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PHASE STRUCTURE OF SELF-SIMILAR MULTIPARTICLE SYSTEMS
AND EXPERIMENTAL DETERMINATION OF INTERMITTENCY PARAMETERS

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ABSTRACT

In view of the possibility that self-similar multiparticle systems may exist in different phases, we investigate the structure of these phases using the random cascade model. It is shown that one phase ("liquid") is populated by many relatively small fluctuations, whereas the other one ("dust") consists of a few very large ones. The "dust" is suspended in the "liquid" like a colloid. Because of the rarity of "dust particles", the measurements of intermittency parameters including the dust phase require unusually large statistics, particularly at high resolution.

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The apparent observation of intermittency [1] in particle production processes at high energy, reported recently by several groups [2], invites one to make further studies of its properties. In the present note we discuss the structure of different phases [3,4] of self-similar multiparticle systems. We use the simplest model of intermittency, i.e., the random cascading model [1,5,6]. This model, although indeed very simple, can describe a rather broad class of the self-similar (intermittent) particle spectra and it is very likely that its consequences are more general than the model itself. In the absence of a theory of multiparticle processes, it is thus interesting to investigate the intermittent behaviour in this model, hoping that the results can be useful in identifying the characteristic properties of the self-similar behaviour in experimentally observed distributions.

In order to keep our discussion as complete as possible, let us begin by defining the model [1,5,6]. It has a tree structure with l branches at each splitting. The average density in any of the bins of size $\delta_{\min}$ at the bottom of the cascade is a product of L positive random factors (w) ascribed to the L links forming a path leading from the origin of the cascade to the given bin. The distributions of these random factors $r(w)$ are the same for all links (self-similarity), and they are independent of each other. For simplicity we normalize w by requiring $\int r(w)dw = 1$. The particle distribution is constructed by requiring that in a bin of size δ , in which the (average) particle density equals ρ , the distribution of particles is given by the Poisson formula with the average $\rho\delta$. Thus, the distribution of particles is determined in terms of the distribution $r(w)$, the number of branches (l) and the total length of the cascade L . The bin width at the bottom is $\delta_{\min} = \Delta/L$, where Δ is the total rapidity interval considered.

This completes the definition of the model: all particle spectra are now well defined in terms of the parameters listed above. We would like to stress that the model is defined (and can be studied) for any finite L . The choice of L is a physical problem. For instance, in the theory of hydrodynamic turbulence, where similar cascade models are used, one should stop cascading before reaching resolution comparable to intermolecular distances. The range of L relevant for the analysis of high energy experiments is an open problem. Anyway, $L \rightarrow \infty$ ($\delta_{\min} = 0$) is unlikely, at least at finite energies.

The interest in the random cascading model stems from the fact that it reproduces the fundamental property of intermittent behaviour, i.e., the power law dependence of the normalized moments of particle density on the bin size:

$$F_2(\delta) = F_2(\Delta) (\Delta/\delta)^{f_2} \quad (1)$$

under the condition

$$1 \gg \delta \gg \delta_{\min} \quad (2)$$

Here

$$F_2(\delta) = \int P_2(\rho) \rho^2 d\rho \quad (3)$$

where $P_2(\rho)$ is the probability distribution for the average density ρ in a bin of size δ [we assume, without loss of generality, $\int P_2(\rho) d\rho = 1$]. The f_2 's are intermittency exponents, which will henceforth be called "local intermittency exponents", to emphasize that they refer to the particle density in a small phase-space region $\delta/\Delta \ll 1$.

The relation between the local intermittency exponents and the parameters of the model is given by [1,6]

$$f_2 = \log \langle w^2 \rangle / \log l \quad \text{for } f_2 \leq 2-1 \quad (4a)$$

$$f_2 = 2-1 \quad \text{otherwise.} \quad (4b)$$

We restrict our further analysis to the region (4a), which contains all experimental data presently available. The experimental verification of these results for integer q is based on the equality [1,5]

$$\int P_2(\rho) \rho^2 d\rho = \lim_{N \rightarrow \infty} \langle k(k-1) \dots (k-q+1) \rangle_N / \langle k \rangle_N^q \quad (5)$$

where k is the number of particles observed in a given bin of size δ and $\langle \dots \rangle_N$ is the average taken over N events. Equation (5) is a straightforward consequence of the assumed Poisson distribution of particles at each fixed density. It is important to realize, however, that since one has always to deal with a finite number of events, this formula can only be used as an approximation. It is therefore necessary to discuss the general conditions under which this approximation is good enough to reflect the true predictions of the random cascade model.

To this end, let us observe that not all bins contribute equally to a moment of a given rank q . Therefore, the accuracy of the approximation depends not so much on the total number N of measured events, as on the total number N_q of bins giving the dominant contribution to the moment of rank q (henceforth we call them q -bins). This number in turn depends on the parameters of the model. To obtain

its estimate we follow the idea of multifractal analysis of Refs. [7] and [8] (cf. also Ref. [3b]). The result is

$$N_2 \sim N \delta^{-2} d\lambda_2/dq \quad (6)$$

where λ_q is related to the local intermittency exponents f_q by

$$\lambda_2 = (f_2 + 1)/2. \quad (7)$$

Proof Following Refs. [7] and [8], we split all bins into classes. Each class is characterized by the scaling behaviour of the density $p(\alpha) \sim \delta^{\alpha-1}$ with some α . The number N_α of bins of class α scales as $N_\alpha \sim \delta^{-f(\alpha)}$. Consequently, the moment of rank q scales as

$$F_2(\delta) \sim \delta^{1-2} \int \delta^{2\alpha-f(\alpha)} d\alpha. \quad (8)$$

For $\delta \ll 1$ the integral is dominated by the α for which $q\alpha - f(\alpha)$ is minimal. This can be either a saddle point $\alpha = \alpha(q)$ which is the solution of the equation

$$q = df(\alpha)/d\alpha \quad (9)$$

or the relevant value of α is at one of the endpoints of the range allowed for α .

