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TESTING FOR HEAVIER VECTOR BOSONS IN e^+e^- AT Z PEAK A COMPARATIVE STUDY OF DIFFERENT MODELS*

G. Altarelli
CERN - Geneva

R. Casalbuoni
Dipartimento di Fisica, Università di Lecce and INFN, Sezione di Lecce, I-73100 Lecce
and
Département de Physique Théorique, Université de Genève, CH-1211 Genève 4

D. Dominici
Dipartimento di Matematica e Fisica, Università di Camerino
I-62032 Camerino, and INFN, Sezione di Firenze, I-50125 Firenze

F. Feruglio[†] and R. Gatto
Département de Physique Théorique, Université de Genève, CH-1211 Genève 4

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The entries of Tables 12 and 13 should be multiplied by 3/4. In Tables 14-16 the entries corresponding to the deviations δA_{FB}^c and δA_{FB}^b should be multiplied by 3/4. Accordingly the vertical scales of Figs. 12-13 should be multiplied by 3/4.

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ABSTRACT

We have analyzed the experimental consequences which that the existence of a heavy neutral gauge vector boson may produce in e^+e^- measurements made at energies around the Z mass. The analysis has been made as model independent as possible and it is mainly based upon the classification of the various models according to the transformation properties of the extra neutral gauge field under $U(1)_{em}$. As specific examples we consider extra $U(1)$ and left-right models, as well as a model with electroweak symmetry broken by a strongly interacting sector (BESS). For the analysis we have used the new data from CDF, UA2 and SLC/LEP1 on W and Z masses. We give analytical expressions for the deviations from the standard model (SM) of the observables relevant at LSC/LEP1 energies, such as widths and asymmetries. The range of the allowed deviations from the SM is explicitly evaluated for each of the models discussed. Small deviations are generally expected in the Z line shape parameters; for extra $U(1)$ and LR models the most sensitive observables are the leptonic peak cross-section and width. Potentially large effects might show up in asymmetry measurements with a pattern distinguishing BESS from the other models. Larger deviations are favoured in BESS by a larger top mass and the contrary holds for the other models.

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1. INTRODUCTION

The existence of new gauge vector bosons is a common feature of many extensions of the standard model (SM) of strong and electroweak interactions. The recent impressive improvement of experimental capabilities, with Tevatron, SLC, and LEP1, not to mention future programs, will allow to set sharp limits on new gauge vector bosons or to observe some hints of their possible presence. Mass limits on heavier vector bosons have already been produced by the CDF collaboration at the Fermilab's Tevatron under assumptions about their coupling strengths [1]. Accurate studies will be possible at e^+e^- machines and this work is devoted to the analysis of the signals by which the presence of heavier gauge vector bosons would manifest itself in e^+e^- collisions around the Z peak. The presence of higher gauge bosons would manifest itself around the Z peak in a sequence of characteristic patterns, and we shall show how to analyze each of them such as to be able at least to put stringent new bounds on the masses and couplings of these bosons. The theoretical possible origins of heavier gauge bosons will be classified systematically according to a general analysis. In absence of deviations, future experimental bounds will directly lead to exclusions or limitations upon the various models. In this way the domain of validity of the gauge structure implied by the standard model will be extended to energies much higher than the Z energy, by exploiting the available high precision of e^+e^- measurements. On the other hand, if some indication of deviations will emerge, one may be able to pin down characteristic features pointing out to a larger gauge structure, rather than for instance to low energy supersymmetry or else. The successful operation of SLC with its first results [2] and of LEP1 [3] leads one to optimism with respect to such a program. The high statistics expected at LEP1, already with a few hundredth of thousands of Z , and, with final expectation of millions of them, will allow accurate analyses which should be able to resolve this issue of higher gauge structures up to high mass regions.

Here we have considered the following theoretical possibilities:

- a) Extensions of the gauge group of the SM.
- b) Electroweak symmetry breaking by strong sector.

Extensions of the gauge group are motivated by the principle of unification of the different interactions. In grand unified schemes [4], leptons and quarks are on the same footing and the various interactions originate from a symmetry breaking pattern of a simple gauge group down to the SM. $SU(5)$, $SO(10)$, E_6 or even bigger groups can be the starting point of this breaking chain. It may well happen that in the symmetry breaking process some new gauge group factor survives together with the SM group, down to energies which are close to the electroweak breaking scale. In this perspective, signals of the presence of new, relatively light, gauge vector bosons would represent an important hint in reconstructing the full unified picture.

Unified theories may be endowed with supersymmetry [5], which provides a consistent context to scalars, the essential ingredients for discussing symmetry breaking. Unified theories also naturally emerge in string theories [6] where, in a more ambitious attempt, one tries to unify all known forces, including gravity. String theories are supposed to answer many unsolved questions like: what is the unified group one must start with? Which is the matter content of the theory? How many generations are there? What are the right Yukawa couplings? Why must the theory be approximately supersymmetric? At the moment these problems have not found definite solutions. However, on phenomenological ground, one can consider some promising low-energy scenarios and look for possible experimental signatures in an accessible energy range.

On the other hand it may happen that the effective current-current interaction remains unmodified at low energy. Even in such a situation new gauge vector bosons may occur if the $SU(2)_L \otimes U(1)_Y$ symmetry is not realized in the usual, linear way. This

happens (see ref. [7]) when the breaking of the electroweak symmetry is due to some strongly interacting sector of the theory [8]. In this case, at low energy, the symmetry breaking sector is more appropriately described by a non-linear model much as in the case of low energy hadron physics where pions are local coordinates in a non linear realization of chiral symmetry [9]. Additional vector particles [7] are naturally expected in this scenario (cfr. vector mesons) and they can be described in terms of massive Yang-Mills fields satisfying two basic phenomenological requirements: exact $U(1)_{em}$ gauge invariance and the $\rho = 1$ relation.

LEP1/SLC operate at energies of the order of the mass of the Z , and moreover most of the time will be running just at the energies of the resonance. In this way it will be possible to determine various quantities related to the physical properties of the Z at a level of high accuracy [10]. Therefore, any effect of new physics related to the properties of the Z will have good chances to be observed. The existence of new neutral gauge bosons may have important effects on the Z due to the fact that a mixing will almost unavoidably be present. Quantitative consequences of such a mixing will be:

1) Modifications of the couplings of the Z to matter.

2) Shifts in the masses of the ordinary gauge bosons.

The measurements of quantities very sensitive to the previous parameters, like asymmetries and widths, can therefore offer a unique way to reveal the effects of new physics.

In the following we will consider SM extensions containing only one extra neutral gauge boson V_3 , which we assume to be a Yang-Mills particle with gauge coupling g_V . We find it useful to classify different models in terms of the transformation properties of V_3 with respect to the electromagnetic $U(1)_{em}$. In fact, denoting by V_3 the extra gauge vector boson remaining after the identification of W_3 and B as the states coupled to

the weak isospin T_{3L} and to the hypercharge Y , one has the following two possibilities:

1. $V'_3 = V_{3\mu}$
2. $V'_3 = V_{3\mu} - \frac{1}{g_V} \partial_\mu \lambda(x)$

under $U(1)_{em}$. The first possibility corresponds to a truly neutral additional gauge vector boson. Typical examples of this scenario are:

- gauge extension of the SM with gauge group $SU(2)_L \otimes U(1)_Y \otimes U(1)_\phi \subset E_6$ [11];
- left-right symmetric models based on the gauge group $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ [12];

with a certain Higgs sector responsible for the symmetry breaking. An explicit realization of the second scenario is given in ref.[7] by a model with a breaking of the electroweak symmetry due to a strongly interacting sector: breaking electroweak symmetry strongly (BESS). In this case the electroweak symmetry breaking is obtained via a non-linear realization and no Higgs particles are present. As we shall discuss more extensively in the next section, the two possibilities listed above lead to significantly different structures. The major difference is exhibited in the mass matrix for the neutral massive gauge vector bosons. In terms of the predicted deviations, this results in a strong dependence on M_V , the mass of the new gauge particle, for the models fitting the first case, whereas the predictions of the models falling into the second scenario are largely independent of M_V .

In studying new physics one is faced with two problems: first of all, it is necessary to parametrize in some way the new effects; second, it is necessary to find criteria to distinguish among the models. One has to be particularly careful about radiative corrections. In fact these corrections can easily be of the same order of magnitude of the new physics effects, and it is necessary to find a way to disentangle the different

contributions. We will discuss these issues in the following sections.

As a general comment, one can observe that the models will contain a priori a lot of parameters such as quark and lepton masses, Higgs masses, vacuum expectation values, and so on. But, for the kind of physics we are interested in, the most important parameters are the ones which are peculiar to the gauge theory itself, that is gauge couplings and masses of the vector bosons. For instance, in this context, the relevant parameters of the SM will be the Fermi coupling constant G_F , the fine structure constant at zero momentum α , and the mass of the Z , M_Z which will be measured at LEP 1 with an accuracy of $20 \div 50 \text{ MeV}$.

Heavier gauge bosons effects at e^+e^- colliders have been extensively studied during the last few years [13-16]. Our analysis differs from those in the existing literature in several aspects. First of all we give a different formulation of the extended models which, being based on $U(1)_{em}$ gauge invariance and on the approximate $\rho = 1$ relation, is particularly physically motivated. Moreover, this new approach allows us to analyze from the same point of view the two quite different theoretical schemes a) and b) mentioned above, unifying to a large extent the underlying formalism. We have made an effort in order to make our presentation as much model independent as possible, so that any other model containing new neutral gauge vector bosons, not explicitly considered here, could be easily studied in terms of our parametrization. Concerning the final Sections, where we discuss the physical results, we have tried to put emphasis on the practical usefulness of the outputs. While the experimental data are rapidly accumulating and possible bounds on the relevant parameters are more or less continuously changing, we have chosen to present not only an update range of the allowed expected deviations, but also simple, explicit analytical formulas which we hope could be used also at future experimental stages.

In Sections 2 and 3 we will discuss the general properties of SM extensions with an

extra gauge vector boson transforming either as $V'_{3\mu} = V_{3\mu}$ or as $V'_{3\mu} = V_{3\mu} - \partial_\mu \lambda / g_V$ under $U(1)_{em}$. Section 4 is devoted to a discussion of radiative corrections and of bounds on masses and mixing parameters that can be derived from recent $p\bar{p}$ collider data. In Section 5 we list the e^+e^- observables on which we base our analysis and we discuss some qualitative features of the expected deviations. Sections 6 and 7 contain our quantitative results presented in the form of analytical expressions for the deviations and maximal discrepancies which might show up in measurements of partial widths and asymmetries. In Section 8 we outline a general procedure for deriving bounds on the relevant parameter space for the various models. In the final Section our conclusions are presented.

2. MODELS WITH $V'_{3\mu} = V_{3\mu}$

In this section we will discuss the general structure of models containing an extra gauge vector boson $V_{3\mu}$ transforming under $U(1)_{em}$ as $V'_{3\mu} = V_{3\mu}$. We will also present as specific examples of this case two extensions of the SM characterized by the gauge invariance under $SU(2)_L \otimes U(1)_Y \otimes U(1)_F \subset E_6$ [11] and $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ [12].

