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UNCONSTRAINED BRST FOR SUPERPARTICLES

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ABSTRACT

By using a gauge condition and a change of variables different from the previous treatment of the ten-dimensional superparticle, we get a gauge-fixed action which is both quadratic and free and gives a correct counting of degrees of freedom. We explicitly show that the gauge-fixed action is BRST-invariant and that the BRST charge is nilpotent off-shell.

In this way we have solved the constraints on the BRST-symmetry, which existed in a previous quantization.

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1. Introduction

The problem of covariant quantization of the ten-dimensional superparticle [1] has attracted a lot of interest over the last few years. The ten-dimensional superparticle describes the dynamics of the zero-modes of the ten-dimensional superstring [2]. The covariant quantization of the superparticle is therefore a necessary step towards a covariant quantization of the superstring.

Many different approaches have been used to covariantly quantize the superparticle. None of them so far has led to a complete and satisfactory solution of the problem. The main problem stems from the presence of a local fermionic so-called "kappa" symmetry [3]. In the Hamiltonian approach this kappa symmetry gives rise to a fermionic constraint which is half first-class and half second-class [4]. It seems problematic to disentangle these first and second-class constraints in a manifest (Lorentz-) covariant way. In the Lagrangian approach, which we will consider in this letter, the problem manifests itself by the difficulty to describe covariantly 8 fermionic degrees of freedom by means of 16-component ten-dimensional Majorana-Weyl spinors. It has been realized that this could be achieved by means of an infinite number of space-time spinors of alternating statistics [5]. Such an infinite number of spinors is expected to arise in a system which is infinite reducible in the language of Batalin and Vilkovisky (BV) [6]. The superparticle is an example of such an infinite reducible system [7].

Recently, a special gauge choice to fix the kappa symmetry was proposed which, after a number of field redefinitions, leads to a remarkably simple quadratic gauge-fixed action [8]. A counting, based on the assumption that the action is not only quadratic but also free, leads to the correct number of 8 bosonic and 8 fermionic degrees of freedom. Unfortunately, it was subsequently shown that the field redefinitions leading to the quadratic action introduce constraints among the new variables [9-11]. Therefore the gauge-fixed action, although quadratic, is not free. Moreover, it was shown that the gauge-fixed action is invariant under a new gauge symmetry [9] which is of the Stueckelberg type [10] indicating that some of the original variables are not present in the gauge-fixed action. In appendix A of [10] we have shown how one can get rid of the Stueckelberg symmetries by making field redefinitions. However, these field redefinitions render the action non-quadratic.

In this letter we propose a new gauge choice for fixing the kappa symmetry which has the following attractive feature. Again our gauge-fixed action is invariant under the Stueckelberg symmetries given in [10]. However, in this case the field redefinitions making the gauge-fixed action quadratic coincide with the field redefinitions which eliminate the Stueckelberg symmetries! Furthermore, the field redefinitions we make do not introduce

any constraints among the new variables. We thus end up with a gauge-fixed action which is both quadratic and free. The action is BRST invariant and we will give the Noether BRST charge which is nilpotent off-shell. Below we will describe in more detail how the new gauge choice leads to a free quadratic gauge fixed action.

2. Covariant Quantization

The ten-dimensional classical superparticle Lagrangian [1] in first order form is given by

$$\mathcal{L}_{cl.} = P_\mu \dot{X}^\mu - \theta \dot{\theta} - \frac{1}{2} g P^2 \quad (2.1)$$

Here (X^μ, θ) are the classical coordinates of the superparticle, P_μ is the momentum conjugate to X^μ , g is the einbein and the dot indicates a time derivative. The X^μ ($\mu = 0, 1 \dots 9$) are vectors of the ten-dimensional Lorentz group and the θ are 16-component Majorana-Weyl spinors of positive chirality*. The action corresponding to (2.1) is invariant under one-dimensional general coordinate transformations (g.c.t.) and local fermionic kappa transformations.

