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INTERMITTENCY IN MULTIPARTICLE DISTRIBUTIONS AND ONE-DIMENSIONAL ISING SYSTEMS

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ABSTRACT

One-dimensional Ising systems are used to simulate particle rapidity distributions. We show that in the one-dimensional Ising model intermittent-like behaviour occurs. We further show that with increasing multiplicities and factorial moments this behaviour tends to decrease.

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I- Introduction

Recently intermittent phenomena have attracted much attention both in hadronic physics and statistical mechanics. The main reason for this interest lies in the fact that, in both these domains, intermittency should be taken as evidence for fractal structures. In multiparticle hadronic production¹, intermittency is expected to stem from QCD evolution and branching, thus defining a fractal whose structure should be more and more visible by looking at the multiplicity distribution in smaller and smaller (pseudo-)rapidity bins. The experimental results of NA22², TASSQ³, and many other experiments⁴, are consistent with the existence of intermittency. Theoretically it is still not clear whether this behaviour is just a "trivial" manifestation of two-particle correlations⁵.

In statistical mechanics the situation is equally confusing. Wosiek⁶ was the first to study intermittency in a 2-dimensional Ising system using the idea that intermittency could be related to critical phenomena. Therefore, at the critical point one should observe a non-zero slope for the logarithm of the scaled factorial moments

$$\langle F_i \rangle = \left\langle \frac{\frac{1}{M} \sum_{m=1}^M k_m (k_m - 1) \dots (k_m - i + 1)}{(\frac{1}{M} \sum_{m=1}^M k_m)^i} \right\rangle_{\text{configurations}} \quad (1)$$

as a function of M , the number of blocks into which one divides the original lattice. The k_m 's are, in Wosiek's approach, the number of spins pointing up in the m -th block, and the average is performed over configurations generated, as usual, with a Monte Carlo method. Wosiek seemed able then to show the existence of intermittency in such systems. However, as was later noticed by Bambah *et al.*⁷, this behaviour was not related to any kind of critical phenomena, but was simply a consequence of the way the F_i 's were measured. At the same time, Satz⁸ also showed that for intermittency to be related to critical phenomena, the normal moments

$$\langle C_i \rangle = \left\langle \frac{\frac{1}{M} \sum_{m=1}^M k_m^i}{(\frac{1}{M} \sum_{m=1}^M k_m)^i} \right\rangle_{\text{configurations}} \quad (2)$$

should behave as

$$\langle C_i \rangle \propto \begin{cases} L^{-i(d-x)} & i \text{ even;} \\ L^{-(i-1)(d-x)} & i \text{ odd.} \end{cases} \quad (3)$$

where L is the linear dimension of a block, d is the number of dimensions of the Ising system and x an exponent related to the critical exponents of the model. Although clear signs of intermittency have been observed in [7], the actual contribution of finite size effects could not be quantitatively measured.

In this paper we show that intermittent-like behaviour can appear in systems having no critical properties. Furthermore, since we are going to work with very simple statistical systems of which analytical solutions exist, the physics behind this behaviour is going to appear in a very transparent manner.

II- Intermittency in particle rapidity distributions and one-dimensional Ising systems

As was stated above, intermittency in high multiplicity hadronic events was experimentally observed in the behaviour of the scaled factorial moments²

$$\langle F_i \rangle = \frac{\langle \frac{1}{M} \sum_{m=1}^M k_m (k_m - 1) \dots (k_m - i + 1) \rangle_{\text{events}}}{(\langle \frac{1}{M} \sum_{m=1}^M k_m \rangle_{\text{events}})^i} \quad (4)$$

where $\langle \dots \rangle_{\text{events}}$ means average over all events, M is the number of bins into which the (pseudo-)rapidity interval under consideration Y has been divided, and k_m is the number of (charged) particles in the m -th bin. Of course, if one divides the rapidity interval into an increasingly larger number of bins we will arrive at a situation where we will have at most one particle per bin. Beyond this limit, no new information is obtained from the scaled factorial moments. However, one can formally define the limit $M \rightarrow \infty$, and look for the behaviour of $\langle F_i(M) \rangle$ at increasingly smaller bins:

$$\langle F_i \rangle \xrightarrow{M \rightarrow \infty} (1/M)^i. \quad (5)$$

If (5) is valid, with $f_i > 0$, i.e., if the $\langle F_i \rangle$ show a power behaviour increase as the bin size decreases, the system is said to be intermittent.

We shall now introduce our one-dimensional Ising model and show how it can be used to simulate multiparticle rapidity distributions. In a rapidity interval Y we consider the number of divisions M_{max} such that there is at most one particle per bin. Let σ_m be a characteristic function such that

$$\sigma_m = \begin{cases} +1 & \text{if one particle is present in the } m\text{-th bin;} \\ -1 & \text{if no particle is present in the } m\text{-th bin.} \end{cases} \quad (6)$$

with $m = 1, 2, \dots, M_{\text{max}}$. This is formally analogous to a one-dimensional spin system on a lattice of $N = M_{\text{max}}$ sites. From this point of view, the study of particle distributions in rapidity is nothing more than the study of a one-dimensional spin statistical system.

