect).

Thus the Higgs effect is not an abstract device, but it is endowed with a precise physical reality. In the electroweak theory it is absolutely necessary in order to give masses to the W 's and Z^0 and to the fermions, as well as to ensure the correct high-energy behaviour required by renormalizability.

However a more profound physical reality could be hidden behind or accompany the Higgs formalism. In fact, the Higgs mechanism is at present without experimental support. Actually the

clarification of the physical origin of the electroweak symmetry breaking is one of the most important problems for experimental particle physics in the next decade. There are arguments indicating that the minimal Standard Model with fundamental Higgs fields cannot be the whole story and that some kind of new physics must necessarily appear near the Fermi scale. The most famous argument of this type is based on the so-called 'hierarchy problem', which we now describe.

There is no unification of the fundamental forces in the Standard Model, because a separate gauge group and coupling is introduced for each interaction. On the other hand, the structural unity implied by the common, restrictive property of gauge invariance strongly suggests the possibility that all the observed interactions actually stem from a unified theory at some more fundamental level. The idea of unification at energies of order $m_{\text{GUT}} \approx 10^{15}$ GeV, below the energy scale where quantum gravity becomes effective at masses of the order of the Planck mass, $m_P \approx 10^{19}$ GeV, has been much studied in recent years. However, the question remains whether unification without the inclusion of quantum gravity is really plausible. It is clear that the inclusion of gravity must induce major changes in the physics of the Standard Model at energies of order m_P and possibly even below.

Thus, at least because of the fact that gravity is not included in the Standard Model, new physics must necessarily emerge at some large energy scale Λ (equivalent to some small distance scale). Then the problem is to understand what order of magnitude can reasonably be expected for Λ . In particular, we can ask whether it is natural to expect that Λ may be as large as m_P or m_{GUT} . In other words, is it possible that the Standard Model holds without any new physical input up to the energy scale of quantum gravity? The answer is probably negative, because then we could not naturally explain the enormous value of m_P/m_W , i.e. the ratio between the Planck and the Fermi scales of masses.

To develop this point further, we recall that in the Standard Model the fermion and vector-boson masses are all specified in terms of the VEV of the Higgs field v , according to Eqs. (3.41) and (3.51). The value of v is determined by the curvature scale of the Higgs potential V :

$$V(\phi) = -\frac{1}{2}\mu^2\phi^\dagger\phi + (\lambda/4)(\phi^\dagger\phi)^2, \quad (4.17)$$

according to

$$v = \mu/\sqrt{\lambda}. \quad (4.18)$$

The observed values of the masses require for v (and, therefore, roughly for μ as well) that $v \approx 10^2$ GeV [see Eq. (3.52)].

If $\Lambda \approx (10^{15} - 10^{19})$ GeV, then we face the problem of justifying the presence of two so largely different mass scales in a single theory (the so-called hierarchy problem). In general, if Λ is very large in comparison with μ , then, even if we set by hand a small value for μ at the tree level, the radiative corrections would make μ increase up to nearly the order of Λ . This problem is particularly acute in theories with scalars, as in the Standard Model, because the degree of divergence of mass corrections is quadratic, whilst the same divergences are only logarithmic for spin- $1/2$ fermions.

One general way out would be that the limit $\mu \rightarrow 0$ corresponds to an increase of the symmetry of the theory. In fact, the observed value of μ^2 can be decomposed as

$$\mu^2 = \mu_0^2 + \delta\mu^2, \quad (4.19)$$

where μ_0 is the tree-level value and $\delta\mu^2$ arises from the loop quantum corrections. If no new symmetry is induced when $\mu \rightarrow 0$ and no other non-renormalization theorem is operative, then a small value for μ^2/Λ^2 can only arise from an unbelievably precise cancellation between μ_0^2/Λ^2 and $\delta\mu^2/\Lambda^2$. If, however, $\mu = 0$ leads to an additional symmetry, then $\delta\mu^2$ must be proportional to μ_0^2 , because for μ_0

= 0 both the tree diagrams and the loop corrections must respect the symmetry. Then, if one starts from a small value of $\hat{\mu}_0^2/\Lambda^2$, the radiative corrections would preserve the smallness of $\hat{\mu}^2/\Lambda^2$.

For fermions, chiral symmetry is added when $m_f \rightarrow 0$, because the axial currents are also conserved in this limit, as their divergence is proportional to the fermion mass. Chiral symmetry and the logarithmic degree of divergence for fermion masses considerably alleviate the hierarchy problem in theories with no fundamental scalars.

In the Standard Model no additional symmetry is gained for $\mu_0 = 0$. This is also seen from the explicit formula for $\delta\mu^2$ at the one-loop level, which shows that $\delta\mu^2$ is not proportional to $\hat{\mu}_0^2$:

$$\mu^2/\Lambda^2 = (\hat{\mu}_0^2/\Lambda^2) + (1/128\pi^2)(d/dv)^2 \sum_j (2J + 1)(-1)^{2j} m_j^2(v) + \dots, \quad (4.20)$$

where terms which vanish with $\Lambda \rightarrow \infty$ are indicated by the dots. The sum over J includes both particles and antiparticles (counted separately) of spin J and mass m_j (expressed as a function of v). Thus we are forced to the conclusion that in the Standard Model the natural value for $\delta\mu^2/\Lambda^2$ is of order one or so. Therefore, as Λ can be interpreted as the energy scale where some essentially new physical ingredient becomes important, we are led to expect that the validity of the present framework cannot be extended beyond $\Lambda \approx (1-10)$ TeV.

The problem of explaining the Fermi scale is seen to be closely connected to the Higgs mechanism and to the consequent presence of scalars, which makes the problem of testing the Higgs sector particularly crucial.

One possible solution is that the Higgses are really scalar fundamental fields, but naturality is restored by supersymmetry.

Supersymmetry (SUSY for short) [20] relates bosons and fermions, so that in a multiplet that forms one representation of supersymmetry there is an equal number of bosonic and fermionic degrees of freedom. This implies that SUSY generators are spin- $1/2$ charges Q_α . SUSY leads to an extension of the Poincaré algebra. Besides the obvious algebraic relations between Q_α and the Poincaré generators, which specify the spinorial transformations of Q_α under Lorentz transformations and its invariance under translations, the essentially new relation is the anticommutator

$$\{Q_\alpha, \bar{Q}_\beta\} = -2(\gamma_\mu)_\alpha\beta P^\mu, \quad (4.21)$$

where P^μ is the energy-momentum four-vector, which generates space-time translations.

If all fundamental symmetries are gauge symmetries, then also SUSY is presumably a local symmetry. This immediately leads to the realm of gravity. In fact, the product of two local SUSY transformations is a translation with space-time-dependent parameters, as follows from Eq. (4.21). But a translation with space-time-dependent parameters is a general coordinate transformation. As ordinary gravity can be seen to arise from gauging the Poincaré group, a similar gauging of the Poincaré algebra enlarged by SUSY generators leads to an extended version of gravity, called supergravity. In fact, supersymmetry and supergravity play a crucial role in most present attempts at constructing a sensible theory of quantum gravity, including the very interesting superstring theories [21] that at present represent the most advanced and promising project for a theory of gravity (and of all particle interactions).

Theorists like SUSY for several reasons. SUSY is the maximum symmetry compatible with a non-trivial S-matrix in a local relativistic field theory. The powerful constraints between couplings and masses implied by SUSY drastically reduce the degree of singularity of field theory as deduced from power counting. In some cases a finite field theory is even obtained. For example, $N = 4$ extended SUSY Yang-Mills theories are finite in four dimensions. In general, more powerful non-renormalization theorems are deduced for SUSY theories. This property may solve many

naturalness problems of the Standard Model. In particular, the hierarchy problem can be solved in SUSY theories because the mass divergences of bosons and fermions become the same, and the quadratic divergences of scalars are reduced to logarithmic divergences as for spin- $1/2$ fermions. Thus in SUSY theories, we automatically obtain

$$(d/dv)^2 \sum_j [(2J + 1)(-1)^{2j} m_j^2(v)] = 0, \quad (4.22)$$

because of a cancellation between bosons and fermions. Of course the cancellation is only exact in the limit of unbroken SUSY. But we know that SUSY must be broken because the SUSY partners of ordinary particles have not been observed. In broken SUSY, the Λ , appearing in Eq. (4.20), can be identified with the mass scale that is typical of SUSY partners of ordinary states. Thus if Λ is of order $G_F^{1/2}$ or at most $O(1)$ TeV, then the hierarchy problem would be solved in a natural way. This is the only known way out of the hierarchy problem compatible with fundamental scalar Higgs fields.

We have seen that most theorists working on quantum gravity and superstrings tend to consider SUSY as 'established' at m_P and beyond. For economy, we are naturally led to also try to use SUSY at low energy to solve some of the problems of the Standard Model, including the hierarchy problem. It is therefore very important that it was indeed shown [22] that models where SUSY is softly broken by gravity do offer a viable alternative. We stress again that the supersymmetric way is very appealing to theorists. In fact, it would represent the ultimate triumph of a continuous line of progress obtained by constructing field theories with an increasing degree of exact and/or broken symmetry and applying them to fundamental interactions. Also the value of the ratio of knowledge versus ignorance would be remarkably large in the case of SUSY: the correct degrees of freedom for a description of physics up to gravity would have been identified, the Hamiltonian would be essentially known, and the theory would, to a large extent, be computable up to m_P .

The alternative main avenue to physics beyond the Standard Model is compositeness or, more generally, the existence of new strong forces. For example, the electroweak symmetry could be broken by condensates of new fermions attracted by a new force with $\Lambda_{\text{new}} \approx m_F \approx G_F^{1/2}$ (Λ_{new} being the analogous quantity of Λ_{QCD}), as in technicolour theories [23]. Or the Higgs scalar can be a composite of new fermions bound by a new force [24]. In the last two cases there are unsolved problems related to the fermion masses. Or the $SU(2) \otimes U(1)$ gauge symmetry can be a low-energy fake [25]. At high energies $\approx m_F$, the W and Z^0 would be resolved into their constituents.

Another possibility is that the Higgs mass becomes very large [26] [$O(1)$ TeV]. Then, as we shall see, the weak interactions become strong and a spectroscopy of tightly bound weak interaction resonances appears (e.g. WW , WZ , or ZZ states).