Following the BV formalism [6], to fix the g.c.t., we introduce an anti-commuting ghost c , an anti-commuting anti-ghost b and a commuting lagrange multiplier λ . Furthermore, to fix the kappa-symmetries, we introduce a set of ghosts and extra ghosts $\{\theta_{p,q}\}$ ($p \geq q = 0, 1, 2 \dots$) and corresponding to the non-minimal and extra ghosts additional lagrange multipliers $\{\lambda_{p,q}\}$ ($p \geq q, q = 1, 2 \dots$). We also introduce a set of anti-ghosts $\{\bar{\theta}^{p,q}\}$ ($p \geq q, q = 0, 1, 2 \dots$) with corresponding Lagrange multipliers $\{\bar{\lambda}^{p,q}\}$ ($p \geq q, q = 0, 1, 2 \dots$). The (anti-) commutativity properties of the fermionic ghosts, extra ghosts, anti-ghosts and lagrange multipliers are given by (we indicate explicit spinor indices in this formula to denote the chirality of the spinors):

$$\begin{array}{ll} (\theta_{p,q})^\alpha, (\bar{\lambda}^{p,q})_\alpha & (p+q \text{ even}) \quad \text{anti - commuting} \\ (\theta_{p,q})_\alpha, (\bar{\lambda}^{p,q})^\alpha & (p+q \text{ odd}) \quad \text{commuting} \\ (\bar{\theta}^{p,q})^\alpha, (\lambda_{p,q})_\alpha & (p+q \text{ odd}) \quad \text{anti - commuting} \\ (\bar{\theta}^{p,q})_\alpha, (\lambda_{p,q})^\alpha & (p+q \text{ even}) \quad \text{commuting} \end{array} \quad (2.2)$$

* We use the following conventions. Indicating explicit spinor indices we denote a positive-chiral spinor, like θ , by $(\theta)^\alpha$, and a negative-chiral spinor, like $\theta_{1,0}$, by $(\theta_{1,0})_\alpha$. In this letter we will never indicate explicitly these spinor indices. Note that a bar above a spinor, like in $\bar{\theta}^{p,q}$, does not refer to the Dirac conjugate but indicates that the corresponding spinor is an anti-ghost.

We propose the following choice of gauge fermion (using the terminology of BV):

$$\Psi = b(g-1) + \sum_{q=0}^{\infty} \sum_{p=q}^{\infty} \bar{\theta}^{p,q} \theta_{p,q} \quad (2.3)$$

To fix the g.c.t. we have imposed the conformal gauge $g = 1$. However, this is not essential for what follows and we could equally well have chosen the harmonic gauge $\hat{g} = 0$. The gauge choice (2.3) is unusual in the sense that each antighost multiplies only one ghost or extra ghost. For the classical field θ eq. (2.3) corresponds to the gauge $\hat{\theta} = 0$. In fact, only half of this equation is a gauge choice, the other half equals the field equation for θ . The gauge choice $\hat{\theta} = 0$ also occurs in the work of [8] and has been proposed before in [12].

Using the gauge fermion (2.3), a straightforward calculation shows that the corresponding gauge-fixed lagrangian is given by:

$$\begin{aligned} \mathcal{L}_{g,f} = & \mathcal{L}_{cl} + b\dot{c} + \lambda(g-1) + 4b\hat{\theta}\theta_{1,0} \\ & + \sum_{p=0}^{\infty} \{ \theta_{p+1,0} \bar{p} - 2\theta_{p+2,0} b \} \bar{\theta}^{p,0} \\ & + \sum_{q=0}^{\infty} \{ \bar{\lambda}^{p,q} \bar{\theta}_{p,q} + \lambda_{p+1,q+1} \bar{\theta}^{p+1,q+1} \} \end{aligned} \quad (2.4)$$

The gauge-fixed action corresponding to this Lagrangian is invariant under the following BRST transformations with constant anticommuting parameter Λ :

$$\begin{aligned} sX^\mu &= cP^\mu + \theta\gamma^\mu s\theta + 2b\theta_{1,0}\gamma^\mu\theta_{1,0} \\ sg &= \dot{c} + 4\hat{\theta}\theta_{1,0} + 2\sum_{p=0}^{\infty} \theta_{p+2,0} \bar{\theta}^{p,0} \\ sc &= -2\theta_{1,0} \bar{p} \theta_{1,0} - 8b\theta_{1,0}\theta_{2,0} \\ sb &= \lambda \\ s\theta_{p,q} &= \lambda_{p,q} \quad q = 1, 2, \dots \\ s\theta_{p,0} &= \bar{p} \theta_{p+1,0} - 2\theta_{p+2,0} b \\ s\bar{\theta}^{p,q} &= \bar{\lambda}^{p,q} \end{aligned} \quad (2.5)$$

