ELECTRON COOLING: THEORY, EXPERIMENT, APPLICATION

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ABSTRACT

As a consequence of Liouville's theorem, the momentum spread and the emittance in charged-particle beams cannot be reduced by means of ion optics. This limits the number of ions that can be accelerated or decelerated in a circular machine and, in turn, the intensity and luminosity available for experiments. Electron cooling can overcome this obstacle. The mechanism underlying electron cooling is equivalent to that of temperature relaxation in a plasma consisting of a hot and a cold component. Initially electron cooling was proposed for the accumulation of antiprotons, but today its application is found mainly in the improvement of light- and heavy-ion beams to be used for precision experiments in atomic and nuclear physics.

Electron cooling reduces the spread in the longitudinal and transverse ion velocities, which means a decrease in the momentum spread, in the diameter, and in the divergence of the ion beam. The time that elapses until equilibrium is reached is usually of the order of seconds. Under favourable conditions, thermal equilibrium between electrons and ions can be obtained, yielding very high phase-space densities. Consequently, more intense and brilliant beams of a wide range of ions can be delivered to atomic, nuclear, and intermediate-energy physics experiments. The cooling beam provides also a clean, free-electron target, and this is especially interesting for atomic physics.

The understanding of electron cooling—both theoretically and experimentally—has now reached such a level that there are only a few questions still to be answered and a few doubts to be removed. This article attempts to give a comprehensive coverage of the subject and summarizes the present knowledge. Possible future developments and refinements of the method are discussed, as well as the application of the merged parallel-beam arrangement for atomic physics.

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CONTENTS

FREQUENTLY USED SYMBOLS	vii
1. INTRODUCTION	1
2. INTRODUCTION TO ELECTRON COOLING IN STORAGE RINGS	2
2.1 A simplified picture of the operation principle	2
2.2 Motivation for electron cooling	3
2.3 The plasma physics aspect of charged-particle beams and electron cooling	4
3. THE COOLING THEORY	8
3.1 A simple derivation of the cooling force and the diffusion tensor	8
3.2 The impact parameter regions relevant to electron cooling	14
3.3 The binary-collision model of Novosibirsk	16
3.4 The dielectric plasma description of electron cooling	18
3.5 Difference in cooling of positive and negative ions by magnetized electrons	21
3.6 Summary of the theory	22
4. EVOLUTION OF THE DISTRIBUTION FUNCTION, THE COOLING TIME AND DECREMENT, AND THE THERMAL EQUILIBRIUM	24
5. OTHER ELECTRON-ION INTERACTIONS	27
5.1 Spontaneous recombination	27
5.2 Dielectronic recombination	29
5.3 Ion excitation	29
5.4 Ionization	29
5.5 Three-body recombinations	30
6. ELECTRON COOLER DESIGN	30
6.1 The electron gun	30
6.1.1 Electron sources	30
6.1.2 Electron acceleration	31
6.2 Electron beam transport	34
6.3 Interaction or cooling region	36
6.4 The electron collector	36
6.5 The electric circuitry	38
6.5.1 High-voltage system	38
6.5.2 Cathode heating	39
6.5.3 Control system	39
6.6 Vacuum system	40
6.7 The magnetic field	41
6.8 Electron beam diagnostics	41
7. RELEVANT ION BEAM PROPERTIES	43
7.1 The reference system	43
7.2 The ion motion in a circular machine	43
7.3 Working point, tune shifts, and resonances	47
7.4 Longitudinal phase space	47
7.5 Instabilities	49
7.6 Intrabeam scattering	50
7.7 Residual gas scattering	51

7.8	Beam lifetime	53
7.9	Vacuum and gauge readings	54
7.10	Emitance and momentum spread scaling with energy	54
7.11	Collective effects and ordering	54
7.12	Spin motion	55
7.13	Beam diagnostics	56
8.	THE EFFECTS OF THE ELECTRON COOLER ON THE ION BEAM	59
8.1	Machine optics effects of the electron cooler	59
8.2	The actions of the frictional force in the cooling section	61
9.	COMPUTER SIMULATION OF ELECTRON COOLING, AND PRACTICAL ESTIMATES	62
9.1	Simulation programs	62
9.2	Transformation to the laboratory system	64
9.3	Thermal equilibrium	64
9.4	Practical formulae	65
9.4.1	Transverse equilibrium between multiple scattering and electron cooling	65
9.4.2	Energy loss	65
9.4.3	Intrabeam scattering	66
9.4.4	Down-charge rate	66
9.5	Optimal lattice functions for electron cooling	67
9.5.1	Dispersion in the cooling region	67
9.5.2	β -functions in the cooling region, and the size of the electron beam	67
9.5.3	Length of the cooling section	68
10.	ELECTRON COOLING EXPERIMENTS	68
10.1	The first electron cooling at Novosibirsk, CERN, and FNAL	68
10.2	The second generation of electron cooling: the LEAR electron cooler	68
10.3	The linear arrangement MOSOL at Novosibirsk	69
10.4	The new electron coolers for light- and heavy-ion rings	69
10.5	Observables and measuring techniques	69
10.5.1	Coasting beam	69
10.5.2	Bunched beam	71
10.6	Results	72
10.6.1	Transverse cooling (cooling of betatron oscillations)	72
10.6.2	Longitudinal cooling (cooling of momentum spread) and frictional force measurements	73
10.6.3	Equilibrium properties	74
10.6.4	Beam lifetime	75
10.6.5	Cooling of bunched beams, and accumulation	75
10.6.6	Instabilities	76
10.6.7	Comparison of the frictional force exerted on positive and negative ions with MOSOL	76
10.7	Discussion of results	77
11.	CONCEIVABLE FUTURE DEVELOPMENTS OF ELECTRON COOLING	78
11.1	New cooling-ring projects	78
11.2	Linear cooler	78
11.3	Circular cooler	79

11.4 Cooling with MeV electrons	80
11.5 Cooling with sub-keV electrons	80
11.5.1 Electron cooling in small storage rings	80
11.5.2 Electron cooling in traps	81
11.6 Subthermic cooling beams	82
11.7 New techniques	82
11.8 Polarization of an ion beam through electron cooling	83
12. APPLICATIONS OF ELECTRON COOLING IN STORAGE DEVICES	84
12.1 Increase of the beam lifetime	84
12.2 Increase in luminosity	84
12.3 Increase of spatial and momentum resolution	85
12.4 Increase of the duty cycle	85
12.5 Accumulation of ions	85
12.6 Ion-beam deceleration with electrons	86
12.7 Provision for internal target operation	87
12.8 Beam extraction and diagnostics	87
12.9 Production of short bunches	88
12.10 Cooling of molecules	88
12.11 Axial loading of traps	88
12.12 Beam dynamics studies, ordering	89
12.13 Special techniques	89
12.14 Limits	90
12.15 Open questions	91
13. ATOMIC PHYSICS APPLICATIONS OF ELECTRON COOLING	91
13.1 Spontaneous radiative electron capture	91
13.2 Induced radiative electron capture	93
13.3 Antihydrogen production	94
13.4 Three-body recombination	94
13.5 Dielectronic recombination	94
14. SUMMARY	94
Acknowledgements	97
REFERENCES	98
APPENDIX I: VALUES AND PARAMETERS FOR THE STANDARD ELECTRON COOLING CASE	109
APPENDIX II: QUASIPOTENTIALS AND FRICTIONAL FORCE	112
APPENDIX III: FREQUENTLY USED RELATIONS	115
FIGURES	123

1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14. 15. 16. 17. 18. 19. 20. 21. 22. 23. 24. 25. 26. 27. 28. 29. 30. 31. 32. 33. 34. 35. 36. 37. 38. 39. 40. 41. 42. 43. 44. 45. 46. 47. 48. 49. 50. 51. 52. 53. 54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 66. 67. 68. 69. 70. 71. 72. 73. 74. 75. 76. 77. 78. 79. 80. 81. 82. 83. 84. 85. 86. 87. 88. 89. 90. 91. 92. 93. 94. 95. 96. 97. 98. 99. 100.

1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14. 15. 16. 17. 18. 19. 20. 21. 22. 23. 24. 25. 26. 27. 28. 29. 30. 31. 32. 33. 34. 35. 36. 37. 38. 39. 40. 41. 42. 43. 44. 45. 46. 47. 48. 49. 50. 51. 52. 53. 54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 66. 67. 68. 69. 70. 71. 72. 73. 74. 75. 76. 77. 78. 79. 80. 81. 82. 83. 84. 85. 86. 87. 88. 89. 90. 91. 92. 93. 94. 95. 96. 97. 98. 99. 100.

Frequently used symbols

A	atomic mass number
A_e	machine acceptance
A_b	bunch area
b	impact parameter
B	magnetic field
B_f	bunching factor
B_{sol}	magnetic field of cooling solenoid
B_{tor}	magnetic field of cooling toroid
c	velocity of light
C_0	nominal ring circumference
d	cathode–anode distance
$d_i(s)$	radial distance of ion from nominal orbit
$D(s)$	ion beam dispersion
D_{mn}	diffusion coefficient
e	elementary charge ($1.6 \times 10^{-19} \text{ A} \cdot \text{s}$)
E_a	binding energy of atomic ground state
E_{ax}	electron energy on the beam axis (laboratory system)
E_e	kinetic energy of electron
E_i	kinetic energy of ion
E_{str}	atomic binding energy, which corresponds to field ionization threshold (stripping field)
E_θ	energy of photons emitted under angle θ
f	distribution function
f_0	revolution frequency of ions on nominal orbit
f_{rev}	revolution frequency of ions in storage ring
f_{RF}	frequency of voltage applied to radio-frequency cavity
f_{syn}	frequency of synchrotron oscillations
F	frictional force
F_0	scaling constant for frictional force
g	gain factor for induced electron capture
h_f	harmonic number of revolution frequency
h	Planck's constant
$\hbar$	Planck's constant $\times 2\pi$ ($= 6.56 \times 10^{-16} \text{ eV} \cdot \text{s}$)
I_e	electron current
I_{loss}	electron loss current
j_e	electron current density
k	Boltzmann constant ($8.617 \times 10^{-5} \text{ eV/K}$)
$\mathbf{k}$	wave vector
ℓ	length of cooling section
ℓ_b	bunch length
L	luminosity
L_C	Coulomb logarithm
m_e	electron mass
m_i	ion mass
n	principal quantum number
n_e	electron number density
n_{gas}	number density of residual gas particles

n_i	ion number density
N_i	number of stored ions
P_{gas}	residual gas pressure
p	ion momentum
p_0	ion momentum on nominal orbit
P_0	scaling constant for perveance ($= 7.3 \times 10^{-6} \text{ A} \cdot \text{V}^{-3/2}$)
P_{coll}	perveance of electron collector
P_{gun}	perveance of electron gun
q	charge of plasma particle
q_y	non-integer fraction of betatron tune
Q_y	betatron tune for direction y
Q_v	velocity flattening coefficient
r	radial distance from beam axis
r_{av}	average particle distance
r_0	electron beam radius
r_{0i}	ion beam radius
r_c	gyration radius
r_e	classical electron radius ($2.818 \times 10^{-13} \text{ cm}$)
r_p	classical proton radius ($1.53 \times 10^{-16} \text{ cm}$)
r_i	classical ion radius ($= r_e m_e / m_i$)
r_{tube}	radius of vacuum chamber
R_a	rate of down-charged ions
R_0	effective radius of nominal ion orbit ($R_0 = C_0 / 2\pi$)
s	spatial coordinate (longitudinal direction)
t	time
t_{pl}	Langmuir period
t_{rev}	revolution time of ions in storage ring
T_{cath}	cathode temperature
T_e	electron temperature
T_i	ion temperature
u	electron-ion velocity difference
U_{cath}	cathode voltage
U_{coll}	electron collector voltage
U_{RF}	voltage applied to radio-frequency cavity
v_{e0}	nominal electron velocity (in laboratory frame)
v_i	ion velocity
x	spatial coordinate (horizontal direction)
y	spatial coordinate (any transverse direction)
z	spatial coordinate (vertical direction)
Z	charge of ion (nuclear charge minus number of attached electrons)
$Z_{\parallel h}$	longitudinal coupling impedance
Z_{gas}	nuclear charge of residual gas
α	fine structure constant
α_p	momentum compaction factor
α_0	quantity that accounts for magnetic field ripple in cooler
α_r	recombination coefficient
$\alpha(s)$	Twiss parameter

$\beta(s)$	Twiss parameter
β	particle velocity in units of c
β_0	nominal particle velocity (in laboratory frame) in units of c
γ	relativistic Lorentz factor [$= (1 - \beta_0^2)^{-1/2}$]
$\gamma(s)$	Twiss parameter
Γ	plasma correlation factor
δ	delta function
Δ	spread of a variable (FWHM)
Δ_e	velocity spread of electrons (r.m.s.)
$\epsilon(k, \omega)$	dielectric function
ϵ_y	beam emittance
ϵ_0	permittivity of free space ($8.854 \times 10^{-12} \text{ A} \cdot \text{s/V} \cdot \text{m}$)
ξ	degree of space-charge compensation
η	off-momentum factor
η_c	ratio of cooling length to ring circumference
θ_{acc}	machine acceptance angle
θ_{ms}	angular spread due to multiple scattering
θ_{spin}	spin rotation angle
θ_{scatt}	scattering angle
λ	cooling decrement
λ_D	Debye length
λ_{IBS}	intra-beam scattering growth rate
λ_{laser}	wavelength of laser photons in laboratory system
λ_s	pitch of electron spiral
λ_y	betatron wavelength
μ_0	permeability of free space [$= 1/(\epsilon_0 c^2)$]
μ_B	Bohr magneton ($= 5.79 \times 10^{-5} \text{ eV/T}$)
μ_P	micropervance ($= 10^{-6} \text{ A} \cdot \text{V}^{-3/2}$)
ν	frequency of photons
ξ	chromaticity
Ξ	quasi-potential for diffusion
ϱ	average magnetic ring radius
σ	r.m.s. spread of a variable
σ_n	cross-section for radiative recombination in atomic state n
σ_{dr}	cross-section for dielectronic recombination
τ	cooling time
τ_{lab}	cooling time in laboratory system
τ_{db}	debunching time
τ_{ms}	multiple scattering beam lifetime
τ_{ss}	single scattering beam lifetime
ϕ	electric potential
ϕ_{an}	anode potential
ϕ_{cath}	cathode potential
ϕ_{coll}	collector potential
Φ	quasipotential for frictional force
ω_{az}	angular azimuthal frequency of electrons
ω_c	angular cyclotron frequency
ω_{pl}	angular plasma frequency

Indices:

$\perp$	transverse components
$\parallel$	longitudinal components
y	either of the transverse components (x or z)
ad	adiabatic (magnetic) component
NM	non-magnetic component

1. INTRODUCTION

The quality of a charged-particle beam is characterized by the product of its radius and divergence—the emittance—and by the momentum spread. They define the part of the phase space that is occupied by the ions in the storage ring. Emittance and momentum spread are directly related to the transverse and longitudinal temperature in the coordinate frame co-moving with the ions.

The search for a particle-beam treatment that would allow compression of the occupied phase space has led to the invention of electron cooling. This method was proposed about 24 years ago by Budker [1]. It was a prerequisite for the accumulation of antiprotons. The first electron cooler was built in Novosibirsk and produced its first beam in 1974. The proton storage ring NAP-M was constructed there for testing this cooling technique. The first electron cooling experiments were performed in 1974, and the results [2] surpassed all expectations. Electron coolers and cooling rings were built also at CERN and at FNAL, and the cooling process was studied in detail. These three experiments [2–4], which were finished in 1981, can be considered as the pioneering investigations of electron cooling. Based on the experience gained with the Initial Cooling Experiment (ICE) at CERN [3], a second-generation electron cooling device [5] was developed for routine operation and was commissioned in the Low-Energy Antiproton Ring (LEAR). Electron coolers are being developed for a number of ion storage rings now under construction all over the world, to be used for atomic, nuclear, medium-energy, and heavy-ion physics. In the early experiments, electron cooling was demonstrated with proton beams at semirelativistic energies. Meanwhile it has been applied to antiprotons and light ions. In the upcoming storage rings it will be used for the cooling of heavy ions up to U^{92+} . In this article we shall review electron cooling theories and summarize the available experimental data, and describe the electron cooling hardware and possible future developments. The present work updates earlier reviews [6–10] and extends their scope. Apart from beam cooling, different applications of cold electron beams will be discussed.

An alternative method—stochastic cooling—was proposed by van der Meer [11] for the cooling of charged-particle beams. Later, it turned out that stochastic cooling is more appropriate for the cooling of high-energy antiprotons, and it was therefore chosen for their accumulation.

The cooling of thermal atoms through optical pumping with lasers [12] has recently yielded intriguing results. It was proposed to apply this method also to stored, not fully stripped, ion beams. These three techniques for ion-beam cooling are largely complementary and their combination could be very useful in particular cases.

For the introduction to electron cooling, a binary collision model will be chosen, which should make it easy for the particle physicist to get into the subject matter, but, from the beginning, the emphasis will be on the plasma physics aspects. However, both aspects—the mechanical picture and the plasma physics—prepare the reader for the sections (3 and 4) on the theory. After the cooling forces and times are derived, other electron-ion interactions will be briefly mentioned, mainly in view of the later section on application. Then the hardware of the electron cooling system and the layout of cooling devices are discussed. Also, the relevant accelerator terms are defined. Thereafter, the effect of electron cooling on a circulating beam is outlined and computer simulations of electron cooling are presented. After this the reader should be well prepared to digest the experimental results of electron cooling. The future developments of electron cooling and its application in accelerator physics will then be considered in some detail. The use of an electron beam having properties similar to those of a cooling beam is discussed in the section on atomic physics application, which precedes the summary.

Note that bold characters are chosen for vectors, the subscript i for ion properties, and the subscript e for electrons, if not otherwise stated. Throughout the paper we describe the physical processes in a frame (rest frame) moving with the average velocity of the ions and electrons. The quantities measured in the laboratory system are usually denoted by the subscript ‘lab’.

2. INTRODUCTION TO ELECTRON COOLING IN STORAGE RINGS

2.1 A simplified picture of the operation principle

Electron cooling is a fast process for shrinking the size, the divergence, and the energy spread of stored charged-particle beams without removing particles from the beam. It is achieved by combining a circulating beam with a monochromatic and well-directed electron beam over a certain distance in a section of a storage ring. The electrons are produced continuously in an electron gun with a cathode heated to T_{cath} . They are accelerated electrostatically to a velocity $v_{e0} = \beta_0 c$ (c is the velocity of light) equal to the average ion velocity, and are inflected into the ion beam. Both beams overlap for a certain length ℓ . At the end of the section, the electrons are separated again from the ion beam (Fig. 1).

A close-up of the overlap region shows the ions traversing, under different angles and velocities, the stream of parallel electrons, all moving with the same velocity (Fig. 1a). However, if observed from a frame moving with the velocity of the electrons (rest frame), the latter are at rest, whilst the ions pass through the electron gas from any direction and with a variety of velocities, resembling the motion of particles in a hot gas (Fig. 1b). In the rest frame, one can associate an ion temperature T_i , which is normally of the order of 10^6 K, with the width of the ion velocity distribution. The ions undergo Rutherford scattering in the gas and lose energy. The heat is transferred from the ions to the electrons through this Coulomb interaction. The motion of the ions in the rest frame is reduced. Simply speaking, the ions that are slower than the electrons are kicked on and will eventually assume the velocity of the electrons; correspondingly, faster ions kick the electrons and lose energy. The electrons are continuously renewed. In this picture, the electron cooling section can be understood as a heat exchanger and the electrons as the cooling medium. Owing to the reduction of the velocity vector, the distribution of the ion velocities in the rest frame will become narrower in all three space directions, which means that the temperature of the ion beam (gas) will decrease. This explains the name 'cooling'.

One may also consider the electrons as being represented by a foil moving with velocity v_{e0} . Ions moving faster than the foil (electrons) penetrate it and lose energy along the direction of their momentum (dE/dx) during each passage, until all transverse components are diminished and their longitudinal velocity is equal to the foil velocity (Fig. 1c). Slower ions traverse the foil from the opposite side, but the effect is the same. Ideally, in the end, all ions will have the same longitudinal velocity as the foil and no transverse velocity component. The energy loss can be calculated from the Bohr equation [13]:

$$-dE/dx = 4\pi Z^2 e^4 n_e / [(4\pi\epsilon_0)^2 m_e v_i^2] \alpha_1, \quad (1a)$$

where m_e is the electron mass; v_i is the ion velocity in the frame where the electrons are at rest; Z is the charge of the ion; e is the elementary charge; n_e is the electron density; ϵ_0 ($= 8.854 \times 10^{-12}$ A·s/V·m) is the permittivity of free space; and α_1 is a material constant. As an effect of multiple scattering (ms), the beam will be blown up as it passes through a foil of thickness x_0 . This will also cause a smearing of the longitudinal energy, known as energy straggling. The r.m.s. width θ_{ms} of the angular distribution is given by

$$\theta_{ms}^2 = 2\pi Z^2 e^4 n_e / [(4\pi\epsilon_0)^2 E_i^2] x_0 \alpha_2 \quad (1b)$$

(E_i is the kinetic energy of the ion, α_2 is another material constant).

We therefore distinguish two effects: i) the energy loss (friction) and ii) the multiple scattering (diffusion). The first effect results in cooling, the second one corresponds to heating. The diffusion is characterized by the coefficient D , defined as

$$D = \frac{d}{dt} (p_i^2 \theta_{ms}^2)$$

(p_i is the momentum of the ion).

The energy loss and the diffusion coefficient are related through [Eqs. (1a) and (1b)]

$$\frac{dE}{dx} = \frac{1}{2m_e} \frac{\partial D}{\partial v_i} \quad (\text{for } \alpha_1 = \alpha_2). \quad (1c)$$

Electron cooling needs a cold electron beam. Cooling is at present performed at particle velocities to which electrons can be accelerated electrostatically. Electrostatic acceleration permits the low temperature given by the thermocathode to be maintained; it can even be reduced in the longitudinal direction through acceleration. Throughout this article, a standard case is used to evaluate a typical situation. The chosen parameters are given in Table A1.1 in Appendix I. Electron cooling is not limited to ion beams but can also be applied to trapped ions. In this case the rest frame coincides with the laboratory frame.

The effect of electron cooling on the size and the momentum spread of a stored beam or trapped ions is illustrated in Fig. 2.

2.2 Motivation for electron cooling

Electron cooling was originally proposed for the accumulation of antiprotons. The required high energy favoured stochastic cooling, which is nowadays applied for this purpose, but electron cooling is still needed for accumulation if one wants to collect rare ions at energies lower than those needed for antiproton accumulation. Usually, rare ions are produced at low intensity and with a moderate beam quality. Injection into a storage ring would soon fill the acceptance of the ring, and further pulses could not be added without throwing the first ones out. Cooling would reduce the beam size and the momentum spread of the circulating beam, and would therefore provide unoccupied space for the injection of a new pulse. This is the condition for accumulation. Rare particles can, for instance, be polarized protons, highly charged heavy ions, or rare isotopes. Cooling with electrons is probably more appropriate than stochastic cooling for this application, because of the short cooling times that are achievable.

The reduction of the beam size, the divergence, and the momentum spread provide a beam for precision experiments. The small beam size facilitates the determination of the interaction vertex. The accurate knowledge of the particle momentum is vital for high-resolution spectroscopy.

The stored beam is blown up by scattering from residual gas molecules or from an internal target. Blow-up leads to rapid beam loss. Electron cooling can counteract gas scattering and conserve a small beam. It therefore increases the beam lifetime and permits the operation of a thin internal target.

In specific cases the stored beam has to be slowed down. Deceleration leads to a blow-up of the beam. Electron cooling can compensate this, and can provide small beams even after deceleration to low energy.

One major application of electron cooling will be linked to the attempt to produce ion beams of such low temperature that a kind of crystalline order appears in the beam. Electron cooling has the potential to provide such a low temperature for heavy ions.

The virtue of electron cooling lies in the fact that it can provide short cooling times, small beams, and low momentum spreads. Its performance does not depend on the number of stored particles. Its range of practical application is, however, limited to low and medium energy ($\beta_0 < 1$).

It turns out that the electron beam used for cooling stored ions provides an excellent electron target for atomic physics experiments, and in this respect the interest in the application of cooling

electron beams is continuously expanding. We therefore distinguish three categories for the application of electron cooling:

- i) reduction of occupied phase space, i.e. reduction of beam emittance and momentum spread,
- ii) counteraction against heating processes,
- iii) free-electron target.

2.3 The plasma physics aspect of charged-particle beams and electron cooling

The electrons and the ions represent a dilute plasma, of which the electrons are the low-temperature component. In the cooling section of the ring, one deals with a two-component plasma, and with a one-component plasma elsewhere. Plasma physics is a well-developed field, and it provides us with a rich set of implements [14–17] for developing a theoretical description of the electron cooling process. In the plasma physics of controlled fusion, the electrons serve as a heating medium to achieve high temperatures for the heavy plasma component, which is just the opposite of what is considered here. However, the description of the underlying processes is equivalent. Before entering into the cooling theory, the basic elements of plasma physics needed for the later discussion are introduced in this subsection.

The particles in the plasma are distributed in coordinate space and in velocity space. Their behaviour is described in a six-dimensional space (phase space), which is generated by the space coordinates $\mathbf{r}$ and the velocity coordinates $\mathbf{v}$. The distribution of the particles represented by sextuplets in the phase space is described by a distribution function f , which also depends on time t :

$$f = f(\mathbf{r}, \mathbf{v}, t), \quad \varrho(\mathbf{r}, t) = \int f d^3\mathbf{v}, \quad n = \int \varrho(\mathbf{r}, t) d^3\mathbf{r}. \quad (2)$$

The integration of f over the velocity space yields the spatial distribution $\varrho(\mathbf{r}, t)$ of the particles, and this, integrated over the coordinate space, gives the particle density n . In a closed system, Liouville's theorem holds:

$$df/dt = 0, \quad (3)$$

which means that the volume density in phase space is constant. Of particular interest is the distribution function, which gives the velocity distribution function $f(\mathbf{v}, t)$. The function $f(\mathbf{r}, \mathbf{v}, t)$ may be factorized into $\varrho(\mathbf{r}, t)$ and $f(\mathbf{v}, t)$. The overall velocity distribution is obtained by integration over the coordinate space:

$$f(\mathbf{v}, t) = \int f(\mathbf{r}, \mathbf{v}, t) d^3\mathbf{r}. \quad (4)$$

In thermal equilibrium, f is time-independent and the velocities are distributed according to a Maxwellian curve:

$$f(\mathbf{v}) = \exp -[(v_x^2/\sigma_{v_x}^2) + (v_y^2/\sigma_{v_y}^2) + (v_z^2/\sigma_{v_z}^2)]/(\sigma_{v_x}\sigma_{v_y}\sigma_{v_z}\pi\sqrt{\pi}). \quad (5)$$

Here $f(\mathbf{v})$ is normalized to unity: $\int f(\mathbf{v}) d^3\mathbf{v} = 1$. For a spherical distribution, $\sigma_{v_x} = \sigma_{v_y} = \sigma_{v_z} = \sigma_v$, one has

$$f(\mathbf{v}) = \frac{e^{-v^2/\sigma_v^2}}{\sigma_v^3 \pi\sqrt{\pi}} = \left(\frac{mc^2}{2kT}\right)^{3/2} \frac{e^{-mv^2/2kT}}{c^3 \pi\sqrt{\pi}}, \quad (6a)$$

with $kT = \frac{1}{2}m\sigma_v^2$ being the temperature of the plasma particles per degree of freedom and m their mass (later on, a subscript e, p, or i will indicate when the mass of the electron, of the proton, or of an ion is meant); k ($8.617 \times 10^{-5} \text{ eV K}^{-1}$) is the Boltzmann constant. For an anisotropic velocity distribution, for instance $\sigma_{v_x} = \sigma_{v_y} = \sigma_{v_\perp}$ and $\sigma_{v_z} = \sigma_{v_\parallel}$, we get

$$f(\mathbf{v}) = \frac{mc^2}{2kT_\perp} \left(\frac{mc^2}{2kT_\parallel} \right)^{1/2} \frac{1}{c^3 \pi \sqrt{\pi}} \exp - [mv_\perp^2 / (2kT_\perp)] \exp - [mv_\parallel^2 / (2kT_\parallel)] , \quad (6b)$$

with $v_\perp^2 = v_x^2 + v_y^2$, $v_\parallel = v_z$, and $2kT_\perp = m\sigma_\perp^2$, $2kT_\parallel = m\sigma_\parallel^2$. Note that with this convention,

$$T_\perp = \frac{T_x + T_y}{2} . \quad (6c)$$

The temperature and the density characterize the domain of plasma physics in which one works. Figure 3 shows the region where charged-particle beams are situated with respect to other types of plasmas.

Let us imagine that a positive test charge is introduced into the plasma. The electrons will group themselves around it and will shield its Coulomb field. As a consequence, the electric field decays exponentially from a certain distance onwards. This distance is called the Debye length (Debye radius) λ_D and is related to the plasma temperature kT and density n by

$$\lambda_D^2 = \epsilon_0 kT / ne^2 . \quad (7)$$

The Debye length can be compared with the average distance r_{av} that separates the plasma particles:

$$r_{av} = (4/3 \pi n)^{-1/3} . \quad (8)$$

The line where $\lambda_D = r_{av}$ is shown in Fig. 3, and $T \propto n^{1/3}$ holds. The ideal plasma is found above this line.

The time t_{pl} it takes for the plasma to rearrange itself after a perturbation (e.g. introduction of the test particle) is given by the plasma frequency $\omega_{pl} = 2\pi/t_{pl}$, defined by

$$\omega_{pl}^2 = ne^2 / (\epsilon_0 m) = 4\pi n r_{cl} c^2 , \quad (9)$$

where $r_{cl} = e^2 / (4\pi \epsilon_0 m c^2)$ is the classical particle radius. (The subscript letters cl are replaced by an i, an e, or a p to denote the corresponding quantity for ions, electrons, or protons, and m by the corresponding mass.) With the above relations, the plasma frequency can be connected with the Debye length and the temperature:

$$\lambda_D^2 = kT / (m \omega_{pl}^2) , \quad \sigma_v = \omega_{pl} \lambda_D \sqrt{2} . \quad (10a)$$

If the electron gas has a very low temperature ($T_e \rightarrow 0$) and the test charge moves with the velocity v_i , the screening length is given by

$$\lambda_{ad} = v_i / \omega_{pl} . \quad (10b)$$

Usually the motions of the plasma particles are uncorrelated. However, if the plasma temperature is low and attains values of the order of the Coulomb potential at the average distance

between two ions, the particles cannot be considered as being independent. The parameter that describes the degree of order is the plasma correlation factor Γ , defined as the ratio of the Coulomb energy at the average distance r_{av} between particles to the plasma temperature:

$$\Gamma = q^2 / (4\pi\epsilon_0 r_{av} kT), \quad (11)$$

where q is the charge of the plasma particles ($q = Ze$). For $\Gamma = \text{const}$, $T \propto n^{1/3}$ and $\lambda_D = r_{av}$ corresponds to $\Gamma/Z^2 = 1/3$. Figure 3 also shows lines with $\Gamma = 1$ for $q = e$ and with $\Gamma = 170$ for $q = 92e$. The latter corresponds to a highly ordered plasma.

