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A UNIFIED THEORY IN HIGHER DIMENSIONS

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Abstract

We present a grand unified model defined in ten dimensions and based on the group $SO(13)$. The model is dimensionally reduced over the non-simply-connected space $(SU(3)/U(1) \times U(1))/Z_2$ giving in four dimensions the standard model.

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Introduction. The unification of all the known interactions has been considered for many years to be one of the major goals of research in elementary particle physics. Despite the qualitative and, to a certain level, quantitative successes of the GUT's [1], Kaluza-Klein theories [2] and superstrings [3], a totally realistic unified theory does not exist. Therefore new ideas or examination of old ideas from a new point of view are always welcome. Here we shall consider a four-dimensional gauge theory obtained through dimensional reduction of another gauge theory originally defined in ten dimensions.

The introduction of gauge theories in higher dimensions was proposed as a solution to some of the problems of the Kaluza-Klein theories [4]. It is very interesting that the introduction of gauge fields in higher dimensions can be considered now not as the price that one has to pay in order to make the Kaluza-Klein program viable, but as a virtue, since they can eventually describe the low energy interactions, as we shall see. We should also recall that the introduction of gauge theories in higher dimensions is very well motivated also in superstring theories.

Given a gauge theory in higher dimensions, an attractive procedure has developed that permits the detailed study of the resulting theory in four dimensions after dimensional reduction. The scheme is called the Coset-Space-Dimensional-Reduction (CSDR) scheme and its essential points are reviewed in the following.

CSDR scheme.[5-12] Consider in D-dimensions a Yang-Mills-Dirac theory with gauge group G defined on a manifold M^D which is the direct product of a four-dimensional spacetime M^4 and a compact coset space S/R, $D=4+d$ where $d=\dim(S/R)=\dim S-\dim R$. The action in D dimensions is:

$$A = \int d^4x d^d y \sqrt{-g} \left\{ -\frac{1}{4} \text{Tr}(F_{MN} F_{KL}) g^{MK} g^{NL} + \frac{1}{2} \bar{\psi} \Gamma^M D_M \psi \right\} \quad (1)$$

where we use Greek letters to denote curved indices (while Latin letters later on will denote flat indices); K, L, M, N run over the D-dimensional space, μ, ν, λ , over M^4 and α, β, γ , over S/R.

The fields A_M, ψ are assumed to be symmetric in the sense that any transformation under symmetries of S/R (which form the isometry group S of S/R) is compensated by gauge transformations. The ψ can be in any representation F of G unless a further symmetry such as supersymmetry is required. So if $g(s), f(s)$ are gauge transformations in the adjoint and F irreps of G corresponding to the transformations acting on S/R, we require:

$$A_\mu(x, y) = g(s) A_\mu(x, s^{-1}y) g^{-1}(s) \quad (2)$$

$$A_\alpha(x, y) = g(s) \sum_\beta A_\beta(x, s^{-1}y) g^{-1}(s) + g(s) \partial_\alpha g^{-1}(s) \quad (3)$$

$$\psi(x, y) = f(s) \Omega \psi(x, s^{-1}y) f^{-1}(s), \quad (4)$$

where $\sum_\beta$ is the Jacobian matrix for the transformation s and Ω is the Jacobian matrix plus a local Lorentz rotation in the tangent space which is needed for the fermions when they transform in a curved space.

These conditions imply certain constraints that the D-dimensional fields have to obey. The solution of these constraints will provide us with the four-dimensional unconstrained fields as well as with the gauge invariance that remains in the theory after dimensional reduction.

From eq.(2) it follows that the components $A_\mu(x, y)$ of the initial gauge field $A_M(x, y)$ become, after dimensional reduction, the four-dimensional gauge fields and furthermore they are independent of y. In addition one can find that they have to commute with the elements of the R subgroup of G. Thus the four-dimensional gauge group H is the centralizer of R in G.