where the s-transformations are related to the BRST-transformations by the relation $\hat{\theta}_{BRST} = (s\hat{\theta})\Lambda$. The remaining variations are zero. Besides the BRST invariance the gauge fixed action is also invariant under the following Stueckelberg transformations with

local parameters $\bar{e}^{p,0}$ which have the same commutativity properties and chirality conditions as $\hat{\theta}^{p,0}$:

$$\begin{aligned} \delta_{ST} X^\mu &= -\sum_{p=0}^{\infty} \theta_{p+1,0} \gamma^\mu \bar{e}^{p,0} \\ \delta_{ST} c &= +2\sum_{p=0}^{\infty} \theta_{p+2,0} \bar{e}^{p,0} \\ \delta_{ST} \bar{\lambda}^{p,0} &= -(-)^p \bar{p} \bar{e}^{p-1,0} + 2\bar{e}^{p-2,0} b \quad p = 2, 3, \dots \\ \delta_{ST} \bar{\lambda}^{1,0} &= \bar{p} \bar{e}^{0,0} \\ \delta_{ST} \bar{\theta}^{p,0} &= \bar{e}^{p,0} \end{aligned} \quad (2.6)$$

Note that these Stueckelberg symmetries are identical to the ones given in appendix A of [10].

The Stueckelberg symmetries (2.6) show that effectively the anti-ghosts $\{\hat{\theta}^{p,0}\}$ do not occur in the gauge-fixed Lagrangian (2.4). To eliminate the $\{\hat{\theta}^{p,0}\}$ we make the following field redefinitions:

$$\begin{aligned} \tilde{X}^\mu &= X^\mu + \sum_{p=0}^{\infty} \theta_{p+1,0} \gamma^\mu \bar{\theta}^{p,0} \\ \tilde{c} &= c - 2\sum_{p=0}^{\infty} \theta_{p+2,0} \bar{\theta}^{p,0} \\ \tilde{\lambda}^{p,0} &= \bar{\lambda}^{p,0} + (-)^p \bar{p} \bar{\theta}^{p-1,0} - 2\bar{\theta}^{p-2,0} b \quad p = 2, 3, \dots \\ \tilde{\lambda}^{1,0} &= \bar{\lambda}^{1,0} - \bar{p} \bar{\theta}^{0,0} \end{aligned} \quad (2.7)$$

Performing these field redefinitions is equivalent to taking the $\{\bar{\theta}^{p,0}\} \equiv 0$ both in the gauge-fixed action and in the transformation rules and defining new BRST transformation rules as follows:

$$\tilde{\delta}(\Lambda) = \delta(\Lambda) + \delta_{ST}(\bar{e}^{p,0} = -\bar{\lambda}^{p,0}\Lambda) \quad (2.8)$$

After making the field redefinitions (2.7) the gauge fixed action is independent of the $\{\hat{\theta}^{p,0}\}$ and almost quadratic. To make the action completely quadratic one must perform the following additional field redefinitions:

$$\begin{aligned} \tilde{\lambda}^{0,0} &= \bar{\lambda}^{0,0} - \bar{p} \theta + 2b\theta_{1,0} \\ \tilde{\lambda}^{1,0} &= \bar{\lambda}^{1,0} - 2b\theta \\ c' &= c + 2b\theta_{1,0} \end{aligned} \quad (2.9)$$

Note that neither the field redefinitions (2.7) or (2.9) introduce constraints among the new variables like the ones discussed in [10].

We now come to the main result of this letter. We have found that after performing the field redefinitions given in (2.7) and (2.9) the gauge fixed action becomes free and quadratic and is given by (to keep the notation simple we have deleted the tildes and primes indicating the new variables):

$$\begin{aligned} \mathcal{L}_{g.f.} = & P_\mu \dot{X}^\mu - \frac{1}{2} g P^2 + b\dot{c} + \lambda(g-1) \\ & + \sum_{q=0}^{\infty} \sum_{p=q}^{\infty} \{ \bar{\lambda}^{p,q} \dot{\theta}_{p,q} + \lambda_{p+1,q+1} \bar{\theta}^{p+1,q+1} \} \end{aligned} \quad (2.10)$$