However, it is fair to say that the compositeness alternative is not at all so neat and clearly formulated as the supersymmetric option. On the contrary, in many respects the compositeness way is not well defined at all and leads to many unsolved problems.

Of course, the two avenues are not necessarily mutually exclusive, and theoretical frameworks where both appear have been considered.

Searching for the standard Higgs particle appears to be a good way to organize the experimental solution of the symmetry-breaking problem. Note that the information we have regarding the mass of the standard Higgs is not very abundant. The experimental lower bound [27] on the Higgs mass from atomic and nuclear physics (absence of long-range forces),

$$m_H \geq 15 \text{ MeV}, \quad (4.23)$$

is rather solid, unless perhaps for $m_H < 2m_e$. The limit $m_H \geq 325 \text{ MeV}$, which was claimed in the past to derive from $K^+ \rightarrow \pi^+ + H$, was later criticized [28]. In a recent reanalysis [29] of the problem, the excluded domain was reported to be $50 < m_H < 140 \text{ MeV}$. However, some model

dependence is clearly unavoidable in this case (from hadronic matrix elements). The most important experimental input is the lower limit on m_H that can be obtained from the search for the decay $T \rightarrow H + \gamma$ [30]. If we assume the validity of the theoretical prediction from the Wilczek [31] formula plus first-order QCD corrections [32], then CUSB data lead to $m_H > 5$ GeV (or $m_H < 2m_p$). However, the QCD corrections are large so that the higher orders can also be of importance. Also, corrections to the non-relativistic approximation could be sizeable. It is therefore difficult to fix a precise value for the lower bound. However, I think that the data are by now good enough to make a minimal standard Higgs of a few GeV unlikely. A much better limit will soon be available from LEP. Of course, it is sufficient to introduce two Higgs doublets in order to evade this limit (by taking suitable values for the ratio of VEVs).

Vacuum stability can be considered as a criterion for deriving theoretical lower limits on the Higgs mass [33]. At tree level the scalar potential is given by

$$V(\phi) = \mu^2 |\phi|^2 - (\mu^2/2v^2) |\phi|^4. \quad (4.24)$$

The quantum corrections can be computed by expanding in the number of loops. At one loop we obtain

$$V(\phi) = \mu^2 |\phi|^2 - (\mu^2/2v^2) |\phi|^4 + \gamma |\phi|^4 [\ln(|\phi|^2/v^2) - (1/2)], \quad (4.25)$$

with

$$\gamma = \left[3 \sum_{\text{vectors}} m_v^4 + \sum_{\text{scalars}} m_s^4 - 4 \sum_{\text{fermions}} m_f^4 \right] / (64\pi^2 v^4). \quad (4.26)$$

It is simple to check that, also in the corrected form, v is an extremum of $V(\phi)$. In the minimal Standard Model with one Higgs doublet and three fermion families, we obtain

$$\gamma = (3m_Z^4 + 6m_W^4 + m_H^4 - 12m_t^4) / (64\pi^2 v^4). \quad (4.27)$$

(The extra factor of three in front of m_t^4 is, of course, due to colour). From the requirements that $|\phi| = v$ be a minimum and that $V(\phi) \rightarrow +\infty$ for $|\phi| \rightarrow \infty$, we obtain the lower bound shown in Fig. 2. In particular, for small m_t we obtain the Linde-Weinberg [34] bound $m_H \approx 7$ GeV. For large m_t , large values of m_H are needed in order to stabilize the potential at $|\phi| \rightarrow \infty$ (the one-loop approximation has to be improved by renormalization group techniques in this region, as was done in the

Fig. 2 The dashed region is the allowed region for the minimal Higgs mass from vacuum stability [33]

Fig. 3 The allowed parameter space of m_H and m_t for various embedding scales Λ (see Ref. [38]). The area around the origin bounded by the various curves is allowed. The curve labelled 'Landau pole' is obtained from Ref. [37]. The horizontal lines come from avoiding triviality, and the vertical lines are determined from $\lambda(t)$ becoming negative at scales lower than Λ .

computations of Fig. 3). These are certainly very interesting results. The problem is that the limit evaporates if a) $m_t \approx 80$ GeV, or b) there are more Higgs doublets, or c) we allow for a metastable vacuum with long enough lifetime. In these cases there is no limit for $80 \leq m_t \leq 200$ GeV [35].

On the other hand, the theoretical upper bounds on m_H are all based, in one form or another, on requiring perturbation theory to hold up to some large energy Λ . But this perhaps appealing requirement is in no way necessary. As is well known, the coupling of the quartic term $\lambda(\phi^\dagger \phi)^2$ in the Higgs potential increases with m_H^2 , because $m_H^2 \propto \lambda/C_F$. In addition, for a given m_H , λ increases logarithmically with energy since the theory is not asymptotically free in the Higgs sector. Then, requiring perturbation theory to be valid up to $\Lambda = m_P$ leads to [36]

$$m_H \approx 200 \text{ GeV}. \quad (4.28)$$

Similarly, from the requirement that problems due to the possible triviality of the $\lambda\phi^4$ theory be avoided, the limit

$$m_H \approx 125 \text{ GeV} \quad (4.29)$$

was obtained [37]. The upper curves of Fig. 3, taken from Ref. [38], are obtained merely by imposing that the theory remains non-trivial up to a given scale Λ . The same picture has been essentially

confirmed by computer simulations of the electroweak theory on the lattice [39]. However, if m_H is made to increase, no physical contradiction is actually met. All that happens is that at $m_H \approx 1$ TeV, the Born amplitudes for longitudinal gauge-boson scattering violate unitarity [26], manifesting the breakdown of perturbation theory. The helicity-zero state of gauge bosons is obviously connected to symmetry breaking, because it does not exist for massless vector bosons. For $m_H \sim 1$ TeV the weak interactions become strong. The Higgs boson becomes very broad,

$$\Gamma_H \approx \frac{1}{2} (m_H)^3 \quad (\Gamma_H, m_H \text{ in TeV}). \quad (4.30)$$

We have already remarked that the search for the standard Higgs particle appears to be a good way to organize the experimental programme for a solution to the problem of the origin of the Fermi scale. On the basis of the previous arguments, it is believed [40] that if a set of experiments are sensitive enough to be able to detect the Higgs with mass up to $O(1 \text{ TeV})$, then a great discovery will in any case be made. Either one finds the Higgs, or new physics, or both. At the very least, one would observe the onset of a new regime with strong weak-interactions.

The best opportunity to find the Higgs in the near future is offered by LEP experiments. At LEP1 or at the SLAC Linear Collider (SLC) we will look for the Higgs in Z decays. The most important modes are

$$Z \rightarrow H\bar{\ell} \quad (4.31)$$

and (to a lesser extent)

$$Z \rightarrow H\gamma. \quad (4.32)$$

The corresponding branching ratios are shown in Fig. 4. With an integrated luminosity of around 500 pb^{-1} , the Higgs can be found if $m_H \approx (40-50) \text{ GeV}$ [41].

Fig. 4 The branching ratios $\Gamma(Z^0 \rightarrow H^0 \mu^+ \mu^-)/\Gamma_{\mu\mu}$ and $\Gamma(Z^0 \rightarrow H^0 \gamma)/\Gamma_{\mu\mu}$ as functions of m_H , for $\sin^2 \theta_w = 0.23$.

At LEP 2 [42], with $\sqrt{s} = 160-200 \text{ GeV}$, the important process is

$$e^+ e^- \rightarrow ZH \quad \xrightarrow{\quad} \quad b\bar{b} \quad (4.33)$$

The discovery range is easily extended up to $m_H \approx 70 \text{ GeV}$. With more difficulty, one could possibly get up to $m_H \approx m_Z$. In this range of mass the additional problem is the confusion of the Higgs signal with the W and Z peaks in the relevant mass distributions.

Only supercolliders can continue the Higgs search beyond $m_H \approx m_Z$: hadronic supercolliders such as the Superconducting Super Collider (SSC) ($\sqrt{s} \approx 40 \text{ TeV}$), or the Large Hadron Collider (LHC) ($\sqrt{s} \approx 16-17 \text{ TeV}$), or $e^+ e^-$ future linear colliders such as the CERN Linear Collider (CLIC) ($\sqrt{s} \approx 2 \text{ TeV}$). The problem of producing and detecting the Higgs particle at supercolliders has been much studied recently in connection with the SSC, the LHC, and CLIC. In particular, the problem of observing the Higgs at the LHC and CLIC was analysed in great detail at the La Thuile-CERN Workshop on Physics at Future Accelerators [43], and a comparison with the SSC was made. The luminosity for the SSC, the LHC, and CLIC was in most cases assumed to be $L = 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$.

The results obtained at the La Thuile Workshop on the Higgs problem can be summarized as follows. The relevant range of Higgs masses must be separated into two intervals: the intermediate-mass Higgs: $m_W/2 < m_H \leq 200 \text{ GeV}$, and the heavy Higgs: $m_H \gg 200 \text{ GeV}$. The problem of observing the Higgs is completely different in the two cases. In fact, for an intermediate-mass Higgs, the main decay mode is into the heaviest pair of quarks allowed by phase space, whilst a heavy Higgs decays mainly into WW or ZZ pairs.