We denote by $W_{3\mu}$ and B_μ the gauge vector bosons coupled to J_{3L} and J_Y , respectively. Once this identification is made, one has an extra neutral gauge vector boson, here for convenience called Z'_0 , which we take to be invariant under $U(1)_{em}$. The photon field A_μ is completely fixed by the requirement of being coupled to the electromagnetic current J_{em} , given by: $J_{em} = J_{3L} + J_Y$. One finds

$$A = W_{3L} \sin \theta + B \cos \theta \quad (2.1)$$

$$\tan \theta = \frac{g'}{g} \quad (2.2)$$

$$e = g \sin \theta \quad (2.3)$$

where g, g', e are the $SU(2)_L$, $U(1)_Y$ and $U(1)_{em}$ gauge coupling constants. The orthogonal combination:

$$Z_0 = W_{3L} \cos \theta - B \sin \theta \quad (2.4)$$

would correspond to the usual Z in the case of the SM, and it is coupled to the current $J_{3L} - \sin^2 \theta J_{em}$. Note that $\sin^2 \theta$ appearing here is defined, at lowest order, through equation (2.3). The extra gauge vector boson Z'_0 is coupled to a new current $J_{\tilde{Y}}$ which for the moment will not be specified. The $U(1)_{em}$ gauge invariance allows for the following general mass terms in the gauge vector boson sector:

$$\mathcal{L}_M = M_W^2 W^2 + \frac{1}{2} [Z_0 \quad Z'_0] \begin{bmatrix} \frac{M_W^2}{\rho_{SB} \cos^2 \theta} & \delta^2 \\ \delta^2 & M_0^2 \end{bmatrix} \begin{bmatrix} Z_0 \\ Z'_0 \end{bmatrix} \quad (2.5)$$

The neutral sector mass matrix depends upon three independent parameters. The quantity $\rho_{SB} = 1 + \Delta \rho_{SB}$ represents a possible isospin breaking contribution coming from the scalar sector of the theory which is responsible for the symmetry breaking (SB). If scalars with non-vanishing VEV's come only in doublet and singlet $SU(2)_L$ representations, then we have $\rho_{SB} = 1$. The mass eigenstates, Z and Z' , are rotated with respect to the basis Z_0, Z'_0 by an angle ξ_0 :

$$\begin{bmatrix} Z \\ Z' \end{bmatrix} = \begin{bmatrix} \cos \xi_0 & \sin \xi_0 \\ -\sin \xi_0 & \cos \xi_0 \end{bmatrix} \begin{bmatrix} Z_0 \\ Z'_0 \end{bmatrix} \quad (2.6)$$

Denoting by M_Z^2 and $M_{Z'}^2$ the corresponding mass eigenvalues, one has the following relations:

$$M_Z^2 \cos^2 \xi_0 + M_{Z'}^2 \sin^2 \xi_0 = \frac{M_W^2}{\rho_{SB} \cos^2 \theta} \quad (2.7)$$

$$M_Z^2 \sin^2 \xi_0 + M_{Z'}^2 \sin^2 \xi_0 = M_0^2 \quad (2.8)$$

$$\sin \xi_0 \cos \xi_0 = -\frac{\delta^2}{M_Z^2 M_{Z'}^2} \frac{M_Z^2}{M_{Z'}^2 - M_Z^2} \quad (2.9)$$

We introduce the ρ parameter through the following definition

$$\frac{M_W^2}{\rho M_Z^2} = \cos^2 \theta \quad (2.10)$$

At lowest order, ρ receives two independent contributions: one from the SB sector and one from the mixing. We write:

$$\rho = \rho_{SB} \rho_M \quad (2.11)$$

By combining eqs. (2.10), (2.11) and (2.7) one obtains:

$$\rho_M = \frac{\frac{M_Z^2}{M_{Z'}^2}}{1 + \tan^2 \xi_0 \frac{M_Z^2}{M_{Z'}^2}} \quad (2.12)$$

Eq. (2.12) expresses the fact that the rotation (2.6) is associated to a shift of the Z mass M_Z^2 with respect to its SM value $M_W^2/(\rho_{SB} \cos^2 \theta)$, resulting in a positive contribution to ρ .

We observe that, in the limit of $M_{Z'}^2 \gg M_Z^2$ and if the mixing term δ^2 remains bounded, equations (2.9) and (2.12) can be approximated by:

$$\xi_0 \simeq c \left(\frac{M_Z}{M_{Z'}} \right)^2 \quad (2.13)$$

$$\Delta \rho_M = c^2 \left(\frac{M_Z}{M_{Z'}} \right)^2 \simeq \rho_M - 1 \quad (2.14)$$

where $c = -\delta^2/M_Z^2$ is a quantity depending on the scalar particles and their VEV's. Equations (2.9) and (2.12) also show that, provided δ^2 remains bounded, in the limit of very large $M_{Z'}$, the effects due to the presence of the new gauge vector boson become negligible. Then the new particle decouples and the theory reproduces the SM.

In principle, one could completely specify the mass matrix by assigning a definite set of scalar multiplets and their vacuum expectation values [14]. On the other hand one can take advantage of having as free parameters of the theory $M_Z, M_{Z'}, c$ and ρ , avoiding the choice of any specific scalar structure in the class of gauge theories considered. The W mass is then given by eq. (2.10). In eq. (2.10), $\cos^2 \theta$ can be eliminated in favour of the Fermi constant G_F , the fine structure constant α and the other mass parameters. By making use of eq. (2.3) and of the relation

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} \quad (2.15)$$

we obtain

$$\sin^2 \theta \cos^2 \theta = \frac{\mu^2}{\rho M_Z^2} \quad (2.16)$$

where

$$\mu^2 = \frac{\pi \alpha}{\sqrt{2} G_F} = 37.280 (GeV)^2 \quad (2.17)$$

Eq. (2.16) can be solved for $\sin^2 \theta$ and $\cos^2 \theta$ and one finds

$$\sin^2 \theta = \left(\frac{1}{2} - \sqrt{\frac{1}{4} - \frac{\mu^2}{\rho M_Z^2}} \right) \quad (2.18)$$

$$\cos^2 \theta = \left(\frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\mu^2}{\rho M_Z^2}} \right) \quad (2.19)$$

Combining eqs. (2.10) and (2.19), one obtains

$$\frac{M_W^2}{\rho M_Z^2} = \left(\frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\mu^2}{\rho M_Z^2}} \right) \quad (2.20)$$

Eqs. (2.18)-(2.20) are tree-level approximations showing how the corresponding SM relations are corrected by the presence of an extra neutral gauge boson. Since the corrections induced by the presence of a new gauge vector boson can compete with radiative corrections, it is important to take into account, at least approximately, also contributions to the previous relations which come from perturbation theory. This problem is addressed to in Section 5.

We now come to two specific realizations of this scenario. To define a certain model it is necessary to give the new current J_Y to which the new gauge vector boson Z'_0 is coupled. As a first example, we choose this extra current to be the one associated to the extra $U(1)_Y$ factor of the gauge group $SU(2)_L \otimes U(1)_Y \otimes U(1)_\varphi \subset E_6$. Since E_6 contains two additional colorless neutral generators Y' and Y'' , we define J_Y as a general linear combination of the corresponding currents $J_{Y'}$ and $J_{Y''}$ (we are using the conventions of ref. [17]):

$$J_Y = J_{Y'} \cos \theta_2 - J_{Y''} \sin \theta_2 \quad (2.21)$$

The neutral current couplings are:

$$\mathcal{L}_{NG} = ig W_{3L} J_{3L} + ig' B J_Y + ig_Y Z'_0 J_Y \quad (2.22)$$

The vector and axial coupling constants of Z'_0 to known fermions are collected in the appendix. In our numerical analysis, we have fixed $M_Z = 91.1 GeV$, $g_Y = g'$, $\rho_{SB} = 1$. In doing so one is left with a three parameter space $(c, M_{Z'}, \theta_2)$.

As a second realization of the present scenario, we have considered a left-right extension of the SM.

The neutral current couplings are given by

$$\mathcal{L} = ig J_{3L} W_{3L} + i \frac{g}{\lambda} J_{3R} W_{3R} + ig' J_{B-L} B \quad (2.23)$$

where $g \equiv g_L$, $\lambda = \frac{g_L}{g_R}$, $W_{3L,3R}$ are the neutral $SU(2)_{L,R}$ gauge fields, while g' and B are the coupling constant and the gauge field for $U(1)_{B-L}$,

$$\begin{aligned} J_{3L,3R}^\mu &= \bar{\psi} \gamma^\mu T_{3L,3R} \psi \\ J_{B-L}^\mu &= \bar{\psi} \gamma^\mu \frac{B-L}{2} \psi \end{aligned} \quad (2.24)$$

The electromagnetic current is given by

$$J_{em} = J_{3L} + J_{3R} + J_{B-L} \quad (2.25)$$

From eqs. (2.23) and (2.25) one can determine the photon field A , by imposing the coupling to J_{em} to be of the form $ie A J_{em}$. One gets

$$A = \sin \theta W_L^3 + \lambda \sin \theta W_R^3 + y B \quad (2.26)$$

where $y = \sqrt{\cos^2 \theta - \lambda^2 \sin^2 \theta}$ and the angle θ is now defined as:

$$g \sin \theta = g' y = e \quad (2.27)$$

It is convenient to work in an orthogonal basis containing explicitly the photon A . Denoting by Z_{0L} and Z_{0R} the other fields of this basis, they can be fixed by the requirement

that Z_{0R} does not couple to J_{3L} . The new basis is given by the following orthogonal transformation:

$$\begin{pmatrix} A \\ Z_{0R} \\ Z_{0L} \end{pmatrix} = \begin{pmatrix} \sin \theta & \lambda \sin \theta & y \\ 0 & \frac{y}{\cos \theta} & -\lambda \tan \theta \\ \cos \theta & -\lambda \sin \theta \tan \theta & -y \tan \theta \end{pmatrix} \begin{pmatrix} W_L^3 \\ W_R^3 \\ B \end{pmatrix} \quad (2.28)$$

Z_{0L} and Z_{0R} are coupled to the currents ($J_{3L} - \sin^2 \theta J_{em}$) and ($y J_{3R} - \lambda^2 \sin^2 \theta J_{B-L}$) respectively. Therefore the combinations Z_{0L} and Z_{0R} can be identified with the fields Z_0 and Z'_0 of our general analysis. The only difference with the general case discussed before is the presence of a new charged vector boson. As far as the neutral sector is concerned, one can take into account the effects of an additional charged state, by modifying the relation (2.12). In fact the physical W mass M_W gets shifted from its SM value due to the occurrence of a mixing in the charged sector, giving rise to a negative contribution to ρ . One finds:

$$\rho_M = \frac{1 + \tan^2 \xi}{1 + \tan^2 \xi_0} \frac{1 + \tan^2 \xi_0}{1 + \tan^2 \xi} \frac{M_{Z'}^2}{M_Z^2} \quad (2.29)$$

where ξ is the mixing angle arising from the mass matrix diagonalization in the W sector. Moreover, assuming $\xi \ll 1$ and a heavy right-handed neutrino, we obtain eq. (2.15), which, combined with eq. (2.27), allows us to express M_W and $\sin^2 \theta$ in terms of G_F , α , M , ρ . The results are listed in eqs. (2.18)-(2.20) with the value for ρ_M shown in eq. (2.29). Radiative corrections are discussed in section 5.

In conclusion, the overall set of parameters we adopt here consists of: G_F , α , $M_{Z'}$, c , $M_{Z'}$, ρ , ξ , $M_{W'}$, and λ . In our quantitative analysis we have taken $\rho_{SB} = 1$ and $\lambda = 1$. It is generally believed that limits on the charged sector are more severe than those on the neutral one. In particular $M_{W'}$ is highly constrained from the $K_L - K_S$ mass difference [18]: $M_{W'} \geq 1.6 - 2.5$ TeV. A stringent bound on ξ is obtained from an analysis of semileptonic-decay data [19]: $|\xi| \leq 0.005$. It must be said that such limits are not model independent. Both of them assume particular relations between the

Cabibbo-Kobayashi-Maskawa matrices of the left and right sectors. Model dependent limits on $M_{W'}$ and ξ , together with a review of other existing constraints, are discussed in ref. [20], where it is shown that values as low as 300 GeV are allowed for $M_{W'}$ and $|\xi|$ could be as large as 0.013. In some particular case a complete discussion of the scalar potential of the theory and the requirements of perturbative unitarity and absence of flavour changing neutral currents leads to a vanishing ξ [21].

In discussing left-right models quantitatively, we have considered two extreme scenarios. In the first one, we have assumed that mixing in the charged sector is negligible with respect to mixing in the neutral one, and we have simply taken $\xi = 0$. In the second one we have supposed that the charged sector effects are such that they are able to compensate for the neutral sector contribution to $\Delta\rho_M$, resulting in $\rho_M = 1$.

Couplings of the new neutral vector boson to fermions are listed in appendix.