The BRST transformations leaving this action invariant simplify in a beautiful way. After redefining them by some on-shell trivial symmetries they can be cast into the following form:

$$\begin{aligned} sX^\mu &= cP^\mu + \sum_{p=0}^{\infty} \bar{\lambda}^{p,0} \gamma^\mu \theta_{p+1,0} \\ sc &= -2 \sum_{p=0}^{\infty} \bar{\lambda}^{p,0} \theta_{p+2,0} \\ sb &= \frac{1}{2} P^2 \\ s\theta_{p,q} &= \lambda_{p,q} \quad q = 1, 2, \dots \\ s\theta_{p,0} &= \bar{P} \theta_{p+1,0} - 2\theta_{p+2,0} b \\ s\bar{\theta}^{p,q} &= \bar{\lambda}^{p,q} \quad q = 1, 2, \dots \\ s\bar{\lambda}^{p,0} &= (-)^p \bar{P} \bar{\lambda}^{p-1,0} + 2\bar{\lambda}^{p-2,0} b \\ s\bar{\lambda}^{1,0} &= -\bar{P} \bar{\lambda}^{0,0} \quad p = 2, 3, \dots \end{aligned} \quad (2.11)$$

The remaining variations are zero.

We have checked that the BRST transformations (2.11) are off-shell nilpotent, i.e. $[\delta(\lambda_1), \delta(\lambda_2)]$ on all fields gives zero, and that they leave the free quadratic action (2.10) invariant. The Noether BRST charge Q corresponding to (2.11) is given by

$$\begin{aligned} Q &= \frac{1}{2} P^2 c + \sum_{q=1}^{\infty} \sum_{p=q}^{\infty} \bar{\lambda}^{p,q} \lambda_{p,q} + \sum_{p=0}^{\infty} \bar{\lambda}^{p,0} \{ \bar{P} \theta_{p+1,0} - 2\theta_{p+2,0} b \} \\ &= s\{X^\mu P_\mu + cb + \sum_{q=1}^{\infty} \sum_{p=q}^{\infty} \bar{\theta}^{p,q} \lambda_{p,q}\} \end{aligned} \quad (2.12)$$

where it is assumed that the s-operator acts from the left. The BRST transformations can be obtained from the BRST charge (2.12) via the relation $\delta_{BRST} \phi = [\Lambda Q, \phi]$. Using the (graded) bracket relations

$$\begin{aligned} [X^\mu, P_\nu] &= \delta_\nu^\mu \\ \{c, b\} &= 1 \\ \{\theta_{r,s}, \bar{\lambda}^{p,q}\} &= \delta_r^p \delta_s^q \\ \{\bar{\theta}^{p,q}, \lambda_{r,s}\} &= \delta_r^p \delta_s^q \end{aligned} \quad (2.13)$$

one can again check that $\{Q, Q\} = 0$ off-shell.

3. Outlook

In this letter we have made a new proposal for a covariant quantization of the ten-dimensional superparticle. By choosing a special gauge we were able by making field redefinitions to avoid the constraint on the infinite sum of Lagrange multipliers, which was present in [10], to eliminate the Stueckelberg symmetries and simultaneously to bring the gauge-fixed action in quadratic form. We thus ended up with a free quadratic BRST invariant action which gives a correct counting of degrees of freedom.

This new quantization shows clearly that in extending the Batalin-Vilkovisky procedure [6] (which was constructed only for theories with finite stage of reducibility) to infinite reducible theories one should be very careful. In particular, in the previous treatment (see eq. (6.11) in [10]) we ended up with constraints on the infinite sum of Lagrange multipliers, which amounts to introducing a new, dependent, branch of Lagrange multipliers $\{\lambda_{p,0}\}$, given by

$$\lambda_{p,0} \equiv \bar{P} \theta_{p+1,0} - 2\theta_{p+2,0} b \quad (3.1)$$

Also it was realized in [10] that the mysterious gauge symmetry, found by Fish and Henneaux [9], is a Stueckelberg symmetry which made it possible to eliminate the extra $\{\bar{\theta}^{p,0}\}$ anti-ghost branch.