The observation of an intermediate-mass Higgs is certainly possible at $e^+ e^-$ linear colliders with $L = 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$ and $\sqrt{s} \approx 1-2 \text{ TeV}$. On the contrary, at hadron-hadron colliders the observation of an intermediate-mass Higgs presents extremely difficult problems because of the QCD background. This is true for both the single-Higgs production ($pp \rightarrow HX$) and the associated Higgs-W production ($pp \rightarrow HWX$). Since the intermediate-Higgs production cross-section (mainly due to gluon fusion) is quite large (at $\sqrt{s} = 40 \text{ TeV}$, $\sigma \approx 10^2 \text{ pb}$ for $m_H \approx 150 \text{ GeV}$ and is 3-4 times smaller at $\sqrt{s} \approx 16 \text{ TeV}$), one can hope to make use of the rare decay modes of the Higgs, e.g. $H \rightarrow \gamma\gamma$ and $\tau^+ \tau^-$, or $\ell\ell\ell\ell$. The possibility of seeing the rare decay modes is for $m_H < 2m_t$. Then the main decay mode of the Higgs is $H \rightarrow b\bar{b}$, its total width is not too large, and the rare branching ratios are enhanced. Even in this favourable case the detection of rare modes is very difficult, the $\gamma\gamma$ mode being the most promising. Concerning the heavy Higgs, the most reliable strategy appears to be the search for the channel $H \rightarrow ZZ \rightarrow \ell^+ \ell^- \nu \bar{\nu}$ (with $\ell = e, \mu$). This method has the advantage of avoiding all problems connected with QCD backgrounds, which are always present with quarks in the final states of WW or ZZ decays. Also, no additional problems are induced if $m_t > m_W$ and the main decay mode of the t-quark is $t \rightarrow Wb$. In this case, for each tt pair produced, there would be a WW pair and the background to $H \rightarrow WW$ would substantially increase. The disadvantage is that there is a reduced range of observable Higgs masses because of the small branching ratio $\text{BR} \approx 8 \times 10^{-3}$ for $H \rightarrow ZZ \rightarrow \ell^+ \ell^- \nu \bar{\nu}$. The conclusion at La Thuile was that the Higgs can be discovered through this channel at the LHC for only $m_H \leq 0.6 \text{ TeV}$ (with $L = 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$).

The discovery range guaranteed by the leptonic modes would be extended if the hadronic modes could be disentangled from the background. Whilst $H \rightarrow 4 \text{ jets}$ is hopeless, it was pointed out at La Thuile that $H \rightarrow WW \rightarrow \ell\nu + 2 \text{ jets}$ could be extracted from the background if an efficient system of quark tagging could be implemented. In fact, the dominant production mechanism for heavy Higgses is through WW fusion (Fig. 5). The two outgoing quarks that radiated the incoming W pair have a transverse momentum of order m_W and large longitudinal momentum. If both these quarks could be detected at very small angles (typically $\theta \approx 5^\circ$ at the LHC), then the background processes could be sufficiently suppressed. Clearly, this poses formidable problems for the calorimetry, because of the

Fig. 5 The Feynman graph for WW or ZZ fusion into an on-shell Higgs boson

enormous level of radiation near the beam pipe. However, a quantitative analysis of the background rejection factor that can be obtained by tagging the quarks was done by Kleiss and Stirling [44]. Their conclusion was that the tagging method is a plausible possibility at the LHC. The discovery range for m_H could then be extended at the LHC to $m_H \approx 0.8$ TeV with $L = 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$ (at least for $m_t < m_W$ which, however, is by now unlikely).

With higher luminosity [45], from $H \rightarrow ZZ \rightarrow \ell\ell\ell\ell$, one could reach at the LHC:

$$m_H \leq 0.7 \text{ TeV} \quad (L = 5 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}, \ell = \mu),$$

$$m_H \leq 0.8 \text{ TeV} \quad (L = 5 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}, \ell = e, \mu),$$

or

$$m_H \leq 0.6 \text{ TeV} \quad (L = 5 \times 10^{33} \text{ cm}^{-2} \text{ s}^{-1}, \ell = e, \mu).$$

A recent reappraisal [46] of the discovery potential of the heavy Higgs at the SSC, with $L = 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$ and no quark tagging, reached the conclusions that a) the Higgs particle can be observed up to $m_H \leq 0.8$ TeV (at most) from $H \rightarrow ZZ \rightarrow \ell\ell\ell\ell$, $\ell = e, \mu$; b) the possibility of reaching larger Higgs masses by using $H \rightarrow ZZ \rightarrow \ell\ell\nu\bar{\nu}$ depends on detailed studies of the calorimeter hermeticity and response (owing to the large Higgs width and the small signal-to-background ratio for m_H large).

It is thus clear that the operation of the LHC at high luminosity, $L \approx 1-5 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$, and/or the possibility of realizing the quark tagging technique, would push the LHC discovery potential for the heavy Higgs substantially closer to that of the SSC in the basic configuration.

Finally, we recall that at CLIC the heavy Higgs can be detected in the hadronic mode $H \rightarrow 4$ jets up to a mass $m_H \leq 0.6-0.8$ TeV with $L = 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$.

5. BASIC RELATIONS FOR PRECISION TESTS

In the standard electroweak theory, there are a number of basic relations that one wants to verify experimentally with the best possible precision. The same quantity $\sin^2 \theta_w$ appears in all these relations.

First, $\sin^2 \theta_w$ can be measured from the value of m_W . Starting from the tree-level relations [Eqs. (3.13) and (3.17)], $\sin^2 \theta_w = e^2/g^2$ and $G_F/\sqrt{2} = g^2/8m_W^2$, we obtain

$$\sin^2 \theta_w = (\pi\alpha/\sqrt{2}G_F)/[m_W^2(1 - \Delta r)] = (37.2802 \text{ GeV})^2/[m_W^2(1 - \Delta r)], \quad (5.1)$$

where $\Delta r \neq 0$ owing to the effect of radiative corrections.

Then $\sin^2 \theta_w$ is also related to the ratio of the vector boson masses. At tree level [see Eq. (3.22)],

$$\sin^2 \theta_w = 1 - m_W^2/\rho_0 m_Z^2, \quad (5.2)$$

where $\rho_0 = 1$ in the Standard Model with only doublets of Higgs bosons. In general, this relation is also modified by radiative corrections:

$$\sin^2 \theta_w = 1 - m_W^2/\rho_{\text{mass}} m_Z^2, \quad (5.3)$$

with $\rho_{\text{mass}} = \rho_0(1 + \delta\rho_{\text{mass}})$. However, as we shall discuss in detail later, Eq. (5.2) with $\rho_0 = 1$ is often adopted as a definition of $\sin^2 \theta_w$ at all orders. Clearly in this case, $\rho_{\text{mass}} = 1$ by definition.

Finally, $\sin^2 \theta_w$ can be obtained for neutral-current couplings. At tree level, the four-fermion interaction from Z exchange is given [see Eq. (3.20)] by

$$M_{if} = [\sqrt{2}G_F m_Z^2/D(s)] \rho_0 (j_i^3 - 2 \sin^2 \theta_w j_{em}^3)(j_f^3 - 2 \sin^2 \theta_w j_{em}^3), \quad (5.4)$$

where $D(s)$ is the Z propagator, and j_i^3 and j_{em}^3 are the weak isospin and electromagnetic currents for the fermion f . Excluding pure QED corrections, electroweak radiative corrections modify M_{if} according to

$$M_{if} = [\sqrt{2}G_F m_Z^2/D(s)] \rho_f (j_i^3 - 2k_i \sin^2 \theta_w j_{em}^3)(j_f^3 - 2k_f \sin^2 \theta_w j_{em}^3) + \dots, \quad (5.5)$$

where $\rho_f = \rho_0(1 + \delta\rho_f)$; $k_a = 1 + \delta k_a$ ($a = i, f$) are in general different for different fermions and depend on the scheme adopted (for example, δk_a depend on the definition of $\sin^2 \theta_w$). The ellipsis indicates possible additional non-factorizable terms.

6. INPUT PARAMETERS

For LEP 1/SLC physics^{*)}, a self-imposing set of input parameters is given by α , α_s , G_F , m_Z , m_t , and m_H . In fact, m_Z has now been (and will be even more) precisely measured. Clearly, $G_F = 1.166389(22) \times 10^{-5} \text{ GeV}^{-2}$ [48] is conceptually less simple than $\alpha_{\text{weak}} = g^2/4\pi$ (which would more naturally accompany $\alpha = 1/137.036$ and α_s) or $\sin^2 \theta_w$ or m_W , but is known with almost absolute accuracy. Among the quark and lepton masses m_i , the main unknown is the t-quark mass m_t . Our ignorance of m_t is at present a serious limitation for precise tests of the electroweak theory because the radiative corrections depend strongly on m_t . The light quark masses are not very accurately known, but their effective values in all the relevant cases [for instance, in the evaluation of Δr in Eqs. (5.1)] can be obtained from low- and intermediate-energy data. The Higgs mass m_H is almost totally unknown. The sensitivity of the radiative corrections to m_H in the Standard Model with doublet Higgses is small. It fixes the required level of accuracy, as the experimental exploration of the symmetry-breaking sector of the theory is of the utmost importance. The best value of α_s at the Z mass is at present given by $\alpha_s(m_Z) = 0.11 \pm 0.01$ [12]. The QCD corrections to processes involving quarks are typically of order α_s/π , so that the present error on α_s leads to a few per mille relative uncertainty in the corresponding predictions.

^{*)} A thorough and updated review of physics at the Z pole in e^+e^- annihilation can be found in the final report [47] of the Workshop on Z Physics at LEP 1 held at CERN during 1989. In the following, I will make large use of the material collected in the various chapters of this book, which is in three volumes.

7. LARGE CONTRIBUTIONS TO RADIATIVE CORRECTIONS

The most quantitative contributions to the radiative corrections arise from large logarithms [e.g. terms of the form $(\alpha/\pi \ln(m_Z/m_f))^n$, where f_l is a light fermion], and from quadratic terms in m_t , i.e. terms proportional to $G_F m_t^2$.

The sequences of leading and close-to-leading logarithms are fixed [49] by well-known and consolidated techniques (β functions, anomalous dimensions, penguin-like diagrams, etc.). For example, large logarithms dominate the running of α from m_e , the electron mass, up to m_Z , with the result that [48]

$$\alpha(m_Z)/\alpha = 1/(1 - \delta\alpha). \quad (7.1)$$

At present, the best value of $\delta\alpha$, obtained by extracting the relevant effective light-quark masses from the data on $e^+e^- \rightarrow$ hadrons, is given by [48]

$$\delta\alpha = 0.0601 + (40/9)(\alpha/\pi) \ln(m_Z/91 \text{ GeV}) \pm 0.0009. \quad (7.2)$$

Large logarithms of the form $(\alpha/\pi \ln(m_Z/\mu))^n$ also enter, for example, in the relation between $\sin^2 \theta_w$ at the scales m_Z (LEP, SLC) and μ (e.g. the scale of low-energy neutral-current experiments).

