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CP violation measurements at the ϕ resonance

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ABSTRACT

We discuss how to determine the parameter ϵ'/ϵ from ϕ factory data, with special attention taming the background coming from decays of $(K^0 \bar{K}^0, C=+)$ pairs produced in association with a low-energy photon.

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$$\frac{\Gamma(\phi \rightarrow \gamma + S^*/f_0)}{\Gamma(\psi' \rightarrow \gamma + \chi_{c0})} = \frac{1}{4} \left(\frac{q_s}{q_c} \right)^3 \left(\frac{m_c}{m_s} \right)^3 \quad (3)$$

where q_s and q_c are the photon energy for a ϕ or a ψ' state decaying at rest, and m_c and m_s are the constituent masses of the charm and strange quarks ($m_c/m_s=3$). In this way one obtains $\Gamma(\phi \rightarrow \gamma + S^*/f_0) = 6 \cdot 10^{-5}$ MeV, ten times more than in Nussinov and Truong. If the confining potential is quadratic, the decay width reaches nearly 10^{-4} MeV.

None of these models is very compelling, therefore one should not trust too much the smallness of the values obtained; we think that it would be helpful to obtain a determination of ϵ'/ϵ free from the uncertainty due to this contribution.

In this letter we show that with a suitable geometrical cut one can suppress almost completely the bad C-even events, while still retaining a very large fraction of good events.

Let us consider the $(\pi^+ \pi^-, \pi^0 \pi^0)$ final state and denote by $d_1(d_2)$ the path length of a neutral kaon decaying into $\pi^+ \pi^- (\pi^0 \pi^0)$. Let N_A be the number of events falling in the rectangle of the (d_1, d_2) plane defined by

$$d_0 < d_1 < d < d_2 < D \quad (4)$$

and N_B the number of those having

$$d_0 < d_2 < d < d_1 < D \quad (5)$$

d_0 and D being the minimum and the maximum paths inside the detector, respectively. As will be discussed presently, it is possible to choose the distance d in such a way that

- i) $N_A (N_B)$ represents almost the total number of events with $d_0 < d_1 < d_2 < D$ ($d_0 < d_2 < d_1 < D$) coming from a C-odd $K^0 \bar{K}^0$ state;

The measurement of CP violating parameters in the coherent decay in $K^0 \bar{K}^0$ of the $(J^{PC} = 1^{--})$ ϕ resonance has recently excited much interest [1], [2], [3], the ϕ mesons being copiously produced in high-luminosity e^+e^- annihilation machines (ϕ factories). As stressed by many people [4] the ϕ meson decays in an antisymmetric $K_S K_L$ state, and from the study of the 1π final states (time asymmetry in $\pi^+ \pi^-$, $\pi^0 \pi^0$ [5]) or different branching ratios in different charge configurations [6]) one can hope to get a clean determination of the ratio ϵ'/ϵ .

Very recently Nussinov and Truong [7] remarked the presence of a dangerous background coming from the following chain of decays:

$$\phi \rightarrow \gamma + S^*/f_0(976) \rightarrow \gamma + (K^0 \bar{K}^0)_{C=+} \rightarrow \gamma + \frac{1}{\sqrt{2}} (K_S K_S - K_L K_L) \quad (1)$$

Due to the small value of ϵ , a contamination at 10^{-3} level will induce a number of 4π events nearly equal to that coming from the $(K^0 \bar{K}^0, C=-)$ state. Since the maximum photon energy is