Processes within the plasma are described by the evolution of the distribution function in time. The charged particles interact through Rutherford scattering. In a single encounter, the direction and the absolute value of the particle velocity changes only a little. The effects of many collisions add up statistically. The process can be considered as a kind of diffusion. In this *diffusion approximation*, one can define a time-independent probability ψ that the particle velocity vector $\mathbf{v}$ is altered by a small amount $\Delta\mathbf{v}$ [15–17]:

$$f(\mathbf{r}, \mathbf{v}, t) = \int f(\mathbf{r}, \mathbf{v} - \Delta\mathbf{v}, t - \Delta t) \psi(\mathbf{v} - \Delta\mathbf{v}, \Delta\mathbf{v}) d\Delta\mathbf{v}. \quad (12)$$

In the approximation of small changes, $f\psi$ can be developed, which in second order gives

$$f\psi = f(\mathbf{r}, \mathbf{v}, t - \Delta t) \psi(\mathbf{v}, \Delta\mathbf{v}) - \Delta\mathbf{v} \left[\psi \frac{\partial f}{\partial \mathbf{v}} + f \frac{\partial \psi}{\partial \mathbf{v}} + \frac{1}{2} \Delta\mathbf{v}_m \Delta\mathbf{v}_n \left(\psi \frac{\partial^2 f}{\partial v_m \partial v_n} + 2 \frac{\partial f}{\partial v_m} \frac{\partial \psi}{\partial v_n} + \frac{\partial^2 \psi}{\partial v_m \partial v_n} \right) \right] \quad (13)$$

($m, n = x, y, z$). The rate of change of f due to collisions is

$$\partial f / \partial t = [f(\mathbf{r}, \mathbf{v}, t) - f(\mathbf{r}, \mathbf{v}, t - \Delta t)] / \Delta t. \quad (14)$$

With Eq. (13), we then get

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial \mathbf{v}_m} \left[-f \frac{\langle \Delta\mathbf{v} \rangle}{\Delta t} + \frac{\partial}{\partial \mathbf{v}_n} \left(f \frac{\langle \Delta\mathbf{v}_m \Delta\mathbf{v}_n \rangle}{\Delta t} \right) \right], \quad (15a)$$

where

$$\langle \Delta\mathbf{v} \rangle = \int \Delta\mathbf{v} \psi(\mathbf{v}, \Delta\mathbf{v}) d\Delta\mathbf{v} \quad \text{and} \quad \langle \Delta\mathbf{v}_m \Delta\mathbf{v}_n \rangle = \int \Delta\mathbf{v}_m \Delta\mathbf{v}_n \psi(\mathbf{v}, \Delta\mathbf{v}) d\Delta\mathbf{v}. \quad (15b)$$

This is the Fokker–Planck equation describing the evolution of the distribution function in time; $\langle \Delta\mathbf{v} \rangle / \Delta t$ denotes the average rate of change of the particle velocity (deceleration/acceleration) due to collisions, and is related to the frictional force F . Correspondingly, $\langle \Delta\mathbf{v}_m \Delta\mathbf{v}_n \rangle / \Delta t$ signifies the average rate of velocity exchange and is related to the diffusion coefficient D_{mn} :

$$F = m \langle \Delta\mathbf{v} \rangle / \Delta t, \quad D_{mn} = \langle \Delta\mathbf{v}_m \Delta\mathbf{v}_n \rangle / \Delta t. \quad (16)$$

When considering all components, the Fokker–Planck equation reads

$$\frac{\partial f}{\partial t} = \sum_{m,n} \frac{\partial \{ -f(F/m) + [\partial(fD_{mn}) / \partial v_n] \}}{\partial v_m}. \quad (17)$$

In Section 3 (on the cooling theory), the frictional force and the diffusion coefficient are derived and can be inserted into the Fokker–Planck equation. The change of the distribution function due to ion

collisions with electrons will be obtained strictly by solving the Fokker-Planck equation. This description is also useful for the modelling of scattering processes in the ion beam itself and in the electron beam.

The introduction of a test charge into a plasma leads to a regrouping of the plasma particles (mainly electrons, if available) around it, as mentioned before. The plasma is polarized locally (Fig. 4). The polarization creates a local field induced by the test charge, which acts back on it. This field $\mathcal{E}_{\text{ind}}$ ($= \mathcal{E}_{\text{tot}} - \mathcal{E}_i$) is the difference between the total plasma field $\mathcal{E}_{\text{tot}}$ and the electric field produced by the ion $\mathcal{E}_i$. It creates a force acting on the ion, and takes into account the properties of an infinite plasma. This approach can be applied to derive the cooling force. If the test particle moves with a velocity v_i through the plasma, its charge density can be written as $Ze[\delta(\mathbf{r} - \mathbf{v}_i t)]$. Since the local electric field changes in time, it is useful to consider the Fourier transform defined for a quantity A as:

$$A(\mathbf{r}, t) = \int \exp[i(\mathbf{k}\mathbf{r} - \omega t)] A(\mathbf{k}, \omega) d^3\mathbf{k} d\omega.$$

The electric potential introduced by the ion, ϕ_i , is related to the total potential ϕ_{tot} through the dielectric function ϵ :

$$\phi_{\text{tot}}(\mathbf{k}, \omega) = \phi_i(\mathbf{k}, \omega) / \epsilon(\mathbf{k}, \omega). \quad (18)$$

The passage of a charged particle through the plasma is comparable to an electromagnetic wave of frequency ω and wave vector $\mathbf{k}$ propagating through the plasma. The dissipation power that the wave transfers to the plasma is directly related to the force acting on the ion and tending to slow it down. The 'optical' properties of the plasma are characterized by a dispersion relation connecting the frequency and the wave vector $\mathbf{k}$ through the dielectric function $k^2 = (\omega/c)^2 \epsilon$. For an electron plasma at rest (zero temperature), the dielectric function has the value

$$\epsilon = 1 - \omega_{e, \text{pl}}^2 / \omega^2. \quad (19)$$

If the plasma is contained in a magnetic field of strength B , the particles perform rotations about the magnetic field lines with the angular frequency ω_c (cyclotron frequency) and a gyration radius r_c (cyclotron radius):

$$\omega_c = eB/(\gamma m) \quad (20a)$$

(for electrons, $\omega_{e, c} = 176 \gamma^{-1} \text{ GHz } B[\text{T}]$, for protons, $\omega_{p, c} = 95.8 \gamma^{-1} \text{ MHz } B[\text{T}]$), and

$$r_c = v_{\perp} / \omega_c, \quad (20b)$$

where $v_{\perp}$ is the particle velocity in the plane perpendicular to the magnetic field and $\gamma = 1/\sqrt{1 - \beta^2}$. (In the Soviet literature on electron cooling, the gyration radius is usually called the Larmor radius.) Note also that the definition of the Debye length [Eq. (7)] is only valid in the absence of a magnetic field. In the presence of a magnetic field, the mobility of the electrons is reduced and the screening of a charge in the electron plasma is different.

When two separated one-component plasmas of unequal temperatures are mixed, they reach equilibrium after a relaxation time. This equilibration time τ_{eq} was calculated by Spitzer [14] on the basis of plasma physics, long before electron cooling was considered:

$$\tau_{eq} = \frac{3}{8\sqrt{2}\pi n_e Z^2 r_{ei} c L_C} \left(\frac{kT_e}{m_e c^2} + \frac{kT_i}{m_i c^2} \right)^{3/2}, \quad (21)$$

where L_C is the Coulomb logarithm. It is amazing that his calculations already provide a good approximation for electron cooling.

3. THE COOLING THEORY

Electron cooling of ion beams can be described by a model where the two beams are considered as plasmas of different temperatures with the electrons being the low-temperature component. The electron-ion interaction proceeds via multiple Rutherford scattering and leads to an equilibration of temperatures. If the density of the electrons is large enough, or if they are cooled separately or are continuously renewed, then the ions will assume their temperature. The differential cross-section $d\sigma/d\Omega$ for Rutherford scattering is

$$d\sigma/d\Omega = z^2 Z^2 e^4 / [(4\pi\epsilon_0)^2 4E_i^2 \sin^4(\theta_{scatt}/2)], \quad (22)$$

where Ze and ze are the charges of the plasma components and θ_{scatt} is the scattering angle. The low-temperature component is not necessarily an electron plasma; it could also consist of other types of ions. Since we will consider only electrons and positrons as a cooling medium, $|z| = 1$, it will therefore be omitted. The differential Rutherford cross-section is strongly forward-peaked. Consequently, the electron-ion interaction is a series of small-angle scatterings, where little momentum and energy is transferred from the ion to the electron in a single scattering event. Furthermore, the initial direction of the ion motion is changed by small angular increments, leading to a distribution for the final direction of the ion motion. Two effects are to be distinguished: one is the momentum transfer to the electrons, reducing the momentum of the ion along the initial (let us call it longitudinal) direction (friction); the other is the momentum distribution into the motion perpendicular to the initial direction (let us call it transverse) and the smearing (straggling) in the longitudinal direction. This is a kind of diffusion. It corresponds to the multiple scattering of a charged-particle beam passing through a degrader [cf. Eq. (1b)].

3.1 A simple derivation of the cooling force and the diffusion tensor

The reference frame is still the system where the electrons are at rest. First a single electron-ion collision is considered. The ion moves with velocity v_i , and scatters from an electron at an impact parameter b (Fig. 4).

The impact parameter is related to the scattering angle θ_{scatt} by

$$b = Ze^2 / (4\pi\epsilon_0 2E_i) \operatorname{ctg}(\theta_{scatt}/2). \quad (23)$$

The differential Rutherford cross-section integrated over angles between θ_{scatt} and 180° yields

$$\Delta\sigma = \int_0^{2\pi} d\phi \int_{\theta_{scatt}}^{180^\circ} \frac{d\sigma(\theta')}{d\Omega} \sin\theta' d\theta' = \pi \left(\frac{Ze^2}{4\pi\epsilon_0 2E_i} \right)^2 \operatorname{ctg}^2 \frac{\theta_{scatt}}{2} = \pi b^2. \quad (24)$$

The momentum transferred from the ion to the electron is

$$\Delta p = \int \mathcal{E} dt = \frac{1}{4\pi\epsilon_0} \int \frac{Ze^2}{x^2 + b^2} dt,$$

with $\mathcal{E}$ being the Coulomb force. Times going from negative to positive infinity are considered. The longitudinal part of the force can therefore be neglected. Hence $\mathcal{E}$ is replaced by its transverse component $\mathcal{E}_\perp$:

$$\mathcal{E}_\perp = \mathcal{E} \frac{b}{\sqrt{x^2 + b^2}}.$$

Using $v_i = dx/dt$, we get

$$\Delta p = \int_{-\infty}^{\infty} \mathcal{E}_\perp dt = \frac{1}{4\pi\epsilon_0} \int_{-\infty}^{\infty} \frac{Ze^2 b}{v_i \sqrt{(x^2 + b^2)^3}} dx = \frac{2Ze^2}{4\pi\epsilon_0 v_i b}.$$

The energy transferred from the ion to the electron is

$$\Delta E_{i \rightarrow e}(b) = (\Delta p)^2 / 2m_e = 2Z^2 e^4 / [(4\pi\epsilon_0)^2 m_e b^2 v_i^2].$$

The energy transferred from the longitudinal to the transverse motion is

$$\Delta E_{\parallel \rightarrow \perp}(b) = (\Delta p)^2 / 2m_i \quad (m_i \text{ is the mass of the ion}).$$

So far we have considered a single collision. Now we extend the picture to the situation where the ion passes through a large number of electrons. The ion undergoes multiple small-angle scatterings. The effect of many single scatters adds up statistically as far as the momentum transfer from the longitudinal to the transverse motion is concerned. It is described by the statistical expectation value of the product of the momentum transfer components [15]:

$$\langle \Delta p_m \Delta p_n \rangle = (\delta_{mn} - v_{im} v_{in} / v_i^2) \Delta p^2 / 2 = (\delta_{mn} - v_{im} v_{in} / v_i^2) \frac{2Z^2 e^4}{(4\pi\epsilon_0)^2 v_i^2 b^2}. \quad (25)$$

The diffusion coefficient [Eq. (16)] is the rate of change of this value multiplied by the area density of the electrons $n_e b$ and the length of the passage $dx = v_i dt$ integrated over all possible impact parameters (scattering angles). Correspondingly, one obtains the energy lost by the ion in the longitudinal direction when it travels a length dx through the electron cloud (Fig. 5):

$$-\frac{dE}{dx} = 2\pi \int_0^\infty n_e b db \Delta E_{i \rightarrow e}(b) = \frac{4\pi Z^2 e^4 n_e}{(4\pi\epsilon_0)^2 m_e v_i^2} \int_0^\infty \frac{db}{b}, \quad (26a)$$

$$D_{mn} = 2\pi \int_0^\infty n_e b db \frac{d\langle \Delta p_m \Delta p_n \rangle dx}{dt} = \frac{4\pi Z^2 e^4 n_e}{(4\pi\epsilon_0)^2} \left(\frac{\delta_{mn}}{v_i} - \frac{v_{im} v_{in}}{v_i^3} \right) \int_0^\infty \frac{db}{b}. \quad (26b)$$

The first expression describes the cooling (or frictional) force $F = dE/dx$ experienced by the ion, and the second the diffusion. These equations correspond to Eqs. (1), and the generalized relation between force and diffusion coefficient is now [16]

$$F_n = \frac{1}{2m_e} \frac{\partial D_{mn}}{\partial v_{im}}. \quad (27)$$

This expression indicates that in this type of statistical process the friction and the diffusion show up simultaneously.

The integral in the above equations reminds us of the collision theory problem of scatterings from an infinitely ranged potential, namely the logarithmic divergence originating from the upper integration limit. The evaluation of this integral poses one of the principal problems for the electron cooling theory. In the literature, two strategies were applied: one was to find a suitable cut-off for the integration (impact parameter); the other was to modify the Coulomb potential in such a way that the integrated Rutherford scattering cross-section remains finite. The first procedure leads to an extensive discussion of the relevant impact parameter ranges for the various situations in electron cooling. It is considered next.

By restricting the integration to an impact parameter range $b_{\min} \leq b \leq b_{\max}$,

$$\int_{b_{\min}}^{b_{\max}} \frac{db}{b} = \ln \frac{b_{\max}}{b_{\min}} = L_C, \quad (28)$$

one obtains the Coulomb logarithm L_C defined as the logarithmic ratio of the maximal to the minimal impact parameter. It is easy to find a minimal impact parameter that is determined by the maximum momentum transfer $\Delta p_{\max} = 2m_e v_i$ from the ion to the electron [scattering of 180° ; see Eq. (23)]:

$$2Ze^2/(4\pi\epsilon_0 b_{\min} v_i) = 2m_e v_i \rightarrow b_{\min} = Ze^2/(4\pi\epsilon_0 m_e v_i^2) = Zr_e \cdot c^2/v_i^2. \quad (29)$$

On the other hand, the adiabatic screening radius λ_{ad} of the electron gas offers a reasonable maximal impact parameter. Therefore, a first approximation for L_C is

$$L_C(v_i) = \ln(4\pi\epsilon_0 \lambda_{ad} m_e v_i^2 / Ze^2) = \ln \left[\frac{\lambda_{ad}}{Zr_e} \left(\frac{v_i}{c} \right)^2 \right]. \quad (30)$$

Of course, this is only meaningful as long as $b_{\max} > b_{\min}$.

For our standard case (Appendix I) and $v_i = 10^{-3} \beta_0 c$, the Coulomb logarithm L_C assumes a value of 10. With these approximations, the cooling force and the diffusion coefficient are

$$F(v_i) = -F_0 L_C(v_i) c^2 \frac{v_i}{v_i^3}, \quad (31a)$$

$$D_{mn} = m_e F_0 L_C(v_i) c^2 \left(\frac{\delta_{mn}}{v_i} - \frac{v_{in} v_{in}}{v_i^3} \right), \quad (31b)$$

with

$$F_0 = \frac{4\pi Z^2 e^4 n_e}{(4\pi\epsilon_0)^2 m_e c^2} = \frac{\omega_{e,pl}^2 Z^2 e^2}{4\pi\epsilon_0 c^2} = \frac{\omega_{e,pl}^2}{c^2} Z^2 r_e m_e c^2 = 4\pi Z^2 n_e r_e^2 m_e c^2. \quad (31c)$$

The force F is analogous to the electrostatic force acting on the ion at a distance

$$r_F = \lambda_{ad} \sqrt{\frac{2m_e}{m_i Z \cdot L_C}}, \quad (32)$$

and usually far inside the Debye sphere.

The frictional force obtained acts on the ion opposite to its velocity vector, tending to reduce it. So far we have neglected the motion of the electrons. In reality, however, the electrons are not at rest but obey a velocity distribution $f(\mathbf{v}_e)$ given by Eq. (5) (in the following, Δ_e is used to denote the r.m.s. width of the electron velocity distribution). In order to account for this, the ion velocity $\mathbf{v}_i$ is replaced by the electron-ion velocity difference $\mathbf{u} = \mathbf{v}_i - \mathbf{v}_e$, and the force and the diffusion coefficient are averaged over the electron velocity distribution:

$$\mathbf{F}(\mathbf{v}_i) = -F_0 c^2 \int L_C(u) f(\mathbf{v}_e) \frac{\mathbf{u}}{u^3} d^3 \mathbf{v}_e, \quad (33a)$$

$$D_{mn}(\mathbf{v}_i) = m_e F_0 c^2 \int L_C(u) f(\mathbf{v}_e) \frac{u^2 \delta_{mn} - u_m u_n}{u^3} d^3 \mathbf{v}_e = m_e F_0 c^2 \int L_C(u) f(\mathbf{v}_e) \frac{\partial^2 u}{\partial v_{i_m} \partial v_{i_n}} d^3 \mathbf{v}_e. \quad (33b)$$

[For the last formula we used the identity $\partial^2 u / (\partial v_{i_m} \partial v_{i_n}) \equiv (u^2 \delta_{mn} - u_m u_n) / u^3$.]

The Coulomb logarithm now depends also on the velocity difference u . The slow variation of the logarithm permits us, however, to put it in front of the integral and to replace $L_C(u)$ by $L_C(\Delta_e)$ for $v_i < \Delta_e$. The expressions (33) can be written, in analogy to electrostatic quantities, as

$$\mathbf{F}(\mathbf{v}_i) = -F_0 c^2 L_C \text{grad}_{\mathbf{v}_i} \Phi, \quad (34a)$$

$$D_{mn}(\mathbf{v}_i) = m_e F_0 c^2 \frac{\partial^2 \Xi}{\partial v_{i_m} \partial v_{i_n}}. \quad (34b)$$

The frictional force $\mathbf{F}(\mathbf{v}_i)$ resembles the Coulomb force of a charge distribution acting on a test charge. The corresponding quasipotentials Φ and Ξ are defined by different moments of the velocity distribution:

$$\Psi(\mathbf{v}_i) = \int f(\mathbf{v}_i) H(u) d^3 \mathbf{v}_e, \quad (35a)$$

with

$$\left. \begin{array}{l} \Phi(\mathbf{v}_i) \\ \Xi(\mathbf{v}_i) \end{array} \right\} = \Psi(\mathbf{v}_i) \left\{ \begin{array}{ll} \text{for} & H(u) = 1/u, \\ \text{for} & H(u) = u. \end{array} \right. \quad (35b)$$

For a *spherical* electron velocity distribution [$f(\mathbf{v}_e) = f(v_e)$], the integral can be written as (Appendix II)

$$\Phi(v_i) = \frac{2\pi}{v_i} \int_0^\infty \{G(u_{\max}) - G(u_{\min})\} f(v_e) v_e dv_e, \quad (36)$$

where

$$G(u) = \int H(u) u du.$$

For a *spherical Maxwellian* electron velocity distribution [Eq. (6a)], we obtain for the potential Φ (Appendix II),

$$\Phi = \frac{\text{erf}(v_i/\Delta_e)}{v_i}, \quad (37)$$

where $\text{erf}(v_i/\Delta_e)$ is the error function defined in Appendix II. The frictional force and the diffusion tensor are obtained by application of Eqs. (34). By choosing one axis (e.g. the z-axis) parallel to the ion velocity vector, we obtain D in diagonal form:

$$\mathbf{F}(v_i) = \mathbf{F}(v_i) = -F_0 c^2 L_C \Phi'(v_i) \frac{v_i}{v_i} = -F_0 L_C \left\{ \text{erf}(v_i/\Delta_e) - \frac{2v_i}{\Delta_e \sqrt{\pi}} e^{-v_i^2/\Delta_e^2} \right\} \frac{c^2}{v_i^2} \frac{v_i}{v_i}, \quad (38)$$

$$D = m_e \Delta_e \begin{bmatrix} L_C F_0 c^2 - \frac{\Delta_e^2}{2v_i} F & 0 & 0 \\ 0 & L_C F_0 c^2 - \frac{\Delta_e^2}{2v_i} F & 0 \\ 0 & 0 & -\frac{\Delta_e^2}{2v_i} F \end{bmatrix} \quad (39)$$

The quasipotential $\Phi(v_i)$, the frictional force, and the components of the diffusion tensor are plotted in Figs. 6 and 7. Whilst the shape of Φ resembles a Lorentzian curve, the force has a shape which is similar to a dispersion curve. The scaling behaviour of the force can be taken from the logarithmic plot (Fig. 6b). Since

$$\frac{\text{erf}(v_i/\Delta_e)}{v_i/\Delta_e} \xrightarrow{v_i \rightarrow 0} \frac{2}{\sqrt{\pi}}, \quad \text{erf}(v_i/\Delta_e) \xrightarrow{v_i \rightarrow \infty} \frac{2}{\sqrt{\pi}} \quad \text{and} \quad e^{-v_i^2/\Delta_e^2} = 1 - \frac{v_i^2}{\Delta_e^2} \dots,$$

we have

$$\mathbf{F}(v_i) = -\frac{2}{\sqrt{\pi}} F_0 L_C \frac{c^2}{\Delta_e^3} v_i \quad \text{for} \quad v_i \rightarrow 0,$$

$$\mathbf{F}(v_i) = -\frac{2}{\sqrt{\pi}} F_0 L_C \frac{c^2}{v_i^2} \frac{v_i}{v_i} \quad \text{for} \quad v_i \rightarrow \infty,$$

$$D(v_i) = \frac{m_e}{\sqrt{\pi}} F_0 \frac{c^2}{\Delta_e} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{for} \quad v_i \rightarrow 0,$$

$$D(v_i) = \frac{m_e}{\sqrt{\pi}} F_0 \frac{c^2}{v_i^2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \Delta_e^2/2v_i^2 \end{pmatrix} \quad \text{for} \quad v_i \rightarrow \infty.$$

At its extreme ($v_i = \Delta_e$), the force attains a value of

$$F(v_i = \Delta_e) \approx 1/2 F_0 L_C (\Delta_e) (c/\Delta_e)^2. \quad (40)$$

The formula for the force given in Eq. (38) can be approximated by

$$\mathbf{F}(v_i) = -3/2 F_0 c^2 L_C \frac{v_i}{|v_i|^3 + 2\Delta_e^3} \frac{v_i}{v_i}. \quad (41)$$

The cooling force is shown in Fig. 8 as a function of the transverse and longitudinal ion velocity. The round potential hill and the dispersion-like curve of the friction force give a first, rather realistic,

picture of the cooling force. In the following, the model will be refined, leading to some changes but without dramatically altering the basic features.

We now abandon the spherical symmetry and assume that we have an electron velocity distribution that is narrower in the longitudinal direction than in transverse directions ($\Delta_{\parallel} \ll \Delta_{\perp}$). The longitudinal direction is the direction parallel to the electron trajectory as seen from the laboratory system. The integral for the potential [Eq. (35a)] can be rewritten for a prolate Maxwellian velocity distribution [Eq. (6b)] as:

$$\Phi(\mathbf{v}_i) = \frac{1}{\Delta_{\parallel} \Delta_{\perp}^2 \pi \sqrt{\pi}} \int \exp \left[-\frac{v_{e\perp}^2}{\Delta_{\perp}^2} - \frac{v_{e\parallel}^2}{\Delta_{\parallel}^2} \right] u^{-1} d^3 \mathbf{v}_e. \quad (42)$$

The evaluation of the integral has to be done numerically. Its solution is illustrated in Fig. 9, where equipotential lines are plotted in the $v_{e\perp}, v_{e\parallel}$ plane. Their elliptical shape reflects the exponent under the integral in Eq. (42). The force vector is perpendicular to the ellipsis at any point. It is directed to the inside but does not point to the origin. Therefore the force is no longer centripetal, which means that transverse and longitudinal components differ in magnitude, hence

$$F_{\perp} = -|F| \cos \phi_F, \quad F_{\parallel} = -|F| \sin \phi_F. \quad (43a)$$

The angle ϕ_F is given by the relation

$$\text{tg } 2\phi_F = \frac{2v_{e\perp} v_{e\parallel}}{r_e^2 + v_{e\perp}^2 - v_{e\parallel}^2}, \quad (43b)$$

where r_e is the focal length of the elliptic potential line under consideration. The potential hill is steeper now, in particular in the longitudinal direction, which results in an increase of the cooling force as compared to the spherical case with velocity spread Δ_e . The shape of the potential (velocity distribution) also has the consequence that the ions move more rapidly to the longitudinal axis than to the transverse one, which means a faster longitudinal cooling.

The frictional force and the diffusion coefficient were evaluated by Derbenev and Skrinisky [18] for a prolate electron velocity distribution. They give the following approximation for the force:

for $v_i \ll \Delta_{\perp}$,

$$F = -\frac{1}{2} \sqrt{\frac{\pi}{2}} F_0 L_C(\Delta_{\perp}) \left(\frac{c}{\Delta_{\perp}} \right)^2 \left\{ \begin{array}{l} \frac{v_{i\perp}}{\Delta_{\perp}} \quad \text{transverse} \\ \quad \quad \quad \text{component} \\ 2 \sqrt{\frac{2}{\pi}} \frac{L_C(v_{i\parallel})}{L_C(\Delta_{\perp})} - 2 \frac{v_{i\parallel}}{\Delta_{\perp}}, \quad v_{i\parallel} > \Delta_{\parallel} \\ \frac{4}{\pi} \frac{L_C(\Delta_{\parallel})}{L_C(\Delta_{\perp})} \frac{v_{i\parallel}}{\Delta_{\parallel}}, \quad v_{i\parallel} < \Delta_{\parallel} \end{array} \right\} \quad \text{longitudinal} \quad \text{component}. \quad (43c)$$

For $v_i \gg \Delta_{\perp}$ and an arbitrary distribution, we had (Eq. 38)

$$F = -F_0 L_C(u) \left(\frac{c}{v_i} \right)^2 \frac{v_i}{v_i}. \quad (43d)$$

The force is nearly constant in the region $\Delta_{\parallel} < v_{i\parallel} < \Delta_{\perp}$. The application of this formula [Eq. (43c)] has to be done with care. Of course, it is valid for positive Coulomb logarithms only. It should also be ensured that the first term remains larger than the second for $v_{i\parallel} > \Delta_{\parallel}$.

In the following, the frictional force responsible for electron cooling and the diffusion coefficient will be derived in a more rigorous way.

The theory of electron cooling was initially developed mainly by the Novosibirsk group [18, 19]. The classical article on the theory is that of Ref. [18]. It invokes the arguments of plasma physics and uses its implements. However, one important experimental detail was omitted in the early theory: the intense electron beam is guided in a longitudinal magnetic field (as described in Section 6) in order to prevent it from blowing up, owing to mutual repulsion of the electrons. The longitudinal field transforms the electron motion, perpendicular to it, into gyrations about the magnetic field lines, which has the important result that the transverse electron velocity spread does not matter for a specific class of collisions. This is amplified by the fact that, because of acceleration, the electron velocity distribution is usually compressed in the longitudinal direction, whilst it remains unchanged in the transverse one, as we will see later. The consequences of this were outlined above (enhancement of longitudinal force component for ion velocities $v_i < \Delta_{\perp}$, see Fig. 9). The influence of the magnetic field was implemented in the theory in a later work by the same authors [19]. The cooling force for a magnetized electron beam was derived, almost at the same time, also by Rosenbluth [20], and later by Bell [21] and by Ogino and Ruggiero [22]. More recently, the classical binary-collision model was abandoned in favour of a continuum description implementing the collective response of the electrons [23]. This description was also partially used in Ref. [20]. In the following, first the original approach based on binary collisions (Novosibirsk Group) [18, 19] is considered, and then the infinite plasma description of Sørensen and Bonderup [23] is explained. However, before we come to the binary-collision model, it will be helpful to discuss in more detail the impact parameter ranges.

3.2 The impact parameter regions relevant to electron cooling

Two classes of limiting impact parameters are of importance. To the first class belong those that are ion-velocity-independent. The others are derived from collision times and therefore depend on the ion velocity. The scaling of the various impact parameters is shown in Fig. 10. One can distinguish the velocity-independent limits as horizontal lines, r_0 (electron beam radius), λ_D (Debye length), and $r_{e,c}$ (gyration radius). The scales in Fig. 10 correspond to the standard case. Relevant times are

$$t_{\text{cooler}} = \ell/\beta_0 c \quad \text{time it takes for an ion to pass through the cooler,} \quad (44a)$$

$$t_c = 2\pi/\omega_{e,c} \quad \text{cyclotron period,} \quad (44b)$$

$$t_{\text{pl}} = 2\pi/\omega_{e,\text{pl}} \quad \text{Langmuir period.} \quad (44c)$$

Impact parameters are obtained by multiplying the collision times by the corresponding velocities. The relative electron-ion velocity is $\mathbf{u} = \mathbf{v}_i - \mathbf{v}_e$. The electrons are bound to the magnetic field lines (this effect is frequently called magnetization of the electrons) as mentioned before (magnetic field parallel to ion trajectories in the laboratory system). Their transverse motion is frozen in gyrations. For collisions with impact parameters larger than the gyration radius, the transverse electron velocity does not appear, and in this case $\mathbf{u}$ can be replaced by $\mathbf{u}_{\text{ad}}$

$$\mathbf{u}_{\text{ad}} = \mathbf{v}_i - \mathbf{v}_{e\parallel} = \begin{bmatrix} v_{i\perp} \\ v_{i\parallel} - v_{e\parallel} \end{bmatrix}. \quad (45)$$

The characteristic impact parameters for these kinds of collisions are

$$b_{\text{cooler}} = u_{\text{ad}} t_{\text{cooler}}, \quad (46a)$$

$$b_{\text{mag}} = u_{\text{ad}} t_c, \quad (46b)$$

$$b_{\text{ad}} = u_{\text{ad}} t_{\text{pl}} \approx \lambda_{\text{ad}}. \quad (46c)$$

They are shown in Fig. 10 by straight lines running from the lower right corner to the upper left corner. Finally, we have the minimal impact parameter b_{min} [Eq. (29)]:

$$b_{\text{min}} = Zr_e \left(\frac{c}{u} \right)^2. \quad (47)$$

For large velocities $v_i \gg \Delta_{\perp} = \Delta_e$ it scales as $u^{-2} \approx v_i^{-2}$. It would rise continuously for decreasing v_i if the electrons were at rest. Since they have finite temperatures $T_{e\perp}$ and $T_{e\parallel}$, the minimal impact parameter has, for $v_i < \Delta_{\perp}$, the value obtained by inserting kT in Eq. (47):

$$b_{\text{min}} = Zr_e c^2 \begin{cases} v_i^{-2} & \text{for } u \gg \Delta_{\perp}, \\ \frac{m_e}{2k\sqrt{T_{e\perp}T_{e\parallel}}} = \frac{1}{\Delta_{\perp}\Delta_{\parallel}} & \text{for } \Delta_{\parallel} < v_i < \Delta_{\perp}, \\ \frac{m_e}{2kT_{e\parallel}} = \frac{1}{\Delta_{\parallel}^2} & \text{for } v_i < \Delta_{\parallel}. \end{cases} \quad (48)$$

The choice of the relevant temperature in the range $v_i < \Delta_{\perp}$ is debatable; sometimes $T_{e\perp}$ is used alone. Note that the minimal impact parameter depends on the charge number Z of the ion. For high ion velocities it reaches values of the order of $a_{0,n}$, which is the Bohr radius of an atomic orbit with principal quantum number n :

$$a_{0,n} = n^2 \hbar c / (Z\alpha m_e c^2) \quad (49)$$

($\hbar = h/2\pi$ is the Planck constant and α is the fine-structure constant).

Also in Fig. 10, practical limits are indicated by vertical lines. The maximum velocity that the ions can have is given by the velocity spread accepted by the storage ring. It is of the order of 2% of the total velocity $\beta_0 c$ of the ions. The probable lower limit is given by the thermal equilibrium, which means that for the longitudinal temperature, $T_{i\parallel} \geq T_{e\parallel} \rightarrow \sigma_{i\parallel} \geq \Delta_{\parallel} \sqrt{m_e/m_i}$.

So far there has been no evidence of a dependence of the frictional force on the product of the charge signs of the particles involved. Owing to the magnetization of the electrons, there should be a sign-dependent effect [24], as will be illustrated in the following. For simplicity, consider the magnetized electrons (infinitely strong magnetic field) as particles fixed on a string (parallel to the magnetic field lines) and able to move only forward or backward along the string (parallel to the longitudinal axis). In Fig. 11a the situation is sketched for an electron scattering from a positive ion. The ion is considered to be at rest. The electron comes in from below and gains energy owing to the electric force component parallel to its trajectory. The electron velocity is indicated by small arrows, and the force that it exerts on the ion, by thick arrows. When the electron has passed the ion, it again loses energy until it reaches its initial velocity.