3. MODELS WITH $V_{3\mu'} = V_{3\mu} - \frac{1}{g_V} \partial_\mu \lambda$.

In this Section we will consider the case of $V_{3\mu}$ transforming as

$$V_{3\mu'} = V_{3\mu} - \frac{1}{g_V} \partial_\mu \lambda$$

under $U(1)_{em}$. A physical motivation to this construction can be given by invoking the occurrence of a strongly interacting electro-weak sector. The idea is that the symmetry breaking part of the electroweak lagrangian corresponds to a strong interaction, and as a consequence resonances may occur. In particular one expects vector-like bound states [7]. These vector bound states can be described in terms of massive Yang-Mills fields, and by some analogy with ordinary strong interactions, we will choose $SU(2)$ for the corresponding gauge group. The full gauge group will be $SU(2)_L \otimes U(1) \otimes SU(2)$, and the theory will describe 6 massive gauge bosons (the usual $W^\pm$ and Z , plus three new ones to be denoted by $V^\pm$ and V^0), and the photon. Indeed the last $SU(2)$ factor

is "hidden", since it does not give rise to any new fermionic current. The electroweak symmetry breaking does not require an elementary Higgs and the model we are going to discuss does not contain any physical scalar. The prize to pay for the absence of scalars is the renormalizability, which the model does not possess due to the presence of massive Yang-Mills fields. However, in its spirit, the model represents an effective description, valid at energies not much larger than the electroweak breaking scale. We will see a posteriori how the particular transformation property of $V_{3\mu}$ really corresponds to breaking the electro-weak symmetry strongly (BESS).

As in the previous section, our starting point is the mass matrix in the gauge vector boson sector. We observe that the transformation properties of the neutral gauge vector bosons under $U(1)_{em}$ are the following

$$\begin{aligned} W'_{3\mu} &= W_{3\mu} - \frac{1}{g} \partial^\mu \lambda \\ B'_\mu &= B_\mu - \frac{1}{g'} \partial^\mu \lambda \\ V'_{3\mu} &= V_{3\mu} - \frac{1}{g_V} \partial^\mu \lambda \end{aligned} \quad (3.1)$$

Ignoring for a while the extra field $V_{3\mu}$, we note that the gauge boson mass terms of the SM lagrangian can be obtained simply by requiring that they are invariant under the transformations (3.1) and that they satisfy the $\rho = 1$ relation. $U(1)_{em}$ gauge invariance allows one to write just the following mass term for the fields $W_{3\mu}, B_\mu$

$$\mathcal{L}_M = -\frac{v^2}{4} \text{tr}(\mathcal{W}^3 - \mathcal{B})^2 \quad (3.2)$$

where $v^2 = 1/(G_F \sqrt{2}) = (246 \text{ GeV})^2$ and we have defined

$$\begin{aligned} \mathcal{W}_\mu^i &= ig \frac{\tau_i}{2} W_\mu^i \quad (\text{not summed}) \\ \mathcal{B}_\mu &= ig' \frac{T_3}{2} B_\mu \end{aligned} \quad (3.3)$$

The mass term in eq. (3.2) is then completed in a unique way to include also the charged sector by demanding $\rho = 1$. One obtains:

$$\mathcal{L}_M = -\frac{v^2}{4} \text{tr}(\mathcal{W} - \mathcal{B})^2 \quad (3.4)$$

where $\mathcal{W}_\mu = \sum_{i=1}^3 \mathcal{W}_\mu^i$. Notice that in the SM the following two tree-level relations hold:

$$\rho = \left(\frac{M_W}{M_Z \cos \theta} \right)^2 = 1 \quad (3.5)$$

and

$$\rho_{eff} = 1 \quad (3.6)$$

where ρ_{eff} is defined as the ratio between charged and neutral current strengths in the effective low-energy lagrangian.

We now proceed in a similar way for the case of an additional triplet of gauge fields $V_{i\mu}$, whose third component transforms as indicated in eq. (3.1). We define:

$$\begin{aligned} \mathcal{V}_\mu^i &= ig_V \frac{\tau_i}{2} V_\mu^i \\ \mathcal{V}_\mu &= \sum_{i=1}^3 \mathcal{V}_\mu^i \end{aligned} \quad (3.7)$$

By requiring a $U(1)_{em}$ invariant mass term which reproduces the $\rho_{eff} = 1$ relation, one obtains

$$\mathcal{L}_M = -\frac{v^2}{4} [\text{tr}(\mathcal{W} - \mathcal{B})^2 + \alpha \text{tr}(\mathcal{W} - \mathcal{B} - 2\mathcal{V})^2] \quad (3.8)$$

The coefficient α appearing in eq. (3.6) is a free real parameter. As we shall discuss in the following, the lagrangian of eq. (3.6) gives automatically rise to $\rho_{eff} = 1$, whereas the relation (3.5) gets modified by mixing terms. We first extract from eq. (3.8) the photon field A_μ as the massless combination, as well as two fields Z_1 and Z_2 to complete a basis. We choose Z_1 as the field which, in the SM, could be identified with the ordinary Z vector boson, and Z_2 as the remaining orthogonal state. One has

$$\begin{aligned} A &= W_3 \sin \theta \cos \psi + B \cos \theta \cos \psi + V_3 \sin \psi \\ Z_1 &= W_3 \cos \theta - B \sin \theta \\ Z_2 &= W_3 \sin \theta \sin \psi + B \cos \theta \sin \psi - V_3 \cos \psi \end{aligned} \quad (3.9)$$

We have defined

$$\begin{aligned}\sin \theta &= \frac{g'}{\sqrt{g^2 + g'^2}} \\ \tan \psi &= \frac{2gg'}{g''\sqrt{g^2 + g'^2}}, \quad g'' = 2g_V\end{aligned}\tag{3.10}$$

By definition A is coupled to the current eJ_{em} , with the electric charge e given by:

$$e = g \sin \theta \cos \psi\tag{3.11}$$

We observe that in the limit $\psi = 0$ the field V_3 decouples from A and one has the SM relation. This indicates that BESS predicts small deviations from the SM in the limit of large g'' , where the additional gauge vector boson has a strong self-interaction.

From eq. (3.8) one can immediately write the square mass matrix in the charged and neutral sectors. One finds:

$$\begin{aligned}M^2 &= \frac{v^2}{4} \left(\begin{bmatrix} g^2 & 0 \\ 0 & 0 \end{bmatrix} + \alpha \begin{bmatrix} g^2 & gg'' \\ gg'' & g''^2 \end{bmatrix} \right) \\ M_0^2 &= \frac{v^2}{4} \left(\begin{bmatrix} G^2 & 0 \\ 0 & 0 \end{bmatrix} + \alpha \begin{bmatrix} \tilde{G}^2 & \tilde{G}\tilde{G}'' \\ \tilde{G}\tilde{G}'' & \tilde{G}''^2 \end{bmatrix} \right)\end{aligned}\tag{3.12}$$

where M^2 and M_0^2 refer to the basis $(W^\pm, V^\pm)$ and (Z_1, Z_2) respectively, and we have defined

$$G = \sqrt{g^2 + g'^2}, \quad \tilde{G} = G \cos 2\theta, \quad \tilde{g}'' = \frac{g''}{\cos \psi}\tag{3.13}$$

The mass matrices of eqs. (3.12) and (3.13) can be readily diagonalized through the transformations:

$$\begin{bmatrix} \tilde{W}^\pm \\ \tilde{V}^\pm \end{bmatrix} = \begin{bmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} W^\pm \\ V^\pm \end{bmatrix}\tag{3.14}$$

$$\begin{bmatrix} Z \\ V_0 \end{bmatrix} = \begin{bmatrix} \cos \xi & \sin \xi \\ -\sin \xi & \cos \xi \end{bmatrix} \begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix}\tag{3.15}$$

The exact expression for the mixing angles ϕ, ξ as well as for the mass eigenvalues can be found in ref [7]. For convenience we list below the results of a first order expansion in g/g'' for ϕ and ξ :

$$\phi \approx -\frac{1}{1 - \frac{M_W^2}{M_V^2}} \frac{g}{g''}\tag{3.16}$$

$$\xi \approx -\frac{\cos 2\theta}{\cos \theta} \frac{1}{1 - \frac{M_Z^2}{M_{V_0}^2}} \frac{g}{g''}\tag{3.17}$$

W and Z masses get corrected by terms of order g/g'' ; the new triplet of states is nearly degenerate in mass, whose lowest order value is $\sqrt{\alpha} g'' v/2$.

Observe that in this case the mixing angles do not vanish any longer when the largest eigenvalues go to infinity, contrarily to what happens in the models described in the previous section. Therefore the effects of the heaviest modes still persist in the low energy limit. We stress the fact that this non decoupling behaviour can be eventually traced back to the peculiar $U(1)_{em}$ transformation property of V_3 . It is this property which allows in the mass terms of eq. (3.8) a mixing term among V_3 and $W_3 - B$ which turns out to be proportional to the mass of the heaviest states, so that the non diagonal terms are no longer bounded for infinitely heavy V .

To discuss the interactions of the vector bosons Z_1 and Z_2 , one must specify the current $\vec{J}$ to which the new triplet of states $\vec{V}$ couples. Here we discuss the illustrative case $\vec{J} = \vec{J}_L$. The interaction lagrangian becomes:

$$\mathcal{L} = ug\vec{W} \cdot \vec{J}_L + vg'B\vec{J}_Y + ig''\vec{V} \cdot \vec{J}_L\tag{3.18}$$

In eq. (3.17) one would naively have written $u = v = 2t = 1$. However the $U(1)_{em}$ transformation law for V_3 forces a peculiar relation among these coefficients:

$$v = u + 2t = 1\tag{3.19}$$

Therefore, the presence of a direct coupling of $\vec{V}$ to $\vec{J}_L$ induces a renormalization of g . Moreover, the phenomenological requirement of having u close to 1:

$$u = \frac{1}{1+b}, \quad b \ll 1\tag{3.20}$$

implies, from eq. (3.18), that t is a small parameter:

$$t = \frac{1}{2} \frac{b}{1+b}\tag{3.21}$$

so that, even in the limit of large g'' , $\vec{V}$ is weakly coupled to $\vec{J}_L$. In the following we will consider a minimal version of this class of models, assuming no direct coupling of $\vec{V}$ to fermions: $b = 0$. A general discussion of the case $b \neq 0$ can be found in ref. [22].

Concerning the states Z_1 and Z_2 introduced in eq. (3.9), when $b = 0$ they are coupled to the currents

$$\begin{aligned} J_1 &= \frac{e}{\sin \theta \cos \theta \cos \psi} \frac{1}{\cos \psi} (J_{3L} - \sin^2 \theta J_{em}) \\ J_2 &= e \tan \psi J_{em} \end{aligned} \quad (3.22)$$

From eqs. (3.12), (3.13) and (3.22) it is now possible to recover the low-energy charged and neutral current lagrangians. One obtains

$$\begin{aligned} \mathcal{L}_{CC} &= 2\sqrt{2}G_F J_L^+ J_L^- \\ \mathcal{L}_{NC} &= 2\sqrt{2}G_F \rho_{eff} [(J_{3L} - \Sigma_\theta^2 J_{em})^2 + cJ_{em}^2] \end{aligned} \quad (3.23)$$

where

$$\rho_{eff} = 1 \quad (3.24)$$

$$\Sigma_\theta^2 = \sin^2 \theta \left(1 + \frac{\cos 2\theta}{2 \sin^2 \theta} \sin^2 \psi \right) \quad (3.25)$$

$$c = \frac{\sin^4 \psi}{4\alpha} \quad (3.26)$$

Eq. (3.26) indicates that, in the large g'' regime, the parameter c is strongly suppressed so that the experimental bound $c < 0.01$ [23] is always satisfied. Note that eq. (3.24) is a direct consequence of the structure of the mass matrices given in eqs. (3.12) and (3.13), for which we have

$$(M^2)^{-1}_{11} = \frac{4}{g'^2 v^2}$$

and

$$(M_0^2)^{-1}_{11} = \frac{4}{G'^2 v^2}$$

We further observe that $\rho \neq 1$, if defined in terms of the vector boson mass ratio. Indeed one finds:

$$\cos^2 \theta = \frac{M_W^2}{\rho M_Z^2} \quad (3.27)$$

with

$$\rho = \rho_Z \rho_W^{-1} \quad (3.28)$$

$$\rho_Z = \frac{1 + \tan^2 \xi}{1 + \tan^2 \xi \frac{M_Z^2}{M_{V_0}^2}} \quad (3.29)$$

$$\rho_W = \frac{1 + \tan^2 \phi}{1 + \tan^2 \phi \frac{M_W^2}{M_{V_0}^2}} \quad (3.30)$$

In the previous equations M_W, M_V, M_Z, M_{V_0} are the physical vector boson masses. Eqs. (3.29) and (3.30) provide for another example of how physical effects due to the heaviest particles V_0 and $\tilde{V}$ do not disappear in the limit of infinite M_{V_0}, M_V , since in general the mixing angles ϕ and ξ are non vanishing. Another useful relation is:

$$\sin^2 \theta \cos^2 \theta = \frac{\mu^2}{\rho' M_Z^2} \quad (3.31)$$

where:

$$\rho' = \rho_Z \cos^2 \psi \quad (3.32)$$

From eq. (3.31) and eq. (3.27) one easily gets

$$\frac{M_W^2}{M_Z^2} = \rho \left(\frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\mu^2}{\rho' M_Z^2}} \right) \quad (3.33)$$

Finally, in the case $b = 0$, the physical Z is coupled to the current

$$J_Z = J_1 \cos \xi + J_2 \sin \xi = \frac{e}{\cos \theta \sin \theta \cos \psi} \cos \xi \left(J_{3L} - \sin^2 \theta \left(1 - \frac{\tan \xi \sin \psi}{\tan \theta} \right) J_{em} \right) \quad (3.34)$$

It may be convenient to express the combination $G = e/(\sin \theta \cos \theta \cos \psi)$ in terms of G_F, α and M_Z . By combining eq. (3.31) and eq. (3.32) one obtains:

$$G^2 = 4\sqrt{2}G_F M_Z^2 \rho_Z \quad (3.35)$$

The vector and axial couplings to the physical Z are explicitly listed in appendix.