Similarly, from eq.(3) the $A_\alpha(x, y)$ components of $A_M(x, y)$, denoted by Φ_α from hereon, become scalar fields at four dimensions. These fields transform under R as a vector v specified by the embedding of $R \subset S$

$$\text{adj}(S) = \text{adj}(R) + v \quad (5)$$

Moreover $\Phi_a(x,y)$ act as intertwining operators connecting induced representations of R acting on G and S/R . This implies, exploiting Schur's lemma that the transformation properties of the fields $\Phi_a(x,y)$ under H can be found if we express the adjoint irrep. of G in terms of reps of (R,H) ,

$$\text{adj}(G) = \sum (r_k, h_k) \quad (6)$$

Then if $v = \sum s_i$, where each s_i is an irrep of R , there is an h_k scalar multiplet for every pair (r_k, s_k) where r_k and s_k are identical irreps.

Turning now to the fermion fields [6, 7] we see from eq.(4) that they act as intertwining operators between induced representations of R acting on G and the tangent space of S/R , $SO(d)$. The non-trivial solution of the constraints can be obtained if we proceed along the same lines as in the case of scalars. Applying Schur's lemma in this case, one has to decompose F , the representation of G in which the fermions are assigned, under $R \times H$

$$F = \sum (r_k, h_k), \quad (7)$$

then decompose the spinorial representations of $SO(d)$ under R

$$\sigma_d = \sum \sigma_k. \quad (8)$$

It turns out that for every pair (r_k, σ_k) where r_k and σ_k are identical irreps there is an h_k multiplet of spinor fields in four-dimensional theory. In order, however, to obtain chiral fermions in the effective theory we have to impose some further requirements. We first impose the Weyl condition in D dimensions. In $D=2+4n$ dimensions, which is the case at hand, the decomposition of the left-handed, say, spinor under $SO(4) \times SO(d)$ is $\sigma_D = (2, 1, \sigma_D) + (1, 2, \bar{\sigma}_D)$. So we have in this case the decompositions $\sigma_d = \sum \sigma_k$ and $\bar{\sigma}_d = \sum \bar{\sigma}_k$. Therefore starting

from complex Weyl fermions in higher dimensions we obtain chiral fermions in four dimensions. Moreover we can even start from a vectorlike representation F and obtain chiral fermions. In that case each term (r_k, h_k) in the decomposition of eq.(7) will be either self-conjugate or it will have a partner $(\bar{r}_k, \bar{h}_k)$. According to the rule described in eqs.(7), (8), if we consider σ_d , the four-dimensional left-handed representation $f_L = \sum h_k^{(L)}$ is not vectorlike since $\sum \sigma_j$ is not self-conjugate. In addition, from $\bar{\sigma}_d$, we have in four dimensions the right-handed representation $f_R = \sum \bar{h}_k^{(R)} = \sum h_k^{(L)}$. Therefore there will appear two sets of Weyl fermions with the same quantum numbers under H . These two sets of Weyl spinors can be identified if we further impose the Majorana condition. Note that in $D=2+4n$ the Majorana condition is diagonal with respect to the Weyl one so there is no problem in imposing both at the same time. Furthermore if F is to be real then we have to have $D=2+8n$, while for F pseudoreal $D=6+8n$.

Next let us obtain the four-dimensional effective action. Assuming that the metric is block diagonal, taking into account all the constraints and integrating out the extra coordinates we obtain in four dimensions the following Lagrangian:

$$A = C \int d^4x \left\{ -\frac{1}{4} \text{Tr}(F_{\mu\nu} F^{\mu\nu}) + \frac{1}{2} \text{Tr}(D_\mu \Phi_a D^\mu \Phi_a) - V(\Phi) \right\}$$

$$+ C \int d^4x \frac{1}{2} \{ \bar{\psi} \Gamma^\mu D_\mu \psi + \bar{\psi} \Gamma^a D_a \psi \} \quad (9)$$

where C is the volume of the coset space and the potential $V(\Phi)$ is given by the formula

$$V(\Phi) = \frac{1}{4} g^{ac} g^{bd} \text{Tr} \{ (f_{ab}^D \Phi_D - [\Phi_a, \Phi_b]) (f_{cd}^E \Phi_E - [\Phi_c, \Phi_d]) \} \quad (10)$$

where the indices D and E run over S. Furthermore D_μ, D_a are the appropriate covariant derivatives, $D_\mu = \partial_\mu - iA_\mu$, $D_a = \partial_a - \frac{i}{4} \Theta_{abc} \Sigma^{bc} \Phi_a$, where Θ_{abc} is the connection of S/R.