The situation for the scattering from a negative ion (Fig. 11b) is different because a slow electron loses energy as it approaches the ion, comes to rest, and runs back down the potential hill to pass its original position 1 again. In this case, the force exerted on the ion points always in the positive longitudinal direction. It can be seen that the action exerted on the ion in the longitudinal direction cancels for a positive ion but adds up for a negative one. Hence, in the latter case, additional momentum is transferred in the longitudinal direction. This effect will be discussed later in more detail.

So far we have not considered effects due to the electron cloud around an ion of a partially stripped atom. This will be done in Section 5. The Rutherford cross-section [Eq. (24)] $\Delta\sigma = \pi b^2$ has to compete with other reaction cross-sections (mainly inelastic) for partially stripped ions having comparable or even higher values. This may modify the cooling of such ions.

3.3 The binary-collision model of Novosibirsk

The fact that the electrons are magnetized has led to a division of the collisions into two classes. Depending on the value of the impact parameter, they are distinguished as

$$\text{fast collisions} \quad b < \min(r_{e,c}, b_{\text{mag}}), \quad (50a)$$

$$\text{adiabatic collisions} \quad b > \max(r_{e,c}, b_{\text{mag}}). \quad (50b)$$

Fast collisions are binary collisions with collision times much smaller than the inverse cyclotron frequency. Hence the electrons can be considered as being free. The situation is equivalent to what has already been discussed. In adiabatic collisions, however, the electrons perform many gyrations during the collision time. The effect of their transverse motion cancels and only the longitudinal motion matters. The longitudinal velocity spread $\Delta_{\parallel}$ is much smaller than $\Delta_{\perp}$, resulting in an increase of the absolute value of the force (above the usual maximal value at $v_i = \Delta_{\perp}$) for decreasing v_i until $v_i = \Delta_{\parallel}$. The extremes—fast and adiabatic collisions—can be treated reasonably well, but the theory for collisions with $b \approx r_{e,c}$ is rather unsatisfactory. The authors of Ref. [19] suggested decomposing the force into a non-magnetic component F^{NM} (fast collisions) and a magnetic or adiabatic part F^{ad} :

$$\mathbf{F} = \mathbf{F}^{\text{NM}} + \mathbf{F}^{\text{ad}}. \quad (51)$$

The non-magnetic force is identified with Eq. (33a). The areas of both force contributions are indicated in Fig. 10. The Coulomb logarithm for the non-magnetic force L_C^{NM} is obtained by taking the maximal [Eq. (50a)] and the minimal [Eq. (48)] impact parameters, as indicated in Fig. 10. The magnetic force contribution is readily derived [19] for $v_i \gg \Delta_{\parallel}$ when replacing $\mathbf{u}$ by $\mathbf{u}_{\text{ad}}$ in Eq. (33b) and applying relation (27):

$$\left. \begin{aligned} F_{\#}^{\text{ad}}(v_i) &= -\frac{1}{2} F_0 c^2 L_C^{\text{ad}} \int \frac{\partial}{\partial u_{\text{ad}\parallel}} \left[f(v_e) \frac{u_{\text{ad}}^2}{u_{\text{ad}}^3} \right] d^3 v_e, \\ &= -\frac{1}{2} F_0 c^2 L_C^{\text{ad}} v_{i\perp} \int \frac{u_{\text{ad}\perp} u_{\text{ad}\#}}{u_{\text{ad}}^3} \frac{\partial f(v_e)}{\partial v_{e\parallel}} d^3 v_e \end{aligned} \right\} v_i \gg \Delta_{\parallel}, \quad (52a)$$

where $\# = \parallel$ or $\perp$,

$$D_{mn}^{\text{ad}}(v_i) = m_e F_0 c^2 L_C^{\text{ad}} \int \frac{u_{\text{ad}}^2 \delta_{mn} - u_{\text{ad}m} u_{\text{ad}n}}{u_{\text{ad}}^3} f(v_e) d^3 v_e. \quad (52b)$$

Note that in Eq. (52b) the integration over $v_{e\perp}$ is straightforward since the integrand depends only on $v_{e\parallel}$.

The evaluation of the integrals for $f(v_{e\parallel}) = \delta(v_{e\parallel})$ leads [6c, 19] finally to

$$F_{\parallel}^{ad}(v_i) = -1/2 F_0 c^2 L_C^{ad}(u_{ad}) 3 \frac{v_{i\perp}^2}{v_i^2} \frac{v_{i\parallel}}{v_i^3}, \quad (53a)$$

$$F_{\perp}^{ad}(v_i) = -1/2 F_0 c^2 L_C^{ad}(u_{ad}) \frac{v_{i\perp}^2 - 2v_{i\parallel}^2}{v_i^2} \frac{v_{i\perp}}{v_i^3}, \quad (53b)$$

$$D_{nm}^{ad}(v_i) = m_e F_0 c^2 L_C^{ad}(u_{ad}) \frac{v_i^2 \delta_{mn} - v_{im} v_{in}}{v_i^3}, \quad (53c)$$

The maximal and minimal impact parameters for the Coulomb logarithm of adiabatic collisions L_C^{ad} can again be taken from Fig. 10 [Eq. (50b) for $b_{\min}$ and Eqs. (46) or (10b) for $b_{\max}$].

Equations (53a) and (53b) can be written (Appendix II) in cylindrical coordinates:

$$F_{\perp}^{ad} = -|F^{ad}| \cos \phi_F^{ad}, \quad F_{\parallel}^{ad} = -|F^{ad}| \sin \phi_F^{ad}, \quad (53d)$$

with

$$\tan \phi_F^{ad} = \frac{2 \cos^2 \phi_v - \sin^2 \phi_v}{3 \cos \phi_v \sin \phi_v} \approx \tan \left[\frac{\pi}{2} \left(1 - \frac{\phi_v}{\arccos 1/\sqrt{3}} \right) \right], \quad \tan \phi_v = \frac{v_{i\perp}}{v_{i\parallel}}, \quad (53e)$$

$$|F^{ad}| = -\frac{1}{2} F_0 \frac{c^2}{v_i^2} L_C^{ad} \sin \phi_v \sqrt{(\cos^2 \phi_v + 1)^2 + 9 \cos^2 \phi_v \sin^2 \phi_v}. \quad (53f)$$

As can be seen from Eqs. (53), the magnetic force and the diffusion coefficient do not depend on the transverse electron temperature [for $v_i \gg \Delta_{\parallel}$ and $f(v_{e\parallel}) = \delta(v_{e\parallel})$]. The individual force components now depend on *both* the longitudinal and the transverse component of the ion velocity in a different functional way. This provokes a coupling of the motion in the different planes. Moreover, the transverse component of the force has the unusual feature that it changes sign at $v_{i\perp} = \sqrt{2}v_{i\parallel}$, corresponding to an angle ϕ_v of 54.7° formed by the velocity vector with the direction of the magnetic field. For larger angles, the ion will be accelerated transversely (Fig. 12a) instead of being slowed (transverse heating). Furthermore, the longitudinal force component is rather small, owing to the velocity difference in the nominator, and it vanishes for $v_{i\perp} \rightarrow 0$.

Using a similar procedure, the magnetic force was derived [6c, 19] for $v_i < \Delta_{\parallel}$, replacing Δ_e in Eqs. (38) by $\Delta_{\parallel}$ and using a prolate electron distribution as in Eq. (42), respectively:

$$F_{\perp}^{ad}(v_i) = -1/\sqrt{2\pi} F_0 c^2 \ln(\Delta_{\parallel}/v_{i\perp}) L_C^{ad}(\Delta_{\parallel}) v_{i\perp}/\Delta_{\parallel}^3, \quad (54a)$$

$$F_{\parallel}^{ad}(v_i) = -1/\sqrt{2\pi} F_0 c^2 L_C^{ad}(v_{i\perp}) v_{i\parallel}/\Delta_{\parallel}^3. \quad (54b)$$

The longitudinal force component for both regions ($v_i \ll \Delta_{\parallel}$ and $v_i \gg \Delta_{\parallel}$) can be approximated by

$$F_{\parallel}^{ad} \approx -9F_0 c^2 L_C^{ad}(u_{ad}) \frac{v_{i\perp}^2 v_{i\parallel}}{6|v_i|^5 + \Delta_{\parallel}^3 v_{i\perp}^2}. \quad (54c)$$

Its dependence on the velocity components (Fig. 12b) differs very much from the spherical symmetric case of the non-magnetic force (Fig. 8). The adiabatic force attains a well-localized maximum in the $v_{i\parallel}, v_{i\perp}$ plane, in contrast to the ridge in Fig. 8.

With Eqs. (38) to (54) we have obtained a viable description of electron cooling in the presence of a longitudinal magnetic field. Whilst the derivation of the frictional force for vanishing magnetic field is straightforward, for the case of a finite magnetic field it is obtained only through the strict division of the collisions into two categories characterized by the impact parameter cut-offs. This is not very satisfactory since the region around $b \approx r_{e,c}$ is certainly not treated correctly. Moreover, the force is derived on the basis of a single-particle model, where the collective effects of the electron plasma are neglected. These are the principal weak points of the original theory. In practice, the ion velocities range around the transverse electron velocity, and the contributions from scatterings with impact parameters around the gyration radius are of particular importance. For the quantitative prediction of the balance of heating processes and of electron cooling, and for the achievable equilibrium values, it is necessary to have a reliable model for low ion velocities. The cooling of highly charged ions also calls for collective effects to be taken into account, since the electron gas will be strongly polarized in the presence of a highly charged ion. The model that will be discussed next permits us to overcome the weaknesses of the previous one, at the cost of giving up an analytical derivation of the force.

3.4 The dielectric plasma description of electron cooling

The previous derivation of the frictional force within the binary-collision model is briefly summarized. The force was identified with the energy loss accumulated by the ion when passing repeatedly through the electron gas. The energy loss averaged over the electron velocity distribution was obtained from the momentum transfer integrated over the impact parameter range. Finally, the momentum transfer was derived from the Coulomb force $\mathcal{E}$ integrated over the interaction time (the arrow ' $\Leftarrow$ ' is used here to indicate the sequence of the derivation of the expressions):

$$\mathbf{F}(\mathbf{v}_i) \Leftarrow \iint dE/dx f(\mathbf{v}_e) db d^3\mathbf{v}_e \Leftarrow \iint (\Delta p)^2 f(\mathbf{v}_e) db d^3\mathbf{v}_e \Leftarrow \iint (\int \mathcal{E} dt) f(\mathbf{v}_e) db d^3\mathbf{v}_e .$$

(Strictly speaking, Derbenev and Skrinksky [18] started from the kinetic equation and derived the force and the diffusion coefficient.)

In the dielectric description the force is derived from the electric field $\mathcal{E}_{ind} (= \mathcal{E}_{tot} - \mathcal{E}_i)$ induced by the ion in the plasma and acting back on it [23]. Instead of considering the electric field, the corresponding potential ϕ [$\mathcal{E} = \text{grad}_r \phi(r,t)$] is used. Since the ion moves with velocity $\mathbf{v}_i$, the potential varies rapidly with time and place. It is therefore reasonable to consider the Fourier transform of the potential $\phi(r,t) \Rightarrow \phi(\mathbf{k},\omega)$. The total potential can be derived from the induced potential with the help of the dielectric function $\epsilon(\mathbf{k},\omega)$: $\phi_{tot}(\mathbf{k},\omega) = \phi_{ind}(\mathbf{k},\omega)/\epsilon(\mathbf{k},\omega)$. The induced potential is nothing else but the Fourier transform of the ion charge density: $Ze \delta(\mathbf{r} - \mathbf{v}_i t) \Rightarrow Ze \delta(\mathbf{k}\mathbf{v}_i - \omega)$. The problem is solved by finding the appropriate dielectric function, integrating it over $\mathbf{k}$ and ω , and solving the Poisson equation:

$$\begin{aligned} \mathbf{F}(\mathbf{v}_i) &= Ze \mathcal{E}_{ind} \Leftarrow Ze \text{grad}_r [\phi_{tot}(r) - \phi_{ind}(r)] \\ &\Leftarrow Ze \text{grad}_r \int e^{i(\mathbf{k}\mathbf{r} + \omega t)} [\phi_{tot}(\mathbf{k},\omega) - \phi_{ind}(\mathbf{k},\omega)] d^3\mathbf{k} d\omega \\ &\Leftarrow i Ze \int Ze \delta(\mathbf{k}\mathbf{v}_i - \omega) e^{i(\mathbf{k}\mathbf{v}_i + \omega)t} [1/\epsilon(\mathbf{k},\omega) - 1] \mathbf{k}/k^2 d^3\mathbf{k} d\omega . \end{aligned}$$

The complete formula for the force is given by [23]

$$\mathbf{F}(\mathbf{v}_i) = \text{Re} \left\{ -\frac{iZ^2e^2}{4\pi\epsilon_0 2\pi^2} \int \delta(\mathbf{k}\mathbf{v}_i - \omega) e^{i(\mathbf{k}\mathbf{v}_i + \omega)t} \left[\frac{1}{\epsilon(\mathbf{k}, \omega)} - 1 \right] \frac{\mathbf{k}}{k^2} d^3\mathbf{k} d\omega \right\} \quad (55)$$

and evaluated at $-\omega = \mathbf{k}\mathbf{v}_i$:

$$\mathbf{F}(\mathbf{v}_i) = -\frac{Z^2e^2}{4\pi\epsilon_0 2\pi^2} \int \text{Im} \left[\frac{1}{\epsilon(\mathbf{k}, -\mathbf{k}\mathbf{v}_i)} \right] \frac{\mathbf{k}}{k^2} d^3\mathbf{k} . \quad (56)$$

(The dielectric description was also applied in recent theoretical works of the Novosibirsk group [24b,c].)

As mentioned before, the moving ion can be considered as an electromagnetic wave travelling through the plasma. The plasma response is contained in the dielectric function. The calculation of the frictional force is equivalent to the derivation of the loss power dW/dt , which the electromagnetic wave transfers to the plasma [$\mathbf{F}(\mathbf{v}_i) = dW/dt$]. An incoming electromagnetic wave would excite the plasma most strongly at the plasma frequency, so a strong contribution to the integral is expected at $\omega = \omega_{e,pl}$. In a magnetically confined plasma, one should, moreover, get contributions to the integral at the cyclotron frequency $\omega_{e,c}$ and its higher harmonics.

In order to give an example, the above integral is evaluated for an *electron plasma at rest and for* $B = 0$. The dielectric function for this case [Eq. (19)] reads as

$$\epsilon(\mathbf{k}, \omega) = 1 - \lim_{\nu \rightarrow 0} \frac{\omega_{e,pl}^2}{\omega^2 - i\omega\nu} . \quad (57)$$

The quantity ν is introduced to provide a small damping and to keep $1/\epsilon$ finite at $\omega = \omega_{e,pl}$ (plasma pole). By noting that

$$\lim_{c \rightarrow 0} \frac{c}{(a-b)^2 + c^2} = \pi \delta(a-b) , \quad (58)$$

we obtain $\text{Im } 1/\epsilon = \pi\omega_{e,pl}^2/\omega$. With $d^3\mathbf{k} = d\phi d\cos\theta k^2 dk$, $\mathbf{k}\mathbf{v}_i = kv_i \cos\theta = -\omega$, and $d\cos\theta = d\omega/kv_i$, the integral [Eq. (56)] becomes

$$\mathbf{F}(\mathbf{v}_i) = -\frac{Z^2e^2}{4\pi\epsilon_0 \pi v_i^2} \iint \omega \text{Im} [1/\epsilon(\mathbf{k}, -\mathbf{k}\mathbf{v}_i)] d\omega (1/k) dk , \quad (59a)$$

which can also be written as

$$\mathbf{F}(\mathbf{v}_i) = -F_0 \frac{c^2}{v_i^2} \int_0^\infty I(k) \frac{dk}{k} \quad (59b)$$

[for F_0 , see Eq. (31c)]. For the case of *electrons at rest*, the integral over the dielectric function,

$$I = 1/(\pi\omega_{e,pl}^2) \int \omega \text{Im} [1/\epsilon(\mathbf{k}, -\mathbf{k}\mathbf{v}_i)] d\omega , \quad (60)$$

yields only a contribution from the plasma pole:

$$I = \frac{1}{\pi\omega_{e,pl}^2} \int \frac{\pi\omega_{e,pl}^2}{\omega} \delta(\omega - \omega_{e,pl}) \omega d\omega = 1 . \quad (61)$$

Since the dielectric function is evaluated at $kv_i \cos \theta = -\omega$ and the condition $\omega = \omega_{e,pl}$ (δ -function) holds, $I(k)$ disappears for decreasing k at $k = \omega_{e,pl}/v_i = 1/b_{ad}$. This reflects the adiabatic screening of the ion charge. The upper limit of the integration over k in Eq. (59b) is given by the minimal impact parameter equal to that given by classical backscattering [Eq. (48)]. Hence we get:

$$F(v_i) = -F_0 \frac{c^2}{v_i^2} \int_0^{k_{max}} I(k) \frac{dk}{k} = -F_0 \frac{c^2}{v_i^2} L_C. \quad (62)$$

The expression for the force is equivalent to what was derived in the binary-collision model for electrons at rest [Eq. (31a)]. The electron temperature is implemented by modifying the dielectric function correspondingly [23]. It again reproduces the results of the binary-collision model. The more interesting case is the implementation of the magnetic confinement of the electron motion. Let us reconsider it for an *electron gas at rest* and a *finite magnetic field*. The dielectric function for this case yields

$$\epsilon(k, \omega) = 1 - \lim_{\nu \rightarrow 0} \frac{\omega_{e,pl}^2}{k^2} \left(\frac{k_{\parallel}^2}{\omega^2 + i\nu\omega} + \frac{k_{\perp}^2}{\omega^2 - \omega_{e,c}^2 + i\nu\omega} \right). \quad (63)$$

In addition to the plasma frequency, now also the cyclotron frequency appears. As before, the quantity I can be derived [23] from the dielectric function, and is plotted in Fig. 13 versus ζ . The variable ζ and the components of I and F are defined by

$$F_{\perp, \parallel}(v_i) = -F_0 \frac{c^2}{\pi v_i^2} \int I_{\perp, \parallel}(\zeta) d\zeta, \quad (64a)$$

$$\zeta = \ln(kv_i/\omega_{e,c}) \quad \text{for} \quad v_i = v_{i\perp}, \quad (64b)$$

$$\zeta = \ln(k_{\perp}v_i/\omega_{e,c}) \quad \text{for} \quad v_i = v_{i\parallel}. \quad (64c)$$

Note that ζ contains the ion velocity. In Fig. 13a,b two regions are distinguished: one region for large ζ (small impact parameters), where I reaches the value of 1 (as before, no magnetic field), and the other with $\zeta < 0$. In the latter case there are contributions from the plasma pole only. In the transverse direction the plasma pole contributes 50% to I , even when $\zeta > 0$. The clear division into these two regions, which one assumes to be valid for the binary-collision model, seems justified for the electron gas at rest.

When allowing for a *finite electron temperature*, the calculation, performed in a similar way [23] as before, yields shapes for I as shown in Fig. 13c,d. There we have $\zeta' = \ln(kr_{e,c})$ for $v_i = v_{i\perp}$ and $\zeta' = \ln(k_{\perp}r_{e,c})$ for $v_i = v_{i\parallel}$. The sharp structures apparent in Fig. 13a,b are now washed out. The asymptotic behaviour of I for large k is still visible, which agrees with what is obtained from the binary-collision model for small impact parameters. So the frictional force can be split up into a contribution from the plasma pole for small k values (shown by the dotted line) and a contribution from the binary collisions. The latter can be taken directly from the binary-collision model. Hence for the case of a finite electron temperature and a finite magnetic field the force can be approximated by

$$F(v_i) = -F_0 \frac{c^2}{v_i^2} \int I^{\omega_{e,pl}} d\zeta' + F^{NM}, \quad (65)$$

where the integration is stopped at $k = \omega_{e,c}/v_i = 1/b_{\text{mag}}$. The two contributions to $\mathbf{F}$ (truncated plasma pole $\mathbf{F}^{\omega_{e,p}}$ and non-magnetic binary force $\mathbf{F}^{\text{NM}}$) in Eq. (65) are marked in Fig. 13c,d by the hatched areas.

The dielectric description has confirmed the results obtained with the binary-collision model in the absence of a magnetic field and taking the velocity spread of the electrons into account. When a magnetic field is present, it allows collisions with arbitrary impact parameters to be treated correctly. However, also in this case it appears that the frictional force can be approximated by the sum of two contributions: one is essentially the non-magnetic force component from the binary-collision model; the other is the contribution of the plasma pole truncated at the impact parameter b_{mag} which is larger than the gyration radius $r_{e,c}$. The dielectric description provides precise results, but in most of the cases it requires numerical methods to obtain the frictional force, whilst in the case of the binary-collision model we have the force in an analytical form. Furthermore, the dielectric description implements the plasma properties of the magnetically confined electron gas, which is of particular interest when describing the cooling of highly charged ions.

3.5 Difference in cooling of positive and negative ions by magnetized electrons

In subsection 3.2 the scattering of magnetized electrons from negative and positive charges was illustrated qualitatively (Fig. 11). An additional momentum transfer arises when electrons are reflected from negative ions. This happens when the kinetic energy of the electrons is smaller than the repulsive potential at the distance of closest approach b_0 ,

$$\frac{m_e u_{\parallel}^2}{2} \gtrless \frac{Ze^2}{4\pi\epsilon_0 b_0}. \quad (66a)$$

The momentum transfer in collisions where this condition is fulfilled is always $\Delta p_{\parallel} = 2m_e u_{\parallel}$. The additional contribution to the force is obtained by integrating the momentum transfer over the relevant impact parameter range [in analogy to Eq. (26a)]:

$$\Delta F_{\parallel}^{-} = - \int 2\pi \int_0^{b_0} n_e b \frac{4m_e^2 u_{\parallel}^2}{2m_e} db f(v_e) d^3 v_e. \quad (66b)$$

After integration over b and elimination of b_0 with the help of Eq. (66a), we get

$$\Delta F_{\parallel}^{-} = -2F_0 c^2 \int \frac{f(v_e)}{u_{\parallel}^2} d^3 v_e. \quad (66c)$$

When comparing Eq. (66c) with Eq. (33a), we find

$$\frac{\Delta F_{\parallel}^{-}}{F^{\text{NM}}} \approx \frac{2}{L_C(v_i)}. \quad (66d)$$

This means that for low ion velocities ($L_C \approx 1$) the non-magnetic force can be up to three times larger for negative ions [$F_{\text{tot}} = F^{\text{NM}} + \Delta F_{\parallel}^{-} \approx 3F_0 c^2/v_i^2$] than for positive ions. However, one must pay attention that no double counting occurs, and therefore take the lower cut-off in L_C at $b_0 = 2b_{\text{min}}$ when evaluating F^{NM} .

The integral of Eq. (66b) was evaluated for the longitudinal frictional force component in Ref. [24b]. In Ref. [24c], a semiempirical formula for the total non-magnetic force was given for the force components parallel and transverse to the ion velocity for the case of an infinitely strong magnetic field. The force components transferred to our standard reference frame (rotation by the angle ϕ_v) are in cylindrical coordinates [cf. Eqs. (53d)]:

$$|F_{\text{emp}}^{\text{NM}}| = F_0 \frac{c^2}{v_i^2} L_C \sin \phi_v \sqrt{\frac{1}{4} + \frac{3}{4} \cos^2 \phi_v + \frac{1+\kappa}{L_C} \cos^4 \phi_v + \left(\frac{1+\kappa}{L_C}\right)^2 \left(\cos^8 \phi_v + \frac{\cos^{10} \phi_v}{\sin^2 \phi_v}\right)}, \quad (67a)$$

$$\text{tg } \phi_{F_{\text{emp}}} = \frac{\cos^2 \phi_v + 2 \frac{1+\kappa}{L_C} \cos^4 \phi_v + 1}{\cos^2 \phi_v + 2 \frac{1+\kappa}{L_C} \cos^4 \phi_v - 1} \text{tg } \phi_v, \quad (67b)$$

where

$$\sin \phi_v = \frac{v_{i\perp}}{v_i}, \quad \kappa = \frac{z \cdot Z}{|z \cdot Z|}.$$

The remarkable feature of this formula is that the absolute value of the non-magnetic force $|F_{\text{emp}}^{\text{NM}}|$ reaches a value greater than zero for vanishing transverse ion velocity ($\phi_v = 0$) and $\kappa = 1$ (electron cooling of negative ions or positron cooling of positive ions) owing to the last term in Eq. (67a). The behaviour of $|F_{\text{emp}}^{\text{NM}}|$ and $\phi_{F_{\text{emp}}}$ is illustrated in Fig. 14.

3.6 Summary of the theory

The frictional force and the diffusion coefficient were derived from a binary-collision model and from a dielectric of a plasma model. Two classes of electron-ion interactions were distinguished: the fast collisions (described by the non-magnetic force) and the adiabatic collisions (described by the magnetic or adiabatic force). These kinds of collisions are approximatively separated from each other by the value of the impact parameter (Fig. 10), which for fast collisions is smaller than the gyration radius or b_{mag} [see Eq. (46b)].

The general results from the binary-collision model are, for the *non-magnetic case*:

$$\mathbf{F}^{\text{NM}}(\mathbf{v}_i) = -F_0 c^2 \int L_C^{\text{NM}}(u) f(\mathbf{v}_e) \frac{\mathbf{u}}{u^3} d^3 \mathbf{v}_e, \quad \mathbf{u} = \mathbf{v}_i - \mathbf{v}_e, \quad (33a)$$

$$D_{mn}^{\text{NM}}(\mathbf{v}_i) = m_e F_0 c^2 \int L_C^{\text{NM}}(u) f(\mathbf{v}_e) \frac{u^2 \delta_{mn} - u_m u_n}{u^3} d^3 \mathbf{v}_e, \quad L_C(u) = \ln(b_{\text{ad}}/b_{\text{min}}), \quad (33b)$$

and for the *magnetic case*:

$$F_{\#}^{\text{ad}}(\mathbf{v}_i) = -1/2 F_0 c^2 L_C^{\text{ad}} v_{i\perp} \int \frac{u_{\text{ad}\perp} u_{\text{ad}\#}}{u_{\text{ad}}^3} \frac{\partial f(\mathbf{v}_e)}{\partial v_{e\parallel}} d^3 \mathbf{v}_e, \quad \mathbf{u}_{\text{ad}} = \mathbf{v}_i - \mathbf{v}_{e\perp}, \quad (52a)$$

$$D_{mn}^{\text{ad}}(\mathbf{v}_i) = m_e F_0 c^2 L_C^{\text{ad}} \int \frac{u_{\text{ad}}^2 \delta_{mn} - u_{\text{ad}m} u_{\text{ad}n}}{u_{\text{ad}}^3} f(\mathbf{v}_e) d^3 \mathbf{v}_e, \quad L_C^{\text{ad}}(u_{\text{ad}}) = \ln(b_{\text{max}}/b_{\text{ad}}) \quad (52b)$$

(see Fig. 10),

where $\# = \parallel$ or $\perp$. These equations could be further evaluated for particular electron velocity distributions and ranges of electron-ion velocities. They are presented below in a way that is analogous to Fig. 10.

$$\begin{array}{c}
\begin{array}{l}
F_{\parallel}^{\text{ad}} \approx -9F_0c^2L_C^{\text{ad}}(u_{\text{ad}}) \frac{v_{i\perp}^2 v_{i\parallel}}{6|v_i|^5 + \Delta_{\parallel}^3 v_{i\perp}^2}, \quad (54c) \\
F_{\#}^{\text{ad}} = -\frac{F_0c^2}{\sqrt{2\pi}} \frac{v_{i\#}}{\Delta_{\parallel}^3} c_{\#}^{\text{ad}} \quad (54a,b) \\
F_{\parallel}^{\text{ad}}(v_i) = -1/2 F_0c^2L_C^{\text{ad}}(u_{\text{ad}}) 3 \frac{v_{i\perp}^2}{v_i^2} \frac{v_{i\parallel}}{v_i^3}, \quad (53a) \\
F_{\perp}^{\text{ad}}(v_i) = -1/2 F_0c^2L_C^{\text{ad}}(u_{\text{ad}}) \frac{v_{i\perp}^2 - 2v_{i\parallel}^2}{v_i^2} \frac{v_{i\perp}}{v_i^3}, \quad (53b)
\end{array} \\
\hline
\begin{array}{l}
F_{\#}^{\text{NM}} = -2 \frac{F_0c^2}{\sqrt{2\pi}} L_C^{\text{NM}}(\Delta_{\#}) \frac{v_{i\#}}{\Delta_{\#} \Delta_{\perp}^2} c_{\#}^{\text{NM}} \quad (43c) \\
F_{\#}^{\text{NM}} \approx -\frac{F_0c^2}{\Delta_{\perp}^2} \begin{cases} \sqrt{\frac{\pi}{8}} L_C^{\text{NM}}(\Delta_{\perp}) \frac{v_{i\perp}}{\Delta_{\perp}} & \text{for } \# = \perp \\ L_C^{\text{NM}}(v_{i\parallel}) - \sqrt{\frac{\pi}{2}} L_C^{\text{NM}}(\Delta_{\perp}) \frac{v_{i\parallel}}{\Delta_{\perp}} & \text{for } \# = \parallel \end{cases} \quad (43c) \\
F^{\text{NM}} \approx -F_0c^2L_C^{\text{NM}}(v_i) \frac{v_i}{v_i^3} \quad (43d) \\
F^{\text{NM}} = -F_0c^2L_C^{\text{NM}} \left\{ \text{erf}\left(\frac{v_i}{\Delta_c}\right) - \frac{2}{\sqrt{\pi}} \frac{v_i}{\Delta_c} e^{-v_i^2/\Delta_c^2} \right\} \frac{v_i}{v_i^3} \approx -\frac{3}{2} F_0c^2L_C^{\text{NM}} \frac{v_i}{|v_i|^3 + 2\Delta_c^2} \quad (\text{spherical}) \quad (38) \\
\end{array}
\end{array}$$

$\Delta_{\parallel}$ $\Delta_{\perp}$ v_i

where

$$c_{\#}^{\text{ad}} = \begin{cases} L_C^{\text{ad}}(v_{i\perp}) & \text{for } \# = \parallel \\ L_C^{\text{ad}}(\Delta_{\parallel}) \ln(\Delta_{\parallel}/v_{i\perp}) & \text{for } \# = \perp \end{cases} \quad c_{\#}^{\text{NM}} = \begin{cases} 1 & \text{for } \# = \parallel \\ \pi/4 & \text{for } \# = \perp \end{cases}$$

and

$$F_0 = 4\pi Z^2 n_e r_e^2 m_e c^2. \quad (31c)$$

It must be ensured that the Coulomb logarithms are positive. The modulus of the longitudinal force component from the binary-collision model [Eqs. (43), (53) and (54)] is plotted in Fig. 15, showing the dominating role of the magnetic force at low ion velocity.

From the dielectric description, we obtained the general expression for the frictional force, which is valid for all types of collisions such as

$$\mathbf{F}(v_i) = -F_0 \frac{c^2}{v_i^2} \int_0^{k_{\text{max}}} \mathbf{I}(\mathbf{k}) \frac{d\mathbf{k}}{k} \quad (62)$$

with

$$\mathbf{I} = 1/(\pi\omega_{e,p}^2) \int \omega \text{Im}[1/\epsilon(\mathbf{k}, -\mathbf{k}v_i)] d\omega. \quad (60)$$

Equation (62) contains all contributions from $k_{\text{max}} = 1/b_{\text{min}}$ and $k_{\text{min}} = 1/b_{\text{ad}}$ (intrinsic cut-off). The adiabatic contributions are projected out by cutting off the integral at $k_{\text{max}} = 1/b_{\text{mag}}$:

$$\mathbf{F}^{\text{ad}}(\mathbf{v}_i) = -F_0 \frac{c^2}{v_i^2} \mathbf{v}_i \int_0^{1/b_{\min}} \mathbf{I}(\mathbf{k}) \frac{d\mathbf{k}}{k}.$$

For computational reasons it is useful to evaluate this integral numerically and to complement it with the non-magnetic force derived in the binary-collision model.