4. RADIATIVE CORRECTIONS AND BOUNDS ON THE PARAMETERS

Since new vector bosons effects at LEP1 are expected to be small, because already constrained from the existing low energy data and from the new data from CDF, it can be a far from trivial task to disentangle them from other small contributions to a given physical observable. In particular in comparing the various models with the SM a careful treatment of the radiative corrections is needed. A major problem arises immediately from the fact that not all the parameters entering the computation of the relevant quantities are known, even in the case of the SM with three fermion generations and a single complex $SU(2)_L$ Higgs doublet. Higgs and top having unknown masses, room is left for potentially large radiative corrections whose exact values are at present only poorly determined. Since the problem deserves a particular attention, we will briefly review the status of radiative corrections in the SM, focusing on $e^+e^- \rightarrow f\bar{f}$ processes at the Z resonance [24]. Here we distinguish two classes of corrections:

- "Pure-QED" corrections (different from the photon vacuum polarization).

- "Weak corrections" (self-energy, vertex corrections, ...).

The first type of corrections include initial and final state radiation, together with the virtual corrections, the sum being free of infrared divergences. For our purposes we shall assume that they have been taken into account, and subtracted from the experimental data.

As far as the second type of corrections is concerned, they can be mostly included within an improved Born approximation [24,25]. In this scheme the ordinary Z couples on the resonance to a fermion-antifermion pair $f\bar{f}$ through the effective current

$$J_{eff}^\mu = (4\sqrt{2}G_F M_Z^2 \rho_f)^{\frac{1}{2}} \bar{f} \gamma_\mu \left[T_{3L}^f (1 - \gamma_5) - 2Q_{em} \sin^2 \theta_f \right] f \quad (4.1)$$

where, in eq. (4.1), ρ_f and $\sin^2 \theta_f$ contain both the universal corrections from vector boson self-energies and the non-universal ones from the vertex corrections. In the case

$f \neq b$, vertex corrections are negligible (they produce modifications of the vector and axial couplings of order $2 \cdot 10^{-3}$ [24]), and if one restricts himself to the potentially largest corrections, one obtains the simple pattern

$$\rho_f = \frac{1}{1 - \Delta\rho_T} \equiv \rho_T \quad (4.2)$$

$$\Delta\rho_T \approx 3 \frac{G_F m_t^2}{8\pi^2 \sqrt{2}} \quad (4.3)$$

$$\sin^2 \theta_f = \frac{1}{2} - \sqrt{\frac{1}{4} - \frac{\mu^2}{\rho_T M_Z^2 (1 - \Delta\alpha)}} \equiv \sin^2 \bar{\theta} \quad (4.4)$$

$$\Delta\alpha = 0.0602 + \frac{40}{9} \frac{\alpha}{\pi} \log \frac{M_Z}{92 \text{ GeV}} \pm 0.0009 \quad (4.5)$$

The effective angle $\bar{\theta}$ [24,25,26], which enters in the current of eq. (4.1), differs from the angle θ_W of the on-shell scheme, defined by $\sin^2 \theta_W = 1 - M_W^2/M_Z^2$. It originates from a modification of the Z vector coupling due to a non diagonal γZ self-energy matrix element. Once expressed in terms of G_F , α and M_Z , $\sin^2 \bar{\theta}$ exhibits a much weaker dependence on the top mass than the original $\sin^2 \theta_W$ parameter. To appreciate this feature, one can approximate eq. (4.4) with

$$\sin^2 \bar{\theta} \approx \frac{1}{2} - \sqrt{\frac{1}{4} - \frac{\mu^2}{M_Z^2 (1 - \Delta\bar{r})}} \quad (4.6)$$

where $\Delta\bar{r} = \Delta\alpha - \Delta\rho_T$. A similar expression also holds for $\sin^2 \theta_W$ but with $\Delta\bar{r}$ replaced by $\Delta\bar{r} = \Delta\alpha - \Delta\rho_T \cos^2 \theta / \sin^2 \theta$. For a top mass ranging between 50 GeV and 200 GeV, taking $M_Z = 91.1 \text{ GeV}$, $\sin^2 \bar{\theta}$ varies of about -0.005, while the variation of $\sin^2 \theta_W$ is more than three times bigger.

For $f = b$ the vertex corrections are no longer negligible due to the top exchange contribution, and one has to make the following substitutions [24,27]:

$$\rho_b = \rho_T \left(1 - \frac{4}{3} \Delta\rho_T \right) \quad (4.7)$$

$$\sin^2 \theta_b = \sin^2 \bar{\theta} \left(1 + \frac{2}{3} \Delta\rho_T \right) \quad (4.8)$$

A final step consists in replacing the tree-level constant width pole structure ($s - M_Z^2 + iM_Z\Gamma_Z$)⁻¹ in the Z propagator with

$$(s - M_Z^2 + i\frac{s}{M_Z^2}\Gamma_Z M_Z)^{-1} \quad (4.9)$$

This replacement takes properly into account the variation of the imaginary part for approximately massless final states. In eq. (4.9) one has:

$$\Gamma_T = \sum_f \Gamma_f(Z \rightarrow ff) \quad (4.10)$$

The partial widths (neglecting final state mass corrections) are given by:

$$\Gamma_f(Z \rightarrow ff) = N_c^f \rho_f \frac{G_F M_Z^3}{24\pi\sqrt{2}} [1 + (1 - 4|Q_{em}|\sin^2\theta_f)^2] \quad (4.11)$$

where $N_c^f = 1$ for leptons and $N_c^f = 3(1 + \alpha_s/\pi) \approx 3(1 + 0.04)$ for quarks. The quality of this approximation is fully discussed in ref. [24]. In the partial widths the above procedure approximates the complete 1-loop results within 1%. In the forward-backward asymmetry the accuracy is of about 0.005, and in the τ -polarization asymmetry an error of about 0.020 can be estimated.

Coming to the problem of including radiative corrections in the extended models considered here, a treatment of the complete one-loop corrections would be desirable, but it has been only partially realized until now [28]. Here we have adopted the following strategy. We have assumed that the loop contributions are dominated by the SM radiative corrections and that the effect of the new particles can be neglected in the loop diagrams in comparison with their tree-level effects. The effects neglected here are generally small [28] and estimated to be of the order of $(\alpha/\pi)(M_Z/M_{Z'})^2$ [14]. The above assumptions define, for each model, an improved Born approximation which consists in modifying the tree-level expressions of the previous sections according to the rules listed before. For instance, the ordinary Z , in a left-right or in an extra $U(1)$ SM extension, couples to a fermion antifermion pair ff ($f \neq b$) with the coupling constant

given in the appendix and an effective weak angle given by:

$$\sin^2\bar{\theta} = \left(\frac{1}{2} - \sqrt{\frac{1}{4} - \frac{\mu^2}{\rho M_Z^2(1 - \Delta\alpha)}}\right) \quad (4.12)$$

where now the ρ parameter of eq. (2.11) has been modified according to:

$$\rho = \rho_{SB}\rho_M\rho_T \quad , \quad (4.13)$$

to include not only the effects of a particular Higgs structure (factor ρ_{SB}) and those of the mixing (factor ρ_M), but also the loop effect of eq. (4.3).

We conclude this section with some remarks on the existing limitations on the parameters of the subsequent numerical analysis. A first bound is directly obtained from the ratio M_W/M_Z , as derived from CDF [1] and UA2 [29] data and from the value of $M_Z = 91.10 \pm 0.06$ GeV obtained by combining the SLC and LEP1 experiments [2,3].

One has:

$$\frac{M_W^2}{M_Z^2} = 0.779 \pm 0.010. \quad (4.14)$$

For the models described in section 2 one obtains $\rho = 1.01 \pm 0.01$. For $m_t = 80$ GeV the top quark contribution to ρ is about $\Delta\rho_T = 2 \cdot 10^{-3}$. It will increase with m_t^2 , so that, assuming $\rho_{SB} = 1$:

$$\Delta\rho_M = \rho_M - 1 < 0.020 - 0.002 \left[\frac{m_t}{80\text{GeV}}\right]^2 \quad (4.15)$$

The limit in eq.(4.15) is a generous one. One could improve on it significantly only by reanalyzing the whole of neutral current data [30] including the effect of Z' [17,31]. However this refinement is not so important for our purposes.

A lower limit on the Z' mass from the CDF data, for the models considered in section 2, was studied in ref. [32]. In the case of models with an extra $U(1) \subset E_6$, their result, for EHLQ1 structure functions [33], is plotted in fig. 1 as function of θ_2 . Here θ_2 is related to the angle θ of ref. [32] by $\theta_2 = \theta - \alpha$ with $\cos\alpha = \sqrt{5/8}$ ($\alpha \approx 37^\circ$, 45°).

The lower limit on $M_{Z'}$, together with the limit on $\Delta\rho_M$ given in eq. (4.15), leads to a corresponding limit on $\tan\xi_0$, obtained by eq (2.12). As an example, a lower bound on $M_{Z'}$ of 300 GeV results in an upper bound on $\tan\xi_0$ of 0.041 (0.028) for a top mass of 100 (200) GeV. Note that these limits are comparable and in some cases even more severe than those derivable from the neutral current data [17,31]. We further observe that a smaller value of m_t will allow in general for larger effects.

In the case of the left-right model, we used the limit $M_{Z'} > 370$ GeV, obtained again in ref. [32] for EHLQ1 structure functions. In view of the fact that $\Delta\rho_M$ for the left-right model may receive a contribution from the charged sector as expressed by eq. (2.9), so that a bound on $\Delta\rho_M$ and on $M_{Z'}$ does not translate directly in a bound on the mixing angle, we use in our analysis the limit $|\tan\xi_0| < 0.05$ obtained from a fit of neutral current data [17,31].

As far as BESS is concerned, for the case $b = 0$ we are interested in, the CDF data give no significant limitation. For $g/g'' \leq 0.22$, $M_{\nu_0} = 250$ GeV is allowed and, provided that $g/g'' \leq 0.14$, even $M_{\nu_0} = 200$ GeV is not ruled out. On the other hand from the ratio M_W/M_Z given in eq. (4.14) and $M_Z = 91.10 \pm 0.06$ one obtains a bound on g/g'' , reproduced in fig. 2 as a function of the top mass, for $M_{\nu_0} = 250$ GeV. For $m_t = 100$ (200) GeV, one has $g/g'' < 0.055$ (0.130). The result does not practically depend from M_{ν_0} , as soon as $M_{\nu_0} \geq 250$ GeV. Observe that, contrary to what happens in the case of extra $U(1)$ models, larger values of the mixing parameter g/g'' are allowed for a heavier top quark.