In the previous treatment of the superparticle, the presence of these *two unusual phenomena* was necessary to make a reasonable dynamical system with an *equal number of canonical coordinates and momenta* in the gauge fixed action. In the new treatment of the superparticle, proposed in this letter, we end up with the gauge-fixed action (2.10) with no constraints on the Lagrange multipliers and no Stueckelberg symmetries. Both the Lagrange multipliers $\{\lambda_{p,0}\}$ and the anti-ghosts $\{\bar{\theta}^{p,0}\}$ are not present and we have again an equal number of coordinates and momenta.

The BRST charge, which is given in terms of unconstrained variables in eq. (2.12), has a surprisingly simple form in terms of constrained variables (see eq. (3.1))

$$Q = \frac{1}{2} P^2 c + \sum_{q=0}^{\infty} \sum_{p=q}^{\infty} \bar{\lambda}^{p,q} \lambda_{p,q} \quad (3.2)$$

An interesting feature of our results is that although in the BV formalism the minimal branch plays a different role than the other branches, in the final gauge fixed action (2.10) all branches occur on the same footing. In the BRST transformations (2.11) the minimal branch $\{\theta_{p,0}\}$ and the branch $\{\bar{\lambda}^{p,0}\}$ of Lagrange multipliers do play a special role but it is clear that other choices of a special θ branch and a special $\bar{\lambda}$ would give rise to other BRST transformation rules which also leave the action (2.11) invariant and which are nilpotent. In fact, it turns out that the action (2.11) has many other (not necessarily nilpotent) global symmetries which together with the BRST transformations form a closed algebra. The full structure of these global symmetries will be presented elsewhere [13].

An obvious question to ask is whether the results of this letter generalize to other cases. One possible generalization is to redo the calculation of this letter for a superparticle in a curved background. One can check that the covariant quantization scheme proposed in this letter also works for the nine-dimensional massive particle (in a flat background) which has been considered in [14]. Basically one ends up with the same result (2.10), (2.11) where now all spinors denote nine-dimensional spinors and where everywhere P^2 has been replaced by $P^2 + m^2$ (m is the mass parameter) and $\not{P}$ by $\not{P} \pm m$. An interesting feature of the nine-dimensional massive superparticle is that the classical action contains a Wess-Zumino term $m\bar{\theta}\theta$ which is absent in the final gauge fixed action. So, although the classical action is only defined in spacetime dimensions which allow a symmetric charge conjugation matrix, like nine dimensions, the quantum action seems not to require these special dimensions.

The most interesting case to consider is of course the ten-dimensional superstring. A preliminary investigation shows that one should expect the gauge fixed action to be of the form

$$\begin{aligned} \mathcal{L}_{g,f} = & \partial_z X^\mu \partial_{\bar{z}} X_\mu - \frac{i}{2} \bar{\psi}^I \partial_z \psi^I + \sum_{q=0}^{\infty} \sum_{p=q}^{\infty} \{ \bar{\lambda}^{p,q} \nabla_z \theta_{p,q} + \lambda_{p+1,q+1} \nabla_{\bar{z}} \bar{\theta}^{p+1,q+1} \} \\ & - \hbar^{zz} \nabla_z c_z - \hbar^{\bar{z}\bar{z}} \nabla_{\bar{z}} c_{\bar{z}} \end{aligned} \quad (3.3)$$

The calculation of conformal anomaly in terms of OSp supergroup invariants, which was performed by one of the authors and by I. Bars in [15], is slightly modified. The

corresponding fields again form a representation of an OSp supergroup, as it was pointed out in [16], but the distribution of conformal weights along the triangle are shifted by -1. The conformal charge can be identified with the eigenvalue of the isospin operator $2J_0$ shifted by -1, i.e. we have

$$p - q = 2m - 1 \quad (3.4)$$

Equation (1) of ref. [15] is modified in an obvious way.

$$c = 2Str\{(6(2J_0 - 1)^2 + 6(2J_0 - 1) + 1)\} \quad (3.5)$$

Fortunately, the only difference is in the term proportional to $Str(J_0)$, which is zero, since the generators of the superalgebra are supertraceless. This means that the calculation of conformal anomaly, performed in [15], remains correct for the Lagrangian, given in eq. (3.3).

This confirms our expectation that it will be possible to find the unconstrained BRST symmetry for the gauge fixed action (3.3) along the lines developed in this paper for the superparticle. We hope to comment in more detail on the superstring case soon [17].

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