Finally, it was shown that in the case of electron (positron) cooling of negative (positive) ions the non-magnetic force has to be complemented with an additional term ΔF^- accounting for collisions where the electrons (positrons) are reflected. The additional contribution is approximately

$$\Delta F_{\parallel}^- \approx \frac{2}{L_c(v_i)} F^{\text{NM}}. \quad (66d)$$

The diffusion coefficients were obtained in a similar way. The force and the diffusion tensor are related by

$$\mathbf{F}_n = \frac{1}{2m_e} \frac{\partial D_{mn}}{\partial v_{i_m}}. \quad (27)$$

4. EVOLUTION OF THE DISTRIBUTION FUNCTION, THE COOLING TIME AND DECREMENT, AND THE THERMAL EQUILIBRIUM

In the previous section, the frictional force and the diffusion coefficient that govern the cooling process were derived. As a result of cooling, the distribution of particles in phase space is changed. The phase-space density is increased; the temperature of the ion plasma is reduced. The evolution of the phase-space population is described by the distribution function (subsection 2.3). Its time dependence is obtained by solving the Fokker-Planck equation [Eq. (17)]. The expressions for the force and the diffusion coefficient are usually complicated, implying that the Fokker-Planck equation has to be solved numerically in most cases. Also, force and diffusion may depend on the distribution itself. An important case can, however, be solved analytically: namely, when the ion beam has a Gaussian velocity distribution of such a low width that the cooling force is directly proportional to the ion velocity [e.g. Eqs. (43c) and (54)]. This case, which is relevant for thermal equilibrium, will be discussed below. We also assume that $B = 0$. Let us recall the velocity distribution and the Fokker-Planck equation given in Section 2:

$$f(\mathbf{v}) = \exp -[(v_x^2/\sigma_{v_x}^2) + (v_y^2/\sigma_{v_y}^2) + (v_z^2/\sigma_{v_z}^2)]/(\sigma_{v_x}\sigma_{v_y}\sigma_{v_z}\pi\sqrt{\pi}), \quad (5)$$

$$\frac{\partial f}{\partial t} = \sum_{m,n} \frac{\partial \{-f(F/m) + [\partial(fD_{mn})/\partial v_n]\}}{\partial v_m}. \quad (17)$$

Now the widths of the distribution are time-dependent. Differentiation of f yields for the left-hand side of Eq. (17) (we now use v_i and m_i):

$$\partial f / \partial t = \sum_m (2v_{i_m}^2 / \sigma_{v_m}^2 - 1) f (d\sigma_{v_m} / dt) / \sigma_{v_m}, \quad m = x, y, z. \quad (68a)$$

Performing the differentiation on the right-hand side of Eq. (17) gives

$$\sum_{m,n} \frac{\partial}{\partial v_{i_m}} \left(-f \frac{F}{m_i} + \frac{\partial}{\partial v_{i_n}} [f D_{mn}] \right) = \sum_{m,n} \left(\frac{2v_{i_m}^2}{\sigma_{v_m}^2} - 1 \right) f \cdot Y + f v_i \frac{\partial Y}{\partial v_{i_n}},$$

where

$$Y = \frac{F_m}{m_i v_{i_m}} + 2 \frac{D_{mn}}{\sigma_{v_m}^2} - \frac{1}{v_{i_m}} \frac{\partial D_{mn}}{\partial v_{i_n}}. \quad (68b)$$

If we now assume that $F_m = -F_{m0} v_{i_m}$ ($F_{m0} = \text{const}$) and $D_{mn} = \text{const}$, the expression Y reduces to

$$Y = -F_{m0}/m_i + 2D_{mn}/\sigma_{v_m}^2, \quad (68c)$$

and from the Fokker-Planck equation we obtain a differential equation for σ_{v_m} :

$$\sum_m \frac{1}{\sigma_{v_m}} \frac{d\sigma_{v_m}}{dt} = \sum_{m,n} \left(-\frac{F_{m0}}{m_i} + 2 \frac{D_{mn}}{\sigma_{v_m}^2} \right). \quad (69)$$

If there is no coupling between the different planes ($D_{mn} = 0$ for $m \neq n$), Eq. (69) has to be satisfied by each component. It can be verified that $\sigma_{v_m}^2 = \sigma_{v_{m0}}^2 \exp(-2\lambda_m t) + \sigma_{v_{m\infty}}^2$ is a solution of Eq. (69):

$$(d\sigma_{v_m}/dt)/\sigma_{v_m} = \lambda_m \sigma_{v_{m\infty}}^2 / \sigma_{v_m}^2 - \lambda_m. \quad (70)$$

Under the above assumptions the width of the velocity distribution shrinks exponentially with the cooling decrement λ and reaches a final value $\sigma_{v_{m\infty}}$ given by

$$\lambda_m = F_{m0}/m_i \quad \text{and} \quad \sigma_{v_{m\infty}}^2 = 2D_{mm}/\lambda_m. \quad (71)$$

For a spherical Maxwellian electron velocity distribution we can use Eqs. (38) and (39) and get

$$F_{m0} = \frac{2}{\sqrt{\pi}} F_0 L_C \frac{c^2}{\Delta_e^3}$$

and

$$D_{mm} = D = m_e \frac{1}{\sqrt{\pi}} F_0 L_C \frac{c^2}{\Delta_e} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

In this case

$$\lambda_m = \lambda = \frac{2}{\sqrt{\pi}} \frac{F_0}{m_i} L_C \frac{c^2}{\Delta_e^3}$$

and

$$\frac{D}{\lambda} = \frac{m_e \Delta_e^2}{2m_i} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \frac{T_e}{m_i} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Since the ion-beam temperature is $1/2 m_i \sigma_{v_m}^2$, we get, in equilibrium ($t \rightarrow \infty$), $T_{i,m} = T_i = T_e$. It can be shown [23] that

$$D = \frac{T_e}{m_i} \lambda_{\perp} \begin{pmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

for a flattened electron velocity distribution. Therefore, under the above conditions, we obtain for the thermal equilibrium and for a flattened distribution, the longitudinal and transverse ion temperatures

$$T_{i\parallel} = T_e \lambda_{\perp} / \lambda_{\parallel} , \quad (72a)$$

$$T_{i\perp} = 1/2 T_e . \quad (72b)$$

In Ref. [23] it was shown that Eqs. (72) are also obtained for arbitrary initial distributions.

Also, for the case where exponential cooling is *not* realized, the notation of λ is retained. Guided by the above relations [Eq. (71)], we define it as

$$\lambda_{\perp, \parallel} = - \frac{F_{\perp, \parallel}}{p} . \quad (73a)$$

The inverse damping decrement is called the cooling time, $\tau_{\perp, \parallel} = 1/\lambda_{\perp, \parallel}$.

We have already seen that the cooling force does not depend on the ion mass; the decrement, however, depends on it inversely.

The above situation is realized only under specific conditions, namely when all heating processes are negligible, which means low beam intensity and ultra-high vacuum. In all other cases the situation is much more complicated and thermal equilibrium is not necessarily reached. Such cases will be discussed in Section 9.

We use the approximate formula for the non-magnetic cooling force [Eq. (41)] to discuss the qualitative features of the cooling time:

$$\begin{aligned} \tau = 1/\lambda &= (6\pi Z^2 r_i r_e c^4 n_e L_C)^{-1} (\sigma_{v_i}^3 + 2\Delta_e^3) \\ &= \sqrt{2} (3\pi Z^2 r_i r_e c n_e L_C)^{-1} \left[2 \left(\frac{kT_e}{m_e c^2} \right)^{3/2} + \left(\frac{kT_i}{m_i c^2} \right)^{3/2} \right] . \end{aligned} \quad (73b)$$

A comparison with Spitzer's formula [Eq. (21)] gives good agreement. He has added the temperatures and raised the sum to the 3/2 power. We have done it in just the inverse sequence, because of the better fit of the cooling force for a spherical distribution.

For $\sigma_{v_i} \gg \Delta_e$ the cooling is proportional to $\sigma_{v_i}^3$ (hot ion beam $\rightarrow$ slow cooling). On the other hand, for $\sigma_{v_i} \ll \Delta_e$ there is exponential damping, since τ is constant. Moreover, this simple model gives the following scaling behaviour:

$$\text{i) } \tau \text{ is independent of } \beta \text{ for } n_e = \text{const}; \quad (74a)$$

$$\text{ii) } \tau \text{ is independent of the ion intensity}; \quad (74b)$$

$$\text{iii) } \tau \propto 1/(r_i Z^2) \propto A/Z^2; \quad (74c)$$

$$\text{iv) } \tau \propto 1/n_e. \quad (74d)$$

Inserting standard values (Table AI.1) and $\sigma_{v_i} = \Delta_e$ in Eq. (73b), we obtain $\tau = 0.03$ s for protons. The cooling time observed in the laboratory is longer since the electron beam overlaps with the ion beam only for a small fraction of the ring circumference, as will be shown later [Eq. (154c)]. The cooling times for fully stripped ions decrease approximately as $1/Z$ (since $A \cong 2Z$). A better estimate for the cooling time can be obtained by using the relevant formulae for the frictional force and accounting for the magnetization and the prolate electron velocity distribution [Eqs. (53) and (54)].

The correct procedure for the determination of cooling times is to insert the cooling force and the diffusion coefficient in the Fokker-Planck equation and to solve it.

5. OTHER ELECTRON-ION INTERACTIONS

The electron cooling of ions is based on the repeated elastic scattering of ions from electrons:

The cooling beam can be considered as a pure electron target. The interaction of the projectile with this target gives rise to other reactions, which are the inelastic channels; the most dominant are

These are the processes that play a fundamental role in a two-component plasma of low density. For a denser plasma of low temperature, many-body processes are also important. Amongst them, the three-body recombination is of particular interest:

These reactions are not of immediate importance for the cooling mechanism. However, some of them are useful for diagnostic purposes. In particular situations they may have an impact on cooling, as we shall see later.

5.1 Spontaneous recombination

This process is important in any plasma. Its cross-section was calculated a long time ago for hydrogen-like ions [25]. The capture into an atomic state with principal quantum number n is given by [26]

$$\sigma_n = \frac{2\pi^2 r_e^2 E_a^2}{n E_e (E_a + n^2 E_e) \alpha} \quad (77)$$

where $E_a = (m_e c^2 / 2) Z^2 \alpha^2$ is the binding energy of the ground state and E_e is the electron energy. The velocity distribution of the particles in the plasma requires an averaging of the cross-section over these particular distributions. The average over the velocity distributions yields the recombination coefficient α_r :

$$\alpha_r = \int f(\mathbf{v}_e) f(\mathbf{v}_i) u \sum_{n=1}^{n_{\max}} \sigma_n d^3 \mathbf{v}_e d^3 \mathbf{v}_i, \quad (78a)$$

or for a cooled ion beam,

$$\alpha_r = \int f(\mathbf{v}_e) v_e \sum_{n=1}^{n_{\max}} \sigma_n d^3 \mathbf{v}_e. \quad (78b)$$

In the case of near thermal equilibrium, the velocity distribution of the ions can be neglected [Eq. (78b)]. The situation in the electron cooler can be understood as depicted in Fig. 16a.

The electrons populate the continuum just above the ionization threshold if the velocities are matched. The width of the continuum band is given by the electron temperature. Electrons will be captured preferentially in low-lying states. The integral can be evaluated for each summand for a spherical Maxwell and a flattened electron velocity distribution, and yields [27] for the first case

$$\alpha_r = 2\pi r_e^2 \alpha c^2 Z^2 \frac{\sqrt{\pi}}{\Delta_e} \sum_{n=1}^{n_{\max}} \frac{1}{n}. \quad (79)$$

When an ion with a laboratory velocity $v_0 = \beta_0 c$ passes through a magnetic dipole field of strength B , perpendicular to its trajectory, it is exposed to a motional electric field E_{mot} :

$$E_{\text{mot}} [\text{MV/m}] = 300 \beta_0 \gamma \cdot B_{\perp} [\text{T}]. \quad (80)$$

Since highly excited states are immediately stripped in this transverse electric field, the sum can be stopped at a value $n_{\max}$ (see subsection 13.1). For protons at 50 MeV a value of $n_{\max} = 4$ seems to be reasonable. One can express Δ_e by the electron temperature, and arrives at [27, 28]

$$\alpha_r = 1 \times 10^{-10} Z^2 (\text{cm}^3 \text{s}^{-1} \text{K}^{1/2}) T_e^{-1/2} = 9.3 \times 10^{-13} Z^2 (\text{cm}^3 \text{s}^{-1} \text{eV}^{1/2}) (\text{kT}_e)^{-1/2} \quad (81)$$

for bare ions. The number of down-charged atoms produced per time interval (rate R_a) is [28]

$$R_a = N_i \eta_c \alpha_r n_e \gamma^{-2}. \quad (82)$$

Here N_i is the number of stored ions and $\eta_c (= \ell/C_0)$ is the ratio of cooling length to ring circumference. The analysis of these atoms is of interest for two reasons. From the rate, we can deduce the temperature of the electrons in the region where the two beams overlap. On the other hand, the direction of motion of the atomic beam is equivalent to that of the ions in the cooling region. The down-charged ions can be separated from the primary ions because their trajectory is different, and their angular distribution can be analysed without disturbing the primary beam. In the case of single-charged stored ions, the electron capture produces neutral atoms that leave the ring tangentially and can be easily detected. In other cases, the down-charged ions will be less bent in the dipole magnets of the storage ring, and they will exit from the ring at a well-defined place. However, they will be subjected to fringe fields, which renders the reconstruction of the original distribution of the ion motion more difficult.

With an appropriate set-up close to the cooling section, also the photons from the spontaneous recombination can be measured. Depending on the direction of emission with respect to the beam, their frequency is Doppler-shifted to

$$h\nu_{\text{lab}} = \frac{h\nu}{(1 - \beta_0 \cos \theta) \gamma}. \quad (83)$$

By doing a precise measurement of the angle θ and a careful line-shape analysis it might be possible to determine the absolute velocity of the beam and to get information about the electron velocity distribution (see subsection 13.1). In previous electron-cooling experiments, the neutral channel has served as a very important means of diagnostics.

5.2 Dielectronic recombination

This reaction will only come into play if the circulating ions are not fully stripped. An electron is captured into an auto-ionizing state, and the energy gained is used to lift one of the core electrons to an excited state [29].

In order to maintain energy conservation, the energy of the incoming electron has to equal the energy of the auto-ionizing state. This resonance condition can be realized if the electrons are faster or slower than the ions (Fig. 16b). The resonance energy E_r is related to the particle laboratory velocities by

$$E_r = \frac{m_r}{2} \left(\frac{v_{i0} \pm v_{e0}}{1 \pm v_{i0}v_{e0}/c^2} \right)^2 = \frac{m_r}{2} v_r^2 \quad (84a)$$

[$m_r = m_i m_e / (m_i + m_e)$]. If this resonance condition is fulfilled, dielectronic recombination can take place. The cross-section attains very high values and can reach $\sigma_{dr} = 10^{-18} \text{ cm}^2$. The auto-ionizing states decay either back into the original channel or into a lower state. In the latter case a photon is emitted. The charge of the ion is changed if states below the ionization threshold are reached. Dielectronic recombination resonances are often narrow, and the evaluation of their width can eventually be used to deduce the effective temperature of the electron beam directly. As for spontaneous recombination, the down-charged ions and the recombination photons can be detected and used for beam diagnostics purposes. The advantage of the process lies in the high cross-section. It may be used when the number of stored ions is rather low and other means of non-destructive diagnostics fail.

The value of the dielectronic recombination cross-section depends on the particular state [30, 31]. However, the down-charge rate caused by dielectronic recombination can be estimated by taking an average value of $\sigma_{dr} \approx 10^{-19} \text{ cm}^2$ and neglecting the finite electron temperature:

$$R_{dr} = N_i \eta_c \sigma_{dr} \cdot v_r n_e \gamma^{-2}. \quad (84b)$$

The use of this process for diagnostics purposes requires the detuning of the electron energy, unless auto-ionizing states fall into the continuum band populated by the velocity-matched electron beam.

5.3 Ion excitation

Partly-stripped ions that circulate in the storage ring are usually in the ground state. In collisions with the cooling electrons they can be excited to a higher state. Their excitation cross-section depends on the electron energy in the ion rest frame, and attains a maximum when the electron energy is about three times greater than the excitation energy. The excited state will decay back into the lower state through the emission of a photon. In this process the ion charge is not changed and the ion continues on its trajectory. However, during excitation a small longitudinal momentum is transferred from the electron to the ion, which slightly slows down or accelerates the ion. The radiative de-excitation will in turn lead to a momentum transfer opposite to the direction of photon emission. This situation is similar to that of laser cooling of an atomic beam.

5.4 Ionization

If the electron configuration of the stored ions is such that it possesses a weakly bound outer electron, it can easily be stripped off in collisions with the electrons of the cooler. The process will occur as soon as the binding energy of the outer electron is exceeded. For positive ions the probability of occurrence is small but it is important for negative ions, since they usually possess a weakly bound electron. In particular, the negative hydrogen ion H^- , where the electron is bound by only 0.75 eV, is easily neutralized when the H^- beam is hot:

$$\text{H}^- + e \rightarrow \text{H} + e + e. \quad (85)$$

In these cases, electron cooling might initially be very lossy until the beam is cold enough and the ionization channel is closed [32]. However, occasional slight detuning of the electron beam energy might be useful in order to produce a neutral beam for diagnostics.

5.5 Three-body recombinations

Three-body or collisional recombinations play an important role in two-component plasmas of low temperature and high density. Their importance stems from the rapidly increasing cross-section with decreasing plasma temperature, and two partners from one plasma component take part. Therefore the recombination rate scales with the *square* of the density. In collisional recombination it is mostly the highly excited states that are populated, in contrast to spontaneous radiative capture, and the result is that they are also easily re-ionized. The net down-charge rate can, however, exceed the radiative recombination by many orders of magnitude for dense and cold electron beams. The coefficient for collisional recombination was given by Ref. [33] for protons:

$$\alpha_{r3} = 3.8 \times 10^{-9} (\text{cm}^6 \text{s}^{-1} \text{K}^{9/2}) T^{-9/2} n_e. \quad (86)$$

The relevance of the three-body recombination involving two electrons for electron cooling was recently investigated [34, 35]. It only plays a role for dense standard electron beams or subthermic ones (Sections 11 and 13). In Ref. [34] it was found that α_{r3} scales with Z^3 . The given value of α_{r3} constitutes only an upper limit when an electron cooling arrangement is considered, since only a fraction of the electrons that are captured in Rydberg states survive the stripping in the motional electric fields in the cooler solenoid. This coefficient has to be added to α_r in Eq. (82) to obtain the total down-charge rate. A comparison of Eq. (86) and Eq. (81), shows that both processes become equally strong for $38Z n_e [\text{cm}^{-3}] = T^4 [\text{K}]$.

6. ELECTRON COOLER DESIGN

The efficiency of electron cooling applied to stored ion beams depends very much on the quality of the electron beam and on the exact matching of both beams. In this section the basic design considerations for the electron cooler hardware will be discussed.

6.1 The electron gun

Electrons for cooling are produced in an electron gun, where they are accelerated to the desired kinetic energy $E_0 = m_e c^2 (\gamma - 1)$. The electron gun is placed in a longitudinal magnetic field (solenoid) to prevent the electron beam from blowing up because of the mutual repulsion of the electrons. The generation of electron beams is a well-established technique, known since many years [36]. Free electrons can be produced in different ways, as will be discussed below.

6.1.1 Electron sources

a) The thermionic cathode

Electrons are emitted from the surface of a thermionic cathode—which is usually heated resistively to a temperature T_{cath} of about 1300 K—and form a cloud in front of it. Their velocity distribution can be assumed to be of Maxwellian shape [Eq. (5)], with the electron temperature T_e equal to T_{cath} . The initial energy of the electrons E_{init} therefore lies in the interval between zero and about 100 meV. Electrons will be extracted from the cloud by one or several ring-shaped anodes. Usually the cathode is on negative high potential ϕ_{cath} and the anode potentials increase towards the potential ϕ_{drift} of the drift tube, which is mostly grounded. In order to minimize transverse electric

field components during acceleration to $E_0 = eU_{\text{cath}} + E_{\text{init}}$ [$U_{\text{cath}} = (\phi_{\text{drift}} - \phi_{\text{cath}})$], the cathode is surrounded by a Pierce shield [36]. This electrode on the cathode potential is modelled to such a shape that, together with the potential given by the electron cloud, it nullifies transverse electric field components on the cathode surface and produces equipotential lines that are parallel to it.

Cathodes can be obtained commercially in different sizes and types. The carrier material is usually tungsten, with an electron-emitting material on the surface (e.g. BaO). If the cathode has been exposed to atmospheric pressure, it has to be activated before it can be used. This is done by heating it smoothly to operation temperature and maintaining it there at a low vacuum pressure ($< 10^{-5}$ Torr) for several hours.

b) The photocathode

In order to reduce the transverse electron temperature to a value around room temperature, or even to cryogenic temperature, the free electrons are created by photoeffect [37]. Recently, for electron cooling it was proposed to use a photocathode for the production of a colder electron beam than is usually available from thermionic cathodes, and a prototype gun is now under construction [38]. The advantage of such an electron source is that much lower ion-beam temperatures can possibly be reached in thermal equilibrium with the electron beam. The problems that have to be solved are i) the production of an electron beam that is sufficiently intense for long operation times, and ii) the acceleration of the electrons to the desired energy without heating them transversely.

c) The electron plasma source

Electrons trapped in a high magnetic field can be cooled, by synchrotron radiation, to liquid-helium temperature, and electron densities of the order of 10^{11} cm^{-3} have been achieved [39]. The use of such cold electrons for cooling purposes could be envisaged in special cases. Again, the major obstacle is the adiabatic acceleration of the electrons to the cooling energy. Moreover, the continuous operation of an electron beam fed by a plasma source would require a recirculation of the electrons after they had passed the cooling region. A recirculation scheme is discussed in Section 11.

The electron guns are embedded in a longitudinal magnetic field (solenoid), which transforms the transverse electron motion into a spiral about the magnetic field lines. The radius of the spiral is given by the cyclotron radius $r_{e,c}$ and the rotation frequency by $\omega_{e,c}$ [Eqs. (20)].

6.1.2 Electron acceleration

The electron current that can be extracted from the space-charge cloud in front of a thermionic cathode is described by Child's law:

$$I_e = P_{\text{gun}} (\phi_{\text{an}} - \phi_{\text{cath}})^{3/2}, \quad (87)$$

where ϕ_{an} is the potential of the first anode. The proportionality factor P_{gun} is called the perveance of the gun. It is determined essentially by the ratio of the beam radius r_0 and the cathode-anode distance d :

$$P_{\text{gun}} = (r_0/d)^2 P_0, \quad (88a)$$

with

$$P_0 = 4\pi\epsilon_0 c/9 \sqrt{e/m_e c^2} = 7.3 \mu\text{P}, \quad (88b)$$

where the unit μP (microperveance) stands for $10^{-6} \text{ A} \cdot \text{V}^{-3/2}$. The extracted electrons are then accelerated to the final energy with the help of further electrodes. This does not change the electron current. The gun perveance is therefore fixed by the geometry and the extraction potential ($\phi_{\text{an}} - \phi_{\text{cath}}$). If the potential ratio between the accelerating electrodes is fixed, then relation (87) holds also when replacing $\phi_{\text{an}} - \phi_{\text{cath}}$ by U_{cath} .

The acceleration has to be done carefully so that no energy is put into the transverse (spiral) motion. Different acceleration techniques can be envisaged; they are described below.

a) The resonant acceleration electron gun

In this type of electron gun the magnetic field strength chosen is such that one cyclotron revolution (or multiple of it) is completed when the electron enters the drift region with a velocity $\beta_0 c$. It can be shown that in this way the effect of transverse electric field components can be minimized [40], otherwise there would be a continuous scalloping of the beam. For this acceleration (resonant acceleration) the traversal time of the electrons through the acceleration region has to be equal to a multiple n_c of the inverse cyclotron frequency. In other words, the pitch of the electron spiral λ_s has to be equal to the length of the acceleration region:

$$\lambda_s = 2\pi n_c \langle \beta \rangle c / \omega_{c,c}, \quad (89)$$

where $\langle \beta \rangle c$ signifies the average electron velocity during acceleration. The magnetic field should therefore have the value

$$B = 2\pi n_c \langle \beta \rangle \gamma m_e c^2 / (e \lambda_s), \quad B [\text{kG}] = 10.7 n_c \langle \beta \rangle \gamma / \lambda_s [\text{cm}]. \quad (90)$$

The electron gun (Fig. 17a) is operated by applying potentials in a fixed relation to the cathode and the anodes. The energy of the electrons is controlled by the cathode potential only. This provides a simple one-parameter (cathode potential) operation if the anode potentials are derived from a voltage divider which is connected to the cathode. The electron-beam current is in this case given by Eq. (87), and the magnetic field has to be changed proportionally to the momentum of the electrons when their energy is altered [Eq. (90)].

In Fig. 17b the electron trajectories computed with the SLAC trajectory program [41] are shown together with the electric potential lines for a constant longitudinal magnetic field of 500 G. The scalloping of the beam envelope during acceleration, which comes to a halt in the drift region, can be seen. At the entrance to the drift region there is a bend in the potential lines. Further downstream they become parallel to the electron trajectories, which is characteristic of a cylindrical beam that is magnetically confined and drifting in a tube of constant diameter. The transverse electron energy in the drift region, as obtained from the trajectory calculations, is plotted versus the distance from the beam axis in Fig. 17c.

b) The separated-function gun

This type of gun consists first of a resonant section, optimized at a well-defined extraction voltage that is kept fixed. The resonant (fast accelerating) part of the gun is followed by an acceleration column with a low longitudinal electric field gradient that brings the electrons smoothly (adiabatically) up to the final energy. The electron gun for CELSIUS is shown in Fig. 18 as an example [42]. The gun delivers a constant electron current, and the magnetic field can be kept fixed for all electron energies. These guns are frequently used in electron cooling devices operating at high energy and for free electron lasers.

c) The diode gun

This gun comprises only a cathode with a Pierce shield, and *one* anode. The nearly hyperbolic shape of the anode is modelled in such a way that it minimizes the transverse field components even for fast acceleration [43]. This type of gun will be used in the first stage of the ESR electron gun, which is also a separated-function set-up; however, the second stage is a single-gap (fast) accelerating section (Fig. 19a) [44].

d) Special configurations

Usually the cathode is planar and the electron beam diameter is equal to the cathode diameter. In the electron gun of the FNAL cooler, a spherically shaped cathode was used and the beam size reduced [45]. This configuration leads to an increase of the electron temperature owing to the compression in space. Special electron gun configurations had been proposed or used for various other purposes. The choice of a particular geometry was motivated, in some cases, by the fact that the toroidal section, where electrons and ions are usually matched, had to be suppressed. In one proposed configuration the cathode is placed perpendicularly to the ion beam, and the electrons are merged with this beam while they are being accelerated [46]. In another scheme, the electron gun is mounted next to the ion beam, and the electrons are inflected by a magnetic field produced with saddle-shaped coils [47]. Cathodes with a central hole for the passage of the ion beam were used successfully in dielectronic recombination experiments [48] and in a single-pass cooling experiment [49]. The application of these configurations is not very suitable for the cooling of a circulating ion beam to low temperature in ultra-high vacuum because of various sources of unwanted effects imposed on the ion beam. To obtain a very homogeneous field in a low-energy gun, a gridded acceleration anode was developed and will be used for electron cooling [50].

e) Longitudinal phase-space compression and temperatures

As mentioned before, the electrons emitted from the cathode obey a spherical velocity distribution [Eq. (6a)] with the temperature T_{cath} . By applying an accelerating voltage, they are brought to the energy E_0 :

$$E_0 = eU_{\text{cath}} + e\tilde{U} + E_{\text{init}}, \quad (91)$$

where $\tilde{U}$ accounts for the ripple of the high-voltage system. While the transverse velocity distribution and energy spread is not affected by acceleration (under ideal conditions), the longitudinal energy of the electrons is now distributed around eU_{cath} . The width of this distribution σ_E is given by the width of the ripple $\sigma_{\tilde{U}}$ and the cathode temperature,

$$\sigma_E = (e\sigma_{\tilde{U}}) + kT_{\text{cath}}. \quad (92)$$

The relative energy spread (σ_E/E_0) is twice as large as the relative longitudinal velocity spread ($\Delta_{\parallel}/v_{e_0}$): $\sigma_E/E_0 = 2\Delta_{\parallel}/v_{e_0}$. In the system moving with the average electron velocity $v_{e_0} = \beta_0 c$, the longitudinal electron temperature is $kT_{e_{\parallel}} = m_e \Delta_{\parallel}^2 / 2$:

$$\begin{aligned} kT_{e_{\parallel}} &= (e\sigma_{\tilde{U}} + kT_{\text{cath}})^2 m_e \beta_0^2 c^2 / 8E_0^2 \\ &= \left(\frac{\sigma_{\tilde{U}}}{U_{\text{cath}}} + \frac{kT_{\text{cath}}}{E_0} \right)^2 \frac{m_e \beta_0^2 c^2}{8}. \end{aligned} \quad (93)$$

For dense and cold electron beams there is another contribution that can determine the longitudinal temperature. This is illustrated in Fig. 20. Before acceleration the electrons have the kinetic energy kT_{cath} , plus the potential energy V_{ee} which results from the Coulomb field of the surrounding electrons and is given by

$$V_{ee} = 2e^2/(4\pi\epsilon_0 r_{av}) = 2m_e c^2 r_e/r_{av}.$$

Acceleration leads to a reduction of the kinetic energy in the accompanying reference frame, as shown above, but the potential energy is not changed if r_{av} remains constant during acceleration. This is the case if the electrons cannot interact during the time of acceleration t_{ac} . The time scale is given by the inverse plasma frequency $t_{pl} = 2\pi/\omega_{e,pl}$. Therefore the potential energy has to be added to expression (93) if $t_{ac} \ll t_{pl}$:

$$kT_{e\parallel} = \left(\frac{\sigma_{\tilde{U}}}{U_{\text{cath}}} + \frac{kT_{\text{cath}}}{E_0} \right)^2 \frac{m_e \beta_0^2 c^2}{8} + 2m_e c^2 \frac{r_e}{r_{av}}. \quad (94)$$

From subsection 2.3, we know that the condition $kT = 1/2 V_{ee}$ corresponds to $\Gamma = 1$, which signifies the starting order. The electrons can interact if the acceleration happens slowly ($t_{ac} \gg t_{pl}$) and the electron plasma can be brought to an ordered configuration which would lower the electron energy below the value given by V_{ee} . Since this is rather difficult to achieve, we take always V_{ee} as the lowest value for $kT_{e\parallel}$ if the other contributions are negligible:

$$\frac{T_{e\parallel}}{T_{e\perp}} = \begin{cases} \frac{kT_{e\perp}}{8E_0} \frac{\gamma + 1}{\gamma^2} & \text{for } kT_{\text{cath}} \gg e\sigma_{\tilde{U}}, V_{ee}, \end{cases} \quad (95a)$$

$$\frac{T_{e\parallel}}{T_{e\perp}} = \begin{cases} \frac{1}{4} \left(\frac{\sigma_{\tilde{U}}}{U_{\text{cath}}} \right)^2 \theta_{e\perp}^{-2} & \text{for } e\sigma_{\tilde{U}} \gg kT_{\text{cath}}, V_{ee}, \end{cases} \quad (95b)$$

$$\frac{T_{e\parallel}}{T_{e\perp}} = \begin{cases} \left(\frac{2}{\beta_0 \theta_{e\perp}} \right)^2 \left(\frac{4\pi}{3} n_e \right)^{1/3} r_e & \text{for } V_{ee} \gg e\sigma_{\tilde{U}}, kT_{\text{cath}}. \end{cases} \quad (95c)$$

Here we have assumed that $kT_{e\perp} = kT_{\text{cath}}$ and introduced $\theta_{e\perp}$ by

$$kT_{e\perp} = \frac{1}{2} m_e \Delta_{\perp}^2 = \frac{1}{2} m_e \beta_0^2 c^2 \theta_{e\perp}^2. \quad (95d)$$

Hence the adiabatic acceleration of the electrons produces a prolate electron velocity distribution, which has important consequences for the frictional force and the equilibrium, as discussed before. The flattening quotient Q_v ,

$$Q_v = \frac{\Delta_{\perp}}{\Delta_{\parallel}} = \sqrt{\frac{T_{e\perp}}{T_{e\parallel}}}, \quad (96)$$

may, in practice, reach values of the order of 20 to 30, given mostly by the limits (95b) or (95c).