5. OBSERVABLES AT LEP 1

The presence of new massive gauge vector bosons will show up through two general mechanisms:

- a modification of the (tree-level) relation between M_W and M_Z (see eq. (2.20)) which

corresponds in turn to a modification of the effective angle entering the neutral current vector couplings (see eq. (2.18)).

- a modification of the ordinary Z couplings. Because of the $Z_0 - Z'_0$ mixing, Z will couple not only to the ordinary current $J_{3L} - \sin^2\theta J_{em}$, with an appropriate effective angle θ , but also to the additional current J_{φ} .

A third mechanism, the additional contribution to a given process coming from Z' exchange, is completely negligible in the LEP1 energy range, because of the dominance of the Z -exchange terms. We observe that, for the models considered in section 2, the mechanisms mentioned above give rise to effects which are proportional to $c^2(M_Z/M_{Z'})^2$ and $c(M_Z/M_{Z'})^2$ respectively, and vanish for both c and $(M_Z/M_{Z'})$ going to zero, independently. This means that precision experiments at LEP1 will not set an absolute bound on either c or $M_{Z'}$, but they will provide for further limitations in the two dimensional parameter space. Different is the case of a model with a well defined scalar structure and known VEV's, where c would be completely fixed and the absence of deviations from SM predictions would give a specific limitation on $M_{Z'}$. Without specifying the Higgs structure of the model, one can still obtain bounds on $M_{Z'}$ by operating at energies away from the Z resonance. In that sense, such a kind of experimental conditions must be considered complementary to the LEP1's and may help in fully investigating the presence of additional gauge vector particles.

Before introducing the observables on which we base our analysis, we will make some remarks on the model dependence of the parameter M_Z , which is one of the fundamental inputs of our analysis. In an idealized situation, M_Z can be determined from the peak position of the total cross section $\sigma(s)$ for the process $e^+e^- \rightarrow f\bar{f}(\gamma)$ ($f \neq e$). In order to assess the influence of an additional neutral vector particle on the determination of M_Z from the analysis of the Z line shape, it will be sufficient to consider the approximation $\sigma_0(s)$ which does not include either the absolute square of the one-photon exchange amplitude or the full initial state bremsstrahlung. One has

[34]:

$$\sigma_0(s) = 12\pi \frac{\Gamma_e \Gamma_f}{|D(s)|^2} \left[\frac{s}{M_Z^2} + R \frac{(s - M_Z^2)}{M_Z^2} + \dots \right] \quad (5.1)$$

where

$$D(s) \approx s - M_Z^2 + iM_Z \Gamma_T \frac{s}{M_Z^2} \quad (5.2)$$

In the SM, at lowest order, the first term of eq. (5.1) is given by the Z -exchange; the second term corresponds to $\gamma - Z$ interference, and R reads:

$$R \approx 32 \sin^2 \theta \cos^2 \theta \frac{(1 - 4 \sin^2 \theta)^2}{[1 + (1 - 4 \sin^2 \theta)^2]^2} \quad (5.3)$$

The line shape $\sigma_0(s)$ has a maximum for:

$$\sqrt{s} \approx M_Z - \frac{(1 - R)}{4} \frac{\Gamma_T^2}{M_Z} \approx M_Z - (1 - R)(17 \text{ MeV}) \quad (5.4)$$

which is independent on Γ_e and Γ_f . However R depends on the model considered. In models with an extra neutral vector boson Z' , R receives a contribution $(\delta R)_{ZZ'}$ from the $Z - Z'$ interference and, correspondingly, the maximum location will be shifted by the amount:

$$\delta(\sqrt{s}) \approx (\delta R)_{ZZ'}(17 \text{ MeV}) \quad (5.5)$$

One finds [34]

$$(\delta R)_{ZZ'} \approx 2k \frac{M_Z^2(M_Z^2 - M_{Z'}^2)}{(M_Z^2 - M_{Z'}^2)^2 + \Gamma_{Z'}^2 M_{Z'}^2} \quad (5.6)$$

where:

$$k = \frac{(v_e v'_e + a_e a'_e)(v_\mu v'_\mu + a_\mu a'_\mu)}{(v_e^2 + a_e^2)(v_\mu^2 + a_\mu^2)} \quad (5.7)$$

and:

$$\begin{aligned} c_f &= g_C^f \cos \xi_0 + g_C^{'f} \sin \xi_0 \\ c_f' &= -g_C^f \sin \xi_0 + g_C^{'f} \cos \xi_0 \end{aligned} \quad (5.8)$$

In eq. (5.8) c_f and c_f' ($c = v, a$ and $f = \mu, e$) are the coupling constants for the physical gauge vector bosons, whereas g_C^f and $g_C^{'f}$ are those before mixing. For the standard model one has:

$$\begin{aligned} g_V^f &= (T_{3L}^f - 2 \sin^2 \theta_f Q_{em}) \\ g_A^f &= -T_{3L}^f \end{aligned} \quad (5.9)$$

For equal Z and Z' coupling constants, one has $k = 1$. A Z' with a mass of 200 GeV and a width of 5 GeV would then produce a shift in the peak position of about $\delta(\sqrt{s}) \approx -8 \text{ MeV}$. Such a variation is well within the experimental uncertainty. For Z' masses not lower than 200 GeV, one would need Z' axial couplings 2.5 times bigger than those of the Z to influence appreciably the Z peak location.

Whereas the location of the maximum is only slightly affected by the presence of an extra gauge vector boson, other parameters of the Z line shape might in principle receive significant corrections. At fixed values of s and M_Z , one has:

$$\frac{\delta \sigma_0(s)}{\sigma_0(s)} \approx \frac{\delta \sigma_f^f}{\sigma_f^f} + 2 \left(1 - \frac{\sigma_0(s)}{\sigma_f^f} \frac{s}{M_Z^2} \right) \frac{\delta \Gamma_T}{\Gamma_T} + \left(\frac{s - M_Z^2}{s} \right) \delta R_{ZZ'} \quad (5.10)$$

In eq. (4.10) we have defined:

$$\sigma_f^f = \frac{12\pi}{M_Z^2} \frac{\Gamma_e \Gamma_f}{\Gamma_T^2} \quad (5.11)$$

The peak cross section given in eq (5.11), obtained after deconvolution of QED radiative corrections, provides a particular combination of widths which is measured with great accuracy and is not much sensitive to weak radiative corrections. On the other hand, mixing effects can conspire to produce relatively large corrections for σ_p .

In section 6 we will discuss the deviations which an additional gauge vector boson might produce in the line shape parameters. In particular we will consider the following observables:

a)	Γ_f	$f = l, \nu$
b)	$\Gamma_h = 2(\Gamma_u + \Gamma_d) + \Gamma_b$	
c)	$\Gamma_T = 3(\Gamma_l + \Gamma_\nu) + \Gamma_h$	
d)	$R = \Gamma_h / \Gamma_l$	
e)	σ_p^f	$f = l, h$

where Γ_f is the Z partial width into $f\bar{f}$ and σ_p^f and σ_p^h are the peak cross sections for muons and hadrons, respectively.

In section 7 we will analyze the deviations which could be induced in the following

$$\begin{aligned} \text{f)} \quad & A_{pol} \\ \text{g)} \quad & A_{FB}^f \equiv A_{FB}^{e^+e^- \rightarrow f\bar{f}} \quad f = \mu, c, b \end{aligned} \quad (5.13)$$

where A_{pol} is the final polarization asymmetry [35], which can be measured for $e^+e^- \rightarrow \tau^+\tau^-$ and A_{FB} is the forward-backward asymmetry [36].

For comparison it is useful to give a list of the expected experimental precisions at LEP1 [10]:

$$\begin{aligned} \frac{\delta\Gamma_f}{\Gamma_f} &= 0.02, \quad \frac{\delta\sigma_f^I}{\sigma_f^I} = 0.02, \quad \frac{\delta R}{R} = 0.01 \\ \delta A_{pol} &= 0.015, \quad \delta A_{FB}^{\mu} = 0.0035, \quad \delta A_{FB}^b = 0.005, \quad \delta A_{FB}^c = 0.007 \end{aligned}$$

6. DEVIATIONS FROM THE WIDTHS OF THE STANDARD MODEL

In this Section we will discuss the deviations of the widths, evaluated for the various models, from those of the SM. The general strategy, following ref. [37], here and in the following Section, will be to expand in a systematic way the deviations from the observables of the SM at the lowest order in the parameters $\Delta\rho_M$ and ξ_0 for the $U(1)$ and LR models, and g/g'' for the BESS model.

Let us start from the $U(1)$ and LR cases. The typical expression we get for the deviations of the observable X associated to a given final state f will be of the form:

$$\delta X^f = X_{\text{model}}^f - X_{SM}^f = \alpha_{X'} \Delta\rho_M + \beta_{X'} \xi_0 \quad (6.1)$$

We will further show that the coefficients $\alpha_{X'}$ are universal, and that all the dependence from the particular model is in the coefficients $\beta_{X'}$. These turn out to be linear in the vector and axial couplings of the unmixed state Z'_0 to the fermions. The forward-backward asymmetry for charged leptonic pairs is an exception, and we will discuss this case separately in the following Section.

Let us now discuss the widths. The deviations have three origins. These are: i) a modification of the effective Weinberg angle (eq. 4.12), ii) a modification of the ρ parameter (eq. 4.13), iii) the mixing effects which manifest themselves in the effective couplings of Z to fermion pairs. In fact these couplings will be a linear combination of the vector and axial couplings $g_{V,A}^f$ and $g_{V,A}'^f$ of the unmixed states Z_0 and Z'_0 to fermion pairs. The couplings of Z_0 are the same as in the SM and they are given in eq. (5.9), whereas the couplings of Z'_0 are given in appendix. The improved Born approximation leads to the following expression for the decay $Z \rightarrow f\bar{f}$

$$\begin{aligned} \Gamma_f &= N_c^f \rho_f \rho_M \frac{G_F M_Z^2}{6\pi\sqrt{2}} \left[\cos^2 \xi_0 \left((g_V^f)^2 + (g_A^f)^2 \right) \right. \\ &\quad \left. + 2 \cos \xi_0 \sin \xi_0 (g_V^f g_V'^f + g_A^f g_A'^f) + \sin^2 \xi_0 \left((g_V'^f)^2 + (g_A'^f)^2 \right) \right] \end{aligned} \quad (6.2)$$

where ρ_f has been defined in eqs. (4.2) and (4.7), ρ_M in (2.12) and (2.29), and N_c^f has the same definition as in (4.11). For our purposes it is enough to write (6.2) at the first order in ξ_0 and $\Delta\rho_M$. As we have already noticed, there are two different contributions to $\Delta\rho_M$. The first one comes from the overall factor $\rho_M \approx (1 + \Delta\rho_M)$. The second contribution comes through the dependence of the couplings from the effective Weinberg angle $\tilde{\theta}_f$ defined in (4.12). The extra label ' f ' introduced here is essentially to distinguish the case of the bottom quark which requires the modifications (4.7) and (4.8) to be applied to eqs. (4.12) and (4.13). The dependence from $\Delta\rho_M$ is given by

$$\sin^2 \tilde{\theta}_f \approx \sin^2 \theta_f - \frac{\sin^2 \theta_f \cos^2 \theta_f}{\cos^2 2\theta_f} \Delta\rho_M \quad (6.3)$$

with $\sin^2 \theta_f$ defined in (4.4). The values for $\sin^2 \tilde{\theta}$ we will use here are (see ref. [24])

$$\begin{aligned} m_t &= 100 \text{ GeV} & m_t &= 200 \text{ GeV} \\ \sin^2 \tilde{\theta} &= 0.234 & \sin^2 \tilde{\theta} &= 0.230 \end{aligned}$$

Then the deviations of Γ_f from the SM values are given by

$$\delta\Gamma_f = \frac{G_F M_Z^2}{24\pi\sqrt{2}} [A_f \Delta\rho_M + B_f \xi_0] \quad (6.4)$$

where

$$A_f = 4N_c^f \rho_f \left[\left(T_{3L}^f - 2 \sin^2 \theta_f Q_{em} \right)^2 + \left(T_{3L}^f \right)^2 + 4 \frac{\sin^2 \theta_f \cos^2 \theta_f}{\cos 2\theta_f} Q_{em} \left(T_{3L}^f - 2 \sin^2 \theta_f Q_{em} \right) \right] \quad (6.5)$$

and

$$B_f = 8N_c^f \rho_f \left[\left(T_{3L}^f - 2 \sin^2 \theta_f Q_{em} \right) g_V^f - T_{3L}^f g_A^f \right] \quad (6.6)$$

The hadronic (Γ_h) and the total width (Γ_T) have an analogous expansion with coefficients $A_{h,T}$ and $B_{h,T}$ given by (see eq. (5.12))

$$\begin{aligned} A_h &= 2(A_d + A_u) + A_b, & A_T &= A_h + 3(A_\nu + A_e) \\ B_h &= 2(B_d + B_u) + B_b, & B_T &= B_h + 3(B_\nu + B_e) \end{aligned} \quad (6.7)$$

A list of the coefficients A_f for the various channels is given in Table 1. As anticipated these coefficients are model independent. The values of A_f are given for two different values of the top mass $m_t = 100, 200 \text{ GeV}$. In fact we will do all our analysis at these two representative values of m_t . The coefficients B_f are given in Table 2 in the case of the $U(1)$ models. Here we leave arbitrary the angle θ_2 because we want to be as more model independent as possible. In Table 3 we give the B_f coefficients for the LR case. It is interesting to notice that the coefficients A_f are all positive and therefore they add together coherently to build up $A_{h,T}$, whereas the coefficients B_f tend to cancel each other giving rise to $B_{h,T}$ of the same order of magnitude of the generic B_f . This has as a consequence that the deviations of the total and hadronic widths are dominantly positive in these models.