The quadratic dependence on m_t [50] (and on other possible widely broken isospin multiplets from new physics) arises because, in spontaneously broken gauge theories, heavy loops do not decouple. On the contrary, in QED or QCD the running of α and α_s at a scale Q is not affected by heavy quarks with mass $M \gg Q$. According to an intuitive decoupling theorem [51], diagrams with heavy virtual particles of mass M can be ignored at $Q \ll M$ provided that the couplings do not grow with M and that the theory with no heavy particles is still renormalizable. In spontaneously broken gauge theories, one important difference is in the longitudinal modes of weak gauge bosons. These modes are generated by the Higgs mechanism, and their couplings grow with masses (as is also the case for the physical Higgs couplings). The upper limit on m_t from radiative corrections arises from this phenomenon. Other subtler sources of non-decoupling are related, for example, to the presence of chiral anomalies [19] (which may not be completely cancelled if heavy particles are removed). Another very important consequence is that precision tests of the electroweak theory are sensitive to new physics even if the new particles are too heavy for their direct production. With the value of m_t being continuously pushed up by experiment, the quantitative importance of the terms of order $G_F m_t^2$ is increasingly large. Both the large logarithms and the $G_F m_t^2$ terms have a simple structure and are to a large extent universal, i.e. common to a wide class of processes. Their study is important for an understanding of the pattern of radiative corrections. One can also derive approximate formulae (e.g. improved Born approximations, see Section 9), which can be useful in cases where a limited precision may be adequate.

8. DETERMINATION OF $\sin^2 \theta_w$ FROM m_Z

Once α , α_s , G_F , m_t , and m_H have been chosen as input parameters, $\sin^2 \theta_w$ is a derived quantity. We now consider the calculation of $\sin^2 \theta_w$ beyond the tree approximation. A precise definition of $\sin^2 \theta_w$ must be specified before its value can be computed. An infinite number of definitions are possible, and many have been actually proposed and studied. Physical results are of course independent of the definition adopted for $\sin^2 \theta_w$. Differences in physical results, obtained from different schemes, can occur only through terms of higher order, because the perturbative series is truncated at a given order. Actually $\sin^2 \theta_w$ is used only as a reference quantity. What really matters for precision tests is the prediction of the different measured quantities P in terms of the input parameters; or, in practice, for fixed m_Z , the value $P = P(m_t, m_H)$ of each given physical quantity in the allowed range of m_t and m_H .

At tree level the relation between $\sin^2 \theta_w$ and m_Z is obtained from Eqs. (5.1) and (5.2) (with $\Delta r = 0$):

$$\sin^2 \theta_w \cos^2 \theta_w = (\pi\alpha/\sqrt{2} G_F)/(q_0 m_Z^2). \quad (8.1)$$

Beyond the tree level, the quantity Δr [52] is introduced by radiative corrections:

$$\sin^2 \theta_w \cos^2 \theta_w = (\pi\alpha/\sqrt{2} G_F)/[q_0 m_Z^2(1 - \Delta r)], \quad (8.2)$$

where $\Delta r \equiv \Delta r(\alpha, \alpha_s, G_F, m_Z, m_t, m_H)$ is of course different for different definitions of $\sin^2 \theta_w$. In the following, some particularly interesting definitions of $\sin^2 \theta_w$ will be discussed. We now set $q_0 = 1$. Let us first consider the usual definition [53]:

$$\sin^2 \theta_w \equiv s_W^2 = 1 - (m_W^2/m_Z^2). \quad (8.3)$$

From now on, the symbol s_W^2 will always refer to this specific definition of $\sin^2 \theta_w$. Clearly, given m_Z from SLC or LEP, s_W^2 is directly equivalent to m_W . In this case Δr specifies the relation between m_Z and m_W :

$$(1 - m_W^2/m_Z^2) m_W^2 = (\pi\alpha/\sqrt{2} G_F)/(1 - \Delta r). \quad (8.4)$$

The value of Δr as a function of the input parameters has been studied in great detail (Fig. 6). In particular, the following results are obtained [48, 52, 54]:

$$1/(1 - \Delta r) = \alpha(m_Z)/\alpha \cdot 1/(1 + (c_W^2/s_W^2)\delta\alpha) + \text{'small'} = 1/(1 - \delta\alpha) \cdot 1/(1 + (c_W^2/s_W^2)\delta\alpha) + \text{'small'}, \quad (8.5)$$

where $c_W^2 = 1 - s_W^2$; $\delta\alpha$ was defined in Eqs. (7.1) and (7.2), and $\delta\alpha \rightarrow \delta\alpha_t$ for large m_t , with $\delta\alpha_t$ given by [50, 55]

$$\delta\alpha_t = (3G_F m_t^2/8\pi^2\sqrt{2}) + (3G_F m_t^2/8\pi^2\sqrt{2})^2 (19 - 2\pi^2/3 + O[(G_F m_t^2)^3]). \quad (8.6)$$

Fig. 6 Δr as a function of the t -quark mass for various m_H ($m_Z = 91 \text{ GeV}$) [52].

By ‘small’ in Eq. (8.5), we mean terms (which at one-loop accuracy are known) without large logs and/or leading powers of $G_F m_t^2$; δq_t is the dominant term, for large m_t , of the famous q -parameter first studied in Refs. [50]. The two-loop term was obtained in Ref. [55] and the geometric series resummation was recently advocated in Ref. [54].

Going back to the Z -exchange amplitude near the resonance and the parameters q_{if} , k_i , and k_f defined in Eq. (5.5) with the definition of $\sin^2 \theta_w \approx \tilde{s}_w^2$, we obtain [48]

$$q_{if} = 1 + \delta q + \text{‘small’} \quad (8.7)$$

and

$$k_f = 1 + (c_w^2/\tilde{s}_w^2) \delta q + \text{‘small’} \quad (8.8)$$

(here $f \neq b$; the b -quark will be reconsidered in the following), where the ‘small’ terms are non-universal (i.e. process-dependent). Note that k_f contains additional ‘large’ terms with respect to those included in Δr and q_{if} . As these ‘large’ terms are also universal, this suggests that $k\tilde{s}_w^2$ could be a better effective $\sin^2 \theta_w$ than $\tilde{s}_w^2$ for physics at the Z pole. Before going into this matter, we will add some comments on δq . For any weak isospin fermion doublet (e.g. new heavy quarks or leptons), the quantity ‘ $3m_f^2$ ’ in δq_t (at one loop) becomes [50, 52]

$$3m_f^2 = N_c [m_f^2 + m_A^2 - 2m_A^2 m_f^2/(m_A^2 - m_f^2) \ln(m_A^2/m_f^2)], \quad (8.9)$$

where N_c is the number of colours. For negligible $m_{A,u}$, ‘ $3m_t^2$ ’ $\rightarrow N_c m_{A,d}^2$, whilst for $m_u = m_d + \epsilon$: ‘ $3m_t^2$ ’ $\rightarrow N_c \epsilon^2$ to leading order in ϵ (the result vanishes for unsplit doublets). Similarly, many more kinds of broken weak isospin multiplets can contribute to ‘ $3m_f^2$ ’ [52, 56] (squarks and sleptons [57], charged Higgses [58], etc). The upper limit on ‘ m_t ’ ≈ 210 GeV (see Section 10) together with the direct lower limit on the t -quark mass, $m_t \gtrsim 77$ GeV, leave little space for heavy multiplets, except for nearly degenerate ones.

The contribution of the neutral Higgs mass to δq is only logarithmic at one loop:

$$\delta q_{Higgs} = - (11 G_F m_W^2 / 24 \pi^2 \sqrt{2}) t g^2 \theta_w \ln(m_A^2/m_W^2) + \dots \quad (8.10)$$

There are no m_A^2 terms but only logs, because the ‘custodial’ $SU(2)$ symmetry is not broken in the Higgs sector [26]. Power terms appear only at two loops [59], but their effect is sizeable only for $m_H \gtrsim 1$ TeV.

We now consider a different class of definitions of $\sin^2 \theta_w$. Assume that we fix $k_f = 1$ [defined by Eq. (5.5)] for one given Z vertex (e.g. $k_f = 1$ in $Z \rightarrow e^+ e^-$ for on-shell Z). Then it follows from the previous discussion that δk is ‘small’ for all neutral-current processes (for $f \neq b$) near the Z pole. Let us introduce the notation

$$\sin^2 \theta_w (\text{from } Z \rightarrow e^+ e^-) \equiv \tilde{s}_w^2. \quad (8.11)$$

In this case, Eq. (5.5) becomes

$$M_{fi} = [\sqrt{2} G_F m_Z^2 / D(s)] q_{if} (J_f^3 - 2\tilde{s}_w^2 J_{em}) (J_f^3 - 2\tilde{s}_w^2 J_{em}) + \text{‘small’}, \quad (8.12)$$

where q_{if} is given by Eq. (8.7) in terms of δq , which is specified in Eqs. (8.6) and (8.10). In the present case,

$$\tilde{s}_w^2 c_w^2 \tilde{c}_w^2 = (\pi \alpha / \sqrt{2} G_F) / [m_Z^2 (1 - \Delta \bar{r})], \quad (8.13)$$

and we find [48]

$$\tilde{s}_w^2 \approx \tilde{s}_w^2 + c_w^2 \delta q + \text{‘small’}. \quad (8.14)$$

In other words, apart from ‘small’ terms, we have

$$\tilde{s}_w^2 \approx 1 - (m_W^2 / q m_Z^2) \quad (8.15)$$

with $q = 1 + \delta q$. It is interesting to note that $\tilde{s}_w^2$ is less dependent than $\tilde{s}_w^2$ on m_t^2 . In fact [48, 52],

$$\Delta r \approx \delta \alpha - (c_w^2 / \tilde{s}_w^2) \delta q, \quad (8.16)$$

$$\Delta \bar{r} \approx \delta \alpha - \delta q, \quad (8.17)$$

so that the amplifying factor $c_w^2 / \tilde{s}_w^2$ in front of δq is missing in $\Delta \bar{r}$. Note that given m_Z , $\tilde{s}_w^2$ is known with an accuracy of about ± 0.002 when m_t varies between 60 and 210 GeV (see Fig. 7). There is a class of definitions of $\sin^2 \theta_w$ in the literature that correspond to $\tilde{s}_w^2$ if ‘small’ terms are neglected: $\tilde{s}_w^2$ of Hollik [60]; $s^2(m_Z)$ of Lynn and Kennedy [61]; $(\sin^2 \theta_w)_{MS}(m_Z)$ [62, 63], etc. Note that because of Eqs. (7.1) and (8.17) one can also write:

$$\tilde{s}_w^2 c_w^2 \tilde{c}_w^2 = \{\pi \alpha (m_Z / \sqrt{2} G_F) / q m_Z^2\}. \quad (8.18)$$

Fig. 7 The behaviour of $\tilde{s}_w^2$ and $\tilde{s}_w^2$ [defined by Eqs. (8.3) and (8.11), respectively] as functions of m_t for $m_Z = 91$ GeV, $m_H = 100$ GeV. The solid [dashed] lines are computed to $O(\alpha^2)$ [O(α)]. From Ref. [48].