As a thought experiment, let us assume that high multiplicity events are generated with the one-dimensional Ising dynamics. Using our previous terminology, the average (over events) multiplicity is

$$\langle n \rangle = \frac{1}{2} \left(\left\langle \sum_{i=1}^N \sigma_i \right\rangle + N \right) = \frac{1}{2} (\overline{M} + 1)N \quad (7)$$

where N is the number of sites (i.e., maximum number of bins of the rapidity interval Y) and $\overline{M}$ is the average magnetization. This can be calculated analytically since the model is solvable. Introducing the action (free boundary conditions)⁹

$$S_F(F, H) = -E \sum_{i=1}^{N-1} \sigma_i \sigma_{i+1} - H \sum_{i=1}^N \sigma_i \quad (8)$$

where E gives the strength of the spin-spin interaction and H is the applied magnetic field. We can construct the partition function

$$Z(\beta, E, H) = \sum_{\{\sigma_1 \dots \sigma_N\}} \exp \left\{ \beta \left[E \sum_{i=1}^{N-1} \sigma_i \sigma_{i+1} + H \sum_{i=1}^N \sigma_i \right] \right\} \quad (9)$$

β being the inverse temperature and $\{\sigma_1 \dots \sigma_N\}$ meaning the sum over all possible spin configurations. This can be written as⁹

$$Z(\beta, E, H) = \lambda_+^{N-1} \{ \cosh \beta H + (\sinh^2 \beta H + e^{-2E\beta}) (\sinh^2 \beta H + e^{-4E\beta})^{-1/2} \} \\ + \lambda_-^{N-1} \{ \cosh \beta H - (\sinh^2 \beta H + e^{-2E\beta}) (\sinh^2 \beta H + e^{-4E\beta})^{-1/2} \} \quad (10)$$

with

$$\lambda_{\pm} = e^{\beta E} \left(\cosh \beta H \pm (\sinh^2 \beta H + e^{-4E\beta})^{1/2} \right) \quad (11)$$

Equation (7) can then be written as

$$\langle n \rangle = \frac{1}{2} \left(\beta^{-1} \frac{\partial \ln Z(\beta, E, H)}{\partial H} \right) \Big|_{\beta} + N \quad (12)$$

For the full system all factorial moments can be calculated using the standard methods. All one needs to know are the quantities

$$\langle n^l \rangle = \frac{1}{2^l} \left\langle \left(\sum_{i=1}^N \sigma_i + N \right)^l \right\rangle \quad (13.a)$$

and

$$\left\langle \sum_{i_1=1}^N \sigma_{i_1} \dots \sum_{i_l=1}^N \sigma_{i_l} \right\rangle = \beta^{-l} \frac{1}{Z(\beta, E, H)} \frac{\partial^l Z(\beta, E, H)}{\partial H^l} \Big|_{\beta} \quad (13.b)$$

For instance,

$$\langle F_2(1) \rangle = \frac{\langle n(n-1) \rangle}{\langle n \rangle^2} = \frac{\langle n^2 \rangle}{\langle n \rangle^2} - \frac{1}{\langle n \rangle} \quad (14)$$

where

$$\langle n^2 \rangle = \frac{1}{4} \left\langle \sum_{i_1=1}^N \sigma_{i_1} \sum_{i_2=1}^N \sigma_{i_2} \right\rangle + \frac{1}{2} N^2 < \overline{M} + \frac{N^2}{4} \quad (14.a)$$

and

$$\left\langle \sum_{i_1=1}^N \sigma_{i_1} \sum_{i_2=1}^N \sigma_{i_2} \right\rangle = \beta^{-2} \frac{1}{Z(\beta, E, H)} \frac{\partial^2 Z(\beta, E, H)}{\partial H^2} \Big|_{\beta} \quad (14.b)$$

In calculations we shall use the exact formulae (13.a-b) when full rapidity interval distributions are to be studied, and Monte Carlo generation for $\delta y = Y/M$, $M > 1$, rapidity bins.

For the Monte Carlo we shall use the Metropolis algorithm with 60000-100000 thermalising iterations and ~ 100000 iterations for measuring (events).

In Fig. 1 we show a scatter plot for a 512-site Ising system in which each point corresponds to an allowed value of $J = E/\beta$ and $K = H/\beta$, for which the system, making use of formulae (13.a-b), is made to agree with the NA22 pp data at $\sqrt{s} = 22 \text{ GeV}$ in a central rapidity interval of length $Y = 1$.

$$\begin{aligned} & \langle n \rangle = 6.96 \pm 0.11 \\ & \langle F_2 \rangle = 1.19 \pm 0.03 \\ & \langle F_3 \rangle = 1.60 \pm 0.07 \end{aligned} \quad (15)$$

One should notice that the magnetic field is negative. This is required in order to reduce the number of particles. One has to remember that the existence of one event in the NA22 experiment with 10 particles in a rapidity interval $\delta y = 0.1$ ($M = 40$) requires, in the language of spins, at least 10 spins in that interval and, consequently, at least 400 spins in total. As the average multiplicity $\langle n \rangle \sim 7$ and the maximal measured multiplicity is 26 this means that the rapidity axis is sparsely populated. A negative magnetic field, defining a preferred direction (spin down) recreates this situation.