$$E_\gamma = \frac{m_\phi^2 - 4m_K^2}{2m_\phi} \simeq 21 \text{ MeV} \quad (2)$$

it will be very difficult to distinguish the direct ϕ decay from that mediated by the S^*/f_0 intermediate state, by reconstructing the $K^0 \bar{K}^0$ mass spectrum. The partial width of this process is strongly model dependent. The value obtained by Nussinov and Truong is $\Gamma(\phi \rightarrow \gamma + K^0 \bar{K}^0) = 1.5 \cdot 10^{-7}$ MeV, which corresponds to $\Gamma(\phi \rightarrow \gamma + S^*/f_0) = 6 \cdot 10^{-6}$ MeV. One could also estimate the partial width for the dipole transition $\phi \rightarrow \gamma + S^*/f_0$ using a non relativistic potential model as done for radiative decays of charmonium and bottomonium. One has for a confining linear potential [8]

equations 4 and 5:

it follows that the value of ϵ'/ϵ can be derived from the asymmetry $A = (N_A - N_B)/(N_A + N_B)$ without problems coming from the C-even contamination.

In terms of the usual CP violating amplitudes

$$\begin{aligned} \eta_{+-} &= \frac{A(K_L^- \rightarrow \pi^+ \pi^-)}{A(K_S^- \rightarrow \pi^+ \pi^-)} = \epsilon + \epsilon' \\ \eta_{00} &= \frac{A(K_L^- \rightarrow \pi^0 \pi^0)}{A(K_S^- \rightarrow \pi^0 \pi^0)} = \epsilon - 2\epsilon' \end{aligned} \quad (6)$$

and for each initial $(K^0, \bar{K}^0, C = -)$ state, with a straightforward calculation one obtains

$$N_A^- = \frac{\Gamma_S^+ \Gamma_L^{00}}{\Gamma_S \Gamma_L} \{ |\eta_{+-}|^2 S_1 + |\eta_{00}|^2 S_2 + \text{Re}(\eta_{+-} \eta_{00}^*) S_3 + \text{Im}(\eta_{+-} \eta_{00}^*) S_4 \} \quad (7)$$

In eq. (7) $\Gamma_S(\Gamma_L)$ is the $K_S(K_L)$ width, whereas Γ_S^{+-} and Γ_S^{00} are the partial widths for K_S decaying into $\pi^+ \pi^-$ and $\pi^0 \pi^0$, respectively. The coefficients S_i are given by

$$\begin{aligned} S_1 &= (e^{-\frac{d}{d_S}} - e^{-\frac{D}{d_S}})(e^{-\frac{d}{d_L}} - e^{-\frac{D}{d_L}}) \quad , \\ S_2 &= (e^{-\frac{d}{d_S}} - e^{-\frac{D}{d_S}})(e^{-\frac{d}{d_L}} - e^{-\frac{D}{d_L}}) \quad , \\ S_3 &= t(d, D) + t(d_0, d) - t(d_0, D) - t(d, d) \quad . \end{aligned} \quad (8)$$

where

$$t(d_0, d) = -\frac{2\Gamma_S \Gamma_L}{\Gamma^2 + (\Delta m)^2} \cos \left[\frac{\Delta m}{\Gamma_S} \left(\frac{d_0 - d}{d_S} \right) \right] e^{-\frac{\Gamma}{\Gamma_S} \frac{d_0 + d}{d_S}} \quad , \quad (9)$$

$d_S \simeq 0.6$ cm and $d_L \simeq 342$ cm are the mean decay paths of K_S and K_L from a ϕ meson at rest, $\Gamma = (\Gamma_S + \Gamma_L)/2$ and $\Delta m = m_L - m_S$.

N_B^- can be obtained from N_A^- (eq. (7)) by interchanging η_{00} and η_{+-} , namely

$$N_B^- = \frac{\Gamma_S^+ \Gamma_L^{00}}{\Gamma_S \Gamma_L} \{ |\eta_{00}|^2 S_1 + |\eta_{+-}|^2 S_2 + \text{Re}(\eta_{+-} \eta_{00}^*) S_3 - \text{Im}(\eta_{+-} \eta_{00}^*) S_4 \} \quad (10)$$