9. IMPROVED BORN APPROXIMATION

For precision tests of the electroweak theory, the complete one-loop radiative corrections (plus higher-order/exponentiated purely photonic corrections) are mandatory and are indeed available for the processes of practical relevance. However, in many cases a less accurate estimate can be enough. For this purpose, it is useful to know [48] that formulae as simple as those of the Born approximation

can be written in a way that takes all 'large' corrections into account. For $e^+e^- \rightarrow f\bar{f}$, with $f \neq e, b$, the amplitude for γ and Z exchange near the resonance can be written in the form

$$M_{f\bar{f}} = Q_e Q_f (4\pi\alpha(m_Z/s)) J_{em}^\dagger J_{em} + \sqrt{2} G_F q m_Z^2 / (s - m_Z^2 + i s \Gamma_Z/m_Z) \bar{J}^\dagger J, \quad (9.1)$$

where $q = 1 + \delta q$, δq being given by Eqs. (8.6) and (8.10), and

$$J_{em}^\dagger = \gamma_\mu, \quad \bar{J}^\dagger = \gamma_\mu [\frac{1}{2}(1 - \gamma_5) - 2Q_f \tilde{s}_W^2], \quad (9.2)$$

with Q_f and I_f^3 being respectively the electric charge and the third component of the weak isospin (e.g. for $f = \mu^-$, $Q = -1$, $I_3 = -1/2$). The important features are the replacement of α -fixed with α -running, the inclusion of the s -dependence for the total width Γ_Z in the resonant denominator, the presence of the factor of q multiplying G_F , and the use in the Z couplings of the effective $\sin^2 \theta_w = \tilde{s}_W^2$, introduced in the previous section. Clearly for $f = e$, i.e. for Bhabha scattering, the t -channel exchange is also to be included. The improved Born approximations include the real parts of self-energies (sometimes [61] called 'oblique' corrections). All large logs and all $G_F m_f^2$ terms are included. What are left out are 'small' corrections from imaginary parts of self-energies, vertices, and boxes.

For $f = b$ there are additional large terms from the vertex corrections [60, 64] of the type shown in Fig. 8. The longitudinal W modes are, also in this case, responsible for the presence of quadratic mass terms. We can simply modify the improved Born approximation in order to include these terms as well. The recipe [48, 65] is as follows: in Eqs. (9.1) and (9.2), replace q by

$$\sqrt{q \bar{q}_b}, \quad \bar{q}_b = q (1 - \frac{4}{3} \delta q), \quad (9.3)$$

and $\tilde{s}_W^2$ by

$$\tilde{s}_W^2 (1 + \frac{2}{3} \delta q) = \tilde{s}_W^2 k_b. \quad (9.4)$$

Note that whilst many sorts of heavy particles can contribute to δq , only the t -quark (or a new t') contributes to $\bar{q}_b$, because the W nearly always turns a b -quark into a t (or a t') quark.

Fig. 8 Vertex corrections that contain large $G_F m_t^2$ terms.

As a final example, the improved Born approximation for the inclusive widths $\Gamma(Z \rightarrow f\bar{f}(\gamma, g))$ (including photons and gluons in the final state), is given by

$$\Gamma[Z \rightarrow f\bar{f}(\gamma, g)] = N_c (G_F q m_Z^2 / 24\pi \sqrt{2}) \{1 - (1 - 4Q_f \tilde{s}_W^2)^2\}, \quad (9.5)$$

where

$$N_c = \begin{cases} 1 [1 + (3\alpha/4\pi) Q_f^2] & (\text{leptons}), \\ 3 [1 + (3\alpha/4\pi) Q_f^2] [1 + \alpha_s(m_Z)/\pi] & (\text{quarks}). \end{cases} \quad (9.6)$$

In the same approximation, the total width is given simply by

$$\Gamma_Z = \Sigma_f \Gamma[Z \rightarrow f\bar{f}(\gamma, g)] + \text{rare decays}, \quad (9.7)$$

where the rare decays [66] (e.g. $Z \rightarrow H\mu^+\mu^-$ for a light Higgs) are numerically unimportant in the Standard Model (for $m_H > 10$ GeV) [67]. For $\Gamma(Z \rightarrow b\bar{b})$, replace [48, 65] q by $\bar{q}_b$ and $\tilde{s}_W^2$ by $\tilde{s}_W^2 k_b$, where $\bar{q}_b$ and k_b are given in Eqs. (9.3) and (9.4). Note that for $f \neq b$, Eq. (9.5) can also be written in the form [by using Eq. (8.18)]:

$$\Gamma[Z \rightarrow f\bar{f}(\gamma, g)] = N_c [\alpha(m_Z) m_Z / (48 \tilde{s}_W^2 \bar{q}_b^2)] [1 - (1 - 4Q_f \tilde{s}_W^2)^2], \quad (9.8)$$

10. STATUS OF PRECISION TESTS OF THE ELECTROWEAK THEORY

After a period of stagnation, a big step forward in precision tests of the electroweak theory has been accomplished with the data presented this summer. Usually, one starts by presenting a table of results on $\tilde{s}_W^2$ extracted from different processes, with radiative corrections computed assuming a given value for m_t and m_H . Over the years, the assumed value of m_t started from 15 GeV, was then set at 30 GeV, then again increased to 45 GeV, and more recently to 60 GeV. This was a sensible procedure, because, for m_t not too large, the radiative corrections are not dominated by m_t . A moderate change of m_t in a range around and below 60 GeV does not affect the radiative corrections very much. This is, for example, evident from Fig. 6, where the value of Δr is plotted as a function of m_t . We now know from new data from CDF [68], UA1 [69], and UA2 [70] that m_t is probably > 70 -80 GeV. For large m_t , the quadratic terms of order $G_F m_t^2$ become a driving term for the radiative corrections, which, as a result, depend strongly on m_t . It seems to me that a better strategy is to discuss the experimental results and the associated radiative corrections needed to extract $\sin^2 \theta_w$ as functions of m_t , with m_t varying in the whole range of allowed values. The requirement that the electroweak theory be consistent with the data, restricts the allowed range of m_t significantly. We shall see that not only is an upper bound obtained on m_t (which is an old result), but also a lower bound can now be obtained. This limit [see Eq. (10.7)] is interesting in itself, although the limits from production experiments are more directly reliable. (Clearly, a lower limit on m_t from radiative corrections can be avoided if heavy new particles contribute to vacuum polarization diagrams, whilst the upper limit is *a fortiori* valid.)

The most important results are the new measurements of the weak gauge-boson masses. The old and new data on m_W and m_Z are summarized in Table 1. The precise determination of m_Z by MARK II [71] (and CDF [72]) are among the highlights of this summer's results. The CDF result is quite impressive if we consider the difficulty of such a measurement at hadron colliders. The MARK II determination of m_Z is a great achievement, not only in itself but also as a promise for the future of e^+e^- linear colliders.

By combining the CDF and MARK II results, we obtain

$$m_Z = 91.12 \pm 0.16 \text{ GeV}. \quad (10.1)$$

There is no theoretical error associated with the measurement of m_Z in e^+e^- annihilation, and also the derivation of $\tilde{s}_W^2$ (or $\tilde{s}_W^2$) from m_Z is very clean, i.e. the theory of Δr (or ΔF) has no ambiguities in the minimal Standard Model, other than our ignorance of m_t and m_H . However, the value of $\tilde{s}_W^2$ obtained from m_Z depends strongly on m_t through Δr . This is seen in Fig. 9 [73], where the value of $\tilde{s}_W^2$ corresponding to a given value of m_Z is plotted versus m_t . As discussed in Section 8, the value of $\tilde{s}_W^2$ extracted from m_Z is less sensitive to m_t . We shall come back later to the determination of $\tilde{s}_W^2$.

The measurement of m_W/m_Z at $p\bar{p}$ colliders is also quite clean, although not as much as that of m_Z . The error on the energy scale drops when taking the ratio. The theoretical error is confined to

Table 1

The numbers in italics refer to errors on the energy scale.
Most of these errors cancel when taking the ratio m_W/m_Z .

	m_W (GeV)	m_Z (GeV)
Old data		
UA1	$82.7 \pm 1.0 \pm 2.7$	$93.1 \pm 1.0 \pm 3.1$
UA2	$80.2 \pm 0.6 \pm 1.4$	$91.5 \pm 1.2 \pm 1.7$
CDF	$80.0 \pm 3.3 \pm 2.4$	
New data		
CDF [71]	$80.0 \pm 0.2 \pm 0.5 \pm 0.3$	$90.9 \pm 0.3 \pm 0.2$
MARK II [72]		91.17 ± 0.18
UA2 [74]	$80.0 \pm 0.4 \pm 0.4 \pm 1.2$	$90.2 \pm 0.6 \pm 1.4$

Fig. 9 Dependence on m_t of $\hat{s}_W^2$ [defined in Eq. (8.3)] extracted from $m_{W,Z}$ (VBM) and from the neutral-current $e-q$, $\nu-q$, or $\nu-e$ sectors (Ref. [74]).

some small uncertainties in the W transverse momentum distribution (e.g. structure functions, A_{QCD} , and related problems), which somewhat affect the determination of m_W from the Jacobian peak in the electron distribution. The values obtained by CDF [72] and UA2 [74] are in excellent mutual agreement:

$$\begin{aligned} m_W/m_Z &= 0.887 \pm 0.009 \quad (\text{UA2}), \\ &= 0.880 \pm 0.007 \quad (\text{CDF}). \end{aligned} \quad (10.2)$$

In this case, there are clearly no problems in obtaining $\hat{s}_W^2$: the ratio m_W/m_Z determines $\hat{s}_W^2$ directly, by definition. The above results correspond to

$$\begin{aligned} (\hat{s}_W^2)_{m_W/m_Z} &= 0.213 \pm 0.015 \quad (\text{UA2}), \\ &= 0.225 \pm 0.012 \quad (\text{CDF}). \end{aligned} \quad (10.3)$$

The resulting combined value $\hat{s}_W^2 = 0.220 \pm 0.009$ is reported in Fig. 10. Clearly, the relatively low value of $\hat{s}_W^2$ obtained from m_W/m_Z when compared with m_Z , pushes the value of m_t towards large

Fig. 10 $\hat{s}_W^2$ [defined in Eq. (8.3)] obtained from m_Z , m_W/m_Z , and $R_\nu = \sigma_{\text{NN}}^{\text{NC}}/\sigma_{\text{NN}}^{\text{CC}}$ as a function of m_t . The error shown for R_ν is only a part of the total; it corresponds to a fixed charm mass $m_c = 1.45$ GeV and $m_H = 100$ GeV (courtesy of G.L. Fogli).

values (within the limited accuracy of the m_W/m_Z measurement). It is important to note that an upper limit on m_t around 260 GeV is already implied by the vector-boson masses alone.