The above expression for $V(\Phi)$ is only formal because Φ_a must satisfy the constraints coming from eq(3)

$$f_{a1}^D \Phi_D \cdot [\Phi_a, \Phi_1] = 0 \quad (11)$$

where the index i runs only in the R-subalgebra of S. These constraints imply that some components Φ_a 's are zero, some are constants and the rest can be identified with components of the genuine Higgs fields. When $V(\Phi)$ is expressed in terms of the unconstrained independent Higgs fields, it remains a quartic polynomial which is invariant under gauge transformations of the final gauge group H, and its minimum determines the vacuum expectation value of the Higgs fields.

The model. An interesting model that we would like to discuss is based on a Yang-Mills-Dirac theory with gauge group $G=SO(13)$ defined in ten dimensions. The theory is assumed to be dimensionally reduced over the coset space $S/R=SU(3)/U(1) \times U(1)$. According to the previous section, the four-dimensional gauge group H is the centralizer of R in G. Therefore we have to specify the embedding of $R=U(1) \times U(1)$ into $SO(13)$. Our choice for this embedding is specified by the following decompositions.

$$SO(13) \supset [SU(2)] \times [SO(10)] \supset [U(1)] \times [SU(5) \times U(1)] \quad (12)$$

i.e. one of the $U(1)$'s in R is the maximal subgroup of $SU(2)$ and the second appears in the maximal subgroup $SU(5) \times U(1)$ of $SO(10)$. Therefore the four-dimensional gauge group is $H = C_{SO(13)}(U(1) \times U(1)) = SU(5)$. In order to find the surviving scalar fields we have to determine the branching rule of $SO(13)$ under $SU(5) \times [U(1) \times U(1)]_R$

$$SO(13) \supset SU(5) \times [U(1) \times U(1)]_R \quad (13)$$

$$78 = 1(0,0) + 1(0,0) + 24(0,0) + 1(6,0) + 1(-6,0) + 10(0,4) + \bar{10}(0,-4) + 5(6,2) + \bar{5}(-6,-2) + 5(-6,2) + \bar{5}(6,-2) + 5(0,2) + \bar{5}(0,-2)$$

The corresponding decomposition of the adjoint of $S=SU(3)$ under $R=U(1) \times U(1)$ is:

$$SU(3) \supset U(1) \times U(1) \quad (14)$$

$$8 = (0,0) + (0,0) + (3,1) + (-3,1) + (-3,-1) + (3,-1) + (0,2) + (0,-2)$$

Therefore according to the rules given in the previous section the surviving scalar fields in the four-dimensional theory transform under $H=SU(5)$ as a 5-plet β_1 .

In order to provide the four-dimensional gauge theory with the appropriate fermionic content we introduce in the ten-dimensional Lagrangian a Dirac part with fermion multiplets transforming as 13, 64 and 78 representations under $G=SO(13)$. Then according to the rules of the previous section, the surviving fermions in four dimensions are determined from the comparison of the decompositions of these representations under $SU(5) \times [U(1) \times U(1)]_R$ given by:

$$SO(13) \supset SU(5) \times [U(1) \times U(1)]_R \quad (15)$$

$$13 = 1(6,0) + 1(0,0) + 1(-6,0) + 5(0,2) + \bar{5}(0,-2)$$

$$64 = 1(\pm 3, -5) + \bar{5}(\pm 3, 3) + \bar{10}(\pm 3, -1) + 1(\pm 3, 5) + 5(\pm 3, -3) + 10(\pm 3, 1)$$

and 78 given by eq.(13), with the decomposition of the spinor of $SO(6)$ under $R=U(1) \times U(1)$:

$$SO(6) \supset [U(1) \times U(1)]_R \quad (16)$$

$$4 = (0,0) + (3,1) + (-3,1) + (0,-2)$$

Starting with Weyl-Majorana spinors in ten dimensions the surviving fermions transform under $SU(5) \times [U(1) \times U(1)]_R$ as $2 \times 5(0, -2)$; $2 \times 10(-3, 1)$. In addition in four dimensions survive fermions transforming according to the representations $1(0, 0)$; $24(0, 0)$, which however obtain superheavy geometrical masses [13]. Therefore we obtain effectively an ordinary $SU(5)$ GUT model with two fermion families and a Higgs multiplet appropriate for electroweak symmetry breaking. The only ingredient which is missing as compared to the usual Georgi-Glashow $SU(5)$ model is the 24 Higgs multiplet which could break the $SU(5)$ down to the standard model. However this can be substituted by the Wilson flux mechanism [14-17] which moreover solves the hierarchy problem of ordinary GUTs.