In Fig. 2.a we show the intermittency plots of $\ln \langle F_i \rangle$, $i = 2, 3, 4, 5$, as a function of $\delta y = Y/M$. The $\langle F_i(\delta y) \rangle$ were computed by Monte Carlo generation of events and applying the experimental procedure of Eq. (4). The values of J and K were 1.05757 and -0.50909, respectively. As can be seen, intermittent-like behaviour is observed (see Eq. (5)), and in fact more than needed, when compared to the NA22 experiment. It is interesting to notice that the simulation produced one single event with 27 particles (maximal multiplicity observed) and 10 particles in the smallest bin ($\delta y = 0.1$), thus with the probability $\sim 10^{-5}$, precisely as it occurred in the NA22 experiment.

The one-dimensional Ising model has no critical behaviour. Intermittency has to do with the sign of J : if $J > 0$, i.e. positive short-range correlations (ferromagnetism), intermittency occurs, if $J < 0$, i.e. negative short-range correlations (antiferromagnetism), intermittency does not occur. This can be seen in Fig. 3 where the calculations for Fig. 2.a were repeated reversing the sign of J .

We next tested the dependence of the model on the various parameters. Concerning the number N of sites, we varied that number from 256 to 1024. The intermittency curves varied (see Fig 2.b-c) but, in the $\delta y \rightarrow 0$ limit, the trend remained practically constant. Intermittency seems then to survive in the thermodynamical limit.[†]

A more interesting situation occurs when we vary the physical parameters J and K and attempt to simulate the increase with energy of both $\langle n \rangle$ and $\langle F_2 \rangle$. Starting from a given pair $(J_0, K_0) = (0.38, -2)$, corresponding to low energy values $\langle n \rangle$ and $\langle F_2 \rangle$, we asked for the pairs (J, K) such that $\langle n \rangle$ and $\langle F_2 \rangle$ continuously increased. The resulting plot is shown in Fig. 4. What is remarkable is that a very narrow trajectory is drawn in the (J, K) plane thus indicating that the constraint of increasing simultaneously

[†] One must remember, however, that the biggest of our one-dimensional systems is of the size of the smallest of the systems studied in [7], where finite size effects have proved to be important. We thank H. Satz for pointing this out to us.

$\langle n \rangle$ and $\langle F_2 \rangle$ is extremely restrictive. It further indicates that in order to simulate the physics of multiparticle production J and K are, in practice, functionally related. This is, of course, what is expected in the theory of strong interactions (QCD) where one single parameter (α_s) occurs.

As $\langle n \rangle$ and $\langle F_2 \rangle$ increase, or as we move along the plot of Fig. 4 from left to right, the intermittent-like behaviour tends to decrease (see Fig. 2.d). If we interpret the increase of $\langle n \rangle$ and $\langle F_2 \rangle$ as an increase in energy, it necessarily follows that intermittency will decrease with energy. The relevance of this result for actual experiments in particle physics at high energies is not at all clear.

What are our conclusions? Clustering is essential to the appearance of intermittency and it is the sign of J ($J > 0$) that determines it. For $J > 0$, if a given spin σ_i flips from -1 to +1 the neighbours will try to follow it and thus generate a cascade phenomena which ends up in a cluster of spins up. If $J < 0$, flipping one spin only amounts to a redistribution of spins and no clusters appear. All the other results are quite suggestive but will require better understanding in the future.¹⁰

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FIGURE CAPTIONS

Fig. 1- Scatter plot showing the points satisfying Eq. (15) for the full (512 sites) one-dimensional Ising system.

Fig. 2- (a) Intermittency plots for $\ln \langle F_i \rangle$, $i = 2, 3, 4, 5$, as a function of $\delta y = Y/M$. The results are obtained by Monte Carlo simulation of a 512-site one-dimensional Ising system. The solid line has been drawn just to guide the eye through the calculated points. The (naïve) error bars on these points are too small to be shown in the figure, but they are probably underestimated; (b-c) Same as (a) but for 256 and 1024 sites; (d) Same as (a-c) but for a different point along the curve of Fig. 4.

Fig. 3- Same simulation as for Fig. 2-a, but with reversed sign for J .

Fig. 4- Scatter plot showing the points (J, K) satisfying the condition that $\langle n \rangle$ and $\langle F_2 \rangle$ (full interval) increase simultaneously.

FIG. 1

FIG. 2.A

FIG. 3

FIG. 2.D

FIG. 4