We now consider neutral-current data. The most precise determination of $\hat{s}_W^2$ is obtained from the ratio R_e of neutral- to charged-current deep-inelastic neutrino scattering. It turns out that R_e imposes an important constraint on the electroweak parameters when compared with m_W or m_Z because $\hat{s}_W^2$, derived from R_e , is nearly independent of m_t in the range of interest [75]. This near independence is apparent in Figs. 9 and 10. It arises because of a largely accidental cancellation of the m_t dependence introduced by defining $\hat{s}_W^2$ from m_W/m_Z , and because of the m_t dependence induced by the q parameter appearing in the neutral- to charged-current ratio. The most precise measurements of $\hat{s}_W^2$ are due to the CHARM [76] and CDHS [77] Collaborations at CERN:

$$\begin{aligned} (\hat{s}_W^2)_{\nu N} &= 0.236 \pm 0.007 \quad (\text{CHARM}), \\ &= 0.228 \pm 0.007 \quad (\text{CDHS}). \end{aligned} \quad (10.4)$$

Note that both $(\hat{s}_W^2)_{\nu N}$ and $(\hat{s}_W^2)_{m_W/m_Z}$ are independent [or nearly so for $(\hat{s}_W^2)_{\nu N}$] of m_t . As a consequence, the direct comparison of Eq. (10.4) with Eq. (10.3) provides a quite remarkable and successful consistency check, independent of m_t , for the electroweak theory. Thus R_e is a good measure of m_W/m_Z (as was recently stressed again in Ref. [78]). Once R_e is fixed, if m_Z is decreased, then m_W has also to be decreased, which means that Δr is decreased, and finally that m_t is increased. The fact that the measured value of m_Z is below the previous world average of about 92 GeV, pushes m_t toward the large values.

The values of $\hat{s}_W^2$ obtained from m_Z , m_W/m_Z , and R_e as functions of m_t with the corresponding experimental errors are displayed in Fig. 10. Clearly, a lower and an upper limit on m_t are implied. For a quantitative determination of the allowed range of m_t and $\hat{s}_W^2$, it must be stressed that the measurement of R_e in neutrino deep-inelastic scattering and the extraction of $\hat{s}_W^2$ from R_e are obviously affected by much larger theoretical uncertainties than those connected with m_Z and m_W/m_Z . In νN scattering, we must cope with uncertainties connected with the validity of the parton model (i.e. the importance of higher twist and other pre-asymptotic corrections), with our relative ignorance of sea densities, with the effects of the onset of the charm threshold and the validity of the slow rescaling procedure (the related uncertainties are roughly parametrized in terms of an effective 'charm mass' parameter m_c), with the modelling of the effect of experimental cuts, and so on. The errors stated by the experiments and quoted in Eq. (10.4) also include an estimate of theoretical errors. The error of 0.007 arises from adding, in quadrature, the experimental and theoretical errors according to $\delta\hat{s}_W^2 = 0.012(m_c - 1.5) \pm 0.005$ (exp.) ± 0.003 (theor.), with $m_c \approx 1.2$ – 1.8 GeV. It should, however, be kept in mind that the determination of $\hat{s}_W^2$ from neutrino-nucleon scattering is quite involved, and that the evaluation of the related theoretical error is to some extent debatable. With the new data on m_W/m_Z and especially on Z physics from SLC/LEP 1, R_e will increasingly become the weak link in the chain.

There are new important results on other neutral-current processes. First, I want to recall the recent measurement of parity violation in atomic caesium that was made at Boulder [79] (by a team of only three physicists!). They quote the remarkably accurate value

$$(\hat{s}_W^2)_{Cs} = 0.219 \pm 0.019 \quad (10.5)$$

obtained for m_t light ($m_t \leq 60$ GeV). For larger m_t , the value of $(\hat{s}_W^2)_{Cs}$ decreases rapidly, as is seen from the curve labelled 'e-q' in Fig. 9. The value of Eq. (10.5) is in agreement with previous, less precise, results on Cs obtained by the Paris group [80]. A decade of continuous progress on atomic 'table-top' experiments has allowed the validity of the electroweak theory at very low energies to be

Table 2
Results on ν -e scattering

	$\nu_e e$	$\bar{\nu}_\mu e$	$\sin^2 \theta_w$ (no rad. corr.)
CHARM 1	83	112	$0.211 \pm 0.035 \pm 0.011$
E734	160	97	$0.195 \pm 0.018 \pm 0.013$
CHARM 2	762	1017	$0.233 \pm 0.012 \pm 0.008$

established with a precision not far from that of experiments, at higher energies, performed with large detectors at big accelerators.

New data on $\bar{\nu}_\mu e$ scattering have been presented this summer, for the first time, by the CHARM 2 Collaboration [81]. As shown in Table 2 (where the numbers of observed events are also displayed), the CHARM 2 values are considerably more precise than in previous experiments. Also, the central value of $\hat{s}_W^2$ is now in better agreement with the results from other processes. The CHARM 2 result, including radiative corrections with $m_t = 100$ GeV (the behaviour at larger values of m_t is shown in Fig. 9), is given by

$$(\hat{s}_W^2)_{\bar{\nu}_\mu e} = 0.232 \pm 0.014. \quad (10.6)$$

The experiment is still running, and the present stage of the analysis is far from achieving the final aim of reducing the error on $\hat{s}_W^2$ to $\delta\hat{s}_W^2 = \pm (6-7) \times 10^{-3}$.

Going back to Fig. 9 showing the dependence, on m_t , of $\hat{s}_W^2$ obtained from different processes, it is seen that on the one hand ν -q is similar to m_W/m_Z , leading to a value of $\hat{s}_W^2$ nearly independent of m_t , whilst on the other hand m_Z , e-q, and $\bar{\nu}_\mu e$ lead to $\hat{s}_W^2$ with a common trend in m_t . Figure 11

Fig. 11 Same as Fig. 10, but with the addition of the data on the e-q and ν -e sectors (courtesy of G.L. Fogli).

differs from Fig. 10 because the constraints from e-q and ν -e have been added to those from ν -q in order to complete the information from neutral-current processes.

Taking all the information together, we can conclude that the ranges of m_t and $\hat{s}_W^2$ indicated by the present data on the electroweak theory are

$$m_t = 130 \pm 50 \text{ GeV}, \quad (10.7)$$

$$\hat{s}_W^2 \equiv 1 - (m_W^2/m_Z^2) = 0.227 \pm 0.006. \quad (10.8)$$

These values are in agreement with the results of a number of complete detailed analyses of the data [73, 82-84] (see, for example, Fig. 12 taken from Ref. [83]), although the errors quoted here are in some cases more conservative. In particular, the upper (lower) limit on the top mass from Eq. (10.7) is about $m_t \approx 210$ GeV ($m_t > 50$ GeV) at 90% CL. Note that the uncertainty in $\hat{s}_W^2$ is actually much smaller now than it was before this summer. In fact, the present error is for all allowed values of m_t and m_H (for m_H we take the range 10 GeV to 1 TeV), whilst in the past the quoted errors referred to assumed fixed values of these masses.

A different question is, What is, at present, the best estimate of $\hat{s}_W^2$? i.e. the effective $\sin^2 \theta_w$ for the on-shell Z couplings introduced in Section 8, Eqs. (8.11) to (8.15). We recall that the value of $\hat{s}_W^2$ obtained from m_Z is less dependent on m_t and m_H than is $\hat{s}_W^2$. If we take for m_Z the value and the error given in Eq. (10.1), for m_t the range in Eq. (10.7), and for m_H the mass range 10 GeV < m_H < 1 TeV, we obtain [63, 78]

$$\begin{aligned} \hat{s}_W^2 &= 0.233 \pm 0.002 \pm 0.0074 \delta m_Z \text{ (GeV)} \\ &= 0.233 \pm 0.002 \pm 0.0012. \end{aligned} \quad (10.9)$$

We see that indeed the Z couplings can be predicted with more accuracy from $\hat{s}_W^2$ than we would naively obtain by going through $\hat{s}_W^2$. Note that $\hat{s}_W^2 \approx (\sin^2 \theta_w)_{\overline{MS}}$ (i.e. apart from 'small' terms), is also the relevant quantity to compare with GUT predictions. The value in Eq. (10.9) is in disagreement with naive SU(5) but in agreement with the minimal supersymmetric extension of SU(5) [85, 86].

Fig. 12 Summary of the present status of electroweak tests. Given m_Z , $m_Z - m_W = m_Z(1 - m_W/m_Z)$ is equivalent to m_W/m_Z or $\hat{s}_W^2$ (courtesy of D. Haidt).

11. THE Z LINE-SHAPE

The forthcoming experiments at the SLC, and especially at LEP, will certainly mark a big step forward in the domain of precision tests of the standard theory. The strategy at LEP 1/SLC is first to study the line shape and to measure m_Z , Γ , and the partial widths, to perform the counting of neutrinos, and to look for possible surprises in the final state. This is the part of the programme that has already started at the SLC and has led to the important results presented this summer. At LEP 1 the same measurements will presumably be completed very soon with considerably more precision'. The exploratory phase will be followed by a long period of running at the Z peak. The goals are to measure, as precisely as possible, a number of asymmetries that allow the most stringent tests of the theory, and to search for interesting rare Z decays, the most important being those involving the Higgs boson (i.e. $Z \rightarrow H_\mu^+ \mu^-$ and $Z \rightarrow H_\gamma$). A thorough and updated review of Z physics in e^+e^- annihilation can be found in Ref. [47]. In the following, I will discuss very briefly some of the highlights of the physics programme outlined above.