6.2 Electron beam transport

After the acceleration stage, the electrons enter a drift region where they move to the cooling section. The electron density n_e after acceleration to the velocity $\beta_0 c$ is

$$n_e = I_e / (\pi r_0^2 e \beta_0 c) = j_e / (e \beta_0 c), \quad (97)$$

where j_e is the electron current density. The inner electrons are less accelerated as they are shielded by the outer electrons (result of space charge). The radial behaviour of the electric field is described by the potential of a homogeneous charge distribution with sharp boundaries. Therefore the longitudinal energy of an electron on a trajectory at a distance r from the axis is given by

$$E(r) = E_{ax} + n_e \pi r_0^2 r_e m_e c^2. \quad (98)$$

Here E_{ax} is the electron energy on the beam axis:

$$E_{ax} = eU_{cath} - n_e \pi r_0^2 r_e m_e c^2 [1 + 2 \ln (r_{tube}/r_0)], \quad (99)$$

where r_{tube} is the radius of the vacuum chamber or the conducting tube surrounding the electron beam. The negative charges might be compensated by trapping positive ions in the electron beam. In this case, one has to replace n_e by $(1 - \zeta)n_e$, where ζ signifies the degree of space-charge compensation and ranges between zero and unity.

The relative difference in the energy of the electrons at the beam centre compared with that at a radial position r is $\Delta E(r)/E_0 = [E(r) - E_{ax}]/eU$, and [using Eqs. (87), (88) and (97)], it yields

$$\frac{\Delta E(r)}{E_0} = \frac{\gamma}{9\sqrt{\gamma+1}} \frac{P_{gun}}{P_0} \left(\frac{r}{r_0}\right)^2. \quad (100a)$$

This value is nearly constant for most of the range of electron cooling ($\gamma \approx 1$) in constant perveance operation and in the absence of space-charge compensation. For constant current operation, the space-charge depression scales inversely to the third power of β_0 :

$$\frac{\Delta E(r)}{E_0} = \frac{I_e r_e}{ec} \frac{\gamma + 1}{\beta_0^3 \gamma^2} \left(\frac{r}{r_0}\right)^2 \approx 1.2 \times 10^{-4} \frac{I_e [A]}{\beta_0^3} \left(\frac{r}{r_0}\right)^2. \quad (100b)$$

The transverse electric field caused by the space charge leads to an $E \times B$ situation in the electron beam, which makes the beam twist azimuthally. The rotation frequency ω_{az} for this coherent motion and the corresponding transverse velocity v_{az} are

$$\omega_{az} = \frac{2\pi r_e n_e c^2}{\gamma \omega_{e,c}}, \quad (101)$$

$$v_{az} = r \omega_{az}. \quad (102)$$

The electron beam is merged with the ion beam by means of a toroidal magnetic field section. Although the electrons are bound to the magnetic field lines, a small dipole field is needed to accomplish the bending by compensating the centrifugal force. At the junction between the solenoids and the toroid, the magnetic field is reduced because of missing windings. If this is not compensated, the change of the magnetic field should be slow in order to allow the electrons to follow the field lines (adiabatic transition). The dipole field is usually matched to the beam axis. A mismatch of the field off-axis can hardly be avoided, provoking a heating of the outer part of the electron beam. This effect is most pronounced for large toroidal angles and high electron energies [51].

For the case of a long drift region and a dense electron beam, the electron–electron interactions are no longer negligible. This intrabeam scattering can reduce the flattening of the velocity distribution in the electron beam [52].

6.3 Interaction or cooling region

In the interaction region the electrons are merged with the ions. Usually, the electron beam size is chosen to be somewhat larger than the initial ion beam size, although ions would, because of their betatron oscillations, penetrate the electron beam even if the latter were smaller. However, the cooling time would be reduced by the fraction of time that the ions would not spend in the electron beam.

Owing to space-charge depression, the velocity profile of the electron beam represents a hot electron beam for an extended ion beam. This situation can be changed by compensating the charges of the electrons with positive ions. Ions with charge Z_{gas} can be trapped by applying a confining potential at either end of the interaction region. The positive ions are produced through ionization of the residual gas. A full compensation of the space charge ($\zeta = 1$) removes the velocity dependence of the electrons on their axial position. The trapped ions represent, however, a local pressure bump, resulting in a vacuum pressure P_{gas} in the trapping region:

$$P_{\text{gas}} = 2.8 \times 10^{-17} \text{ Torr cm}^{-3} n_e \zeta / Z_{\text{gas}} . \quad (103)$$

An unstable ion-trapping situation causes variations in the electron energy, which in turn leads to a variation in the ion energy. For a satisfactory performance of the electron cooling, the net amount of charges in the cooling region has to be precisely controlled. Ions can be removed through clearing electrodes. One has also to keep in mind that intense positive ion beams partly compensate the electron charge in the interaction region. The degree of compensation depends on the size of the ion beam for a constant number of stored ions and therefore changes with ion-beam temperature.

Another possible way of changing the velocity profile of the electrons would be by subdividing the active cathode surface into cylindrical rings and connecting them to different potentials, but owing to the high cathode temperature this is a difficult enterprise.

The energy of the electrons can be altered if there is a change of tube radius r_{tube} , or if the electron beam axis does not coincide with the tube axis. For the achievement of low ion-beam temperatures, the mechanical tolerances and the beam alignment error have to be correspondingly small. The drift tube in the interaction region can be isolated from the vacuum chamber and connected to a potential. In this way the electron energy can be changed locally. For instance, this technique makes it possible to reduce the ripple on the longitudinal electron velocity—caused by the acceleration voltage—via a feedback system.

At the end of the cooling section the electrons are again separated from the ions by a toroidal magnetic field, and drift in a solenoidal field to the collector.

6.4 The electron collector

The task of this device is to collect the electrons of the cooling beam with maximum efficiency. Owing to the high power $I_e U_{\text{cath}}$ stored in the beam, the electrons have to be decelerated to low energy before they impinge on a surface. Therefore the collector is at a potential Φ_{coll} close to that of the cathode. The remaining beam power $I_e(\Phi_{\text{cath}} - \Phi_{\text{coll}})$ is dissipated in the collector.

The requirement for the deceleration of the electrons renders their efficient collection more difficult. Secondary electrons, created by the impact of primary electrons on the collector surface, can be accelerated back into the drift tube. If the collector potential is too close to the cathode potential, primary electrons can be reflected from the collector entrance. Electrons reaccelerated into

the drift tube move back to the gun. At the gun entrance they are again decelerated and reflected. They bounce back and forth in the cooler, are heated transversely, and eventually hit a limiting aperture.

Electrons that hit the drift tube are lost at nearly full energy. They represent a loss current I_{loss} and impose a load on the high-voltage power supply. Upon impact with the surface they liberate gas and deteriorate the vacuum.

The quality of a collector is therefore characterized by its lowest operation voltage $U_{\text{coll}} = \Phi_{\text{cath}} - \Phi_{\text{coll}}$ and by its electron collection efficiency. Moreover, the electron current density j_e at the collector surface is of importance as far as thermal load and outgassing is concerned. The important features can be characterized by the collector perveance $P_{\text{coll}} = I_e(U_{\text{coll}})^{-3/2}$ and the current collection efficiency $\eta_e = 1 - I_{\text{loss}}/I_e$. In the past, a number of studies have been performed to obtain an optimal collector for electrons [53–55]. The most simple electron collector is the Faraday cup type.

Ideally, for an uncompensated beam, the collector voltage U_{coll} could assume the value of the potential on the beam axis. With Eqs. (97) and (99), we get

$$\begin{aligned} U_{\text{coll,min}} &= (\Phi_{\text{coll}} - \Phi_{\text{ax}}) = \frac{n_e(1 - \zeta)\pi r_0^2[1 + 2 \ln(r_{\text{tube}}/r_0)]e}{4\pi\epsilon_0} \\ &= \frac{I_e}{\beta_0 c} \frac{(1 - \zeta)[1 + 2 \ln(r_{\text{tube}}/r_0)]}{4\pi\epsilon_0}. \end{aligned} \quad (104)$$

When the electrons run against the collector potential they are decelerated to a velocity $\beta_{\text{min}}c$, with $\beta_{\text{min}}c = \sqrt{(2eU_{\text{coll}}/m_e c^2)}$. Replacing β_0 in Eq. (104) by β_{min} gives

$$U_{\text{coll,min}} = \frac{I_e}{4\pi\epsilon_0 c} \sqrt{\frac{m_e c^2}{2eU_{\text{coll,min}}}} (1 - \zeta) [1 + 2 \ln(r_{\text{tube}}/r_0)]. \quad (105)$$

Moving $U_{\text{coll,min}}$ to one side, raising it to the $-3/2$ power, and multiplying by I_e , yields

$$P_{\text{coll,max}} = I_e U_{\text{coll,min}}^{-3/2} = \frac{9\sqrt{2} P_0}{(1 - \zeta)[1 + 2 \ln(r_{\text{tube}}/r_0)]}. \quad (106)$$

In this way we have defined a universal collector perveance P_{coll} . It depends only weakly on the ratio of the vacuum chamber radius to the beam radius. For $r_{\text{tube}} = 2r_0$ and no space-charge compensation, we get

$$P_{\text{coll,max}} \approx 39 \mu\text{P}. \quad (107)$$

A number of studies were performed, with the aim of increasing the collector perveance. One possibility is to compensate the space-charge effects by trapping positive ions in front of the collector in a potential valley created with a collector electrode. This technique was successfully applied in the FNAL cooler [56]. Another possibility is to transform the shape of the electron beam. This was studied in detail in Novosibirsk [53], and yielded very high collector perveances. The different types of collectors are shown in Figs. 21 to 23.

In order to minimize the heat load on the collector surface, the electron beam has to be distributed over a large area. This is achieved by decreasing the magnetic field, which leads to a blow-up of the electron beam. This blow-up can be amplified with the aid of a central electrode (at

the cathode potential) pointing into the beam (see ICE collector, Fig. 22), or with a magnetic field opposite to the primary solenoidal field. Moreover, the deceleration of the electron beam before it enters the region where the magnetic field falls off has proved to be useful.

The electron collection efficiency depends strongly on the geometry of the collector. Electrons, impinging on the collector surface, produce a spectrum of secondary electrons which is peaked at low energy. A potential well can prevent these slow electrons from being accelerated back to the drift region. The fast electron component is suppressed by a surface treatment, which makes it possible to modify the angular distribution of the secondaries. Moreover, a special geometry can reduce the solid angle under which the collector entrance is seen from the point of primary electron impact on the inner collector surface.

The electron trajectories in the collector have to be carefully matched to the magnetic field lines in order to prevent the electrons from being heated up transversely owing to an $E \times B$ configuration. Auxiliary coils installed around the collector help to shape the magnetic field correspondingly. It is also useful to increase the magnetic field just before the tapering off in the collector, in order to introduce a magnetic bottleneck that reflects secondaries with high transverse energy back into the collector.

The surface of the collector is usually water-cooled, which requires the cooling water to be brought from ground to high potential. For a high collector perveance and moderate vacuum requirements, it might be possible to have radiation and convection cooling only.

The collectors used in the first electron coolers at Novosibirsk and FNAL reached a loss current I_{loss}/I_e of about 10^{-4} . Special collectors developed in Novosibirsk reached perveances of the order of $150 \mu\text{P}$ [53]. Recently, with a collector developed by the Karlsruhe group [54], a loss current in the upper 10^{-6} range was reached for a standard perveance and low beam current. A perveance of $160 \mu\text{P}$ was obtained in that configuration when the efficiency was permitted to decrease to values between 0.999 and 0.99. For a high-perveance collector, the efficiency can be increased by installing an additional collector for the backward-bouncing electrons [53].

A number of studies were aimed at compensating the electron space charge in front of the collector. So far, opinion is divided regarding the usefulness of such compensation for routinely operating devices, because it results in a rather delicate equilibrium.

6.5 The electric circuitry

6.5.1 High-voltage system

From the previous discussion it is evident that the cathode and the collector are nearly at the same potential. Usually the gun and collector are at negative high potential. The primary current flows between these two components. Only the lost electrons flow from negative high potential to ground. The schema of a typical high-voltage system is shown in Figs. 24.

The collector power supply provides U_{coll} and conducts the full electron current. It is on a high-voltage platform, which is biased by a highly stabilized power supply to cathode potential. Depending on the gun design and the operation mode, the acceleration and deceleration anodes are connected to separate power supplies (e.g. the gun in Fig. 24b), or their potential is derived from a voltage divider (e.g. the deceleration stage in Fig. 24b). Usually the drift tube is grounded, but it can also be connected to a small power supply. The electric potential is depicted in Fig. 24a.

The connection from the high-voltage platform to the gun and the collector can be made with standard high-voltage cables for potentials up to about 120 kV. For higher voltages it becomes necessary to have the connections in an SF_6 gas atmosphere, as realized in Refs. [44], [57], and [58].

The recipe for producing a cold electron beam is to use electrostatic acceleration. As the electron energy increases, the installations become bulkier. Most electron coolers that are already operating,

or will come into operation, work below 300 keV. A 1 MeV device with electron transport in a solenoid was developed in Novosibirsk [43]. The collector and the gun were housed together in the pressurized high-voltage dome. An electron beam of 1 cm diameter and up to 1 A was generated, and transported to the collector in a U-shaped solenoidal magnetic field, where it was collected with an efficiency of 0.999.

Even higher energies were obtained in a device developed in conjunction with free-electron-laser projects [59]. The electron gun was placed in the terminal of a Pelletron. The electrons were accelerated to 3 MeV and transported in a beam line consisting of dipoles and quadrupoles. After passing through a straight section, which could be used for electron cooling or lasing, the electrons were transported to the collector, which is also in the terminal of the Pelletron. This device delivered a beam of 100 mA at 3 MV [60].

Another device, which should deliver electrons of 750 keV, is under construction at Frascati [61]. This development is linked also to free-electron-laser work.

For low and intermediate energies, electron acceleration could be achieved by grounding the cathode and connecting the drift tube to positive high potential [62]. This avoids having the bulky high-voltage platform and permits all power supplies to be operated from ground potential. High-power and high-tension feedthroughs are decoupled. Also, the cooling of the collector is less problematic. Ions might be trapped in the electric field between the drift tube and the vacuum chamber, all within the magnet; however, this can be avoided by adding a small extraction potential at the end of the solenoid.

6.5.2 Cathode heating

The cathode is usually heated resistively to operation temperature (1300 K). The current is delivered by a power supply on the high-voltage platform. The high temperature that is needed requires also an efficient cooling of some of the parts around the cathode (current lead, anodes). Usually their temperature can be kept at a tolerable level by means of gas cooling. The heating current is between 10 and 20 A for a similar voltage range. The temperature of the cathode can be determined pyrometrically, and a calibration between temperature and heating power can be established. The large heating current to be supplied at high voltage requires a big high-voltage feedthrough, even for feeble electron beams. It is also possible to heat the cathode by bombarding it with electrons from the rear. For particular applications, it may be useful to limit the electron emission and control the electron current by reducing the cathode temperature. This is, however, a rather slow process. It can also increase the transverse electron temperature for resonantly accelerating guns.

6.5.3 Control system

The simple operation of the electron cooler requires an elaborate control system that sets and reads parameters. The system is logically divided into two parts, one on terminal potential and the other on ground. Communication between them is established with fibre-optic links. For data-acquisition and controls, commercial electronics (DAC, ADC, registers, scalars, etc.) can be used in conjunction with standardized systems such as CAMAC or VME. The system may be connected to a stand-alone computer or directly to the control computer for the storage ring. In addition to the control system, an interlock system is needed to ensure that water cooling is provided, and that the electric and magnetic fields and the vacuum are within given limits.

Once the basic settings of the electron cooler are determined, most of the parameters can be derived from the desired electron energy on the axis by appropriate scaling. In practice, it is then only the value of the high voltage, and possibly of a current-controlling electrode, that is varied.

6.6 Vacuum system

Electron coolers are used in storage rings operating under ultra-high-vacuum conditions (10^{-10} – 10^{-12} Torr). Consequently, the vacuum system of the cooler has to fulfil the corresponding requirements. It should be bakeable *in situ* at 300°C , and materials of low outgassing rate have to be used. In standard coolers, pumping ports are available in the toroids, in the gun, and in the collector. Careful selection and treatment of the materials enables the vacuum system of an electron cooler to be built in such a way that a pressure limit in the low 10^{-12} Torr range can be achieved with hydrogen as the main residual gas component [63] when the electron beam is turned off. It is more difficult to maintain a low vacuum in the presence of the electron beam. The main gas flow in an electron cooler under operation is produced by

- a) the hot cathode,
- b) the collector,
- c) the vacuum walls hit by lost electrons with nearly full energy.

The generated gases are mainly hydrogen and carbon monoxide.

In order to keep the vacuum as low as possible in the cooling region, a differential pumping stage has to be provided between the gun and the cooling section, and between the collector and the cooling section. The restricted access due to the system containment in the solenoid makes this difficult. In the drift region of the gun solenoid and of the collector solenoid, integrated pumps can be installed in order to obtain a high pressure gradient. Suitable pumps are non-evaporable getters (NEG) and cryopumps. Titanium sublimation pumps might also be used if there is no risk of sublimating Ti vapour on insulated surfaces. Heating jackets have to be wrapped around the entire vacuum system so as to permit bakeout of the vacuum chamber at 300°C *in situ*. Around the heating jackets themselves, a water-cooled heat shield is needed to keep the outside temperature below 100°C . Usually, a bakeout is done for 48 h at 300°C while the system is being pumped by turbomolecular pumps. The vacuum system of the LEAR electron cooler (Fig. 25) is discussed as an example.

In the LEAR system, the outgassing rate of the hot cathode, the collector, and the gas produced by lost electrons was measured. The gas flow generated by the cathode heated to 1300 K was initially about 10^{-6} Torr·l·s $^{-1}$, decreasing to about 2×10^{-7} Torr·l·s $^{-1}$ after a few hours of operation. The vacuum pressure is shown in Fig. 26 as a function of the electron energy for a perveance of $0.56 \mu\text{P}$ soon after activation. The gas analysis (inset in Fig. 26) shows mainly the presence of CO and H $_2$.

The main gas flow from the gun and the collector is pumped away by low-temperature NEG pumps (ST701). Strips of NEG material are formed into a sort of collar (a ruff of V-shaped folds) and inserted into the vacuum chamber. The pumps are activated during system bakeout by heating the housing chamber to 400°C . They provide a pumping speed of 1000 to 1500 l·s $^{-1}$. In the toroids, high-temperature NEG pumps (ST101) are installed. These pumps provide a pumping speed of 2000 l·s $^{-1}$, and are activated by resistive heating to about 700°C for 20 min.

At the entrance and exit of the ion beam and at the downstream end of the collector, ion pumps are installed that provide a pumping speed of 100 l·s $^{-1}$. Also at the collector, a turbomolecular pump and a forepump can be connected by means of a bakeable valve. This group is used only during bakeout. With this configuration, a pressure of 4×10^{-11} Torr was measured during operation at 28 keV electron energy and at 2.6 A [64]. The electron collection efficiency was of the order of 0.997.

The pressure can certainly be further reduced if a small electron beam is used (lower cathode outgassing, lower conductance). A low electron loss and a low collector voltage are also very helpful when aiming for a low vacuum. Finally, if the distance separating the gun and the collector from the

cooling section is long, and if there are some bends, then the gas conduction can be reduced further. These aspects are important for electron cooling of heavy ions at low energy.

6.7 The magnetic field

The purpose of the magnetic field is to confine the electron beam in a cylinder and to transform the transverse motion of the electrons into a rotation about the field lines. The freezing of the transverse motion leads to an increase of the cooling force at low relative electron-ion velocities. Variations in the magnetic field should be smooth, so that the electrons can follow them (adiabatic changes). However, this is not a good enough condition for the cooling section. There, the angle α_0 between the magnetic field lines and the ion trajectories should be smaller than the transverse angle $\theta_{e\perp}$ associated with the transverse electron temperature,

$$\alpha_0 \ll \theta_{e\perp} = \Delta_{\perp}/(\beta_0 c) \simeq 2.5 \times 10^{-4}/\beta, \quad (108)$$

so that the introduction of an apparent transverse velocity component can be avoided. Solenoids can be built with the required precision. A special technique for achieving a high level of homogeneity was developed in Novosibirsk [65]. After field mapping, the remaining magnetic field errors can be corrected by auxiliary windings, which are adapted in shape to the field deviations. Particular attention has to be paid to both ends of the solenoid [66]. The transition to the toroid, and the missing windings at the junction, deform the field. If this deformation is not corrected, it reduces the effective length of the cooling section and may even lead to beam heating. The correction is easily made by introducing additional coils at the junction between toroid and solenoid.

For optimization of the electron cooler properties, it is necessary to have steering coils all along the electron beam path. This is usually achieved by putting pairs of Helmholtz coils inside the solenoids and toroids to provide small horizontal and vertical dipole fields. In the toroids this can be combined with the dipole action that is needed to bend the electron beam. At the entrance of the collector and along it, auxiliary coils are installed to shape the magnetic field for optimal collection efficiency. Usually the three solenoids and two toroids are fed by one power supply via a shunt, to which the correction coils can also be connected. The steering coils are fed by separate power supplies. To trace the magnetic field, it is very helpful to produce a small low-energy electron beam and follow its trajectory through the magnet system.

The magnetic field that is needed to prevent the electron beam from blowing up is small. The field strength is dictated by the gun geometry and the requirement for minimal transverse heating during acceleration. Typical values lie between a few hundred and a few thousand gauss.

6.8 Electron beam diagnostics

In order to optimize the electron beam for cooling, efficient diagnostics are needed. We distinguish between destructive methods and methods that do not change the beam properties noticeably. The quantities to be determined are

- i) cathode temperature T_{cath} ,
- ii) electron current I_e ,
- iii) electron energy E_e ,
- iv) loss current I_{loss} ,
- v) beam axis at different positions along the cooler $h(s)$, $v(s)$,
- vi) electron density distribution $n_e(r)$,
- vii) electron velocity profile across the beam $v_{e0}(r)$,
- viii) transverse electron temperature $T_{e\perp}$,

- ix) longitudinal electron temperature $T_{e\parallel}$,
- x) amount of space-charge compensation ξ .

The cathode temperature T_{cath} can be measured once pyrometrically and then determined from the cathode heating power. The electron current can be deduced directly from the current passing through the collector power supply. The electron energy is obtained from the negative high voltage (high-precision voltage divider) applied to the cathode, if the space-charge compensation is known. It is difficult to reach an absolute accuracy below 10^{-4} . The beam position can be measured with electrostatic pick-ups to a precision of about 1 mm. This requires a modulation of the electron current, which is easily achieved through a modulation of the voltage applied to the cathode or to a gun anode. For initial beam tracing, destructive methods such as the use of a scintillation screen, movable scrapers, or wires [67] might be useful. The latter method allows the electron density distribution to be measured approximately. Furthermore, the vacuum can be worsened locally by injecting gas through a nozzle. The electrons ionize the gas, and the recombination light makes the electron beam visible. There is also the possibility of investigating some properties of the electron beam by analysing the X-ray spectra produced in the cooler [68].

It is more difficult to measure the electron beam temperatures. The overall transverse electron temperature in the cooling section can be determined by observing the microwave radiation emitted by the electrons spiralling in the magnetic field [69]. This technique was applied in the LEAR electron cooler [70]. A cylindrical tube inserted in the drift tube was used as a kind of cavity, and the microwave radiation coupling to it was received by an antenna at one end of the drift tube. The spectral density of the radiation (Fig. 27) was measured with a spectrum analyser and compared with theoretical calculations. The peaks could be identified with the modes of the cavity, whose properties were determined by independent measurements. The overall electron temperature was determined from the excitation of the cavity modes by the radiation emitted from the moving electrons.

The method permits us to determine the electron temperature only as an average over the whole beam (radially and longitudinally along the cavity). This results in an upper limit, since the colder axial part of the electron beam is underestimated.

The longitudinal temperature, the density distribution, and the velocity profile can be measured non-destructively by Thomson scattering of laser light from the electrons [71]. This method was successfully applied for the first time for a cooling electron beam in the LEAR cooler [72]. A laser beam, antiparallel to the electron beam, was directed into the cooling section (Fig. 28a). The wavelength λ_{bs} of the back-scattered light was shifted to shorter values owing to the Doppler effect:

$$\lambda_{\text{bs}} = \lambda_{\text{laser}}(1 + \beta_0 \cos \theta') / (1 - \beta_0 \cos \theta), \quad (109)$$

where θ' and θ are the angles between the light (incoming and outgoing) and the electron beam. This effect makes it possible to determine the absolute energy of the electron beam and the longitudinal velocity distribution by measuring the width and the centroid of the spectral distribution of the back-scattered light. Since the laser beam has a small diameter, this information can be obtained at different radial positions in the electron beam, and the velocity profile of the electrons can be deduced. Also, the measurement of the absolute wavelength of the back-scattered light (Fig. 28b) and the radial velocity dependence in the electron beam enables the degree of space-charge compensation to be determined. The rate R_{ph} of detected back-scattered photons is given by [72]

$$R_{\text{ph}} = r_e^2 [(1 + \beta_0)/(1 - \beta_0)] N_{\text{laser}}(n_e/\gamma)\eta_t\epsilon_{\text{PM}} \int \Delta\Omega d\ell_{\text{obs}}, \quad (110)$$

where N_{laser} is the number of laser photons, ℓ_{obs} is the length of the observation volume, η_t is the transmission of the back-scattered photons through the analysing system, ϵ_{PM} is the detection

efficiency of the photomultiplier, and $\Delta\Omega$ is the observation solid angle. The signal rate in the experiment [72] was 1 for 10 J laser energy (about one photon per minute). This low signal rate was mainly due to the low laser power and repetition rate. There are possibilities of improving the count rates by several orders of magnitude [72], which would make this non-destructive diagnostic very interesting.

7. RELEVANT ION BEAM PROPERTIES

For a discussion of the effect of electron cooling on the ion beam and of the experimental results, we shall need to have some practical notions of accelerator physics, which will be defined in this section. We follow as much as possible the notation of Ref. [73], and most of the relations are taken from there.

7.1 The reference system

As stated in subsection 2.3, the ion beam can be described by a distribution function defined over the six-dimensional phase space. Ion motion in a circular structure is considered, and we have chosen a coordinate system with one axis perpendicular to the plane of revolution [vertical (v) or z-axis], one axis in the plane of revolution that has the direction of the bending radius [horizontal (h) or x-axis], and one that is oriented along the momentary direction of revolution [longitudinal ($\parallel$) or s-axis]. The x-z plane is called the transverse plane ($\perp$) and y is used for a general axis in this plane. The position of a particle in space is defined by its three coordinates (x, z, s) and in phase space by (x, x'; z, z'; s, s'), which includes the derivatives (e.g. $x' = dx/ds$) of the coordinates. Frequently, a projection on one of the three planes generated by a coordinate and its derivative (e.g. x, x') is considered. In the transverse plane a Gaussian distribution is generally assumed:

$$f(y, y') = 4(\pi \sigma_y \sigma_{y'})^{-1} \exp -(y/\sigma_y)^2 \exp -(y'/\sigma_{y'})^2, \quad \int f(y, y') dy dy' = 1. \quad (111a)$$

In the longitudinal phase space the particles are homogeneously distributed in s for a coasting (unbunched) beam. In s' they again obey a Gaussian distribution:

$$f(s, s') = 2(\sqrt{\pi} \sigma_{s'})^{-1} \exp -(s'/\sigma_{s'})^2. \quad (111b)$$

7.2 The ion motion in a circular machine

The ions move periodically around the ring and are kept on orbit by magnetic fields. The time a particle needs to complete a full turn in the ring is the revolution time t_{rev} , and its inverse is the revolution frequency f_{rev} . The most important effects of the magnets on the ions are bending (dipoles) and focusing/defocusing actions (quadrupoles, solenoids). The deflection angle θ_{dip} of a particle in a dipole field of strength B and length ℓ_{dip} is

$$\theta_{dip} \text{ (rad)} = B\ell_{dip}/B\rho, \quad (112a)$$

where the beam rigidity $B\rho$ is related to the beam momentum p by

$$ZB\rho = pcA \quad \text{or} \quad B\rho \text{ [T} \cdot \text{m]} = 3.13 \beta\gamma \frac{A}{Z}, \quad (112b)$$

where A is the mass number of the ion. The circumference of the orbit of a nominal particle continuously exposed to this dipole field is $2\pi\rho$. In reality, the circumference is longer in a synchrotron owing to gaps between the dipoles. Therefore an effective orbital radius R_0 is defined by

$R_0 = C_0/2\pi$, where C_0 is the circumference of the orbit of a particle with momentum p_0 . Different momenta lead to different orbital radii R , and they are related by the momentum compaction factor α_p (independent of s):

$$\alpha_p = \frac{dC/C_0}{dp/p_0}. \quad (113a)$$

The radial distance d_i of the test particle from the nominal orbit at a given position in the ring is related to its momentum p_i by the dispersion function $D(s)$:

$$d_i(s) = D(s) (p_i - p_0)/p_0. \quad (113b)$$

The horizontal position of a particle starting at an orbital position $d_i(0)$ with the correct momentum p_i and zero angle is always given by Eq. (113b) if the ion is not disturbed on its way around the ring. If it starts with a finite angle or is deflected somewhere on its way around the machine, it will oscillate about the orbit corresponding to its momentum (betatron oscillations). The oscillations are due to the restoring effects of the quadrupoles, which act like cylinder lenses. Its horizontal position $h_i(s)$ is therefore given by

$$h_i(s) = x_i(s) + d_i(s). \quad (114)$$

The transport of the ions through the machine is described by a mapping of their positions in phase space from one point to another. Each transformation signifies the action of a ring element (a multipole or a drift region), and is characterized by a matrix. The elements of the transport matrix are given by the magnetic field strength and the gradients. The condition for having a stable beam is that, after one turn, the particles arrive at the same radial position they had at the beginning [$d_i(0) = d_i(2\pi R)$] (closed orbit). From the elements of the transport matrix, one can derive universal functions that characterize the machine structure (lattice) and the motion of the particles. These are the so-called Twiss parameters or lattice functions: $\alpha(s)$, $\beta(s)$, $\gamma(s)$, which are interrelated by

$$\beta(s)\gamma(s) = 1 + \alpha^2(s) \quad \text{and} \quad \alpha(s) = -\frac{1}{2}\beta'(s).$$

These functions describe the transverse motion of the ions in the ring:

$$y(s) = y_a(s) \cos [\psi_y(s) + \delta_y], \quad (115a)$$

$$y'(s) = -y'_a(s) \{ \alpha_y(s) \cos [\psi_y(s) + \delta_y] + \sin [\psi_y(s) + \delta_y] \}, \quad (115b)$$

$$y_a(s) = \sqrt{\epsilon_y \beta_y(s)}, \quad (115c)$$

$$y'_a(s) = \sqrt{\epsilon_y / \beta_y(s)}, \quad (115d)$$

where δ_y is the initial phase angle. The quantity $\epsilon_y = y_a y'_a$ will be explained below. The value of the phase function $\psi(s)$ —which is related to the β -function by $\psi_y(s) = \int_0^s ds / \beta_y(s)$ —after one revolution defines the betatron tune $Q_y = \psi_y(2\pi R_0)/2\pi$, i.e. the number of betatron oscillations per turn. The betatron wavelength is correspondingly $\lambda_y = 2\pi R_0 / Q_y$. The betatron tune has a non-integer value $Q_y = n_y + q_y$ (where q_y is the non-integer fraction of Q_y , and $n_y = 1, 2, 3, \dots$), in order to prevent the particles from arriving at a given position with the same phase after a number of turns. Otherwise, a perturbation at fixed position in the machine (e.g. alignment error) would periodically kick the

particle in the same direction, leading to an increase of the betatron oscillations until the beam is lost. The phase coordinates after ν turns have the values

$$y(s + \nu C_0) = y_a(s) \cos(2\pi Q_y \nu + \delta_y), \quad (115e)$$

$$y'(s + \nu C_0) = y'_a(s) [\alpha_y(s) \cos(2\pi Q_y \nu + \delta_y) + \sin(2\pi Q_y \nu + \delta_y)]. \quad (115f)$$

The following approximation holds for the ring-averaged beta function $\langle \beta_y \rangle$:

$$\langle \beta_y \rangle \approx R_0 / Q_y. \quad (116)$$

Generally, the betatron tune can depend on the momentum of the particles. The quantity that relates the two is called the chromaticity ξ :

$$\xi_y = \frac{dQ_y/Q_y}{dp/p_0}. \quad (117)$$

By adding y and y' quadratically, we obtain the equation for an ellipse:

$$\gamma_y(s)y^2 + 2\alpha_y(s)yy'^2 + \beta_y(s)y'^2 = \epsilon_y. \quad (118)$$

The area of the ellipse, which encircles 63% of all particles in the phase-space plane, is the 1σ emittance ϵ_y . The largest phase-space area that can be transferred through the ring is called the machine acceptance A_ϵ . The projection of the population of the ellipse onto one of the axes gives the spatial distribution and the angular distribution of the particles at position s in the ring. Since the ellipse rotates as the beam progresses around the ring, the widths (σ_y , $\sigma_{y'}$) of these distributions change with s . By a proper choice of the phase, and assuming Gaussian distributions, we can define a relation between the emittance, the r.m.s. radius, and the r.m.s. divergence of the beam:

$$\sigma_y(s) = \sqrt{\frac{\epsilon_y}{\pi} \beta_y(s)}, \quad (119a)$$

$$\sigma_{y'}(s) = \sqrt{\frac{\epsilon_y}{\pi} \gamma(s)} \approx \sqrt{\frac{\epsilon_y}{\pi} \frac{1}{\beta_y(s)}}, \quad (119b)$$

$$\epsilon_y \approx \pi \sigma_y \sigma_{y'}, \quad (119c)$$

$$\beta_y(s) \approx \sigma_y(s) / \sigma_{y'}(s). \quad (119d)$$

The volume V_{beam} occupied by the beam in the storage ring is

$$V_{\text{beam}} = \int_0^{C_0} \pi \sigma_x \sigma_z ds = \pi \int_0^{C_0} \sqrt{\frac{\epsilon_x}{\pi} \beta_x(s)} \sqrt{\frac{\epsilon_z}{\pi} \beta_z(s)} ds.$$

Using Eq. (116), we obtain approximately

$$V_{\text{beam}} \approx \frac{1}{2} \frac{C_0^2}{\bar{Q}} \frac{\bar{\epsilon}}{\pi} \approx C_0 \bar{\epsilon} \bar{\beta}_y, \quad (119e)$$

where $\bar{\epsilon} = \sqrt{\epsilon_x \epsilon_z}$, $\bar{\beta}_y = \sqrt{\langle \beta_x(s) \rangle \langle \beta_z(s) \rangle}$, and $\bar{Q} = \sqrt{Q_x \cdot Q_z}$.