Using (6.4) we can evaluate the deviations of the other related observables we are interested in. Namely, the hadronic and leptonic peak cross-section, and the ratio among hadronic and leptonic widths. From eq. (5.11) for $\sigma_p^{h,l}$, (5.12) for R and (6.4) we obtain the relative deviations of these observables

$$\frac{\delta \sigma_p^i}{\sigma_p^i} = \frac{\delta \Gamma_e}{\Gamma_e} + \frac{\delta \Gamma_i}{\Gamma_i} - 2 \frac{\delta \Gamma_T}{\Gamma_T} = \alpha_{\sigma_i^i} \Delta \rho_M + \beta_{\sigma_i^i} \xi_0 \quad (6.8)$$

with $i = h, l$, and

$$\frac{\delta R}{R} = \frac{\delta \Gamma_h}{\Gamma_h} - \frac{\delta \Gamma_e}{\Gamma_e} = \alpha_R \Delta \rho_M + \beta_R \xi_0 \quad (6.9)$$

The expressions for the coefficients are easily evaluated and their numerical values are given in Tables 4 and 5. Once again the α -type coefficients are model independent. In this evaluations we have used the widths of the SM as given in (4.11). Here and in the following we have used $M_Z = 91.1 \text{ GeV}$. Their values are summarized in Table 6.

We are now in the position to discuss quantitatively our results. What we do here is to present, for any given model and any observable, the allowed range of deviations from the SM. In the case of the $U(1)$ models the results are given graphically (as functions of θ_2) in Figs. 3)-9). In order to get the maximal deviations one has to look at the relative sign between $\alpha_{X'}$ and $\beta_{X'}$. To illustrate the point, let us suppose $\alpha_{X'} > 0$. In this case we get the maximal positive deviation by taking $\Delta \rho_M$ just at the border of the physical region. From Section 4 we have

$$\begin{aligned} m_t &= 100 \text{ GeV} & m_t &= 200 \text{ GeV} \\ \Delta \rho_M &= 0.0169 & \Delta \rho_M &= 0.0075 \end{aligned}$$

and $\xi_0 = \sqrt{\Delta \rho_M (M_Z/M_{Z'})}$. For any θ_2 the value of $M_{Z'}$ is obtained from Fig. 1). The maximal positive deviation $\delta X_{(+)}^f$ is then given by

$$\delta X_{(+)}^f = \alpha_{X'} \Delta \rho_M + |\beta_{X'} \xi_0| \quad (6.10)$$

To discuss the maximal negative deviations it is useful to rewrite eq. (5.1) in the form

$$\delta X^f = \left(\frac{M_Z}{M_{Z'}} \right)^2 [\alpha_{X'} c^2 + \beta_{X'} c] \quad (6.11)$$

where we have used (2.13) and (2.14). This expression has a minimum at $c_{\min} = -\beta_{X'}/(2\alpha_{X'})$ where it holds

$$\delta X^f \Big|_{\min} = - \left(\frac{M_Z}{M_{Z'}} \right)^2 \frac{\beta_{X'}}{4\alpha_{X'}} \quad (6.12)$$

If $c_{\min}$ lies within the limits of the physical region, that is if

$$|c_{\min}| \leq \sqrt{\Delta \rho_M} \frac{M_{Z'}}{M_Z} \quad (6.13)$$

the maximal negative deviation $\delta X_{(-)}^f$ is

$$\delta X_{(-)}^f = \delta X^f \Big|_{\min} \quad (6.14)$$

Otherwise we get

$$\delta X_{(-)}^f = \alpha_{Xf} \Delta \rho_M - |\beta_{Xf} \xi_0| \quad (6.15)$$

with the parameters evaluated at the border of the physical region. An analogous analysis can be made for $\alpha_{Xf} < 0$.

From the plots we see that the deviations from the SM can be as large as $+6 \div 7\%$ for the partial widths into charged leptons and neutrinos, while the hadronic and total widths are at most changed by about 3%. The maximal deviations are smaller for heavier m_t , because the limits on $\Delta \rho_M$ and ξ_0 are sharpened in this case. Note that only very small (Γ_l, Γ_ν) or negligible (Γ_h, Γ_T) negative variations are possible. Actually the terms in B_f are proportional to ξ_0 and could lead to deviations of either signs (as observed in ref. [16]). However, as already noticed the coefficients of the $\Delta \rho_M$ term are large and positive (see Table 1), so that the resulting values of $\delta \Gamma_f$ are predominantly positive [37]. In order to cancel the effects of $\Delta \rho_M$ other new physics terms with negative signs should be present (e.g. from ρ_{SB} , see eq. (2.11)). On σ_p and R the effects of ρ_M cancel in the ratio, and consequently the pattern of possible deviations is almost symmetric (only a small effect coming from $\bar{\delta}_f$ remains, see eq. (6.3)). We see that effects up to $\pm 10\%$ in the charged leptonic channel and up to $\pm 6\%$ in the hadronic channel can be produced.

Let us now consider the LR models. Here the mass and the mixing angle of W' , the right-handed charged vector-boson appear as additional parameters. Thus one can either assume that the W' is very heavy and its effects can be neglected, or take advantage of these additional parameters to enlarge the allowed domain for the predicted values of the physical quantities under study. The first option is suggested by the existing lower bounds on $M_{W'}$, discussed in Section 2. If the W' effects are neglected, the

situation reproduces that of a model with an extra $U(1)$. We used in this case the limit $M_{Z'} > 370 \text{ GeV}$ as discussed in Section 4. The resulting allowed ranges of $\delta \Gamma_f$ and $\delta \sigma_p^i / \sigma_p^i$ are reported in Table 7. As expected the results are quite similar, in magnitude and sign, to those obtained for the models with an extra $U(1)$. Alternatively one can take advantage of the mixing in the charged sector. For example, by using the fact that the shifts of M_W and M_Z with respect to their SM values induce opposite effects on $\Delta \rho_M$ (see eq. (2.29)), according to

$$\frac{\Delta \rho_M}{\rho_M} \approx \frac{\delta M_W^2}{M_W^2} - \frac{\delta M_Z^2}{M_Z^2} \quad (6.16)$$

one can make $\Delta \rho_M \approx 0$. In this case we loose the bound on ξ_0 coming from CDF and UA2, and we will use the general bound coming from low-energy (see refs. [17,31]), $|\xi_0| \leq 0.05$. Therefore, the maximal deviations will be obtained by taking $\Delta \rho_M$ between zero and the maximum value given by eq. (4.15), and $\xi_0 = \pm 0.05$. The deviations are given in Table 8 and we see that somewhat larger effects can be tolerated.

Finally let us discuss the BESS model. Because we restrict our discussion to the case $b = 0$, that is to the case in which no direct couplings between the new vector bosons and fermions are present, the deviations come only from the mass effects. Using eq. (3.35) for G and the couplings given in appendix we find

$$\delta \Gamma_f = \frac{G_F M_Z^3}{24\pi\sqrt{2}} C_f \left(\frac{g}{g''} \right)^2 \quad (6.17)$$

with

$$C_f = -16 N_c^f \rho_f \frac{\sin^2 \theta_f}{\cos 2\theta_f} Q_{em} \left(T_{3L}^f - 2 \sin^2 \theta_f Q_{em} \right) \quad (6.18)$$

Due to the bounds on g/g'' (remember, from Section 4, $g/g'' \leq 0.055$ (0.13) for $m_t \leq 100$ (200) GeV) the width deviations are completely negligible. They range from a few MeV for the hadronic and total widths to tenths of MeV for the leptonic ones. Analogously the R and the peak cross-section deviations are very small $\approx 10^{-3}$. Notice from (6.18) that these deviations are always negative.

7. DEVIATIONS FROM THE ASYMMETRIES OF THE STANDARD MODEL

Following the same lines of the previous section, we will now discuss the deviations for the asymmetries. To this end we introduce the quantities:

$$\mathcal{A}_f = \frac{2a_f v_f}{a_f^2 + v_f^2} \quad (7.1)$$

The coupling constant a_f and v_f of Z to fermions were defined in eq. (5.8). It turns out that, for all practical purposes, the exchange of an additional neutral gauge vector boson can be neglected in the evaluation of asymmetries measured at the energies of the resonance. Then the tau polarization asymmetry and the forward-backward asymmetries can all be expressed in terms of $\mathcal{A}_f$:

$$A_{pol} = -\mathcal{A}_l \quad (7.2)$$

$$A_{FB}^{(l,c,b)} = \frac{3}{4} \mathcal{A}_e \mathcal{A}_{(l,c,b)} \quad (7.3)$$

Therefore it is convenient to evaluate the deviations of the quantities $\mathcal{A}_f$ from the standard model predictions. In this case there is no contribution from the ρ_M factor.

However there is still the effect coming from the modification of the effective Weinberg angle (eq. (4.12)) and there are the mixing effects. Using eqs. (5.8) and (6.3) one obtains:

$$\delta \mathcal{A}_f = K_f (D_f \Delta \rho_M + E_f \xi_0) = \alpha_{\mathcal{A}_f} \Delta \rho_M + \beta_{\mathcal{A}_f} \xi_0 \quad (7.4)$$

where:

$$K_f = -2 \frac{(T_{3L}^f)^2 - (T_{3L}^f - 2 \sin^2 \theta_f Q_{em}^f)^2}{[(T_{3L}^f)^2 + (T_{3L}^f - 2 \sin^2 \theta_f Q_{em}^f)^2]} \quad (7.5)$$

$$D_f = 2 \frac{\sin^2 \theta_f \cos^2 \theta_f T_{3L}^f Q_{em}^f}{\cos 2\theta_f} \quad (7.6)$$

$$E_f = (T_{3L}^f - 2 \sin^2 \theta_f Q_{em}^f) g_A^f + (T_{3L}^f) g_V^f \quad (7.7)$$

Again, the coefficients $\alpha_{\mathcal{A}_f}$ are model independent. It is interesting to notice that $\beta_{\mathcal{A}_f}$ contains the combination of g_V^f and g_A^f orthogonal to the one contained in β_{Γ_f} (see eq. (6.6)). Therefore the asymmetries give informations about the couplings of Z_0^f to fermions which are complementary to the ones obtained from the widths. The coefficients $\alpha_{\mathcal{A}_f}$ and $\beta_{\mathcal{A}_f}$ are given in Tables 9 and 10. From these expressions we can now evaluate the deviations for the physical quantities:

$$\delta \mathcal{A}_{pol} = -\delta \mathcal{A}_l \quad (7.8)$$

$$\delta A_{FB}^{(c,b)} = \frac{3}{4} \delta \mathcal{A}_e \mathcal{A}_{(c,b)} + \frac{3}{4} \mathcal{A}_e \delta \mathcal{A}_{(c,b)} = \alpha_{A_{FB}^{(c,b)}} \Delta \rho_M + \beta_{A_{FB}^{(c,b)}} \xi_0 \quad (7.9)$$