A considerable amount of work has deservedly been devoted to the theoretical study of the Z line-shape [67]. The experimental accuracy on m_Z , which is planned at LEP, is $\delta m_Z = \pm 50$ MeV. The error on m_Z can eventually go down to $\delta m_Z = \pm 20$ MeV if the transverse polarization of the beams measured by adequate polarimeters will be used to calibrate the energy by spin-resonance methods [87]. Similarly, a measurement of the total width to an accuracy $\delta \Gamma = \pm 30$ MeV is now considered feasible. The prediction of the Z line-shape in the Standard Model to such an accuracy has posed a formidable challenge to theory, which has been successfully met. For the inclusive process $e^+e^- \rightarrow f\bar{f}X$, with $f \neq e$ (for simplicity, we leave Bhabha scattering aside) and X including γ 's and gluons, the physical cross-section can be written in the form of a convolution [67]:

$$\sigma(s) = \int_0^1 dz \hat{\sigma}(zs) G(z, s), \quad (11.1)$$

where $\hat{\sigma}$ is the reduced cross-section, and $G(z, s)$ is the radiator function that describes the effect of initial-state radiation; $\hat{\sigma}$ includes the purely weak corrections, the effect of final-state radiation (of both γ 's and gluons), and also non-factorizable terms (initial- and final-state radiation interferences, boxes, etc.) which, being small, can be treated in lowest order and effectively absorbed in a modified $\hat{\sigma}$. The radiator $G(z, s)$ has an expansion of the form [67]

$$G(z, s) = \delta(1-z) + \alpha/\pi (a_{11}L + a_{10}) + (\alpha/\pi)^2 (a_{22}L^2 + a_{11}L + a_{20}) + \dots + (\alpha/\pi)^n \sum_{i=0}^n a_{ni}L^i, \quad (11.2)$$

where $L = \ln s/m_Z^2 \approx 24.2$ for $\sqrt{s} = m_Z$. All first- and second-order terms are known exactly. The sequence of leading and next-to-leading logs can be exponentiated (closely following [88] the formalism of structure functions in QCD). For $m_Z \approx 91$ GeV, the convolution displaces the peak by +110 MeV, and reduces it by a factor of about 0.74. The exponentiation is important in that it amounts to a shift of about 14 MeV in the peak position.

A model-independent analysis [89] of the reduced cross-section $\hat{\sigma}$ leads to the following general expression (here $m \equiv m_Z$):

$$\hat{\sigma}(s) = [12\pi\Gamma_e\Gamma_f/|D(s)|^2] [s/m^2 + R_f(s - m^2)/m^2 + \Gamma/m L_f + \dots] + [4\pi\alpha^2(m^2)Q_f^2N_c/3s], \quad (11.3)$$

* This has indeed been the case. At the moment of this write-up, after three weeks of running at LEP, the combined values from the four LEP experiments are $m_Z = 91.10 \pm 0.033 \pm 0.045$ GeV = 91.10 ± 0.06 GeV, $\Gamma_Z = 2.584 \pm 0.075$ GeV, and $N_c = 3.17 \pm 0.21$.

with N_e given by Eq. (9.6) and

$$D(s) = s - m^2 + i m \Gamma [s/m^2 + \epsilon (s - m^2)/m^2] + \dots \quad (11.4)$$

This form of the resonant term was obtained by starting from a general renormalizable field theory (the Standard Model being a particular case). Near the resonance, $(s - m^2)/m^2 \approx \Gamma/m$ is of order α (or $\alpha\omega$). As only a perturbative calculation of δ is possible in α or $\alpha\omega$, at the same level of accuracy we can expand vertices, propagators, etc., in $(s - m^2)/m^2$ near the resonance. For example, for the inverse propagator, we can write the expansion

$$\begin{aligned} D(s) &= s - m^2 + \Pi(s) \\ &= s - m^2 + \text{Re } \Pi(m^2) + (s - m^2) \text{Re } \Pi'(m^2) + i \text{Im } \Pi(m^2) + i(s - m^2) \text{Im } \Pi'(m^2) + \dots \\ &= [1 + \text{Re } \Pi'(m^2)] \{s - m^2 + i m \Gamma [s/m^2 + \epsilon (s - m^2)/m^2 + \dots]\}, \end{aligned} \quad (11.5)$$

with

$$m\Gamma = \text{Im } \Pi(m^2)/[1 + \text{Re } \Pi'(m^2)], \quad (11.6)$$

and $\text{Re } \Pi(m^2) = 0$ because of the specific definition of $m \equiv m_Z$ that is adopted. The overall factor $1 + \text{Re } \Pi'(m^2)$ is reabsorbed in the numerator. The parameter ϵ measures the deviation from the scaling behaviour of the s -dependent width; ϵ is noticeably different from zero only if in the final state of Z decays there are important channels with massive particles. For example, for $Z \rightarrow A\bar{A}$, $\epsilon_A \approx (4m_A^2/m^2)B(Z \rightarrow A\bar{A})$. The following identity is valid:

$$\begin{aligned} D(s) &= s - m^2 + i m \Gamma [s/m^2 + \epsilon (s - m^2)/m^2 + \dots] \\ &= [1 + i\gamma(t + \epsilon)](s - \bar{m}^2 + i\bar{m}\bar{\Gamma}), \end{aligned} \quad (11.7)$$

with $\gamma = \Gamma/m$ and

$$\bar{m} = m[1 - (\gamma^2/2)(1 + \epsilon) + \dots], \quad (11.8)$$

$$\bar{\Gamma} = \Gamma[1 - (\gamma^2/2)(1 + 3\epsilon) + \dots]. \quad (11.9)$$

As the factor $1 + i\gamma(1 + \epsilon) = \exp i\gamma(1 + \epsilon)$ cannot be observed from the absolute square of $D(s)$, we see that, on the one hand, at $\epsilon = 0$ the replacement of the constant Γ by the s -dependent width $s\Gamma/m^2$ leads to a variation of the apparent mass by

$$\delta m = -1/2 \gamma^2 \gamma^2 m \approx -34 \text{ MeV}, \quad (11.10)$$

which is clearly an important effect [90, 91]. On the other hand, the additional effect from ϵ ,

$$\delta m_\epsilon = -1/2 \gamma^2 \epsilon m \approx -34 \epsilon \text{ MeV}, \quad (11.11)$$

is certainly small because ϵ cannot exceed 10% or so at most. Note that the effect of ϵ cannot be disentangled from m in the fit, so that it leads to an ambiguity in the determination of m . In conclusion, ϵ can be safely neglected at the price of allowing an error of a few MeV on m and Γ .

Of the two parameters R_ℓ and I_ℓ which appear in Eq. (11.3) for δ (in addition to the inclusive partial widths Γ_e and Γ_γ), I_ℓ , like ϵ , depends only on the spectrum of particles below the Z . In fact, I_ℓ

is determined by absorptive parts of vertices and boxes. Thus large deviations from the Standard Model value cannot occur for I_ℓ . In the Standard Model, I_ℓ is very small [for $\mu^+ \mu^-$, $I_\mu \approx -(1-2) \times 10^{-3}$], and as it is multiplied by Γ/m , it can be neglected. The main contribution to R_ℓ is already present at the Born level and arises from γ - Z interference. Higher-order corrections are relatively important, especially for muons. New physics, for example a heavy Z' , can modify R_ℓ because the non-resonating background is changed. In the Standard Model, R_ℓ is small [for muons, $R_\mu \approx (4.5-6) \times 10^{-3}$, for hadrons $R_h \approx (0.75-1) \times 10^{-1}$]. Determining R_ℓ from a fit is difficult. In first approximation, it can be fixed at its Standard Model value. Realistic deviations from the Standard Model would not appreciably affect the determination of m and Γ .

In conclusion, a model-independent analysis of the reduced cross-section proves that a modified Breit-Wigner with s -dependent width plus photon exchange and interference is a perfectly adequate basis for the experimental study of the line shape. Whilst in the Minimal Standard Model all the parameters Γ_e , Γ_γ , R_ℓ , and I_ℓ can be obtained from m_Z , given m_t and m_H , the general expression of δ will, in principle, allow a model-independent measurement of the Z parameters. Starting from Eq. (11.3) for δ , approximate analytic solutions of the convolution integral can be found [89, 92-94]. The resulting compact analytic expressions are sufficiently precise for most applications.

An indicative set of theoretical predictions on the line shape for $m_Z \approx 91 \text{ GeV}$ is reported in Tables 3 to 5 taken from Ref. [67]. The agreement on the line shape, obtained by different calculations using slightly different procedures or schemes, is better than 0.1% for $\mu^+ \mu^-$, corresponding to an error of a few MeV on m_Z . For hadrons, the uncertainty on Γ introduced by $\alpha_s(m_Z) = 0.11 \pm 0.01$ [12] is $\delta\Gamma \approx \pm 6 \text{ MeV}$. We see from Tables 4 and 5 that the visible cross-section at the peak for $N_\gamma = 3$ is expected to be about 35 nb (for reasonable cuts on Bhabha events near the forward direction). This implies that the number of visible Z is expected to be

$$N_Z = \sigma_{\text{vis}}^{\text{peak}} \int L dt = 3.5 \times 10^6 \left(\int L dt / 10^{38} \text{ cm}^{-2} \right). \quad (11.12)$$

At the project luminosity of LEP, $L = 1.7 \times 10^{31} \text{ cm}^{-2} \text{ s}^{-1}$, in a 'year' of 10^7 s , with an overall efficiency $\epsilon \sim 1/2$, we expect to have for each experiment

$$N_Z/\text{year} \approx 3.5 \times 10^6 \times 1.7 \times 10^7 \times 0.5 \approx 3 \times 10^6. \quad (11.13)$$

Table 3

Partial and total Z widths for $m_Z = 91 \text{ GeV}$ and $\alpha_s = 0.12$ obtained from Ref. [67].

m_t (GeV)	m_H (GeV)	$\sin^2 \theta_w$	Γ_Z (MeV)	$\Gamma_{Z \rightarrow e^+ e^-}$ (MeV)	$\Gamma_{Z \rightarrow \mu^+ \mu^-}$ (MeV)	$\Gamma_{Z \rightarrow d\bar{d}}$ (MeV)	$\Gamma_{Z \rightarrow b\bar{b}}$ (MeV)
60	100	0.2366	2460	82.7	292.7	378.3	376.0
90	10	0.2312	2463	82.7	293.4	379.0	375.8
90	100	0.2328	2466	82.9	293.6	379.3	376.2
90	1000	0.2360	2458	82.7	292.3	377.9	374.8
150	100	0.2257	2479	83.3	295.9	382.0	375.5
200	100	0.2181	2493	83.7	298.6	385.2	374.5
230	10	0.2106	2501	83.8	300.4	387.2	373.3
230	100	0.2123	2504	84.0	300.6	387.6	373.7
230	1000	0.2158	2497	83.8	299.4	386.2	372.5

Table 4

The total cross-section including all corrections for $e^+e^- \rightarrow \mu^+\mu^-$, using a Z mass of 91 GeV and a minimum for $\sqrt{s}$ of 0.2 GeV.