The two quantities, N_A^- and N_B^- , differ because of the different dependence of η_{+-} and η_{00} on ϵ'/ϵ . The two most recent experiments, however, both say that ϵ'/ϵ is small (and probably real): we have in fact ϵ'/ϵ ranging from the value published by the CERN NA31 Collaboration [9]

$$\frac{\epsilon'}{\epsilon} = (3.3 \pm 1.1) \cdot 10^{-3} \quad , \quad (11)$$

to the recent preliminary result of the Chicago-Fermilab Collaboration [10]

$$\frac{\epsilon'}{\epsilon} = (-0.5 \pm 1.5) \cdot 10^{-3} \quad . \quad (12)$$

It follows that apart from small terms $O(\epsilon'/\epsilon)$ we have the same number of events in the two regions, i.e.

$$N_A^- \simeq N_B^- \simeq \frac{\Gamma_S^+ \Gamma_L^{00}}{\Gamma_S \Gamma_L} |\epsilon|^2 S \quad , \quad (13)$$

with the "signal" S given by

$$S = S_1 + S_2 + S_3 \simeq S_2 \quad , \quad (14)$$

as can be seen from Table I, where the four coefficients S_i are given for several choices of d_0 , d and D .

In Fig. 1 we report the normalized distribution of the events C -odd vs. the distance from the vertex of the "second" K -decay (in d_S units). By taking a given value d , all the events characterized by a smaller distance represent all that we lose by applying the cut of eq. (4): i.e. the events having $d_0 < d_1 < d_2 < d$ coming from a C -odd state, defined in the following as

$$N_C^- \simeq \frac{\Gamma_S^+ \Gamma_S^{00}}{\Gamma_S \Gamma_L} |\epsilon|^2 C^- \quad (15)$$

If the cut is chosen to be appreciably larger than d_S , then the remaining distribution is "practically" that of the events N_A^- (or N_B^-) (the number of events with $d < d_1 < d_2$ is quite negligible), to be further cut in correspondence with D , the maximum path inside the detector (in the figure a small vertical arrow indicates the position of $D = \frac{1}{2} d_L$).

In the case of a $(K^0 \bar{K}^0, C = +)$ pair, for symmetry reasons there is no difference between the two regions A and B . The corresponding quantities, N_A^+ and N_B^+ , are equal and can be written in the form

$$N_A^+ = N_B^+ = \frac{\Gamma_S^+ \Gamma_S^{00}}{\Gamma_S \Gamma_L} |\epsilon|^2 B \quad (16)$$

where we have explicitly extracted the $|\epsilon|^2$ dependence, typical of N_A^- and N_B^- , in order to make the comparison easier. We give also the explicit expression for the "background" making the approximations $\epsilon' \simeq 0$ (and therefore $\arg(\eta_{+-}) \simeq \arg(\eta_{00}) \simeq \arg(\Gamma + i\Delta m)$) and $e^{-\frac{d}{d_S}} \simeq 0$,

$$B = \frac{\Gamma_L}{\Gamma_S |\epsilon|^2} \left(e^{-\frac{d_0}{d_S}} - e^{-\frac{d}{d_S}} \right) e^{-\frac{d}{d_S}} + \frac{\Gamma_S |\epsilon|^2}{\Gamma_L} \left(e^{-\frac{d_0}{d_S}} - e^{-\frac{d}{d_S}} \right) \left(e^{-\frac{d}{d_S}} - e^{-\frac{D}{d_S}} \right) - \frac{2\Gamma_S \Gamma_L}{\Gamma^2 + \Delta m^2} \left\{ e^{-\frac{d_0}{d_S}} \cos \left[\frac{\Delta m}{\Gamma_S} \left(\frac{d_0 + d}{d_S} \right) \right] - e^{-\frac{D}{d_S}} \cos \left[\frac{\Delta m}{\Gamma_S} \left(\frac{2d}{d_S} \right) \right] \right\} \quad (17)$$

It is easy to verify from eqs. (8) and (17) that B can be very much suppressed if d is chosen to be much larger than d_S but still much smaller than D , in order to retain a large fraction of the signal. This is shown in Fig. 2, where the signal S , the background B and the number C^- of "good" (C -odd) events that we lose by applying the cut in eq. (4) are plotted as functions of d for $d_0 = 0$ and $D = \frac{1}{2} d_L = 171$ cm: it can be seen that for $10 d_S < d < 20 d_S$ one has at the same time a strong suppression of the C -even background B and a large fraction of good events retained ($C^- \ll S$).