In order to implement the model with the flux breaking mechanism, we should substitute the simply connected coset space $B_0 = SU(3)/U(1) \times U(1)$ under which the theory is reduced with the $B = B_0/F^{B_0}$, where F^{B_0} is a freely acting discrete group on B_0 . The space B is non-simply connected since $\pi_1(B_0) = F^{B_0}$. This means that there will be contours not contractible to a point due to holes in the manifold. It turns out that the surviving fields have to be invariant under the diagonal sum $F^{B_0} \oplus T^G$, where T^G is the subgroup of G which is generated by the Wilson loops [14-17]. In addition the final unbroken gauge group is the centralizer of T^G in G . Fortunately all the freely acting discrete symmetries on all the six dimensional coset spaces S/R with $\text{rank } S = \text{rank } R$ have been classified [16]. We refer to ref(16) for the details of the transformation properties of the surviving Higgs and fermions. Here we recall the action of the following $Z_2^{B_0}$

$$Z_2^{B_0} = \begin{cases} (-3, 1) \leftrightarrow (3, -1) \\ (-0, 2) \leftrightarrow (3, 1) \\ (-3, -1) \leftrightarrow (0, -2) \end{cases} \quad (17)$$

Here we use the above $Z_2^{B_0}$ which we choose to embed in the $Z_2^{SU(5)}$ subgroup of the hypercharge in the decomposition $SU(5) \supset SU(3)_c \times SU(2)_L \times U(1)_Y$ and in such a way that all components of the fundamental representation transform evenly under $Z_2^{SU(5)}$. In this way we have achieved

(a) Spontaneous symmetry breaking of the $SU(5)$ down to the standard model using the topology of the coset space instead of the usual 24-plet Higgs whose mixing with the 5-plet Higgs is responsible for the hierarchy problem in GUTs. Here we avoid the hierarchy problem to the extent that the existing 5-plet Higgs in the theory remains light.

(b) The 5-plet Higgs under $SU(5)$, being an eigenstate of $Z_2^{B_0}$, remains invariant under the action of $Z_2^{B_0 \oplus SU(5)}$ and therefore survives in the theory.

(c) Similarly the fermions $\bar{5}$, 10 under $SU(5)$ are invariant under the action of $Z_2^{B_0 \oplus SU(5)}$ and survive too.

Therefore we have obtained in four dimensions the standard model $SU(3)_c \times SU(2)_L \times U(1)_Y$ with the usual fermionic content, since the fermions result from the $\bar{5}$ and 10 of $SU(5)$. Finally there exists a 5-plet Higgs β_1 at the level of $SU(5)$ and our next task is to construct and examine its potential.

In order to construct the unconstrained scalar fields we have to introduce the $G=SO(13)$ and $S=SU(3)$ generators according to the decompositions under $R=U(1) \times U(1)$. In correspondence with the decomposition given in eq.(14) we introduce the generators:

$$Q_{SU(3)} = (Q_0, Q'_0, Q_1, Q_2, Q^1, Q^2, Q_3, Q^3) \quad (18)$$

Similarly corresponding to the decomposition of eq.(13), we introduce the generators

$$T_{SO(13)} = (T_0, T_+, T_-, T_1, T_2, T_3, T_4, T_5, T_6, T_7, T_8, T_9, T_{10}, T_{11}, T_{12}, T_{13}, T_{14}, T_{15}, T_{16}, T_{17}, T_{18}, T_{19}) \quad (19)$$

where $i, j = 1, \dots, 5$ and $a = 1, \dots, 24$. The commutation relations of $Q_{SU(3)}$'s and $T_{SO(13)}$'s are given in tables I and II. Then in order to express the formal Higgs potential of eq.(10) in terms of the genuine Higgs fields β_i we need the explicit expressions for the scalar fields Φ_S

$$\Phi_S = (\Phi_0, \Phi'_0, \Phi_1, \Phi_2, \Phi^1, \Phi^2, \Phi_3, \Phi^3) \quad (20)$$