The results are obtained from Ref. [67].

m_t (GeV)	m_H (GeV)	Γ_Z (MeV)	σ_{max} (nb)	$\sqrt{s_{\text{max}}}$ (GeV)	$\sqrt{s_-}$ (GeV)	$\sqrt{s_+}$ (GeV)
60	100	2460	1.492	91.090	89.802	92.656
90	10	2463	1.490	91.091	89.801	92.657
90	100	2466	1.492	91.091	89.800	92.659
90	1000	2458	1.493	91.090	89.804	92.654
150	100	2479	1.493	91.091	89.794	92.668
200	100	2493	1.497	91.091	89.786	92.678
230	10	2501	1.499	91.092	89.782	92.682
230	100	2504	1.500	91.092	89.781	92.685
230	1000	2497	1.500	91.091	89.785	92.680

Table 5

The total cross-section including all corrections for $e^+e^- \rightarrow \text{hadrons}$, using a Z mass of 91 GeV and a minimum for $\sqrt{s}$ of 10 GeV.

The results are obtained from Ref. [67].

m_t (GeV)	m_H (GeV)	Γ_Z (MeV)	σ_{max} (nb)	$\sqrt{s_{\text{max}}}$ (GeV)	$\sqrt{s_-}$ (GeV)	$\sqrt{s_+}$ (GeV)
60	100	2460	30.652	91.091	89.820	92.637
90	10	2463	30.653	91.092	89.819	92.639
90	100	2466	30.660	91.092	89.817	92.641
90	1000	2458	30.658	91.091	89.821	92.635
150	100	2479	30.699	91.092	89.811	92.649
200	100	2493	30.761	91.093	89.805	92.659
230	10	2501	30.818	91.093	89.801	92.665
230	100	2504	30.817	91.093	89.800	92.667
230	1000	2497	30.796	91.093	89.803	92.662

At LEP—if everything works well—neutrino counting [95] can be completed very quickly. The fastest procedure is to extract N_ν from the peak value of the cross-section assuming the Standard Model. This method, which is already followed by MARK II [71], and similar ones based on the partial widths, would allow us to measure N_ν in the next few months, the expected accuracy being $\delta N_\nu = \pm 0.2$ with 3 pb^{-1} . A similar precision, $\delta N_\nu = 0.25$, can also be obtained by collecting a few pb^{-1} at a number of GeV above the resonance and looking for $e^+e^- \rightarrow \gamma + \text{nothing}$.

12. FUTURE PERSPECTIVE ON PRECISION TESTS

The goals of LEP and the SLC over a longer term are to measure several asymmetries that provide the optimal precision tests at the Z peak. With no longitudinal polarization of the beam, the interesting quantities are the forward-backward asymmetries: A_{FB}^f for $e^+e^- \rightarrow f\bar{f}$, with $f = \mu, b, c, \dots$, and the τ -polarization asymmetry $A_{FB}^{\tau\text{pol}}$, measured in $e^+e^- \rightarrow \tau^+\tau^-$ by reconstructing the τ helicity from the $\tau \rightarrow \pi\nu$ decay. At LEP with $\approx 100 \text{ pb}^{-1}$ of integrated luminosity, the accuracies reported in Table 6 can be expected [96–98]. Note that the quoted error on $\sin^2 \theta_w$ actually refers to the quantity $\tilde{s}_w^2$ defined in terms of effective couplings at the Z. A more significant representation of the planned accuracy is in terms of plots such as those in Figs. 13 and 14, where the predicted values

Table 6
Experimental errors on asymmetries and $\tilde{s}_w^2$ expected at LEP with $> 100 \text{ pb}^{-1}$ of integrated luminosity

Measurement	Error	$\delta\tilde{s}_w^2$
A_{FB}^{μ}	0.0035	0.0017
A_{FB}^b	0.007	0.0012
A_{FB}^c	0.007	0.0015
A_{FB}^{τ}	0.005	0.0009
$A_{FB}^{\tau\text{pol}}$	0.010	0.0020
$A_{FB}^{\tau\text{pol}}$	0.011	0.0014

Fig. 13 A_{FB}^{μ} versus m_Z for given m_t and m_H (from Ref. [97]). The experimental error expected at LEP is also shown for comparison.

Fig. 14 A_{pol}^* versus m_Z for given m_t and m_H (from Ref. [98]). The experimental error expected at LEP is also shown for comparison.

of A_{FB}^* and A_{pol}^* are shown as functions of m_Z for different m_t and m_H . The curves are obtained by using the most up-to-date and complete sets of electroweak radiative corrections. In fact, approximations like the improved Born formulae of Section 9 are not sufficient for precision tests at the level of these asymmetries. The pure QED corrections, being detector-dependent, have to be subtracted away [the asymmetries shown are computed from the analogues of the reduced cross-section $\hat{\sigma}$ in Eq. (11.1)]. The size of the experimental error is also shown. We see that the expected accuracy is of the order of the effect corresponding to a variation of m_H in the range 10 GeV $\leq m_H \leq 1$ TeV. This means that the measurements are indeed probing the fine structure of radiative corrections.

Going back to Table 6, we note that the forward-backward asymmetry for the b-quark is really promising. The error shown in Table 6 for A_{FB}^b (which requires $\sim 10^7$ Z events) also includes the uncertainty associated with the correction [65] demanded by B^0 - $\bar{B}^0$ mixing. In particular, the mixing parameter averaged over all produced b particles, has to be precisely measured at LEP (e.g. from the ratio of equal- to opposite-sign dileptons) in order to use this information for extracting A_{FB}^b . An updated analysis can be found in Ref. [65]. Note that A_{FB}^b is also important in another respect. The other asymmetries all measure essentially the same vacuum polarization effects. For example, if we plot A_{pol}^* versus A_{FB}^b , the resulting curve is nearly independent of m_t , as can be seen from Fig. 15. On the contrary, A_{FB}^b is also strongly dependent on large corrections from vertex diagrams, as was discussed in Section 9.

It is well known that having implemnet longitudinally polarized beams is a great advantage for precision tests of the electroweak theory. This is because the left-right asymmetry A_{LR} has the same

Fig. 15 A_{pol}^* versus A_{FB} (for μ , u, and d final states). The curves for $m_t = 50$ GeV and 230 GeV are almost coincident (from Ref. [97]).

sensitivity as A_{pol}^* to weak effects but is much easier to measure [96]: one simply measures the difference of the totally inclusive cross-sections from left- and right-handed electrons and then divides by the sum. The structure of radiative corrections is also much simpler. At LEP with 40 pb^{-1} of integrated luminosity, and polarization $P = 0.5$ fixed with an accuracy of $\delta P = 1\%$, it is possible to obtain [96] $\delta A_{\text{LR}} \approx \pm 0.003$, corresponding to $\delta \hat{\sigma}_W = \pm 0.0004$. This is a very exciting possibility, and longitudinal polarization is actually under serious consideration at CERN. But implementing longitudinally polarized beams at LEP is difficult (because of the length of time needed to build up the polarization, compared with the beam lifetime, and because of depolarizing effects due to the spin time and the energy spread, etc.). Hence, such implementation is costly. Also, the necessary polarization rotators have to be provided for all four LEP experiments. On the contrary, polarized beams are more easily obtained at the SLC. Thus the most obvious development of the experimental programme at the SLC with the SLD detector is to implement polarization and measure A_{LR} . Even with only 10^4 Z, one can expect [96] $\delta A_{\text{LR}} \approx \pm 0.025$, corresponding to $\delta \hat{\sigma}_W = \pm 0.003$, which is a quite respectable result.

At LEP 2, the measurement of m_W with a precision of $\delta m_W = \pm 100$ MeV will be possible [99]. Hadron colliders can hardly approach a comparable accuracy. In any case, the improvements needed, in terms of integrated luminosity and/or detectors, are such that the time spent on carrying them out would be more or less as long as the wait for LEP 2. Given m_Z , a precision of $\delta m_W \approx \pm 100$ MeV is equivalent to $\delta \hat{\sigma}_W = \pm 0.002$. This accuracy is not only of the same order as that obtained from the measurement of asymmetries; it also has the additional advantage of carrying information of a very different qualitative nature.

We repeat here that the search for the Higgs boson [100] (see Section 4) is one of the most important goals of LEP. There is no better accelerator than LEP for discovering a Higgs with mass

$m_H \approx 40\text{--}50\text{ GeV}$ at LEP 1 and $m_H \approx 70\text{--}90\text{ GeV}$ at LEP 2. Actually, the Higgs search is a reference problem. More generally, one looks for a way to solve the problem of the origin of the Fermi scale of mass $\sim G_F^{1/2}$. Indeed, LEP 2 is a formidable discovery machine, and I am confident that the low-lying fringes of the rich spectroscopy associated with all conceivable scenarios for new physics will already be observed at LEP 2.

In conclusion, a very exciting period for phenomenology lies ahead of us. We all hope that the wealth of data that are about to be produced at the CERN $p\bar{p}$ Collider and LEP, at the SLAC SLC and the FNAL Tevatron, at the DESY HERA, etc., will lead to new important breakthroughs in our understanding of particle physics.

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