In Table II we give the values of the signal S , the background B and the lost events C^- for several values of d_0 , d and D . In the same Table the values of C^+ are listed too, C^+ being related, analogously to C^- in eq. (15), to the fraction of "bad" ($C=+$) events N_C^+ falling in the excluded region $d_0 < d_1 < d_2 < d$.

We come now to the determination of ϵ'/ϵ from the asymmetry between N_A and N_B induced by the different dependence on ϵ' of η_{+-} and η_{00} . If the initial state contains a fraction α of $(K^0 \bar{K}^0, C = +)$ pairs one has

$$A = \frac{N_B^- + \alpha N_B^+ - (N_A^- + \alpha N_A^+)}{N_B^- + \alpha N_B^+ + (N_A^- + \alpha N_A^+)} \simeq 3 \operatorname{Re} \left(\frac{\epsilon'}{\epsilon} \right) \cdot \frac{1}{(1 + \alpha \rho)} \quad (18)$$

where $\rho = N_A^+/N_A^- \simeq B/S$. In deriving eq. (18) we take into account that the background is symmetric (see eq. (16)) and make use of the S_i of Table I. It is easily seen that the coefficient of $\operatorname{Re}(\epsilon'/\epsilon)$ is 3 to a very high precision and independent of the cut we choose. Conversely, that of $\operatorname{Im}(\epsilon'/\epsilon)$ is cut-dependent and negligible ($\sim 10^{-3}$ or several orders of magnitude smaller) for all the cases of interest here. Expected to be zero from CPT invariance, as confirmed by the new experimental result [11] on the phases ϕ_{+-} and ϕ_{00} , also if not zero, $\operatorname{Im}(\epsilon'/\epsilon)$ cannot be appreciated in the present approach.

The C -even background would induce a systematic error $\Delta(\epsilon'/\epsilon)/(\epsilon'/\epsilon) \simeq \alpha \rho$. For the cuts given in Table I, this is less than $4 \cdot 10^{-2}$, and therefore quite tolerable even if α were to be near to 1. In any case, for not too negligible values of α , we can attempt to determine it by measuring the $K^0 \bar{K}^0$ decays with both decay paths larger than d . Let $D^+(X_1, X_2)$ and $D^-(X_1, X_2)$ be the fraction of events falling in this region and coming

from the decay in the $|X_1, X_2\rangle$ final state of a $(K^0 \bar{K}^0, C = +)$ or $(K^0 \bar{K}^0, C = -)$ pair, respectively. The values of D^+ and D^- for different final states and for $d = 10 d_S$ or $d = 20 d_S$ are reported in Table III. If $d = 20 d_S$, essentially all the events in the most external regions of the detector come from the C-even component of the initial state and their number is

$$N_D(X_1, X_2) \simeq N^+ D^+(X_1, X_2) = \alpha N^- D^+(X_1, X_2) \quad , \quad (19)$$

where N^- is the total number of $(K^0 \bar{K}^0, C = -)$ pairs produced. Clearly the K_L allowed decay channels are the most suitable for the determination of α .