The solution of the constraints given in eq.(11) in terms of the genuine field β_i is:

$$\Phi_0 = T_0, \Phi'_0 = 2T'_0, \Phi_1 = 0, \Phi_2 = 0, \Phi^1 = 0, \Phi^2 = 0, \Phi_3 = \beta_1, \Phi^3 = \beta_1 T_1 \quad (21)$$

A final ingredient needed in the construction of the Higgs potential is to specify the metric that we use. The metric of the coset space is not necessarily the usual Cartan-Killing metric of S restricted on S/R . The most general, S -invariant metric on S/R satisfies

$$f_{ab}^b g_{db} + f_{id}^b g_{ab} = 0, \quad (22)$$

where $i = 1, \dots, \dim R$, $a = \dim R + 1, \dots, \dim S$. This condition allows the introduction of different scales (radii) for each R -irreducible representation of the coset.

Now we find that the potential taking into account the solution of the constraints and the possibility to introduce three radii takes the form:

$$V(\Phi, R_1, R_2, R_3) = - \left(\frac{1}{2 R_1^4} + \frac{1}{2 R_2^4} \right) \text{Tr}(3\Phi_0^2 + \Phi_0'^2) - \frac{2}{R_1^2 R_2^2} \text{Tr}(\Phi_3 \Phi^3) - \frac{1}{2 R_3^4} \text{Tr}((2\Phi_0' + [\Phi_3, \Phi^3])^2) \quad (23)$$

Expressing the Φ in terms of the genuine Higgs fields β_i according to eq.(21), we find the following potential in four dimensions:

$$V_{4\text{-dim}} = \frac{1}{g^2} V(\beta, R_1, R_2, R_3) = \frac{7}{g^2} \left(\frac{1}{R_1^4} + \frac{1}{R_2^4} \right) + \frac{16}{g^2 R_3^4} + \frac{4}{g^2 R_1^2 R_2^2} \left(\beta_1 \beta_1^1 \right) + \frac{5}{g^2 R_3^4} (\beta_1 \beta_1^1)^2 \quad (24)$$

From the above potential we conclude that the electroweak symmetry breaking of the theory takes place, even if we do not use the freedom to introduce more than one radius in the coset space $SU(3)/U(1) \times U(1)$, i.e. $R_1 = R_2 = R_3$. However in that case the order parameter of the electroweak symmetry breaking would be comparable to the dimensional reduction scale and the order parameter introduced by the flux breaking mechanism, which is not phenomenologically acceptable. Therefore we are led to choose the relation $R_3^2 = \sqrt{2} R_1 R_2$ between the radii, which obviously introduces at the classical level a vanishing order parameter in addition to the dimensional reduction scale. Then since the four-dimensional theory is considered renormalizable one can rely on the radiative corrections in order to achieve spontaneous symmetry breaking of the electroweak sector of the theory.

Therefore, starting from a gauge theory with $G = SO(13)$ in ten dimensions, we have obtained in four dimensions the standard model via a geometrical breaking of $SO(13)$ down to $SU(5)$ which subsequently breaks down to $SU(3)_C \times SU(2)_L \times U(1)_Y$ due to Wilson flux mechanism. The final breaking of the theory down to $SU(3)_C \times U(1)_e$ is due to Higgs fields of geometrical origin.

A final question concerns proton decay. Indeed protons should decay as fast as in the usual SU(5) GUT unless a mechanism is employed which forbids or suppresses the decay. Unfortunately we did not find an attractive way to realize this.

In conclusion we have presented a realistic grand unified model defined in ten dimensions and based on the gauge group SO(13). Dimensional reduction of the theory over the coset space $B_0 = \text{SU}(3)/\text{U}(1) \times \text{U}(1)$ provides us with the gauge group $H = \text{SU}(5)$. Substitution of the coset space by $B = B_0 / Z_2^{B_0}$ can break the SU(5) gauge group down topologically to the standard model $\text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y$ at the dimensional reduction scale. The final step of spontaneous symmetry breaking is due to Higgs fields which appear in the four-dimensional theory via the dimensional reduction procedure. The order parameter of the last breaking can be naturally small at the cost of a fine tuning of the coset space radii. Then two naturally distinct scales emerge in the theory and the hierarchy problem of usual GUTs is avoided.