Using the SM values of $\mathcal{A}_f$ as given by eq. (7.1) (see Table 11), we can evaluate the coefficients $\alpha_{\mathcal{A}_f}$ and $\beta_{\mathcal{A}_f}$. The result is given in Tables 12 and 13. The observable $A_{FB}^{(l)}$ requires a different treatment. In fact, for values of the parameters $\Delta \rho_M$ and ξ_0 at the border of the physical region, we get $\delta \mathcal{A}_l \approx \mathcal{A}_l$ (see Table 11 for a list of the values of $\mathcal{A}_f$ in the standard model). Therefore it is not justified to throw out the terms of order $(\delta \mathcal{A}_l)^2$ in the expression for $\delta A_{FB}^{(l)}$, and we get:

$$\delta A_{FB}^{(l)} = \frac{3}{2} \mathcal{A}_l \delta \mathcal{A}_l + \frac{3}{4} (\delta \mathcal{A}_l)^2 \quad (7.10)$$

For the case of the extra $U(1)$ we present the maximal deviations of the asymmetries versus the angle θ_2 in figs. 10-13. The maximal deviations are obtained using the same procedure described in the previous section, with the exception of $\delta A_{FB}^{(l)}$. In this case the maximal positive deviation is easily obtained by using in eq. (7.10) the maximal negative deviation for $\delta \mathcal{A}_l$. For the negative deviations one has to look at the minimum located at $\delta \mathcal{A}_l|_{min} = -\mathcal{A}_l$. In practice such a value of $\delta \mathcal{A}_l$ is never reached for any possible choice of the parameters $\Delta \rho_M$ and ξ_0 within the physical range (see figs. 10-13). Therefore the maximal negative deviation is obtained by putting in the left hand side of eq. (7.10) the maximum positive value of $\delta \mathcal{A}_l$. From the plots we see that all the deviations follow the same trend, that is we have substantial positive deviations of

the order of 9% for $A_{\rho\pi}$, 2% for $A_{FB}^{(l)}$ and 6-8% for $A_{FB}^{(c,b)}$, but negative deviations are very small.

In the case of the left-right models the maximal deviations are given in Table 14, under the assumption of no contribution of W' to $\Delta\rho_M$, and in Table 15 in the case of a sizeable W' contribution making $\Delta\rho_M = 0$. The deviations are a little smaller than those obtained in the case of an extra $U(1)$.

Finally, coming to the BESS model, we have:

$$\delta A_f = -2K_f \frac{\sin^2 \theta_f}{\cos 2\theta_f} Q_{em} T_{3L}^f \left(\frac{g}{g''} \right)^2 \quad (7.11)$$

with K_f given in eq. (7.5). Notice that in this case the deviations (summarized in Table 16) have definite sign. Also, as observed in section 4, the deviations increase with m_t . For $m_t = 200 \text{ GeV}$ we get deviations comparable to the ones obtained for the other models.

8. BOUNDS ON THE PARAMETER SPACE

From the general expressions given for the deviations of the observables in the previous section, it is easy to discuss the bounds on the parameter space. Here we outline the procedure which allows to derive such bounds in a simple analytic form. Suppose that the observable X^f is measured with an experimental error $\Delta_{exp}^{X^f}$ and no deviations from the standard model are observed. Then we can use the equation:

$$|\delta X^f| \leq \Delta_{exp}^{X^f} \quad (8.1)$$

to get a restriction on the parameter space.

The simplest model to analyze is BESS. The general expression for the deviation is:

$$\delta X^f = \gamma_{X^f} \left(\frac{g}{g''} \right)^2 \quad (8.2)$$

from which one gets:

$$\left| \frac{g}{g''} \right| \leq \left| \frac{\Delta_{exp}^{X^f}}{\gamma_{X^f}} \right|^{\frac{1}{2}} \quad (8.3)$$

For the extra $U(1)$ and left-right models it is convenient to use the expression given in eq. (6.11) for the deviations (with the exclusion of $A_{FB}^{(l)}$):

$$\delta X^f = \left(\frac{M_Z}{M_{Z'}} \right)^2 [\alpha_{X^f} c^2 + \beta_{X^f} c] \quad (8.4)$$

Then we get the limitation:

$$|\alpha_{X^f} c^2 + \beta_{X^f} c| \leq \left(\frac{M_{Z'}}{M_Z} \right)^2 \Delta_{exp}^{X^f} \quad (8.5)$$

As stressed in section 5, we see that LEP1 cannot set an absolute bound on c and on $M_Z/M_{Z'}$, but rather it will put a limitation on the two parameter space.

As a simple example we take Γ_T in the left-right model. Assuming $\Delta_{exp}^{\Gamma_T} = 50 \text{ MeV}$, from table 1 and 3 we get, approximately:

$$-0.49 \left(\frac{M_{Z'}}{370} \right) \left(1 - 0.22 \left(\frac{370}{M_{Z'}} \right) \right) \leq c \leq +0.49 \left(\frac{M_{Z'}}{370} \right) \left(1 + 0.22 \left(\frac{370}{M_{Z'}} \right) \right) \quad (8.6)$$

where masses are expressed in GeV. In an analogous way one can discuss all other cases. The only exception is $A_{FB}^{(l)}$ which cannot be reconducted to a similarly simple analysis.

9. CONCLUSIONS

In this paper we have analyzed the experimental consequences that the existence of a heavier neutral gauge vector boson may produce in e^+e^- measurements done at energies around the Z mass. We have performed this analysis in such a way as to include a rather large class of the models existing in the literature. Namely we have considered models of two kinds:

- a) extensions of the gauge group of the SM;

b) electroweak symmetry breaking by a strong sector.

In class a) we have included models such as gauge extensions of the SM with gauge group $SU(2)_L \otimes U(1)_Y \otimes U(1)_{Y'} \subset E_6$, and left-right symmetric models based on the gauge group $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$. In class b) we have considered a model where the breaking of the electroweak symmetry is due to a strongly interacting sector giving rise to new vector bound states.

In order to emphasize differences and similarities among the classes a) and b), we have found extremely useful to characterize the models according to the transformation properties of the extra neutral vector boson field, $V_{3\mu}$, with respect to the electromagnetic gauge group. In particular, in models of type a) $V_{3\mu}$ is invariant under $U(1)_{em}$, whereas in models of type b) we have $V'_{3\mu} = V_{3\mu} - \partial_\mu \lambda / g_V$. This different behaviour is reflected in the structure of the neutral vector bosons mass matrix. The use of the approximate $\rho = 1$ relation then constrains the mass matrix of the charged vector bosons. In this way we were able to get a simple parametrization of the various models, as discussed in Sections 2 and 3.

The different behaviour of the two classes of models under $U(1)_{em}$ has a very peculiar effect on the dependence of the deviations from the SM of the physical observables. In fact one finds that in models of class a) the deviations are very sensitive to the mass of the new vector boson, whereas in the second case the predictions are largely independent from this mass. This is because, contrarily to the case a), in models of class b) the invariance under $U(1)_{em}$ allows for a mixing mass term proportional to the mass of the heaviest state. As a result the effects of the heaviest mode persist also in the limit of $M_V \rightarrow \infty$.

In view of the fact that the effects of new vector bosons at LEP1/SLC are expected to be small, it is necessary to treat carefully the radiative corrections. According to

the estimates of refs. [14,28] one expects that the effects of new vector bosons can be neglected in the loop diagrams compared to their tree-level effects. Starting from this assumption, we have given (in Section 4) a set of practical rules in order to construct, for each model, an improved Born approximation which takes into account the main part of the radiative corrections coming from the SM.

In the final Sections we have constructed simple analytical expressions for the deviations of the relevant physical quantities with respect to their SM value. The observables we have taken into account are: partial and total widths, peak cross-sections, polarization asymmetry, and forward-backward asymmetry in various channels. The deviations are obtained by means of a lowest order series expansion in terms of the $\Delta\rho_M$ parameter, which controls the shift of the Z mass due to the mixing, and the mixing angle ξ_0 . The coefficients in front of these parameters can be completely expressed in terms of known quantities and in terms of the fermionic couplings of the new vector bosons. In order to facilitate calculations and comparisons, a tabulation of the coefficients is given in a number of Tables. From our analysis it appears quite clear the relevance, for this type of physics, of measuring both widths and asymmetries. In fact the expressions for the respective deviations contain orthogonal linear combinations of the couplings of the new vector bosons.

Finally we have presented a range of the allowed deviations from the SM, updated to present experimental information, and we have shown how to obtain from our formulation, in a simple analytical way, the limitations on the parameter space of these models.

Using constraints coming from new data on W and Z masses at CDF, UA2 and LEP1/SLC we have found that, for models of class a), negative deviations to the total and hadronic widths are generally strongly suppressed (except in those LR models which allow for important contributions from W'). This follows from the fact that the term

proportional to $\Delta\rho_M$ generally dominates with respect to the term proportional to the mixing angle. This is not the case for the peak cross-sections and for the ratio between hadronic and charged leptons widths, because in this case the $\Delta\rho_M$ contribution tends to be cancelled out. The typical positive relative deviations are not bigger than some percent for the single widths, whereas they can be as large as 10-12% for particular combinations, like the peak cross-section into charged leptons. The maximal absolute deviations for the asymmetries are 8-9% for A_{pol} and $\mathcal{A}_{FB}^{c,b}$, whereas they are at most 2% for $\mathcal{A}_{FB}^l$. In the case of the BESS model (case b)) we have found no appreciable deviations for the widths (limiting the analysis to the case of no direct coupling to fermions), but the maximal deviations for the asymmetries are comparable to the ones of the models of class a), except for the fact that here they are always negative. Larger deviations are generally favoured by a smaller top mass in models of class a). The contrary happens for BESS.

We have presented our results in a somewhat unconventional way, but our feeling is that, due to the forthcoming fantastic accumulation of new results at LEP1, the most efficient way of using the data is to have a formulation allowing for an almost immediate updating of the bounds on the relevant parameters of these models. In this way we hope that our results can turn out to be useful also at future experimental stages.

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APPENDIX

In this appendix we give a complete list of fermion couplings. In the text we have used the following expressions with the angle θ evaluated at the value θ_f (f is the appropriate fermionic channel) defined in eqs.(4.4) and (4.8).

Vector and axial couplings of fermions to the unmixed new vector boson Z'_0 , in the case of extra $U(1)$ and LR models, are given by the following expressions:

Extra $U(1)$ Models

$$\begin{aligned}
 g_V^u &= 0 \\
 g_A^u &= -\frac{2}{3} \sin \theta \cos \theta_2 \\
 g_V^d &= \frac{1}{2} \sin \theta (\cos \theta_2 + \sqrt{\frac{5}{3}} \sin \theta_2) \\
 g_A^d &= \sin \theta (-\frac{1}{6} \cos \theta_2 + \frac{1}{2} \sqrt{\frac{5}{3}} \sin \theta_2) \\
 g_V^e &= -\frac{1}{2} \sin \theta (\cos \theta_2 + \sqrt{\frac{5}{3}} \sin \theta_2) \\
 g_A^e &= g_A^d \\
 g_V^\nu &= -\sin \theta (\frac{1}{6} \cos \theta_2 + \frac{1}{2} \sqrt{\frac{5}{3}} \sin \theta_2) \\
 g_A^\nu &= -g_V^\nu
 \end{aligned} \tag{A.1}$$

Left-Right Models

$$\begin{aligned}
 g_V^f &= \frac{e}{\sin \theta \cos \theta} \left[\frac{\cos^2 \theta}{\lambda y} (T_{3R} - 2\lambda^2 \sin^2 \theta Q_{em}) + \frac{\lambda \sin^2 \theta}{y} (T_{3L} - 2 \sin^2 \theta Q_{em}) \right] \\
 g_A^f &= \frac{e}{\sin \theta \cos \theta} \left[\frac{\cos^2 \theta}{\lambda y} T_{3R} - \frac{\lambda \sin^2 \theta}{y} T_{3L} \right]
 \end{aligned} \tag{A.2}$$

where in the text we have put $\lambda = g_L/g_R = 1$. As a consequence $y = \sqrt{\cos 2\theta}$.