We come now to discuss the minimum number of ϕ (N_ϕ) to be produced in order to observe ϵ'/ϵ to n standard deviations accuracy when we consider only the events selected in the regions A and B. For a small asymmetry ($A \ll 1$),

$$N_\phi = \frac{1}{\text{BR}[\phi \rightarrow (K^0 \bar{K}^0)] \cdot \text{BR}[(K^0 \bar{K}^0) \rightarrow \pi^+ \pi^- + \pi^0 \pi^0]} \cdot \frac{2}{S} \cdot \left(\frac{\pi^2}{2A^2} \right) \quad , \quad (20)$$

where $S/2$ represents the fraction that we observe of the produced events,

$$\begin{aligned} \text{BR}[(K^0 \bar{K}^0)_{C=-} \rightarrow \pi^+ \pi^- + \pi^0 \pi^0] = & \text{BR}(K_S \rightarrow \pi^+ \pi^-) \text{BR}(K_L \rightarrow \pi^0 \pi^0) \\ & + \text{BR}(K_S \rightarrow \pi^0 \pi^0) \text{BR}(K_L \rightarrow \pi^+ \pi^-) \quad , \end{aligned} \quad (21)$$

and $\text{BR}(\phi \rightarrow K^0 \bar{K}^0) = (34.4 \pm 0.9)\%$ [12]. In Table II one can find N_ϕ for different choices of d_0 , d and D and for $\text{Re}(\epsilon'/\epsilon) = 10^{-3}$, when a sensitivity to the asymmetry of three standard deviations is required. It is seen that N_ϕ grows strongly for $d_0 \neq 0$ so that the maximum effort must be made to reduce d_0 . We note that one does not need the precise position of the vertex but only to know if it is inside or outside a circle of radius d : for this reason we think that $d_0 \sim 0$ is not an unrealistic hope.

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Table I. The time-flight integrated factors which parametrize the coherent decay rate of a $(C^-) K_L K_S$ pair into a $\pi^+ \pi^-$, $\pi^+ \pi^0$ state (see eq. (8)), for different values of the cut d and of the minimum (d_0) and the maximum (D) path inside the detector. As units the mean decay paths of K_S (d , ≈ 0.6 cm) and K_L ($d_L \approx 342$ cm) are used.

d_0/d_S	d/d_S	D/d_L	S_1	S_2	S_3	S_4
0	10	$\frac{1}{2}$	$7.76 \cdot 10^{-7}$	0.38	$-2.60 \cdot 10^{-6}$	$-4.80 \cdot 10^{-5}$
0	10	1	$7.76 \cdot 10^{-7}$	0.62	$-2.60 \cdot 10^{-6}$	$-4.80 \cdot 10^{-5}$
0	20	$\frac{1}{2}$	$6.99 \cdot 10^{-11}$	0.36	$3.19 \cdot 10^{-7}$	$-3.90 \cdot 10^{-8}$
0	20	1	$6.99 \cdot 10^{-11}$	0.60	$3.19 \cdot 10^{-7}$	$-3.90 \cdot 10^{-8}$
2	10	$\frac{1}{2}$	$6.20 \cdot 10^{-7}$	$5.09 \cdot 10^{-2}$	$1.41 \cdot 10^{-5}$	$-1.11 \cdot 10^{-5}$
2	10	1	$6.20 \cdot 10^{-7}$	$8.32 \cdot 10^{-2}$	$1.41 \cdot 10^{-5}$	$-1.11 \cdot 10^{-5}$
2	20	$\frac{1}{2}$	$6.28 \cdot 10^{-11}$	$4.87 \cdot 10^{-2}$	$7.94 \cdot 10^{-8}$	$8.74 \cdot 10^{-8}$
2	20	1	$6.28 \cdot 10^{-11}$	$8.10 \cdot 10^{-2}$	$7.94 \cdot 10^{-8}$	$8.74 \cdot 10^{-8}$

Table II. In terms of the same distances used in Table I, we report here the signal S , the background B , the C-odd and C-even events falling in the cut region and finally the number of ϕ (N_ϕ) required to observe an asymmetry A with a sensitivity of 3 standard deviations. N_ϕ is estimated assuming $\text{Re}(\epsilon'/\epsilon) = 10^{-3}$; for different values one has to divide by the square of $\text{Re}(\epsilon'/\epsilon)$ in units of 10^{-3} .