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Table I: SU(3) commutation relations under the decomposition $\text{SU}(3) \supset \text{U}(1) \times \text{U}(1)$

$[Q_1, Q_0] = \sqrt{3}Q_1$	$[Q_1, Q'] = -\sqrt{3}Q_0 - Q'_0$
$[Q_1, Q'_0] = Q_1$	$[Q_2, Q^2] = \sqrt{3}Q_0 - Q'_0$
$[Q_2, Q_0] = -\sqrt{3}Q_2$	$[Q_3, Q^3] = 2Q'_0$
$[Q_2, Q'_0] = Q_2$	$[Q_1, Q_2] = \sqrt{2}Q_3$
$[Q_3, Q_0] = 0$	$[Q_2, Q^3] = \sqrt{2}Q^1$
$[Q_3, Q'_0] = 2Q_3$	$[Q^1, Q_3] = \sqrt{2}Q_2$

Table II: SO(13) commutation relations under the decomposition

SO(13) $\supset$ SU(5) $\times$ U(1) $\times$ U(1)

$[T_+, T_0] = \sqrt{5} \bar{T}_+$	$[T_+, T_0] = 0$	$[T_+^i, T_0] = 0$
$[T_+^i, T_0] = 2\sqrt{5} \bar{T}_+^i$	$[\tilde{T}_+^i, T_0] = \sqrt{5} \tilde{T}_+^i$	$[\tilde{T}_+^i, T_0] = \tilde{T}_+^i$
$[\bar{T}_+^i, T_0] = -\sqrt{5} \bar{T}_+^i$	$[\bar{T}_+^i, T_0] = \bar{T}_+^i$	$[T_+^i, T_0] = 0$
$[T_+^i, T_0] = \bar{T}_+^i$	$[T_+^a, T_+] = 0$	$[T_+, \tilde{T}_+] = \sqrt{5} \bar{T}_+$
$[T_+^a, \bar{T}_+] = \sqrt{\frac{5}{2}} \{ (T^a)_k^i T^{ki} - (T^a)_j^i T^{kj} \}$	$[T_+^a, \tilde{T}_+] = \sqrt{\frac{5}{2}} (T^a)_j^i \tilde{T}_+^j$	$[T_+^a, \tilde{T}_+] = \sqrt{\frac{5}{2}} (T^a)_j^i \tilde{T}_+^j$
$[T_+^i, T_{kl}] = -2\sqrt{5} \delta_{kl}^i T_+^i + \sqrt{\frac{5}{2}} (\delta T^a)_k^i T_+^a$		
$[T_+, T_-] = -\sqrt{5} T_0$	$[T_+, T_+] = -\sqrt{5} \tilde{T}_+$	$[T_+^i, \tilde{T}_+] = \sqrt{5} \delta_{kl}^i \tilde{T}_+^i$
$[T_+^i, \bar{T}_k] = \sqrt{5} \delta_{kl}^i \tilde{T}_+^i$	$[\tilde{T}_+^i, \bar{T}_+] = -\sqrt{5} \bar{T}_+^i$	$[\tilde{T}_+^i, T_+] = -\sqrt{5} \delta_{kl}^i \bar{T}_+^i$
$[\bar{T}_+^i, T_+] = \sqrt{5} \delta_{kl}^i \bar{T}_+^i$	$[T_+, \bar{T}_+] = -\sqrt{5} \bar{T}_+^i$	$[T_+, T_+] = \sqrt{5} \bar{T}_+$
$[\tilde{T}_+^i, \tilde{T}_+] = -\sqrt{5} \delta_{kl}^i T_+^i + \sqrt{\frac{5}{2}} (T^a)_j^i T_+^a$		
$[\tilde{T}_+^i, \bar{T}_+] = \sqrt{5} \delta_{kl}^i T_+^i - \delta_{kl}^i T_+^i - \sqrt{\frac{5}{2}} (T^a)_j^i T_+^a$		
$[T_+^i, T_+] = -\delta_{kl}^i T_+^i + \sqrt{\frac{5}{2}} (T^a)_j^i T_+^a$		

where $(\delta T^a)_{kl}^i = (T^a)_k^i \delta_l^j + (T^a)_j^i \delta_k^l - (T^a)_i^j \delta_k^l - (T^a)_k^i \delta_l^j$ and $\delta_{kl}^i = \delta_k^i \delta_l^j - \delta_k^j \delta_l^i$

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