In this case it is simpler to give directly the expression of the couplings of the mass eigenstate Z :

$$v_f = \frac{AT_{3L} + 2BQ_{em}}{(4\sqrt{2}G_F M_Z^2)^{\frac{1}{2}}}, \quad a_f = -\frac{AT_{3L}}{(4\sqrt{2}G_F M_Z^2)^{\frac{1}{2}}} \quad (A.3)$$

In these formulae:

$$A = G \cos \xi \quad (A.4)$$

$$B = -G \sin^2 \theta \cos \xi \left(1 - \frac{\tan \xi \sin \psi}{\tan \theta}\right) \quad (A.5)$$

$$C = G \sin \xi \quad (A.6)$$

$$D = -G \sin^2 \theta \sin \xi \left(1 + \frac{\cot \xi \sin \psi}{\tan \theta}\right). \quad (A.7)$$

where (see eqs. (3.35) and (3.29))

$$G = \sqrt{g^2 + g'^2} = \frac{1}{(4\sqrt{2}G_F M_Z^2)^{\frac{1}{2}}} \left[1 + \frac{\frac{1}{2} \left(\frac{g}{g'}\right)^2 \cos^2 2\theta}{\cos^2 \theta} \frac{1}{1 - \left(\frac{M_W}{M_V}\right)^2 \frac{1}{\cos^2 \theta}} \right] \quad (A.8)$$

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FIGURE CAPTIONS

Fig. 1 Lower bounds on $M_{Z'}$ vs. θ_2 in the extra $U(1)$ models as obtained from CDF data, see refs. [1,32].

Fig. 2 Upper bounds on the parameter g/g'' vs. m_t in the BESS model as obtained from CDF and UA2 data on $M_W/M_{Z'}$.

Fig. 3 Maximal deviations in MeV for the charged leptonic width vs. θ_2 in the extra $U(1)$ models.

Fig. 4 Maximal deviations in MeV for the neutrino width vs. θ_2 in the extra $U(1)$ models.

Fig. 5 Maximal deviations in MeV for the hadronic width vs. θ_2 in the extra $U(1)$ models.

Fig. 6 Maximal deviations in MeV for the total width vs. θ_2 in the extra $U(1)$ models.

Fig. 7 Maximal deviations for the charged leptonic peak cross-section vs. θ_2 in the extra $U(1)$ models.

Fig. 8 Maximal deviations for the hadronic peak cross-section vs. θ_2 in the extra $U(1)$ models.

Fig. 9 Maximal deviations for the ratio Γ_h/Γ_l vs. θ_2 in the extra $U(1)$ models.

Fig. 10 Maximal deviations for the polarization asymmetry vs. θ_2 in the extra $U(1)$ models.

Fig. 11 Maximal deviations for the forward-backward asymmetry in $\mu^+\mu^-$ vs. θ_2 in the extra $U(1)$ models.

Fig. 12 Maximal deviations for the forward-backward asymmetry in $b\bar{b}$ quarks vs. θ_2 in the extra $U(1)$ models.

Fig. 13 Maximal deviations for the forward-backward asymmetry in $c\bar{c}$ quarks vs. θ_2 in the extra $U(1)$ models.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

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Fig. 9

Fig. 10

Fig. 11

Fig. 12

Fig. 13

Table 1 - The coefficients A_f .

	$m_t = 100 \text{ GeV}$	$m_t = 200 \text{ GeV}$
A_e	1.18	1.23
A_ν	2.01	2.03
A_d	6.55	6.59
A_b	6.52	6.49
A_u	5.69	5.77
A_h	31.0	31.2
A_T	40.5	41.0

Table 2 - The coefficients B_f for U(1) models.

	$m_t = 100 \text{ GeV}$	$m_t = 200 \text{ GeV}$
B_e	$-0.261 \cos \theta_2 + 1.33 \sin \theta_2$	$-0.246 \cos \theta_2 + 1.35 \sin \theta_2$
B_ν	$-0.647 \cos \theta_2 - 2.51 \sin \theta_2$	$-0.648 \cos \theta_2 - 2.51 \sin \theta_2$
B_d	$-3.09 \cos \theta_2 + 1.22 \sin \theta_2$	$-3.11 \cos \theta_2 + 1.20 \sin \theta_2$
B_b	$-3.08 \cos \theta_2 + 1.22 \sin \theta_2$	$-3.06 \cos \theta_2 + 1.19 \sin \theta_2$
B_u	$4.04 \cos \theta_2$	$4.04 \cos \theta_2$
B_h	$-1.19 \cos \theta_2 + 3.66 \sin \theta_2$	$-1.21 \cos \theta_2 + 3.60 \sin \theta_2$
B_T	$-3.92 \cos \theta_2 + 0.140 \sin \theta_2$	$-3.89 \cos \theta_2 + 0.133 \sin \theta_2$

Table 3 - The coefficients B_f for LR models.

	$m_t = 100 \text{ GeV}$	$m_t = 200 \text{ GeV}$
B_e	-1.45	-1.47
B_ν	1.29	1.27
B_d	-0.50	-0.51
B_b	-0.50	-0.50
B_u	-3.35	-3.36
B_h	-8.21	-8.24
B_T	-8.71	-8.84

Table 4 - The coefficients $\alpha_{e_f^{(L,R)}}$ and α_R .

	$m_t = 100 \text{ GeV}$	$m_t = 200 \text{ GeV}$
$\alpha_{e_f^L}$	-0.360	-0.285
$\alpha_{e_f^R}$	-0.0529	-0.0212
α_R	0.307	0.263

Table 5 - The coefficients $\beta_{\sigma^+_{\mu\lambda}}$ and β_R for U(1) and LR models.

U(1)	$m_t = 100 \text{ GeV}$	$m_t = 200 \text{ GeV}$
$\beta_{\sigma^+_{\mu\lambda}}$	$-0.258 \cos \theta_2 + 2.64 \sin \theta_2$	$-0.227 \cos \theta_2 + 2.65 \sin \theta_2$
$\beta_{\sigma^+_{\mu\lambda}}$	$-0.0552 \cos \theta_2 + 1.49 \sin \theta_2$	$-0.0421 \cos \theta_2 + 1.49 \sin \theta_2$
β_R	$0.203 \cos \theta_2 - 1.15 \sin \theta_2$	$0.185 \cos \theta_2 - 1.16 \sin \theta_2$
LR	$m_t = 100 \text{ GeV}$	$m_t = 200 \text{ GeV}$
$\beta_{\sigma^+_{\mu\lambda}}$	-2.31	-2.30
$\beta_{\sigma^+_{\mu\lambda}}$	-1.25	-1.25
β_R	1.05	1.05

Table 6 - The widths in the SM (MeV).

	$m_t = 100 \text{ GeV}$	$m_t = 200 \text{ GeV}$
Γ_l	83.3	84.3
Γ_ν	166	167
Γ_h	1733	1754
Γ_T	2481	2509

Table 7 - Maximal deviations of Γ_f (MeV), σ_p^f and R in LR models. (No W' contribution to $\Delta\rho_M$)

	$m_t = 100 \text{ GeV}$		$m_t = 200 \text{ GeV}$	
$\delta\Gamma_t$	+5.5	-2.2	+3.4	-1.8
$\delta\Gamma_\nu$	+6.2	-1.0	+3.5	-1.0
$\delta\Gamma_h$	+65	-2.7	+34	-2.7
$\delta\Gamma_T$	+80	-2.3	+41	-2.4
$\frac{\delta\sigma_p^f}{\sigma_p^f}$	+0.068	-0.080	+0.047	-0.051
$\frac{\delta\sigma_p^h}{\sigma_p^h}$	+0.039	-0.041	+0.027	-0.027
$\frac{\delta R}{R}$	+0.039	-0.028	+0.025	-0.021

Table 8 - Maximal deviations of Γ_f (MeV), σ_p^f and R in LR models. (With W' contribution to $\Delta\rho_M$)

	$m_t = 100 \text{ GeV}$		$m_t = 200 \text{ GeV}$	
$\delta\Gamma_t$	+7.7	-6.0	+6.8	-6.1
$\delta\Gamma_\nu$	+8.1	-5.3	+6.5	-5.2
$\delta\Gamma_h$	+77	-34	+53	-34
$\delta\Gamma_T$	+93	-36	+62	-37
$\frac{\delta\sigma_p^f}{\sigma_p^f}$	+0.12	-0.12	+0.12	-0.12
$\frac{\delta\sigma_p^h}{\sigma_p^h}$	+0.063	-0.063	+0.063	-0.063
$\frac{\delta R}{R}$	+0.058	-0.052	+0.055	-0.053

Table 9 - The coefficients α_{A_j} .

	$m_t = 100 \text{ GeV}$	$m_t = 200 \text{ GeV}$
α_{A_t}	-2.66	-2.57
α_{A_c}	-1.18	-1.13
α_{A_s}	-0.219	-0.212

Table 10 - The coefficients β_{A_j} for $U(1)$ and LR models.

$U(1)$	$m_t = 100 \text{ GeV}$	$m_t = 200 \text{ GeV}$
β_{A_t}	$-0.976 \cos \theta_2 - 1.15 \sin \theta_2$	$-0.966 \cos \theta_2 - 1.12 \sin \theta_2$
β_{A_c}	$0.320 \cos \theta_2$	$0.318 \cos \theta_2$
β_{A_s}	$0.182 \cos \theta_2 + 0.513 \sin \theta_2$	$0.178 \cos \theta_2 + 0.504 \sin \theta_2$
LR	$m_t = 100 \text{ GeV}$	$m_t = 200 \text{ GeV}$
β_{A_t}	-0.266	-0.329
β_{A_c}	-1.04	-1.04
β_{A_s}	-0.703	-0.695

Table 11 - The values of $\mathcal{A}_f$ in the standard model.

	$m_t = 100 \text{ GeV}$	$m_t = 200 \text{ GeV}$
$\mathcal{A}_t$	-0.127	-0.159
$\mathcal{A}_c$	-0.659	-0.673
$\mathcal{A}_b$	-0.934	-0.935

Table 12 - The coefficients $\alpha_{A_{PB}^f}$.

	$m_t = 100 \text{ GeV}$	$m_t = 200 \text{ GeV}$
$\alpha_{A_{PB}^t}$	1.91	1.91
$\alpha_{A_{PB}^b}$	2.51	2.44

Table 13 - The coefficients $\beta_{A'_{FB}}$ for $U(1)$ and LR models.

$U(1)$	$m_t = 100 \text{ GeV}$	$m_t = 200 \text{ GeV}$
$\beta_{A'_{FB}}$	$0.602 \cos \theta_2 + 0.761 \sin \theta_2$	$0.599 \cos \theta_2 + 0.752 \sin \theta_2$
$\beta_{A''_{FB}}$	$0.888 \cos \theta_2 + 1.01 \sin \theta_2$	$0.875 \cos \theta_2 + 0.965 \sin \theta_2$
LR	$m_t = 100 \text{ GeV}$	$m_t = 200 \text{ GeV}$
$\beta_{A'_{FB}}$	0.308	0.387
$\beta_{A''_{FB}}$	0.338	0.418

Table 14 - Maximal deviations of the asymmetries in LR models.
(No W' contribution to $\Delta\rho_M$)

	$m_t = 100 \text{ GeV}$		$m_t = 200 \text{ GeV}$	
δA_{pol}	+0.054	0.000	+0.026	-0.001
δA^L_{FB}	+0.012	0.000	+0.007	0.000
δA^S_{FB}	+0.042	0.000	+0.023	-0.001
δA^B_{FB}	+0.053	-0.001	+0.027	-0.001

Table 15 - Maximal deviations of the asymmetries in LR models.
(With W' contribution to $\Delta\rho_M$)

	$m_t = 100 \text{ GeV}$		$m_t = 200 \text{ GeV}$
δA_{pol}	+0.058	-0.013	+0.036 -0.016
δA_{FB}^l	+0.014	-0.002	+0.010 -0.004
δA_{FB}^e	+0.048	-0.015	+0.034 -0.019
δA_{FB}^b	+0.059	-0.017	+0.039 -0.021

Table 16 - Maximal deviations of asymmetries for BESS.

	$m_t = 100 \text{ GeV}$	$m_t = 200 \text{ GeV}$
δA_{pol}	-0.011	-0.057
δA_{FB}^l	-0.002	-0.011
δA_{FB}^e	-0.008	-0.042
δA_{FB}^b	-0.010	-0.054