d_0/d_S	d/d_S	D/d_L	S	B	C^-	C^+	N_ϕ
0	10	$\frac{1}{3}$	0.38	$1.51 \cdot 10^{-3}$	$1.35 \cdot 10^{-3}$	$1.66 \cdot 10^2$	$5.88 \cdot 10^9$
0	10	1	0.62	$1.51 \cdot 10^{-2}$	$1.35 \cdot 10^{-2}$	$1.66 \cdot 10^2$	$3.60 \cdot 10^9$
0	20	$\frac{1}{3}$	0.36	$3.74 \cdot 10^{-5}$	$3.03 \cdot 10^{-3}$	$1.66 \cdot 10^2$	$6.15 \cdot 10^9$
0	20	1	0.60	$6.18 \cdot 10^{-5}$	$3.03 \cdot 10^{-3}$	$1.66 \cdot 10^2$	$3.70 \cdot 10^9$
2	10	$\frac{1}{3}$	$5.09 \cdot 10^{-2}$	$2.05 \cdot 10^{-3}$	$1.35 \cdot 10^{-3}$	3.04	$4.35 \cdot 10^{10}$
2	10	1	$8.32 \cdot 10^{-2}$	$2.06 \cdot 10^{-3}$	$1.35 \cdot 10^{-3}$	3.04	$2.66 \cdot 10^{10}$
2	20	$\frac{1}{3}$	$4.87 \cdot 10^{-2}$	$3.31 \cdot 10^{-5}$	$3.64 \cdot 10^{-3}$	3.04	$4.55 \cdot 10^{10}$
2	20	1	$8.10 \cdot 10^{-2}$	$5.50 \cdot 10^{-5}$	$3.64 \cdot 10^{-3}$	3.04	$2.73 \cdot 10^{10}$

Table III. Number of events D^- and D^+ coming from a ($C = -1$) and a ($C = +1$) state respectively, and falling in the most external region of the detector ($d < d_1 < d_2 < D$), reported for the different final states and two values of the cut d in units of the mean decay path of K_S . The limits for the three π 's decays are obtained by using the upper values of the $K_S \rightarrow 3\pi$ branching ratios given in ref. [12].

final states	D^-		D^+	
	$d = 10 d_S$	$d = 20 d_S$	$d = 10 d_S$	$d = 20 d_S$
$\pi^+ \pi^- \pi^0 \pi^0$	$2.1 \cdot 10^{-6}$	$9.2 \cdot 10^{-13}$	$2.3 \cdot 10^{-7}$	$2.1 \cdot 10^{-7}$
$\pi^+ \pi^- \pi^+ \pi^-$	$4.7 \cdot 10^{-6}$	$2.1 \cdot 10^{-12}$	$5.2 \cdot 10^{-7}$	$4.7 \cdot 10^{-7}$
$\pi^+ \pi^- \pi^0 \pi^0$	$2.4 \cdot 10^{-6}$	$1.0 \cdot 10^{-12}$	$3.1 \cdot 10^{-6}$	$2.9 \cdot 10^{-5}$
$\pi^+ \pi^- \pi^0 \pi^0$	$2.1 \cdot 10^{-12}$	$9.1 \cdot 10^{-17}$	$1.9 \cdot 10^{-3}$	$1.7 \cdot 10^{-3}$

Fig. 1 - Normalized distribution of the events C-odd as a function of the decay distance d of the "second" K . The distance d is expressed in units of the mean decay path d_S of K_S . The small arrow indicates $\frac{1}{2}$ of the mean decay path d_L of K_L .

Fig. 2 - Signal S , lost events C^- and background B in arbitrary units (see eqs. (12), (14) and (15), respectively) vs. the distance of the cut d in units of the mean decay path d_S of K_S .