Sometimes the emittance $\epsilon_{2\sigma} (= \pi 2\sigma_y 2\sigma_{y'})$, containing 95% of all particles, is used. Relation (118) holds also for the phase-space coordinates (y_i, y'_i) of an ion when defining a single-particle emittance $\epsilon_{iy}(s)$. The angle $y'_i(s)$ formed by the trajectory of a particle with the longitudinal axis is related to its velocity v_y in the transverse plane by

$$y'_i(s) = v_{iy}/\beta_0 c. \quad (120a)$$

The longitudinal angle is retarded owing to the Lorentz contraction

$$s' = v_{i\parallel}/(\gamma\beta_0 c). \quad (120b)$$

The r.m.s. width $\sigma_{s'}$ is related to the r.m.s. width σ_p by

$$\sigma_{s'} = \frac{\sigma_p}{p} \frac{1}{\gamma}. \quad (120c)$$

Throughout this paper, we use the r.m.s. values for the characterization of the spread of a Gaussian distribution. In situations where the distribution of a variable is not of Gaussian shape, the FWHM (full width at half-maximum) value is used (e.g. $\Delta p/p$ for the momentum spread characterizing an arbitrary symmetric distribution). The relation between the FWHM value and the r.m.s. value [for a distribution as defined in Eq. (6a)] is

$$\Delta x = \sigma_x \sqrt{2 \ln 4} \approx 1.667 \sigma_x. \quad (120d)$$

With these definitions we can assign temperatures to the ensemble of beam particles, and relate them to the divergence and to the momentum spread:

$$kT_{iy}(s) = 1/2 m_i \langle v_{iy}(s) \rangle^2 = 1/2 m_i c^2 \beta_0^2 \gamma^2 \sigma_{y'}^2(s), \quad (121a)$$

$$kT_{i\parallel} = 1/2 m_i \langle v_{i\parallel} \rangle^2 = 1/2 m_i c^2 \beta_0^2 \gamma^2 \sigma_{s'}^2 = 1/2 m_i c^2 \beta_0^2 \left(\frac{\sigma_p}{p_0} \right)^2. \quad (121b)$$

We define a transverse temperature $T_{\perp}$ as the average of $T_x(s)$ and $T_z(s)$:

$$kT_{i\perp}(s) = 1/4 m_i c^2 \beta_0^2 \gamma^2 [\sigma_{x'}^2(s) + \sigma_{z'}^2(s)]. \quad (121c)$$

Note that the transverse temperature varies along the circumference of the ring. This is clear because the spatial extension of the beam changes and leads to a change in volume. The longitudinal temperature is constant because σ_p does not depend on s . A ring-averaged transverse temperature can be defined by using Eqs. (119) and (116):

$$\langle kT_{i\perp} \rangle = 1/2 m_i c^2 \beta_0^2 \gamma^2 (\epsilon_x Q_x + \epsilon_z Q_z)/(2\pi R_0). \quad (121d)$$

In the case of pure dipole and quadrupole fields, the motion of the particles is uncorrelated in the three phase-space planes. However, a solenoidal field and a skew quadrupole lead to a coupling of the motion in the horizontal and the vertical plane, because a longitudinal magnetic field component tends to twist the beam. This is taken into account by the relevant transport matrix.

7.3 Working point, tune shifts, and resonances

The properties of a storage ring are characterized by its betatron tunes Q_x and Q_z , and the pair (Q_x, Q_z) is called the working point of the machine. As already mentioned, the tunes should have non-rational values in order to avoid the build-up of betatron oscillations (resonances). In real machines there is always a coupling between the transverse planes. Therefore one has also to avoid that linear combinations of Q_x and Q_z assume integer values:

$$nQ_x + mQ_z \neq k, \quad k, m, n = 1, 2, 3.$$

The Q -values are somewhat smeared out because of ripples in the magnet power supplies. A tune spread could also be due to a non-vanishing chromaticity. The Q -values depend not only on magnetic fields but also on fields generated by the ion beam itself. One major effect comes from the field screening axial ions from the external guiding field (space charge); this depends on the ion density. The shift of the working point due to this effect is

$$\Delta Q_{\text{sp.ch.}} = ZN_i(2\pi\epsilon_y\beta_0^2\gamma^3)^{-1} r_i B_f, \quad (122)$$

where B_f is the bunching factor describing the fraction of the ring circumference that is occupied by the ions ($B_f = 1$ for coasting beams).

The electron beam acts as a lens and also causes a tune shift, which is given by

$$\Delta Q_e = Z(2\beta_0^2\gamma^3)^{-1} n_e r_i \ell \langle \beta_y \rangle. \quad (123)$$

Note that for $n_i = N_i/V_{\text{beam}} = \eta_c n_e$ these tune shifts are equal.

There are a number of other effects that can cause a displacement of the working point and shift it into a resonance, leading to rapid growth of the transverse beam dimension and eventually to beam loss. A detailed discussion of this can be found in the appropriate literature [74].

7.4 Longitudinal phase space

In subsection 7.1 we assumed a Gaussian distribution of the longitudinal ion velocity [Eq. (111b)]. The momentum is distributed correspondingly. It is related to the revolution frequency by

$$\eta = \frac{df/f_0}{dp/p_0}, \quad (124)$$

where $f_0 = \beta_0 c / 2\pi C_0$. The off-momentum function η depends on the machine parameters and on the ion velocity, and is related to the momentum compaction factor by

$$\eta = 1 - \beta_0^2 - \alpha_p. \quad (125)$$

Usually α_p is smaller than unity, which implies that η is positive for low ion energies ($\beta_0 \ll 1$). Consequently, for a circulating beam, particles with momentum higher than p_0 arrive earlier than less energetic ones (i.e. after one revolution). When the beam is accelerated and β_0 approaches unity, the particles cannot gain sufficient velocity to compensate for the larger circumference of the outer orbits. At the unstable point (transition energy) where $\alpha_p = 1 - \beta_0^2 = \gamma_{tr}^{-2}$, the revolution frequency and the momentum are no longer correlated, which means that particles on any orbit have the same revolution frequency, irrespective of their momentum. Above this transition energy, η is negative; thus ions on outer orbits (higher momentum) have lower revolution frequencies.

When a certain voltage U_{RF} of radio frequency f_{RF} (usually a sine wave) is applied across an acceleration gap (cavity), the effect is null for ions with a revolution frequency equal to f_{RF} . Ions with a higher revolution frequency arrive earlier and will be decelerated by the negative amplitude of the voltage; the others will be accelerated correspondingly. This is so for operation below transition. Above transition it is just the opposite. As in the transverse planes, the particles perform oscillations (synchrotron oscillations). The frequency of the synchrotron oscillation is (for f_{RF} constant)

$$f_{syn} = \frac{c}{C_0} \sqrt{\frac{Ze U_{RF} h_f |\eta|}{2\pi m_i c^2 \gamma}}. \quad (126)$$

If the energy deviation ΔE_i of a particle is lower than the peak amplitude of U_{RF} it enters into a phase relation with the cavity frequency (radio frequency). The region with stable phase condition is called the RF bucket. The application of an RF voltage at a multiple h_f of the revolution frequency results in h_f buckets. A constant f_{RF} (stationary bucket) leads to oscillations around the zero-crossing of U_{RF} . The phase ϕ_i relative to U_{RF} and the energy deviation ΔE_i from the nominal energy $m_i \gamma c^2$ can be chosen as the coordinates of the longitudinal phase plane. The stable synchrotron oscillation of a particle corresponds to an ellipse with the half-axes

$$\phi_i/2 \quad \text{and} \quad \beta_0 \sqrt{\frac{Ze U_{RF} m_i c^2 \gamma}{2\pi h_f |\eta|}} \phi_i/2.$$

The ensemble of particles fills an elliptical area in the bucket which is called the bunch. The energy spread ΔE_b of the bunch has the value

$$\Delta E_b = \beta_0 \sqrt{\frac{Ze U_{RF} m_i c^2 \gamma}{2\pi h_f |\eta|}} \Delta \phi_b, \quad (127a)$$

and the phase spread $\Delta \phi_b$ is related to the spatial extension of the bunch ℓ_b (i.e. the bunch length)

$$\Delta \phi_b = \frac{\ell_b}{C_0} 2\pi = 2\pi f_0 \Delta t_b, \quad \Delta t_b = \frac{\ell_b}{\beta_0 c}. \quad (127b)$$

The bunch area A_b is a constant of motion and is defined as

$$A_b = \frac{\pi}{4} \Delta E_b \Delta t_b = \frac{\pi}{4} \beta_0 \gamma m_i c \ell_b \frac{\Delta p}{p} \quad (127c)$$

[since $\Delta E_b/m_i \gamma c^2 = \beta_0^2 (\Delta p/p)$], and is measured in eV·s. Note that for a coasting beam $\ell_b = C_0$. From Eq. (127a), it follows that bunch length, momentum spread, and RF voltage are related by

$$\frac{Ze U_{RF}}{m_i c^2} = \frac{1}{2\pi} h_f |\eta| \beta_0^2 \gamma \left(\frac{\Delta p}{p}\right)^2 \left(\frac{C_0}{\ell_b}\right)^2. \quad (128a)$$

When bunching a coasting beam adiabatically, the bunch area remains constant, and it is useful to rewrite Eq. (128a) using the bunch area

$$\frac{Ze U_{RF}}{m_i c^2} = \frac{8}{\pi^3} \frac{h_f |\eta|}{\gamma} \left(\frac{A_b}{m_i c}\right)^2 \frac{C_0^2}{\ell_b^4}. \quad (128b)$$

In this case (adiabatic bunching) the applied RF voltage determines the bunch length directly.

A change of frequency will accelerate or decelerate the ions, and if the magnetic field is kept constant it will displace the orbit of the particles. If the beam has to be accelerated or decelerated over a wide range, the magnetic field is changed simultaneously with the radio frequency in order to keep the particles on the nominal orbit.

7.5 Instabilities

There exist a variety of instabilities (transverse and longitudinal) that can occur in a storage ring; they are discussed extensively in the literature [74]. The ones that are relevant here are mostly connected with the high phase-space density attained with electron cooling. In this context, the most dangerous are the transverse instabilities since they frequently lead to immediate particle losses. When the beam emittance is reduced, the working point shifts [Eq. (122)] and can approach a resonance, which may lead to rapid transverse beam growth and loss. The severity of the instability depends on the blow-up rate, which occasionally can be balanced by cooling or other kinds of damping. The tune shift induced by the electron beam [Eq. (123)] can be corrected provided the charge density in the cooling region is constant—which is not necessarily the case during beam acceleration/deceleration or when working with a space-charge compensated electron beam. Ions trapped in the electron beam change the potential on the axis and lead to a change in the electron velocity. This can cause oscillations in the ion beam energy. Electrons or ions from residual gas ionization can also be trapped in the ion beam. If the latter is small and intense, it presents a potential valley for electrons (positive ion beam) or for positive ions (negative ion beam). The trapped particles can cause beam instabilities because their motion in this potential can couple to that of the ions and lead to resonances. Provision has to be made to remove the trapped particles from the ion beam.

The cooling of the momentum spread to very low values can possibly produce longitudinal instabilities. They originate from the electric field of the ions, which is induced in the surrounding surfaces and which acts back on them (resistive-wall instabilities). The electric field moving with the beam can be regarded as an electromagnetic wave. The way in which this wave propagates along the beam tube is of importance in relation to the way it acts back on the ion beam. It may cause local acceleration or deceleration, and thus change the homogeneous ion distribution in a coasting beam and eventually lead to self-bunching. The quantity that characterizes these effects is $Z_{\parallel h}$, the longitudinal coupling impedance for the h_f^{th} harmonic of the revolution frequency given, for a perfectly conducting circular beam pipe, by

$$Z_{\parallel h} = i h_f Z_0 [1 + 2 \ln (r_{\text{tube}}/r_{0i})] (2\beta_0\gamma^2)^{-1}; \quad Z_0 = \mu_0 c = 377 \Omega, \quad (129)$$

where μ_0 is the permeability of free space and r_{0i} is the ion beam radius. The threshold for longitudinal beam instabilities is given by the Keil-Schnell criterion [75]

$$\left(\frac{\sigma_p}{p_0}\right)_{\text{Keil-Schnell}}^2 \geq \frac{Z^2 e^2 N_i c}{m_i C_0^2 \beta_0 \gamma |\eta|} \left| \frac{Z_{\parallel h}}{h_f} \right| = \frac{4\pi Z^2 r_i N_i}{\beta_0 \gamma |\eta| C_0} \left| \frac{Z_{\parallel h}/h_f}{Z_0} \right|. \quad (130a)$$

The equals sign in Eq. (130a) defines the critical number of stored particles N_{crit} :

$$N_{\text{crit}} = \frac{\beta_0 \gamma \eta C_0}{4\pi Z^2 r_i} \left(\frac{\sigma_p}{p_0}\right)^2 \left(\frac{Z_{\parallel h}/h_f}{Z_0}\right)^{-1}. \quad (130b)$$

Note that N_{crit} can assume negative values depending on the sign of η and $Z_{\parallel h}$. If the beam momentum spread is cooled to below this value, or if N_i is increased above N_{crit} , instabilities build up, which would lead to a clustering of particles in the longitudinal direction. Above transition, this effect is amplified by the negative mass instability. It is due to a momentary perturbation of the coasting beam, which causes particles to be accelerated or decelerated, depending on their phase with respect to this distortion. This results in a decrease of the revolution frequency for the accelerated ions and in an increase for the others. If the perturbation persists for many turns, then the effect is multiplied and leads to a self-bunching of the beam.

7.6 Intrabeam scattering

The ability to increase the phase-space density enormously through electron cooling has the consequence that scattering processes within the ion beam become important. Ions scatter from ions and transfer momentum from one degree of freedom to another. The underlying dynamics is equivalent to that of electron cooling. Intuitively, one understands that intrabeam scattering (IBS) leads to an equalization of beam temperatures in all three directions. Since electron cooling tends to reduce the beam temperature to a much lower value in the longitudinal direction than in the transverse one [Eqs. (72)], intrabeam scattering could be considered as being useful for reducing the transverse temperature to lower values. Unfortunately, energy can also be extracted from the total longitudinal motion, leading to deceleration of the particles and introducing heat into the motion appearing in the accompanying coordinate system.

The theory of intrabeam scattering was first developed by Piwinski [76]; it was later considered by Bjorken and Mtingwa [77] and by Martini [78]. Piwinski derived growth rates (longitudinal and transverse) for coasting and bunched beams. These increments are complicated functions of the machine parameters (lattice functions), of the beam emittances, and of the momentum spread itself. Computer codes are required [78] in order to evaluate the growth rates for specific cases. However, we can make a simple model for the counteraction between intrabeam scattering and cooling, by using Spitzer's formula for the temperature equalization time in a two-component plasma [Eq. (21)]. With a slight modification, we obtain

$$\tau_{IBS} = \frac{3}{8} \frac{1}{\sqrt{2\pi}} \frac{(kT_i/m_i c^2)^{3/2}}{r_i^2 c Z^4 L_C n_i} \quad (131a)$$

In order to account for the different temperatures in the longitudinal and the transverse direction, we split $T_i^{3/2}$ into $T_{i\perp} \cdot T_{i\parallel}^{1/2}$. For electron-cooled ion beams we usually have $T_{i\parallel} \ll T_{i\perp}$; therefore only the longitudinal blow-up matters. We make the following ansatz for the longitudinal increment:

$$\lambda_{IBS\parallel} = \frac{1}{\tau_{IBS\parallel}} = \frac{8\sqrt{2\pi}}{3} \frac{r_i^2 Z^4 c L_C n_i (m_i c^2)^{3/2}}{k T_{i\perp} (k T_{i\parallel})^{1/2}} \left(\frac{T_{i\perp}}{T_{i\parallel}} \right)^{\Upsilon} \quad (131b)$$

The exponent Υ has to be found from experiments or numerical analyses. With Eqs. (116), (119e), and (121), we get for $\epsilon_x = \epsilon_z$ and $Q_x = Q_z$:

$$\lambda_{IBS\parallel} = \frac{32}{3\sqrt{\pi}} \frac{r_i^2 Z^4 L_C n_i}{C_0 (\epsilon/\pi)^2 \beta_0^3 \gamma^2 \sigma_p/p} \left(\frac{\epsilon/\pi}{(\sigma_p/p_0)^2} \right)^{\Upsilon} \left(\frac{\gamma^2}{\langle \beta_y \rangle} \right)^{\Upsilon} \quad (131c)$$

Sørensen [79] calculated λ_{IBS} for a number of machines and a wide range of $T_{i\perp}/T_{i\parallel}$. From these numerical calculations he found that λ_{IBS} divided by $\mu_{IBS} L_C$ has a universal behaviour for most machines and phase-space quantities (Fig. 29). He chose the normalization factor μ_{IBS} , as

$$\mu_{IBS} = \frac{N_i}{(m_i c \beta_0 \gamma)^3 (\epsilon/\pi)^2 (\sigma_p/p_0) C_0}.$$

With this, we obtain from Fig. 29, for $(\epsilon/\pi)/(\sigma_p/p_0)^2 > 50$ rad/m:

$$\frac{\lambda_{IBS\parallel}}{\mu_{IBS} L_C} = C_{IBS} \left(\frac{\epsilon/\pi [\text{rad} \cdot \text{m}]}{(\sigma_p/p_0)^2} \right)^{3/4}, \quad (131d)$$

where

$$C_{\text{IBS}} = 0.56 \left(\frac{\text{eV} \cdot \text{m}}{c} \right)^3 \text{s}^{-1}.$$

We use this result and choose Υ as follows

$$\begin{aligned} \Upsilon &= 0 & \text{for} & \quad T_{i\perp} \leq T_{i\parallel}, \\ \Upsilon &= 3/4 & \text{for} & \quad T_{i\perp} \geq T_{i\parallel}. \end{aligned} \quad (131\text{e})$$

Incidentally, the factor C_{IBS} in Eq. (131d) coincides approximately with the numerical value of $r_p^2 c (m_p c)^3$. We therefore propose the following semiempirical formula for the blow-up by intrabeam scattering in the longitudinal direction for $T_{i\perp} \geq T_{i\parallel}$:

$$\lambda_{\text{IBS}\parallel} = \frac{32}{3\sqrt{\pi}} \frac{r_i^2 c Z^4 L_C N_i}{C_0 \beta_0^3 \gamma^{1/2}} \left(\frac{\epsilon}{\pi} \right)^{-5/4} \left(\frac{\sigma_p}{p_0} \right)^{-5/2} \langle \beta_y \rangle^{-3/4} = \frac{32}{3\sqrt{\pi}} \frac{r_i^2 c Z^4 L_C N_i}{C_0 \beta_0^3 \gamma^{1/2}} \langle r_{0i} \rangle^{-5/2} \left(\frac{\sigma_p}{p_0} \right)^{-5/2} \langle \beta_y \rangle^{1/2}. \quad (131\text{f})$$

In this expression we used $\langle r_{0i} \rangle^2 = (\epsilon/\pi) \langle \beta_y \rangle$ for the average ion beam radius. In Fig. 29, two domains can be clearly distinguished: to the right (longitudinally cold beam), where momentum blow-up dominates; and to the left, where the horizontal blow-up is most important. The separation point is at about $\epsilon_y \approx \langle D^2/\beta_y \rangle (\sigma_p/p)^2$ [79]. Note also the strong dependence of the growth rate on the charge of the ion.

7.7 Residual gas scattering

The other important process that presents a heat source is the scattering of the ion from residual gas particles (with nuclear charge Z_{gas}). The composition of the residual gas in the vacuum system of an accelerator depends on many factors (materials, material treatment, types of pumps, etc.). The pressure is measured with vacuum gauges, which have different sensitivities for different gases (subsection 7.9). To make a precise evaluation of the relevant vacuum pressure, the gas composition has to be known. Usually, the vacuum pressure read from the gauges is then converted into a nitrogen-equivalent pressure, i.e. the pressure that would produce the same Rutherford scattering effect as that given by the measured pressure originating from the contribution of the different gases in the vacuum system. For instance, a residual gas pressure of $P_{\text{H}_2} = 4.9 \times 10^{-11}$ Torr, composed entirely of hydrogen, corresponds to $P_{\text{N}_2} = 10^{-12}$ Torr $= P_{\text{H}_2}/Z_{\text{gas}}^2$.

The scatterings are described by the differential Rutherford cross-section [Eq. (22)], which is often also given in the form

$$d\sigma/d\Omega = \frac{1}{4} \frac{Z_{\text{gas}}^2 Z^2 (m_e c^2 / \beta_0 p_0 c)^2 r_e^2}{\sin^4(\theta_{\text{scatt}}/2)}. \quad (132)$$

The cross-section for scatterings that are larger than the machine acceptance angle θ_{acc} is

$$\begin{aligned} \Delta\sigma &= \int_0^{2\pi} \int_{\theta_{\text{acc}}}^{\theta_{\text{max}}} \frac{d\sigma}{d\Omega} d\phi \sin\theta d\theta \\ &= \pi Z_{\text{gas}}^2 Z^2 (m_e c^2 / \beta_0 p_0 c)^2 r_e^2 [(\theta_{\text{acc}}/2)^{-2} - (\theta_{\text{max}}/2)^{-2}] \end{aligned} \quad (133\text{a})$$

(replacing $\sin \theta$ by θ). In most cases, the contribution from the large angles can be neglected because of the rapid drop in the cross-section:

$$\Delta \sigma = 4\pi Z_{\text{gas}}^2 Z^2 [m_e c^2 / (\beta_0 p_0 c)]^2 r_e^2 \theta_{\text{acc}}^{-2} = 4\pi Z_{\text{gas}}^2 Z^2 r_e^2 \beta_0^{-4} \gamma^{-2} \theta_{\text{acc}}^{-2}. \quad (133b)$$

When neglecting the finite beam emittance and momentum spread, θ_{acc} can be approximated for a rectangular beam chamber by

$$\theta_{\text{acc}} = \sqrt{\pi / \langle H \rangle}; \quad \langle H \rangle = \Sigma (H \Delta s) / (\Sigma \Delta s), \quad (134a)$$

where $\langle H \rangle$ is an average of the expression H over the machine circumference,

$$H = (ab)^{-1} [1 + a/b \arctan (a/b) + b/a \arctan (b/a)]$$

[with $a = \sqrt{A_x / (\pi \beta_x)}$ and $b = \sqrt{A_z / (\pi \beta_z)}$], and Δs is the fraction of the circumference where H can be considered to be constant. A finite momentum spread and a finite emittance reduce θ_{acc} .

In the latter case we have to replace a by $a[1 - (\epsilon_x / A_x)]$, and b correspondingly. For a round vacuum chamber, $\beta_x \approx \beta_z \approx \langle \beta_y \rangle$ and $\epsilon_x \approx \epsilon_z \approx \epsilon$, the ring acceptance angle is approximately given by

$$\theta_{\text{acc}} = \sqrt{\frac{A_\epsilon}{\pi \langle \beta_y \rangle}}. \quad (134b)$$

The finite beam emittance is taken into account by replacing θ_{acc}^2 by

$$\theta_{\text{acc}}^2 \rightarrow \frac{A_\epsilon}{\pi \langle \beta_y \rangle} \left(1 - \frac{\epsilon_y}{A_\epsilon}\right)^2. \quad (134c)$$

Whilst single scatterings with angles larger than θ_{acc} lead to immediate beam loss, scatterings with smaller angles result in an increase of the angular spread of the beam, which can be calculated with Eq. (1b):

$$\theta_{\text{ms}}^2 = 8\pi Z_{\text{gas}}^2 Z^2 n_{\text{gas}} r_e^2 c L_{\text{res}} \beta_0^{-3} \gamma^{-2} t, \quad (135a)$$

where we replaced α_2 by the Coulomb logarithm responsible for this scattering ($L_{\text{res}} \approx 10$) and x_0 by $\beta_0 c$. Emittance growth leads to the filling of the acceptance, and it reduces the average acceptance angle. When the emittance equals the acceptance, all further scatterings lead to immediate beam loss. The emittance growth rate is obtained from Eq. (135a):

$$\frac{\dot{\epsilon}_{\text{ms}}}{\pi} = 8Z_{\text{gas}}^2 Z^2 n_{\text{gas}} r_e^2 c \langle \beta_y \rangle \beta_0^{-3} \gamma^{-2} L_{\text{res}}. \quad (135b)$$

The number density of the residual gas n_{gas} can be calculated from the number of particles per mol ($6 \times 10^{23} / 22.4$ l) multiplied by the residual gas pressure P_{gas} :

$$n_{\text{gas}} = 3.5 \times 10^{16} \text{ Torr}^{-1} \text{ cm}^{-3} P_{\text{gas}}. \quad (136)$$

The residual gas scattering results also in an energy loss of the ions, which can be calculated from Eq. (1a) when including binding effects for the residual gas molecules:

$$-dE/dx = 4\pi Z_{\text{gas}}^2 Z^2 n_{\text{gas}} r_e^2 m_e c^2 \beta_0^{-2} L_{\text{res}}, \quad (137a)$$

where we put $\alpha_1 \approx \alpha_2 \approx L_{\text{res}}$. We can compare the emittance growth rate with the energy loss:

$$-\frac{dE/dt}{E_i} = \frac{\dot{\epsilon}_{\text{ms}}}{\langle \beta_y \rangle} \frac{m_i}{m_e} . \quad (137b)$$

If not compensated, this loss leads to a deceleration of the beam. The rate of deceleration can be used to determine the average vacuum pressure. These formulae can also be applied to estimate the effect produced on the ion beam by a thin internal gas target.

7.8 Beam lifetime

There are many effects that determine the lifetime of a beam in a storage ring (resonances, instabilities, etc.). In the following, we shall consider only the decay of the beam intensity due to residual gas scattering, since this can be influenced by electron cooling. The number of stored particles decreases in time as

$$N_i = N_{i_0} e^{-t/\tau_{\text{beam}}} , \quad (138)$$

where N_{i_0} is the number of stored particles at $t = 0$. The beam lifetime τ_{beam} depends on the scattering cross-section and the number density of the residual gas particles. For single-scattering processes we have

$$\tau_{\text{ss}} = (\Delta\sigma n_{\text{gas}} \beta_{0c})^{-1} .$$

With Eq. (133b), we obtain

$$\tau_{\text{ss}} = (4\pi Z_{\text{gas}}^2 Z^2 r_1^2 n_{\text{gas}} c)^{-1} \beta_0^3 \gamma^2 \theta_{\text{acc}}^2 . \quad (139a)$$

For a residual gas with two components, a and b, in the ratio a:b, the total lifetime is obtained by adding the partial lifetimes inversely:

$$\tau_{\text{ss}}^{-1} = a\tau_a^{-1} + b\tau_b^{-1} .$$

The partial lifetimes are obtained from Eq. (139a) by inserting the corresponding pressures. In Ref. [80] the beam lifetime with respect to multiple scattering was derived for a proton beam,

$$\tau_{\text{ms}} = k_0 \frac{A_\epsilon}{\dot{\epsilon}_{\text{ms}}} , \quad (139b)$$

where the constant k_0 assumes a value of 0.85 (2.16) for a round (rectangular) vacuum chamber. In the latter case, the uniform machine acceptance A_ϵ has to be replaced by $(A_x^{-1} + A_z^{-1})^{-1}$. The ratio of single-scattering to multiple-scattering lifetime is found by comparing Eqs. (139a) and (139b) and using (135b)

$$\frac{\tau_{\text{ss}}}{\tau_{\text{ms}}} = 2\theta_{\text{acc}}^2 (k_0 A_\epsilon)^{-1} \langle \beta_y \rangle L_{\text{res}} , \quad (139c)$$

which simplifies for a round vacuum chamber and beam to

$$\frac{\tau_{\text{ss}}}{\tau_{\text{ms}}} \approx \frac{3}{4} L_{\text{res}} . \quad (139d)$$

Taking into account the finite beam emittance yields finally

$$\frac{\tau_{ss}}{\tau_{ms}} \approx \left(1 - \frac{\epsilon_y}{A_e}\right)^2 \frac{3}{4} L_{res}. \quad (139e)$$

This shows that the multiple-scattering lifetime is usually at least one order of magnitude smaller than the single-scattering lifetime.

In rings storing positive ions at a moderate vacuum, the electron pick-up and (for partly stripped ions) the electron stripping in the residual gas lead also to losses and may be the limiting factor for the beam lifetime.

7.9 Vacuum and gauge readings

In ultra-high vacuum systems the precise determination of the pressure is a difficult and frequently controversial point. We therefore describe how the real residual gas pressure is derived from the gauge reading P_{gr} . First the background value P_{bgd} has to be subtracted (for specific gauges a modulation voltage is applied), yielding $P_{gauge} = P_{gr} - P_{bgd}$, which then has to be divided by the gas-independent calibration factor k_{gauge} of the specific gauge to give the corrected gauge reading $P_{cor} = P_{gauge}/k_{gauge}$. A residual gas analysis provides the gas composition. For the determination of the real gas pressure, we have to take into account the sensitivity η_{gas} of the gauge for the different gases by having $P_{gas} = P_{cor}/\eta_{gas}$. For instance, a gauge reading of $P_{gr} = 1.5 \times 10^{-11}$ Torr at a background level of $P_{bgd} = 5 \times 10^{-12}$ Torr yields a true residual gas pressure of $P_{H_2} = 2.5 \times 10^{-11}$ Torr (pure hydrogen residual gas) and $P_{N_2} = 1.25 \times 10^{-11}$ Torr (pure nitrogen). Here we have assumed that $k_{gauge} = 0.8$, $\eta_{H_2} = 0.5$, and $\eta_{N_2} = 1$.