Let us consider the saddle point. Using (1) and (8), we have

$$\lambda_2 \equiv (f_2 + 1)/2 = 1 - \alpha(q) + f(\alpha(q))/2 \quad (10)$$

which, by differentiation with respect to q and by using (9), gives

$$f(\alpha(q)) = -q^2 d\lambda_2/dq \quad (11)$$

Since Eq. (9) gives a one-to-one correspondence between α and q , we conclude that N_α is equal to $N_{\alpha(q)} \sim \delta^{-f(\alpha(q))}$ which, using (11), gives (6) QED.

Equation (6) implies that the (average) number of q -bins for $\delta \ll 1$ depends crucially on the sign of the derivative $d\lambda_q/dq$. It was shown, for example in Ref. [4], that λ_q can have at most one minimum and cannot have a maximum in the region $q > 1$. Denoting the minimum point by q_c , we see that $d\lambda_q/dq < 0$ for $q < q_c$ and then N_q becomes large in the limit $\delta \ll 1$. Thus, many bins contribute to the average value $\langle k(k-1) \dots (k-q+1) \rangle_N$. In this case one may expect that Eq. (5) can be well approximated by replacing the limit $N \rightarrow \infty$ by a large but fixed number of

events N . On the other hand, $d\lambda_q/dq > 0$ for $q > q_c$, and then N_q is small in the limit $\delta \ll 1$, unless the total number of events is of the order

$$N \sim \delta^{-2} d\lambda_2/dq \quad (12)$$

which becomes a very large number for small δ .

To summarize: for $q > q_c$, Eq. (5) is valid in the limit $N \rightarrow \infty$, δ fixed and not valid in the limit N fixed, $\delta \rightarrow 0$. In general, these limits cannot be interchanged. For $q < q_c$ both limits give the same result. At this point we would like to stress again that, as already explained, we believe that the limit $\delta \rightarrow 0$, N fixed is, strictly speaking, not the proper one: at finite energy, there must be a finite $\delta_{\min}$ beyond which the system is no longer self-similar.

It follows from this argument that the measurement of the intermittency exponents is fairly straightforward for $q < q_c$, but rather tricky for large $q > q_c$: it is then necessary to study the moments (5) as functions of the number of available events. For $q > q_c$ and δ small enough, the moments are expected to fluctuate violently with increasing statistics before saturating at very large N .

Since the actual value of $\delta_{\min}$ is not known and may perhaps be very small, it is also interesting to see what happens for $q > q_c$ in the limit $\delta \rightarrow 0$. In this case one can prove (see, e.g., Ref. [9]) that the minimum of $q\alpha - f(\alpha)$ is attained at α_0 such that $f(\alpha_0) = 0$ and is therefore equal to $q\alpha_0$. But from Eq. (11) we can conclude that $\alpha_0 = \alpha(q_c)$. Consequently, we obtain (cf. also Ref. [3b])

$$\lambda_2 \equiv (f_2 + 1)/2 = 1 - \alpha(q_c) \quad (13)$$

where we have denoted by λ_{q_c} the value of the local intermittency exponent in the limit N fixed, $\delta \rightarrow 0$. The λ_q given by (13) is smaller than the true local intermittency exponent given by Eq. (4a). This is illustrated in Fig. 1.

It was already shown in Refs. [3] and [4] that self-similar multiparticle systems behave differently in the regions $q < q_c$ and $q > q_c$. The present investigation allows us to formulate more precisely the difference between the structure of the system in these two regions. The region $q < q_c$ appears to be dominated by numerous fluctuations containing not much more than q_c particles in one bin. The region $q > q_c$ is governed by a small number of sometimes very large fluctuations. This situation is reminiscent of a mixture of a "liquid" of many (relatively) small fluctuations and a "dust" consisting of a few "grains" of very high density. The "liquid" and the "dust" coexist. When the system is probed by a moment of rank $q < q_c$, one sees only the dominating liquid phase. When the system is probed by a

moment of rank $q > q_c$, one sees only the dust phase, because these moments are insensitive to small fluctuations: non-vanishing contributions to the average value $\langle k(k-1) \dots (k-q+1) \rangle_N$ come only from bins which contain at least q particles. Furthermore, since the number of grains of dust is rather small, one needs large statistics to obtain a good measurement. In this interpretation there is no "phase transition" at the point $q = q_c$ (as suggested, for example, in Ref. [3]), but rather the "measuring apparatus" (i.e., the selected moment of the density) is sensitive to one or the other phase.

It would, of course, be of great interest to find out experimentally whether two such phases do indeed coexist in Nature. As seen in Fig. 2, the present data [10-12] are not yet sufficient to decide whether λ_q does have a minimum at some value of q . One way to study this problem was suggested in Ref. [4]. It consists of measuring the global fluctuations of the system. They were proved to have qualitatively different scaling properties in the two phases we have been discussing. Of course, the present analysis of measurement applies also to the results of [4]. In particular, the scaling formulae of [4] for $q > q_c$ are valid in the limit $N \rightarrow \infty$, δ fixed and not in the limit N fixed, $\delta \rightarrow 0$.

Finally, let us remark that if the existence of the two phases is proved experimentally, it will be an exciting challenge to find their physical interpretation in terms of the underlying theory of strong interactions.

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FIGURE CAPTIONS

Fig. 1 : λ_q as a function of q ($q > 1$) for the "weak intermittency" region (4a). One sees different behaviour of one- and two-phase systems.

Fig. 2 : λ_q measured in recent experiments [10-12].

Fig. 1

Fig. 2