The scattering cross-sections scale as Z_{gas}^2 . Different sensitivities of the gauges for the various gases may introduce another factor when using the gauge reading as a reference. In the above example, the scattering cross-sections and corresponding beam lifetimes for nitrogen and hydrogen would differ by a factor of 25 instead of the expected 49.

7.10 Emittance and momentum spread, scaling with energy

At a fixed energy, the beam emittances and the momentum spread are constants of motion and do not change when non-conservative forces are absent. However, when the beam energy is changed, they vary as

$$\epsilon' = \frac{\epsilon}{\beta\gamma} \quad \text{and} \quad \left(\frac{\sigma_p}{p}\right)' = \left(\frac{\sigma_p}{p}\right) \frac{1}{\beta\gamma}. \quad (140)$$

Here the prime denotes the final values. This behaviour is called adiabatic shrinkage (for acceleration) and adiabatic blow-up (for deceleration). It is the adiabatic blow-up that makes electron cooling very important at low energy and for decelerated beams. The products $\beta\gamma\epsilon$ and $\beta\gamma(\sigma_p/p)$ are normalized phase-space quantities: they remain constant for adiabatic acceleration and deceleration.

7.11 Collective effects and ordering

So far we have considered the beam ions as being independent particles moving on individual orbits. As long as no external forces are exerted on the beam, and the beam temperature is high enough, this is a reasonable assumption. A perturbation of the beam, caused either by a kick (deflection) given by a ring element or by self-induced fields (see subsection 7.5), can lead to joint motion of a large portion of the beam (coherent motion). For instance, the application of a transverse deflecting field during a certain duration of the revolution period would result in coherent betatron oscillations. It would be misleading to include these coherent motions in the beam

temperature, which we have already defined [Eqs. (121)]. This has to be kept in mind when measuring beam emittances and converting them into temperatures.

From plasma physics we know that ordering will set in smoothly when the plasma parameter Γ [Eq. (11)] increases to values above unity. Cooling reduces the beam size and the divergence, which means that it increases the spatial density and decreases the temperature at the same time. This means that in the transverse plane, Γ depends only on the beam emittance. Using Eqs. (8), (119e), and (121d), we can write Eq. (11) for the transverse plane:

$$\Gamma_{i\perp} = 2Z^2 r_i \left(\frac{4\pi \langle \beta_y \rangle^2}{3C_0} \right)^{1/3} N_i^{1/3} \left(\frac{\epsilon}{\pi} \right)^{-4/3} \beta_0^{-2} \gamma^{-2}. \quad (141a)$$

This shows that $\Gamma_{i\perp}$ is directly proportional to the third root of the number of stored ions and to the $-4/3$ power of the emittance. For the longitudinal phase space, Γ depends, in addition, on the beam momentum spread. Using Eq. (121b), we arrive at

$$\Gamma_{i\parallel} = 2Z^2 r_i \left(\frac{4\pi}{3C_0 \langle \beta_y \rangle} \right)^{1/3} N_i^{1/3} \left(\frac{\epsilon}{\pi} \right)^{-1/3} \left(\frac{\sigma_p}{p_0} \right)^{-2} \beta_0^{-2}. \quad (141b)$$

Evidently $\Gamma_{i\parallel} / \Gamma_{i\perp} = T_{i\perp} / T_{i\parallel}$ holds.

Since the equilibrium temperatures depend very strongly on intrabeam scattering, the actual plasma correlation parameters that can be obtained in electron cooling should be derived from calculations taking into account heating and cooling processes (see Section 9). Because electron cooling usually results in ion beam temperatures that are much lower longitudinally than transversely, it is expected that longitudinal ordering sets in first [81]. The ordering and crystallization of ion beams has recently attracted considerable interest, and a number of theoretical calculations were performed [82], claiming that a very cold ion beam could crystallize in cylindrical or helical structures. We want to point out here that ordering can possibly be achieved without removing the heat completely from the incoherent motion of the ions, which is done by transferring it to coherent motion. However, one has to be sure that these coherent motions do not lead to instabilities.

7.12 Spin motion

This aspect is of relevance here, since it is planned to store polarized ions in cooling rings equipped with electron coolers. The theory of spin motion in circular accelerators is outlined in detail elsewhere [83], and the stability of polarization in the presence of electron cooling is discussed in Ref. [84]. Depolarization is caused mainly by the spin rotation in the longitudinal magnetic field of the cooler. Usually, the stable direction of polarization in a circular machine is vertical. Hence the spin σ_i would be rotated when passing through the solenoid. The spin precession frequency in a magnetic field perpendicular to the spin orientation is

$$\omega_{spin} = \omega_{e,c} \begin{cases} 1 + G & \text{for } \mathbf{v}_i \parallel \mathbf{B}, \mathbf{B} \perp \sigma_i \text{ (longitudinal magnetic field, transverse polarization)} \\ \gamma G & \text{for } \mathbf{v}_i \perp \mathbf{B}, \mathbf{B} \perp \sigma_i \text{ (transverse magnetic field, transverse or longitudinal polarization)} \end{cases} \quad (142a)$$

where G defines the magnetic moment of the ion μ_i in units of the nuclear magneton μ_N : $\mu_i = (1 + G)\mu_N$ for protons, $G = 1.793$). This spin rotation has to be compensated by correction solenoids, or the spin has to be oriented along the beam axis before the beam passes through the solenoid. Also, the action of the toroid magnets has to be corrected. The depolarization due to electron-ion hyperfine interaction in the cooler was found to be negligible [84]. Polarized beams are usually

obtained from polarized ion sources. For antiprotons—where such sources are not available—a number of ways of polarizing them have been discussed in the literature [85–87].

7.13 Beam diagnostics

Here we consider only the diagnostics that are of immediate relevance for electron cooling. A summary of beam-diagnostic means can be found elsewhere [88–90]. The following observables of the ion beam have to be determined when studying electron cooling:

- a) number of stored particles,
- b) lifetime,
- c) position and direction in the cooling region,
- d) amplitudes of coherent betatron oscillations,
- e) betatron tunes,
- f) coupling between transverse planes,
- g) bunch length and longitudinal particle distribution for bunched beams,
- h) rate of down-charged ions,
- i) emittance (beam size and divergence),
- j) momentum spread,
- k) absolute beam momentum,
- l) beam polarization,
- m) ordering.

The *number of stored particles* is usually determined with the aid of a current transformer; this has a sensitivity of the order of $1\ \mu\text{A}$. The number of ions can also be deduced from the longitudinal Schottky signal if it is well understood (see below). This is particularly useful at low beam intensity. For very low beam intensity the number of ions has to be measured destructively by dumping the beam on a calibrated counter. Possibly the residual gas ionization in a local pressure bump can be used for non-destructive low-beam intensity measurements. The number of circulating ions is related to the ion-beam current I_i by

$$I_i = Ze\beta c N_i/C_0. \quad (142b)$$

The *beam lifetime* is derived from the decrease of the beam intensity in time, measured as described before. It yields the total lifetime and does not allow a distinction to be made between the various types of losses. This method fails when the aim is to detect very fast losses. Counters (scintillator, channel plates, or semiconductor detector), at positions where ions hit the vacuum chamber first, could detect transverse beam growth with a time resolution that is much shorter than the revolution time of the ions. Detectors at positions where there is a large dispersion would be more sensitive to longitudinal instabilities.

The *position* (centre of gravity) is derived, with a precision of about 1 mm (absolute reproducibility; the relative accuracy is about 0.2 mm), from the signals induced in position pick-up electrodes by the bunched beam. With two pick-up stations in the cooling region, the beam direction therein is obtained.

When the beam is deflected coherently in the transverse plane, the *betatron tunes* can be derived from the envelope of the amplitudes of the position pick-up signals. Another method—beam transfer function—is to measure the beam response on a transverse perturbation and to analyse its Fourier transform [91]. This method provides also the possibility of measuring the coupling between horizontal and vertical phase space and the strength of possible coupling resonances [92]. The coupling between horizontal and vertical betatron oscillation is caused, for instance, by the solenoid of the cooler. If it is not compensated, it transfers energy from one plane to the other and usually

leads to a reduction of the machine acceptance from the horizontal value to the value of the (smaller) vertical acceptance. Therefore it is important to measure the coupling and to compensate it with correction solenoids or skew quadrupoles.

For a bunched beam, the *bunch length* has to be measured in order to determine the longitudinal emittance. The width of the pick-up signal (if fast electronics are used) provides this information. However, for very short bunches it may be difficult to extract the longitudinal ion distribution from this signal. The electron capture by positive beam ions, or the stripping of negative ions in the cooling region, results in a beam component that is separated from the primary beam, and the ions can be seen in fast detectors. The arrival time of the ions measured therein with respect to the RF phase yields the distribution of the beam particles in the bunch. Time resolutions below 1 ns are achievable.

The change of the charge state through *electron capture* in the cooler depends on the electron-ion velocity difference. The capture rate can be calculated from Eqs. (77) to (82) in subsection 5.1. The measurement of this rate provides a certain amount of information: firstly, the apparent temperature of the electron beam in the region defined by the overlap with the ions can be obtained; secondly, a possible misalignment of both beams, or wiggles in the magnetic field, also contributes. A rule-of-thumb says that the highest down-charge rate corresponds to the best matching of electron cooling. Once calibrated, the down-charge rate can be used to monitor the beam intensity and extend the sensitivity of beam-current measurements below the 1 μ A range.

The measurement of the spatial distribution of the lower-charge-state beam provides also the possibility of determining the *emittance* of the cooled beam if the focusing properties of the ring are well known for the trajectory of the lower-charge component. Its transverse distribution can be measured at several positions downstream from the cooler. If the β -functions are known, the divergence and the size of the cooled beam, and hence its emittance, can be determined. This method is not available with antiprotons. The transverse beam distribution can be derived, however, from the residual gas ionization in a region of the storage ring where there is bad vacuum pressure. The electrons and the ions from this process, or the recombination radiation, can be used to extract this information. A pressure in the region of 10^{-9} Torr is sufficient for obtaining the beam distribution within about 1 s. If the vacuum has to be better, a local pressure bump can be created by a pulsed (gas or vapour) atomic beam. This type of profile monitor was used in the initial cooling experiments at Novosibirsk, CERN, and FNAL, and proved to be very important for the observation of transverse cooling. The *beam size* can be measured by moving thin fibres or wires across the beam, or vice versa. This leads to a slight blow-up or a partial destruction, depending on the beam energy. Scrapers moved into the beam would eventually result in its full destruction. The beam can also be extracted for emittance measurements.

A very important non-destructive diagnostic for coasting beams is the Schottky noise picked up by electrodes [89, 93]. The *Schottky signal* corresponds to statistical density (current) fluctuations in the beam. For a circulating beam, the frequency spectrum shows peaks of amplitude $\langle A \rangle$ at the revolution frequency and its harmonics (Fig. 30). The electrical power contained in such a band ($\propto \langle A^2 \rangle$) is proportional to the number of stored particles if the signal is purely incoherent and free from collective effects. The relative spectral width of a line σ_f/f ($\Delta f/f$ is the most frequently used notation) is related to the beam *momentum spread* σ_p by Eq. (124):

$$\sigma_f/f_0 = \eta \sigma_p/p_0, \quad \Delta f/f_0 = \eta \Delta p/p_0, \quad (143)$$

and allows the determination of the latter. The absolute spectral width increases with the harmonic number h : $\sigma_{fh} = h_f \sigma_{f_0}$, and the signal height decreases since the power remains constant. The linear relation between peak power and the number of stored particles N_i is guaranteed [94, 95] only if $N_i \ll N_{\text{crit}}$ [for N_{crit} , see Eq. (130b)]. Collective effects contribute also to the signal. Their importance

depends on the beam temperature and on the machine properties [94, 95]. At low temperature the ions cannot be considered as being independent of one another, and collective motions build up (see longitudinal instabilities in subsection 7.5). In the longitudinal phase space they can be understood as waves moving both with and against the beam direction (for instance, negative mass instability). As the collective contributions increase, the intensity is removed from the centre of a Schottky line to its tails, resulting in two peaks shifted from the revolution frequency by $\pm f_c$ (Fig. 31). The frequency f_c is related to the number of stored particles and to the longitudinal coupling impedance for the machine by

$$(f_c/f_0)^2 = Z^2 e^2 \left| \frac{Z_{\parallel b}}{h_f} \right| \frac{|\eta| N_i}{p_0 C_0} = \frac{4\pi Z^2 r_i |\eta| N_i}{\beta_\gamma C_0} \left| \frac{Z_{\parallel h}/h_f}{Z_0} \right|. \quad (144a)$$

With Eq. (130b), we get

$$f_c/f_0 = (\eta \sigma_p/p_0) \sqrt{N_i/N_{\text{crit}}}, \quad (144b)$$

which shows that $N_i \rightarrow N_{\text{crit}}$ results in $f_c \rightarrow \sigma_f$. The behaviour of the power $\langle A^2 \rangle$ of a Schottky line as a function of the number of stored particles N_i depends on the machine parameters (η and $Z_{\parallel h}$). Below transition ($\eta > 0$) and purely inductive longitudinal impedance ($\text{Im } Z_{\parallel h}/h_f < 0$), we have $N_{\text{crit}} > 0$ and the power is initially ($N_i \ll N_{\text{crit}}$) proportional to N_i (shot noise) but levels off for large N_i ($\gg N_{\text{crit}}$) at a value which is determined by the longitudinal beam temperature (Fig. 32). In the other case (e.g. above transition and capacitive impedance), collective effects enhance the density fluctuations significantly and lead to a dramatic rise when N_i approaches N_{crit} . Whilst for $N_{\text{crit}} > 0$ the beam remains longitudinally stable, it becomes unstable (self-bunching) in the case where N_i approaches N_{crit} . A rigorous evaluation of the longitudinal Schottky signal is given in Refs. [94] to [96]. The perturbed Schottky signal can still be used to determine the momentum spread of the beam by measuring f_c and the ratio of peak height to central dip. At the same time, it yields a good check of the value of the longitudinal coupling impedance.

A pick-up made of electrodes, split into two parts, symmetric about the beam axis, delivers additional peaks (Fig. 33) separated from the revolution frequency peak $h_f f_0$ by $\pm h_f q_\gamma$ (non-integer fraction of the betatron tune, see subsection 7.2). They originate from the off-axis position of particles (owing to betatron oscillations) when passing the pick-up. The area under these peaks is proportional to $\epsilon_y N_i$ ($N_i \ll N_{\text{crit}}$) [89, 93]. Therefore the change in the beam emittance can be deduced by simultaneous measurement of the transverse and longitudinal Schottky signal, at least in relative values. For an absolute measurement, the signal has to be calibrated.

The Schottky noise analysis presents a very sensitive diagnostic for normal beam intensities, and its sensitivity can be considerably increased by using resonant pick-ups (narrow band) tuned to a harmonic of the revolution frequency. With such a system a beam intensity as low as 80 circulating antiprotons was measured in the ICE ring at CERN [97].

Collective motions also change the frequency dependence of higher harmonics; they enhance their amplitude. The coherent signal from a bunched beam covers the Schottky signal completely and makes a measurement of the momentum spread nearly impossible. In addition, it shows satellites corresponding to the synchrotron oscillations of the particles in the bunch. In this case, a crude estimate of the momentum spread can be obtained from the distribution of the synchrotron oscillation peaks.

The momentum spread can also be derived from the debunching time, i.e. the time it takes for a pulsed beam to smear out into a coasting beam in the absence of an RF voltage. The debunching time is given by

$$\tau_{ab} = (\pi - \Delta\phi) [2\pi f_0 \eta \sigma_p/p_0]^{-1}, \quad (145)$$

where $\Delta\phi$ is the azimuthal half-length of the bunch. This method can also be used for initially coasting beams. The bunch structure can be produced by kicking a hole into the beam and measuring the time that elapses until it is filled up. Of course, the momentum spread can be deduced from the longitudinal Schottky spectrum, as mentioned before. Finally, nuclear and atomic physics methods can be applied; these will be discussed in Section 13.

The *absolute value of the beam momentum* can be derived from the revolution frequency (peak position of the Schottky signal) if the orbit length is known. The accuracy achievable here is of the order of 10^{-3} to 10^{-4} . With electron cooling, the beam momentum can be determined from the electron acceleration voltage with about one order of magnitude better precision. Still higher accuracy is again obtained with nuclear and atomic physics methods.

The determination of the *beam polarization* is difficult and takes a long time. This is because the particles have to be scattered from a target, and the scattering asymmetries have to be measured. The low nuclear cross-section renders this procedure time-consuming. For the polarization measurement, either an internal target, or a beam extraction and an external target, is needed. In specific cases, again atomic physics methods might be used.

The unambiguous detection of ordering in the stored ion beam will be difficult. The methods to be applied are not clear at the moment; a suitable choice will be dictated by the type of ordering. In an ordered beam the Schottky signal is expected to vanish [94], so other methods have to be looked for. One line of thinking is directed towards the development of micron-wavelength pick-up stations; another direction could possibly be the measurement of beam transfer functions, i.e. the beam response to well-defined perturbations. If the beam ions still have at least one electron attached, possibly also atomic physics methods could be used.

8. THE EFFECTS OF THE ELECTRON COOLER ON THE ION BEAM

8.1 Machine optics effects of the electron cooler

The electron cooler represents an additional optical element in the ring lattice. The three major effects are produced by

- a) the solenoid magnet in the cooling region,
- b) the toroid magnets,
- c) the electric field of the electron beam.

The longitudinal magnetic field of the solenoid B_{sol} causes a cyclotron rotation about the longitudinal axis for ions with finite transverse velocity (betatron amplitude). The azimuthal angle θ_{iaz} of rotation per passage is

$$\theta_{iaz} = \frac{B_{sol} \ell}{B_Q}. \quad (146a)$$

The x- and z-coordinates of an ion after its passage are $x = y_0 \cos(\alpha_0 + \theta_{iaz})$ and $z = y_0 \sin(\alpha_0 + \theta_{iaz})$, with $\tan \alpha_0 = z_0/x_0$ and $y_0 = \sqrt{x_0^2 + z_0^2}$. The subscript zero denotes the coordinates before the passage. This twist leads to a coupling of the two transverse phase-space planes. For a coherent betatron oscillation, it shows up as a beating of the coupled oscillations. When using Eq. (115e), the variation of the transverse position after ν turns can be described by

$$y(\nu C_0) = \sqrt{\epsilon_y \beta_y(\text{cooler})} \sin(\alpha_0 + \nu \theta_{iaz}) \cos(2\pi \nu Q_y + \delta_y). \quad (146b)$$

The rapid betatron oscillations and the slow coupling frequency $\theta_{iaz}/2\pi$ can be seen.

Usually the vertical acceptance of a storage ring is smaller than the horizontal one. The coupling of the betatron oscillations reduces the latter to the smaller limit. The coupling may be useful for the reduction of the equilibrium emittance of the ion beam to the value given by the smaller β -function in the cooler. The smaller β -function magnifies the transverse angle (velocity), which helps to increase the cooling force for small beam emittances (cooled beam).

The spin of an ion is rotated by the longitudinal magnetic field according to Eq. (142a). For a transverse polarized beam passing through the cooler solenoid, the spin rotation angle θ_{spin} is [see Eqs. (112) and (142a)]

$$\theta_{\text{spin}} = \frac{(1 + G) B_{\text{sol}} \ell}{B_Q} = (1 + G) \theta_{\text{iaz}}. \quad (147)$$

This has to be compensated, or the spin has to be rotated by a multiple of π to maintain a stable polarization. Possibly, the cooler solenoid can also be implemented in a spin rotation scheme.

The magnetic field in the toroids can be decomposed into a longitudinal and a vertical component $B_z = B_{\text{sol}} \sin \phi$ if the gun and the collector are mounted upright (ϕ defines the angular position in the toroid). Consequently, for an S-shaped magnetic field configuration the ion beam is deflected horizontally to the same side, at the entrance and exit of the cooler. In a U-shaped cooling device, the deflection at the entrance is opposite to the deflection at the exit. The horizontal deflection angle θ_x amounts to

$$\theta_x = \int \frac{B_z ds}{B_Q}. \quad (148a)$$

With $ds = r_{\text{tor}} \cos^{-2} \phi d\phi$, we find

$$\theta_x = (\cos^{-1} \phi_0 - 1) \frac{B_{\text{sol}} r_{\text{tor}}}{B_Q}, \quad (148b)$$

where r_{tor} is the toroid bending radius and ϕ_0 is the value of ϕ at the ion exit of the toroid (Fig. 34). Simultaneous ramping of the ring and the cooler magnets does not change θ_{iaz} , θ_{spin} , and θ_x for resonant gun optics because B_z , B_{sol} , and the beam rigidity scale as $\beta\gamma$. The deflections of the toroids have to be corrected by dipole magnets, close to the cooler. It should be noted that the deflection is less severe for heavy ions of corresponding velocity, since the beam rigidity increases as A/Z [Eq. (112b)].

At this point a remark concerning the shape of the cooler (S or U) is appropriate. For a U-shape, the deflection of the ion beam, if not corrected, leads to its parallel displacement in the cooler (Fig. 35a). For an S-shape, the ion beam will be deflected twice in the same direction. This produces an offset of the orbit and a deflection angle downstream from the cooler (Fig. 35b).

The effect of the magnetic field shape on the electron beam is just the opposite. If the electron beam is not deflected correctly in the first toroid, the error is compensated in the second one and the electrons are brought back on axis for an S-shape (assuming equal fields in the toroids). For a U-shape the errors add up, leading to a displacement of the electron beam axis. This renders the electron collection more sensitive to mis-steering. However, the overall pros and cons are balanced for both options. Usually the cooler is installed with the vertical plane being the bending plane for the electrons (upright installation). If mounted in the horizontal plane, the ion beam is deflected vertically by the toroids, which is not always desirable because of the smaller vertical acceptance of the storage ring. This produces also an additional rotation of the spin, which is usually aligned in the vertical direction.

Finally, the electron beam presents an electrostatic lens for the ion beam. It causes the β -functions in the cooler to change slightly, and produces a tune shift [Eq. (123)]. Note that for a

constant-perveance gun ($I_e = P_{\text{gun}} U_{\text{cath}}^{3/2}$), this tune shift is almost independent of the electron cooling operation energy. With Eqs. (87), (88), (97), and (123), we get

$$\Delta Q_{y,e} = \frac{1}{18} \frac{Z}{(1 + \gamma)^{3/2}} \frac{m_e}{m_i} \frac{\ell \langle \beta_y \rangle}{\pi r_0^2} \frac{P_{\text{gun}}}{P_0}. \quad (149)$$

8.2 The actions of the frictional force in the cooling section

The situation in the cooling section is illustrated in Fig. 36a, where the velocity profile of the electrons (parabola) and the dispersion of the ion beam (straight line) are plotted versus the horizontal coordinate h_i in the cooling section. In phase space, normally distributed ions would have the positions indicated by the dots. The origin of the coordinate system corresponds to the velocity and the horizontal position of an ion circulating on the nominal orbit. The projection of the dot distribution on the momentum and on the horizontal axes is given below and to the left in Fig. 36a, and the situation for the vertical direction (zero dispersion) is shown in Fig. 36b. The distance of a dot from the dispersion line corresponds to the betatron excursions of the ion. Betatron oscillations are represented by lines parallel to the vertical axis centred on the dispersion line.

The effect of longitudinal cooling can be understood as follows. When an ion passes through the cooler at a position $h_i = d_i$ and its velocity is greater than the electron velocity at this position $v_e(d_i)$ (dots above the parabola), the drag force will decelerate it a little (vertical step in Fig. 36a). Particles to the left of the dispersion line in Fig. 36a will thus have their betatron amplitude reduced simultaneously, whilst the other particles will have it increased. During the following turn in the ring, the particle performs Q_x betatron oscillations and arrives in the cooling section at a different position d_i' the next time [Eq. (115e)]. The action of the frictional force will again lead to a change of its energy. Simultaneously, transverse cooling will lead to a deflection that results in a reduction of the horizontal elongation of the betatron oscillation. If the dots are below the parabola (ions slower than corresponding electrons), the ions will be slightly accelerated when they pass through the cooler. These repeated kicks will finally move all particles with positions to the left of point B in Fig. 36a to point A . Ions to the right of point B are slower than the electrons. Their acceleration leads to a displacement further to the right of B . These particles are therefore lost after repeated acceleration, because they leave the momentum acceptance of the machine. The stable region in Fig. 36a is hence to the left of B . For a finite emittance beam, B is not a sharp boundary but is more a diffuse region created by the interplay of transverse and longitudinal cooling. The stable region can be extended by lowering the electron energy (acceleration voltage) shown by the dashed parabola. Then the ions are cooled to point A' .

The velocity dispersion in electron and ion beams changes the shape of the longitudinal cooling force, because the electron-ion velocity difference (i.e. the vertical distance between the dispersion line and the parabola for a zero emittance beam) is a function of the radial position. The cooling in the vertical plane is similar; however, there the dispersion is usually zero. Non-vanishing dispersion leads to a coupling of longitudinal and transverse phase space, as explained before. It also causes a horizontal spread in the beam size if the electron acceleration voltage fluctuates (high-voltage ripple).

The shape of the longitudinal cooling force modified by a finite dispersion (Fig. 37) shows that the ions between points A and B are cooled rapidly. Those that are not so far to the left of A will also be cooled quickly, but the strong decrease of the force for still slower ions can cause a low-energy tail in the momentum distribution.

A misalignment of electron and ion beams (Fig. 38a) has the consequence that ions meet electrons of different longitudinal velocities on their way through the cooler. This leads to a smearing of point A (Figs. 38b and c) and limits the lowest attainable momentum spread (and beam size, for non-zero dispersion).

For well-aligned beams and zero dispersion in the cooler, point A moves along the ordinate when the electron voltage is changed. The revolution frequency on the nominal orbit corresponds to the electron high-voltage setting given by

$$f_0 = \frac{c}{C_0} \sqrt{\left\{ \frac{eU_{\text{cath}}}{m_e c^2} - n_e(1-\zeta)\pi r_0^2 r_e \left[1 + 2 \ln \left(\frac{r_{\text{tube}}}{r_0} \right) \right] \right\} \frac{\gamma + 1}{\gamma^2}} \quad (150)$$

{see Eq. (99) and $E_0 = m_e c^2 [(\beta_0^2 \gamma^2)/(\gamma + 1)]$ }, where the second term under the square root is due to the space-charge depression. When the dispersion D has a finite value in the cooler, A moves along the dispersion line. A change in the high-voltage dU is related to a change in the revolution frequency df through

$$\frac{df}{f_0} = \frac{\eta}{c_2} \left(\sqrt{1 + c_2 \frac{eU_{\text{cath}}}{E_{\text{ax}}} \frac{dU}{U_{\text{cath}}}} - 1 \right) \quad (151a)$$

[using Eqs. (98), (99), (113b), and (124)], with $c_2 = n_e(1-\zeta)\pi r_e(\gamma-1)^{-1} D^2 - 1$. For small changes in dU we get approximately

$$\frac{df}{f_0} = \frac{\eta}{2} \frac{eU_{\text{cath}}}{E_{\text{ax}}} \frac{dU}{U_{\text{cath}}} \quad (151b)$$

By making a careful measurement of these relations [Eqs. (151)], we can extract the potential depression on the electron beam axis ($eU_{\text{cath}} - E_{\text{ax}}$) and hence the amount of space-charge compensation, even without comparing the absolute values of the high voltage and the revolution frequency. Whilst in the first case (zero dispersion) the ion beam has to be moved radially across the electron beam in order to measure the parabolic velocity profile in that beam, in the latter case (finite dispersion) the potential depression shows up directly in the dependence of the revolution frequency on the high voltage [Eq. (150)]. If the position of the cooled beam in the cooler can be measured precisely, then the electron velocity profile should show up according to

$$e dU = -\frac{c_2}{D^2} E_{\text{ax}} r^2 + \frac{2}{D} E_{\text{ax}} r. \quad (152)$$

9. COMPUTER SIMULATION OF ELECTRON COOLING, AND PRACTICAL ESTIMATES

The detailed analysis of electron cooling experiments and the prediction of expected cooling performances for specific configurations calls for a precise description of the processes involved, including the properties of the electron cooler and the storage ring. Programs that simulate electron cooling exist at varying levels of complication. Most of them use cooling times rather than frictional forces, and do not take into account machine and beam properties. They are limited to special cases. The simulation program should provide the interface between experiment and theory. Measurements of beam emittances (beam size and divergence) and momentum spread, their evolution in time, and the equilibrium values, have to be compared with calculations in order to draw conclusions about the cooling force and the heating processes or to optimize machine settings.

9.1 Simulation programs

Several simulation programs have been developed [98–103] that partly fulfil these requirements. The effect of an internal target on beam emittance and momentum distribution counteracted by

electron cooling is simulated by a program [102] that evaluates the average energy loss and transverse scatterings per particle and passage, and modifies the beam emittances and momentum distribution accordingly. In this program, the electron cooling is taken into account by a constant transverse cooling time and a longitudinal force consisting of a linear part, a region where it is constant, and a gradually decreasing part. One of the results delivered by this program is the prediction of a long low-energy tail for beam energy distribution when using a thick internal target with a coasting beam.

A simple program for the study of ultra-slow extraction of an ion beam in the presence of electron cooling [103] simulates the longitudinal ion-beam blow-up and the diffusion into an extraction resonance through stochastic heating. Electron cooling is added by using the force approximation given in Eq. (41). It calculates the longitudinal distribution function in a steady state by solving the Fokker-Planck equation.

The most advanced program is SPEC, the code developed by the KfK Group [99–101], which makes use of the magnetic cooling force formulae derived by Sørensen and Bonderup [23] and of the non-magnetic force in the form given by the Novosibirsk group [19]. It is partly based on the program developed by Bell [98], and simulates electron cooling in the presence of intrabeam and residual gas scattering. In this simulation, a hypothetical ring is filled with 100 model particles (this number is variable) representing ensembles of real particles. The initial distributions in emittance and momentum spread are chosen according to the input parameters. The position of each particle in the cooler is calculated from these initial phase-space coordinates (Fig. 36). This fixes the corresponding electron velocity [parabolic velocity profile, Eq. (98)]. The transverse electron temperature, the azimuthal drift at this radial position, and the acceleration voltage ripple are also taken into account. The momentary cooling forces are calculated from the relative ion-electron velocity difference vector. The cooling forces (magnetic and non-magnetic) are tabulated for a wide range of values of the two independent parameters: the value of the ion velocity in the rest frame, and the angle between the velocity vector and the longitudinal direction. The determination of these parameters from the value of the single-particle emittance of each model particle [Eq. (120a)] makes it possible to control the effects of different contributions (Fig. 39) and to study their impact on the cooling process.

Once the cooling force is determined from the tables (interpolation), the velocities of the model particles are changed correspondingly (time base: multiple of turns). Then the particles are transferred around the ring, using the lattice functions and the procedure described in subsection 7.2. Before the next cooling action is applied, the phase space is blown up according to the rates for intrabeam scattering calculated from the formulae given by Piwinski [76], and reduced to a simpler form by Evans and Zotter [104]. The actual phase-space population is taken for the blow-up rates. Residual gas scattering is implemented by rescaling the emittances according to Eq. (135b). The changes in the phase-space coordinates are made after a preselected number of turns in order to save computer time for a sufficiently long observation of the evolution of the phase-space population (Figs. 40a–c). The effects are scaled accordingly. In this way the momentary cooling and blow-up rates for all three degrees of freedom are calculated, and the evolution of the emittances (Fig. 41a) and momentum spread (Fig. 41b) in time is obtained. This treatment allows a detailed study of the beam properties and their dependence on the machine parameters to be made. Moreover, it provides us with the possibility of investigating specific contributions from the cooling force and of modifying them according to the experimental data.

In Fig. 41a, two regions of distinct damping rates and a region where the equilibrium is reached can be seen. In the part with the smaller damping rate, essentially only the non-magnetic cooling force plays a role, whilst after initial compression of the phase space, the onset of the magnetic force leads to faster cooling. The equilibrium values reflect the contribution from intrabeam scattering, from residual gas scattering, from the alignment of the beams, and from the residual temperature of the electron beam. However, the program does not include diffusion, which means that for negligible

residual gas and intrabeam scattering the thermal equilibrium is not simulated correctly. We shall again make use of the simulation program for the interpretation of the experimental data at the end of the next section.

9.2 Transformation to the laboratory system

The forces and cooling times mentioned in Sections 3 and 4 were derived in the system moving with the average velocity of the ions. In the laboratory system they are obtained through a Lorentz transformation:

$$F_{\perp \text{lab}} = \gamma F_{\perp} , \quad (153a)$$

$$F_{\parallel \text{lab}} = F_{\parallel} , \quad (153b)$$

$$\tau_{\text{lab}} = \gamma \tau . \quad (153c)$$

Replacing the electron density in the rest frame by the laboratory electron density $n_{e \text{lab}} = \gamma n_e$ yields another factor of γ . Finally, the ratio η_c of the active cooling length to the ring circumference has to be taken into account. This then yields

$$F_{\perp \text{lab}} = \eta_c F_{\perp} , \quad (154a)$$

$$F_{\parallel \text{lab}} = \eta_c / \gamma F_{\parallel} , \quad (154b)$$

$$\tau_{\text{lab}} = \gamma^2 / \eta_c \tau . \quad (154c)$$

The above formulae for the laboratory system indicate that electron cooling at highly relativistic energies ($\gamma \gg 1$) gives long cooling times because of the γ^2 / η_c dependence. Therefore it is unlikely to be of great interest at high energies.

The relation between the velocity of the ion in the rest frame and its deviation $\delta p = (p_i - p_0)$ from the nominal momentum measured in the laboratory is

$$v_{i \parallel} = \beta_0 c (\delta p / p_0) .$$

The relation of $v_{i \parallel}$ to a change of electron energy can be found from Eqs. (124) and (151). The relation between the transverse ion velocity and the transverse angle is

$$v_{i \perp} = \beta_0 \gamma c \theta_{i \perp} .$$

9.3 Thermal equilibrium

The prediction of theory for thermal equilibrium in the case of $B = 0$ is $T_{i \parallel} = (\lambda_{\perp} / \lambda_{\parallel}) / T_e$ and $T_{i \perp} = 1/2 T_e$ [Eqs. (72)]. The longitudinal ion temperature (velocity spread) results from the momentum distribution and does not change within the ring:

$$\frac{\sigma_p}{p} = \frac{\lambda_{\perp}}{\lambda_{\parallel}} \sqrt{\frac{m_e}{m_i}} \frac{\Delta_e}{\beta_0 c} \approx \sqrt{\frac{m_e}{m_i}} \frac{\Delta_{\parallel}}{\beta_0 c} . \quad (155)$$

(For the approximation we assumed $\lambda_{\perp} / \lambda_{\parallel} = \Delta_{\parallel} / \Delta_{\perp}$.) The transverse velocity spectrum depends, however, on the β -function in the cooler [Eqs. (115) and (121)]. The ion beam divergence σ_y adjusts itself to the electron beam 'divergence' $\Delta_e / \beta_0 c$ as

$$\sigma_{y'} = \frac{1}{2} \sqrt{\frac{m_e}{m_i}} \frac{\Delta_e}{\beta_{0c}} = \frac{1}{2} \sqrt{\frac{m_e}{m_i}} \theta_{e\perp} . \quad (156)$$

In the case of a thermocathode and of ideal electron beam acceleration and transport, $\Delta_e/\beta_0 = c \sqrt{0.22 \text{ eV}/m_e c^2} = 2 \times 10^7/\beta_0 \text{ cm/s}$, which means that the transverse velocity spread in a proton beam could be reduced to, at best, $2.3 \times 10^5 \text{ cm/s}$ or a divergence of $7.6 \mu\text{rad}$. Hence in principle, a beam emittance of about $6 \times 10^{-5} \pi \text{ mm}\cdot\text{mrad}$ ($\beta_y \approx 1 \text{ m}$) could be reached. Longitudinally, the velocity spread could assume values in the range of 10^3 cm/s in thermal equilibrium. In practice, however, machine errors (misalignment, ripple) usually prevent such low values from being reached.

9.4 Practical formulae

9.4.1 Transverse equilibrium between multiple scattering and electron cooling

Repeated small-angle scattering from residual-gas particles leads to an emittance growth $\dot{\epsilon}_{ms}$ (subsection 7.7), which has to be balanced by the emittance reduction $\dot{\epsilon}_c$ through cooling. The equilibrium emittance is obtained by solving the differential equation

$$\dot{\epsilon}_{ms} + \dot{\epsilon}_c = 0 . \quad (157)$$

Let us assume, for simplicity, that the cooling is exponential; then $\dot{\epsilon}_c = -2/\tau_{lab}\epsilon_y$ (the factor of 2 comes from $\epsilon_y \propto \sigma_y^2$). Inserting this in Eq. (157), we get for the equilibrium emittance approximately

$$\epsilon_{eq} = \tau_{lab}/2\dot{\epsilon}_{ms} . \quad (158)$$

The emittance growth rate can be obtained from Eq. (135b). If the multiple scattering is due to scattering from an internal gas target, $\dot{\epsilon}_{ms}$ has to be multiplied by $\beta_{targ}/\langle\beta_y\rangle$, where β_{targ} is the value of the β -function at the target position.

9.4.2 Energy loss

When the ions traverse the residual gas or an internal target, they are subjected to an energy loss dE/dx [Eq. (137a)]. This deceleration is compensated by electron cooling as long as $dE/dx < F_{\parallel lab}^{max}$. When equality is reached, the ion beam loses energy continuously and a low-energy tail develops, which will be lost when the longitudinal machine acceptance is reached. The inspection of Eq. (54c) and Fig. 12b shows that the longitudinal adiabatic force reaches its maximum around $v_{i\parallel} \approx 0.2\Delta_{\parallel}$. It is however doubtful whether this is a reasonable upper limit for practical considerations. It might be safer to take $F_{\parallel}^{ad}$ at $v_{i\parallel} = \Delta_{\parallel}$ and $v_{i\perp} = 2v_{i\parallel}$. Then we get

$$|F_{\parallel lab}^{max}| \approx 0.1 F_0 \frac{c^2}{\Delta_{\parallel}^2} L_C^{ad}(\Delta_{\parallel}) \frac{\eta_c}{\gamma} , \quad (159)$$

which results in about $3 \text{ eV/m} \times \eta_c/\gamma$ for our standard case. We therefore obtain

$$n_{gas} \leq 0.1 \frac{L_C}{L_{res}} \frac{\eta_c}{\gamma Z_{gas}^2} \left(\frac{\beta_{0c}}{\Delta_{\parallel}^2} \right)^2 n_e ,$$

which means that the gas density can be many orders of magnitude higher than the electron density.

9.4.3 Intrabeam scattering

Similarly, the equilibrium between intrabeam scattering and electron cooling can be estimated by using the model derived in subsection 7.6. In practice, the beam is usually colder in the longitudinal direction than in the transverse one. Consequently, intrabeam scattering leads to a blow-up of the momentum spread. If thermal equilibrium is reached, then it is $\sigma_p/p \ll \Delta_{||}$ [see Eq. (155)] and the longitudinal cooling decrement $\lambda_{||} = -F_{||}^{ad}/(m_i v_{i||})$ is independent of $v_{i||}$ ($\propto \sigma_p/p$). With an increasing number of stored ions, intrabeam scattering becomes important and the equilibrium is determined by the condition $\lambda_{IBS||} = \lambda_{||lab} = \tau_{||lab}^{-1}$. With Eq. (131f) this results in:

$$\left(\frac{\sigma_p}{p}\right)_{IBS} = \left\{ \frac{32}{3\sqrt{\pi}} \frac{r_i^2 Z^4 L_C N_i \tau_{||lab}}{C_0 \beta_0^3 \gamma^{1/2}} \right\}^{2/5} \left\{ \frac{\epsilon}{\pi} \langle \beta_y \rangle \right\}^{-1/2} \langle \beta_y \rangle^{1/5}. \quad (160)$$

We may evaluate this formula further by using Eqs. (54c) and (154c) assuming, for the Coulomb logarithm for electron cooling and intrabeam scattering, $4.5 L_C^{ad} = L_C$. For $\sigma_p/p \leq \Delta_{||}$ we then get

$$\left(\frac{\sigma_p}{p}\right)_{IBS} = \left(\frac{4}{3\sqrt{\pi}} \frac{m_e}{m_i} \frac{\Delta_{||}^3}{c^3} \frac{Z^2 \gamma^{1/2}}{\beta_0^3 n_{el}} \right)^{2/5} \frac{\langle \beta_y \rangle^{1/5}}{\langle r_{oi} \rangle} N_i^{2/5}. \quad (161)$$

This formula indicates the following scaling behaviour for a constant beam emittance (radius):

$$(\sigma_p/p)_{IBS} \propto \begin{cases} (Z^2/A)^{2/5} \\ N_i^{0.4} \end{cases} \quad (162)$$

Since $\Delta_{||}$ may also depend on β_0 [Eqs. (95)] the equilibrium momentum spread scales with β_0 as

$$(\sigma_p/p)_{IBS} \propto \begin{cases} \text{independent of } \beta_0 & \text{for } \sigma_U/U_{cath} \gg kT_{cath}, V_{ec} \\ \beta_0^{-6/5} & \text{for } V_{ec} \gg \sigma_U/U_{cath}, kT_{cath}. \end{cases}$$

This scaling holds as long as $\lambda_{||} = \text{const.}$, i.e. $\sigma_p/p \leq \Delta_{||}$. For a higher number of stored ions, σ_p/p may become larger than $\Delta_{||}$ and $\lambda_{||} \propto v_{i||}^{-3}$. Consequently $(\sigma_p/p)_{IBS}$ should increase more rapidly than $N_i^{0.4}$. The equilibrium momentum spread calculated from Eqs. (160) and (161) is plotted in Fig. 42 versus the number of stored ions for a few typical cases. One distinguishes the region of thermal equilibrium (horizontal lines) and the domain where intrabeam scattering leads to a higher momentum spread. We shall see later how these estimates, in particular the scaling with N_i , compare with experiments and simulations of electron cooling in storage rings.

9.4.4 Down-charge rate

As discussed in Section 5, the pick-up of electrons by ions in the cooler leads to a down-charge of the ion. Depending on the state in which the electron is captured, it is again stripped off in the motional electric field produced by the cooler toroids or by the ring bending-magnets. These ions usually remain in the ring. However, when the captured electron is bound strongly enough, the ions are lost unless the storage ring can accommodate several charge states. A change in the charge-to-mass ratio is equivalent to a change in momentum by approximately the same amount. Therefore the fate of the down-charged ion depends on the momentum acceptance of the ring.

The down-charge rate for naked ions can be calculated from Eq. (82). By comparing Eqs. (73b), (79), and (82), we find that the ratio of beam decay time τ_r ($\approx N_i/R_a$) due to electron capture, to cooling time ($\sigma_{v_i} \ll \Delta_e$), is

$$\frac{\tau_r}{\tau} = \frac{3}{4\sqrt{\pi}\Sigma(1/n)} \frac{m_e}{m_i} \frac{L_c}{\alpha} \frac{m_e c^2}{kT_e} \approx \frac{1}{3A} \frac{m_e c^2}{kT_e}. \quad (163)$$

Since $m_e c^2/kT_e$ is of the order of 5×10^6 , the recombination time for all ions is much longer than the cooling time.

9.5 Optimal lattice functions for electron cooling

Of course it is impossible to give a recipe for the optimal matching of cooler and ring-lattice functions because a particular choice depends very much on the application of electron cooling. However, a few general considerations might provide helpful guidelines.

9.5.1 Dispersion in the cooling region

A finite dispersion in the cooling section leads to a coupling of the longitudinal and transverse phase space through the cooling action. Consequently, a ripple of the electron acceleration voltage smears out the crossing between the parabola and the dispersion line (point *A* in Fig. 34), which leads to a horizontal beam spread:

$$\Delta x = \frac{D}{2} \frac{\sigma_{\tilde{U}}}{U_{\text{cath}}}. \quad (164)$$

On the other hand, a finite dispersion increases the longitudinal cooling force for ions above point *A* in Fig. 36, because the velocity difference between an ion and an electron at a given radial position *r* is smaller than for $D = 0$. It probably also increases the transverse cooling force. This could help to reduce the cooling time for initially hot beams. A similar reduction of the cooling time for large momentum spreads is obtained, even for zero dispersion, by sweeping techniques: the electron voltage is first put to the lowest voltage and then slowly increased. The parabola moves up, and the bottom part of it is filled by preventing ions from falling substantially below point *A*. All in all, it is advantageous to have a small dispersion in the cooling region. Zero dispersion, however, is mandatory for reaching low beam temperatures.

9.5.2 β -functions in the cooling region, and the size of the electron beam

The values of the β -functions and the desirable electron beam size are related. There are two criteria to be fulfilled:

- i) the initial ion beam size should be smaller than that of the electron beam:

$$\frac{\epsilon_y \beta_y}{\pi} + \left(D \frac{\sigma_p}{p} \right)^2 \leq r_0^2; \quad (165)$$

- ii) the transverse ion velocity should be smaller than the corresponding one for the electrons:

$$\sigma_{y'} \leq \frac{\Delta_e}{\beta_{0c}}. \quad (166a)$$

With Eq. (119b) we therefore get the condition

$$\frac{\epsilon_y}{\pi} \left(\frac{\Delta_e}{\beta_{0c}} \right)^{-2} \leq \beta_y \leq \frac{r_0^2 - [D(\sigma_p/p)]^2}{\epsilon_y/\pi}. \quad (166b)$$

The minimal radius with guaranteed overlap is obtained when the equality is reached in Eq. (166c):

$$r_0^2 = \left(\frac{\epsilon_y}{\pi} \right)^2 \left(\frac{\Delta_e}{\beta_{0c}} \right)^{-2} + \left(D \frac{\sigma_p}{p} \right)^2. \quad (166c)$$

This then fixes also the range of β_y . The lowest β_y within this range should be chosen because it leads to the lowest ion-beam temperature.

9.5.3 Length of the cooling section

The cooling time in the laboratory depends linearly on the length of the cooling section. In initial cooling experiments, it was between 2% and 4% of the ring circumference. The length of the cooler defines approximately the lower limit of the β -function. Therefore it should be

$$\ell \leq \frac{\epsilon_y}{\pi} \left(\frac{\Delta\epsilon}{\beta_0 c} \right)^{-2}. \quad (166d)$$

On the other hand, the deterioration of the magnetic field properties at both ends of the cooler solenoid reduces the useful cooling section. Therefore ℓ should not be much below 1 m.

10. ELECTRON COOLING EXPERIMENTS

10.1 The first electron cooling at Novosibirsk, CERN, and FNAL

The first test of electron cooling and the study of the cooling performances were made in the NAP-M ring at Novosibirsk [2] from 1975 to 1979, in the ICE ring at CERN [3] from 1979 to 1980, and in the cooling ring at FNAL [4] from 1980 to 1982. A summary of these experiments can be found in Ref. [9]. In all three laboratories the initial aim of the cooling tests was to demonstrate the principle of this new method and to study its practicability. It was soon followed by a systematic investigation of the cooling force and times, and of their dependence on various parameters. The most detailed studies were done in the NAP-M ring. The first coolers were experimental set-ups that were not suitable for routine operation in a regular storage ring for physics experiments. The parameters of the first storage rings and their coolers are given in Table 10.1 and a schematic layout is shown in Figs. 43 and 44. As mentioned in Section 1, the original idea was to use electron cooling for antiproton accumulation, but it turned out that stochastic cooling was more appropriate for this task. However, an electron cooler was developed for the cooling of *low-energy* antiprotons. Based on the ICE electron cooler, the first electron cooling device for cooling antiprotons routinely [5] was developed for LEAR [105].

10.2 The second generation of electron cooling: the LEAR electron cooler

The original motivation for the application of electron cooling in LEAR was to improve the properties of slow antiproton beams for external experiments, to enable the operation of an internal target, and possibly to provide a low-emittance beam for collider experiments [106–108]. As LEAR was to be equipped with stochastic cooling at injection energy (175 MeV) and at some other energies, the emphasis was put on electron cooling at low energy, below 76 MeV. It was also planned to store H^- ions at low energy. These conditions required that the electron beam be operated in ultra-high vacuum to avoid excessive stripping of H^- ions and loss of antiprotons when operating at the lowest energy. The requests for reliable routine operation and for transport of the electron beam in an ultra-high vacuum were the two most important of the many challenges to be faced. The LEAR electron cooling parameters are listed in Table 10.2. The LEAR electron cooler came into operation in 1985 but was installed in LEAR only in 1987, when it was first tested with protons. The main goals of the first electron cooling experiments were to demonstrate that this device holds the promise of smooth operation in the specified energy range, and to provide cooling times of the order of one second. The first performance tests with the LEAR electron cooler were done at three energies: 50, 21, and 10 MeV [109]. A number of investigations of electron cooling were made during these experiments. In particular, the longitudinal Schottky signal and its splitting was investigated in detail.

Also, instabilities showing up at low beam temperature were studied. Recently also the cooling of antiprotons was successfully performed at LEAR [110].

10.3 The linear arrangement MOSOL at Novosibirsk

When the first cooling experiments at Novosibirsk were terminated abruptly by a fire that destroyed the NAP-M ring, the group built up an arrangement, the Model Solenoid (MOSOL) [49], with which they measured the cooling force exerted by electrons on protons and on H^- ions (Fig. 45). A comparison was made of the force between like charges and between unlike charges.

The H^- ion beam was accelerated electrostatically to 850 keV and bent into the direction of the solenoid containing the 460 eV electron beam of up to 15 mA. There it overlapped with the electron beam over a length of 2.4 m. Then the 1 mm electron beam was decelerated and collected. The ion beam escaped through a hole in the collector, and its energy and size were analysed further downstream. Also the neutralization of the H^- beam was measured. The H^- ions could be stripped to protons before they were bent to enter MOSOL, which allowed the measurements to be repeated with protons. The frictional force was measured for magnetic fields between 1 and 4 kG. The results of the measurements will be presented in subsection 10.6.

10.4 The new electron coolers for light- and heavy-ion rings

As a consequence of the success of the first cooling experiments — and possibly also triggered off by the development of the electron cooling for LEAR — a number of electron cooling projects were started. Their parameters will be summarized in subsection 11.1. Two of these projects have already reached the operational stage and have delivered first cooling results: the high-energy cooler at the Indiana University Cyclotron Facility (IUCF) in Bloomington, Indiana [111], and the low-energy Test Storage Ring (TSR) facility in Heidelberg [112]. The major new results from these projects are: the cooling at highest proton energies; the cooling of polarized protons [113]; and the cooling of light ions [112]. More results will be given in subsection 10.6.

10.5 Observables and measuring techniques

Before summarizing the main results from all cooling experiments, we will briefly identify the observables that are of importance for the cooling studies, and describe how they are measured. The most important quantities are

- i) momentum distribution,
- ii) transverse beam distributions and emittances,
- iii) beam lifetime,
- iv) electron pick-up rate,
- v) bunch area.

Our knowledge of these quantities and their evolution in time enables most of the cooling characteristics to be extracted. For example, the longitudinal frictional force can be directly determined by observing the time evolution of the momentum distribution. In the transverse direction, it is mostly the cooling time that is deduced.

10.5.1 Coasting beam

For coasting beams, the important Schottky diagnostic is available for the measurement of the longitudinal and transverse phase-space quantities.

A) Longitudinal cooling

i) *Cooling after injection:* Usually a freshly injected beam has a high temperature in all three directions, and from the evolution of the Schottky signal (Fig. 46) the longitudinal cooling time for a

high-emittance beam can be deduced. If the initial momentum spread of the injected beam is low (as in the case of an electrostatic injector), the longitudinal frictional force can be determined from the time it takes the injected beam to be accelerated (decelerated) to the velocity of the electrons (method used at the TSR).

ii) Longitudinal blow-up: A cold beam is blown-up longitudinally to a well-defined distribution (for instance by imposing stochastic noise or repeated fast bunching and debunching) and is then cooled (Fig. 47). This provides the possibility of measuring the longitudinal cooling time as a function of initial momentum spread. If coupling between the longitudinal and the transverse direction is avoided during blow-up, the beam remains cold in the transverse direction, which makes the evaluation simple.

iii) Equilibrium between longitudinal heating and cooling: If the cooling is activated while heating continues, then equilibrium between heating and cooling is obtained (Fig. 48). The shape of the cooling force can be deduced from the equilibrium distribution if the characteristics of the heating source are known [114]. This method provides access to the measurement of the cooling force at very small ion velocities.

iv) High-voltage step, ramp, or modulation: The longitudinal cooling force can be measured directly by rapidly detuning the velocity of the electron beam (a step in the acceleration voltage of the electron beam) and measuring the rate of momentum-change from the Schottky spectrum (Fig. 49) or from the variation of the horizontal beam position in a region with high dispersion (Fig. 50). With this procedure, the longitudinal cooling force can be measured as a function of the ion velocity. The arrival time of the ion beam at the new position after a voltage step can also be measured by having a small RF bucket at the corresponding frequency position. The filling of the bucket can be seen from the rise of the coherent Schottky signal or from an electrostatic pick-up. This method (used at FNAL) is particularly interesting for low beam intensities because the coherent signal is much larger than the Schottky noise signal. A continuous high-voltage ramp permits the maximum cooling force to be determined by varying either the step height or the frequency and finding the maximum drag rate. This can also be achieved by imposing a low-frequency modulation on the high voltage (≈ 1 Hz) and observing how long the ion beam follows the oscillation of the electron energy when the modulation frequency is increased.

B) Transverse cooling

The evolution of beam emittances can be deduced from the sidebands appearing in the transverse Schottky spectrum. This requires a simultaneous measurement of the longitudinal Schottky spectrum. Another possibility is to measure the beam profiles at positions with low dispersion. The beam profiles can be obtained from residual gas ionization [3, 4] or by passing the ion beam through an atomic beam [2] or by sweeping it over a wire [113]. The divergence can be deduced from the angular distribution of the down-charged ion beam, which is particularly simple for single charged ions because the electron pick-up delivers a neutral beam and renders beam tracking unnecessary [109].

i) Cooling after injection: As in the longitudinal cooling studies, one measures either the time evolution of the emittances deduced from the sidebands of the transverse Schottky signal (Fig. 33) or the development of the beam size after injection (Fig. 51). This provides the transverse cooling times. They can be measured as a function of transverse velocity spread if the beam size is defined.

ii) Transverse blow-up: The cold ion beam can be heated in the transverse direction, either by using a method that is similar to the one used in the longitudinal direction (stochastic heating) or by multiple scattering in a bad vacuum region. Also, transverse instabilities can be used to blow up the beam. Coherent betatron oscillations may be excited by a transverse kick. In the first two cases, the transverse cooling time can be determined from the evolution of emittances or sizes; the transverse force can be deduced from the damping of coherent betatron oscillations. In these measurements, possible contributions from Landau damping have also to be taken into account.

C) Equilibrium properties and beam temperatures

Equilibrium momentum spread and beam sizes are determined by the heating processes and by the quality of the electron beam. In the absence of internal targets, the major heating source is intrabeam scattering. For a comparison with theory, the equilibrium momentum spread has to be measured versus the number of stored particles. The unambiguous determination of this quantity is of great importance for evaluating the possibility of increasing the region of longitudinal stability and of reaching ordering in the ion beam. The longitudinal Schottky signal has to be carefully analysed because of the collective effects discussed in subsection 7.11. The momentum spread can be determined differently by measuring the debunching time.

The transverse temperatures can be determined either from the beam emittance or from the beam size and divergence, measured with sufficiently high resolution. In the case of single charged ions, the neutral beam offers an excellent means for the determination of these quantities.

10.5.2 Bunched beam

The cooling of a bunched beam requires the electron acceleration voltage to be set so that the revolution frequency (or its higher harmonic) to which the beam is cooled equals the RF value. For a bunched beam the momentum spread and the bunch length have to be measured in order to fix the longitudinal phase space. The measurement of the transverse-beam properties is done as described in subsection 10.5.1 for the coasting beam. The Schottky noise signal is covered by the coherent bunch signal (orders of magnitude larger) and is therefore not available for the measurements. On the other hand, the signals produced by the bunched beam in electrostatic (position) pick-ups are useful for the measurements.

A) Longitudinal cooling

i) Adiabatic bunching and longitudinal cooling: The coasting beam is captured in a bucket without increasing the longitudinal emittance. The activation of electron cooling results in an increase of the signal from the electrostatic pick-up and in a reduction of the bunch length. The observation of this signal amplitude as a function of time after turning on the electron cooling (Fig. 52) provides the longitudinal cooling time for a bunched beam.

ii) Injection in the presence of RF: Injection of a beam pulse when the RF and cooling are on, yields the same behaviour of the electrostatic pick-up signal as that described in (i). The longitudinal cooling time is again obtained from the evolution of the pick-up amplitude and width.

B) Transverse cooling

In addition to the possibilities described above for the coasting beam, here there is the chance to measure the beam position with a position pick-up and, from its change, to deduce the transverse cooling force when coherent betatron oscillations are excited by a transverse kick.

C) Equilibrium values

The bunch length can be determined either from the width of the pick-up signal or from the arrival time of the down-charged ions measured (in a scintillator, channel plates, or similar detector) with respect to an electrostatic pick-up or the RF signal. Although the width of the pick-up signal does not give us precise information about the phase distribution of the ions in the bunch, we can get this from the time-of-flight signal.

The momentum spread can be approximately determined from the envelope of the coherent longitudinal Schottky spectrum (Fig. 53) or from the decrease of the synchrotron oscillation peaks (Fig. 54). If the RF voltage can be rapidly reduced to zero and if the momentum spread is low, the latter can be deduced from the debunching time. The transverse equilibrium values are measured as for the coasting beam.

Also, information about the electron beam can be obtained from the cooling measurements, in particular about its temperatures. The flattening ratio Q_v can be estimated from the longitudinal (adiabatic) frictional force and from the equilibrium values of the ion beam, if thermal equilibrium is reached. The transverse electron temperature in the region that is traversed by the ion beam is directly obtained from the down-charge rate [Eqs. (81) and (82)]. In practice, it is useful to heat up the electron beam (HT modulation, magnetic field variation) and measure the increase of the ion beam equilibrium values [109a].

10.6 Results

10.6.1 Transverse cooling (cooling of betatron oscillations)

The results of the transverse cooling-time measurements obtained in the three laboratories (Novosibirsk, CERN, and FNAL) are compiled in Fig. 55. The data comprise measurements at 114, 65, 46, 35, and 1.5 MeV proton energy and are normalized to an electron density $n_e = 10^8 \text{ cm}^{-3}$ and to $\eta_c = 0.02$. It is amazing how well the measured cooling times follow a potential law. A fit to the data (full straight line in Fig. 55) results in

$$\tau_{\perp \text{lab}} [\text{s}] = 5.6 \times 10^5 (\sigma_{v_{\perp}}/c)^{1.7}. \quad (167)$$

If the cooling was governed entirely by the *non-magnetic* force, we would expect a constant cooling time ($\propto \sigma_{v_{\perp}}/F$) for $\sigma_{v_{\perp}} < \Delta_c = \Delta_{\perp}$ [Eqs. (41) and (43c)] and a scaling proportional to $\sigma_{v_{\perp}}^3$ elsewhere. The steady decrease of the transverse cooling time with decreasing velocity spread is an unambiguous sign of the contribution of the *magnetic force*. The binary collision model predicts a scaling with the third power of the velocity (spread) in the range $\Delta_{\parallel} < \sigma_{v_{\perp}} < \Delta_{\perp}$ [Eq. (53b)]. This velocity region has to be investigated in more detail with a suitable simulation program, taking into account the force from the dielectric description, before a final conclusion can be drawn. The relative weights of the different contributions depend also on the ratio of transverse to longitudinal ion velocity, on the transverse electron temperature, on the flattening of the electron beam temperatures, and on the magnetic-field properties. These dependences were studied in detail by the Novosibirsk Group and resulted in the establishment of a semiempirical formula for the transverse damping rate $\lambda_{\perp}$ [115]:

$$\tau_{\perp \text{lab}}^{\text{emp}} = 1/\lambda_{\perp} = (12\pi\sqrt{\pi}r_e r_p \gamma n_e c^4 \eta_c)^{-1} [(\alpha_0 \beta_0 c)^2 + \sigma_{v_{\perp}}^2 + 11\sigma_{v_{\parallel}}^2] \sqrt{\Delta_c^2 + 2\sigma_{v_{\parallel}}^2} \quad (168)$$

($\sigma_{v_{\perp}}^2 + \sigma_{v_{\parallel}}^2 = 2\sigma_{v_i}^2$). Here, α_0 is a quantity that accounts for misalignment of the beams and for magnetic-field errors [Eq. (108)]. In Fig. 52 the prediction of Eq. (168) is compared with the experimental data. Formula (168) shows characteristic features of the transverse cooling (Fig. 55):

$\tau_{\perp \text{lab}}^{\text{emp}}$ remains almost ($\alpha_0 \ll 1$) constant if the beam momentum is changed and $n_e = \text{const}$;

$\tau_{\perp \text{lab}}^{\text{emp}}$ scales as $v_{i\perp}^2$ for $v_{i\perp} \ll \Delta_e$ and as $v_{i\perp}^3$ for $v_{i\perp} \gg \Delta_e$;

$\tau_{\perp \text{lab}}^{\text{emp}}$ is constant for $v_{i\perp}, v_{i\parallel} \ll \alpha_0$.

The flattening of the electron velocity distribution is not explicitly contained in Eq. (168). However, it can be plugged into the value for α_0 .

10.6.2 Longitudinal cooling (cooling of momentum spread) and frictional force measurements

The measurements in the longitudinal direction again yielded cooling times or decrements, and, directly also the frictional force.

In the initial experiments (NAP-M, ICE, FNAL) the effect of the magnetic frictional force was clearly seen as a consequence of the continuous rise of the longitudinal frictional force for $v_{i\parallel} \ll \Delta_{\perp}$, but the ultimate maximum of the cooling for ion velocities close to $\Delta_{\parallel}$ was not accessed and the region where the force decreases with decreasing ion velocity was not reached (Fig. 56a). The decrease of the ICE data with decreasing velocity is probably an effect due to beam misalignment or magnetic field ripple. The Novosibirsk group has studied the frictional force dependence on different parameters and has again derived a semiempirical formula for the frictional force [115]

$$F_{\parallel \text{lab}}^{\text{emp}} = \frac{-3F_0 \gamma \eta_c c^2}{\sqrt{\left[\left(\frac{\alpha_0 \beta_0 c}{2}\right)^2 + v_i^2\right] \left[\left(\frac{\Delta_e}{2}\right)^2 + v_i^2\right]}}. \quad (169)$$

It has the following scaling behaviour:

- i) $F_{\parallel \text{lab}}^{\text{emp}} = \text{constant}$ for $v_{i\parallel} \ll \Delta_{\perp}$ and $v_{i\parallel} \ll v_{i\perp}$,
- ii) $F_{\parallel \text{lab}}^{\text{emp}} \propto 1/v_{i\parallel}$ for $\Delta_{\parallel} \ll v_{i\parallel} \ll \Delta_{\perp}$ and $v_{i\perp} \ll v_{i\parallel}$,
- iii) $F_{\parallel \text{lab}}^{\text{emp}} \propto 1/v_{i\parallel}^2$ for $\Delta_{\perp} \ll v_{i\parallel}$ and $v_{i\perp} \ll v_{i\parallel}$.

The experimental data are in good agreement for high ion velocities and also the TSR results obtained with C^{6+} and divided by Z^2 coincide with the proton data. (At the TSR the beam was rather hot in the transverse direction during these measurements; a transversely colder beam would give a higher longitudinal cooling force.) All data are described rather well by the semiempirical formula for ion velocities above some 10^6 cm/s (dashed curve).

With new experimental techniques it recently became possible, at LEAR and at IUCF, to extend the measurements to lower velocities. Whilst the data from the IUCF cooler show again a decrease of the force, the data from LEAR for an optimized system exhibit a dramatic rise to a maximum around 8 eV/m and then a fall-off as $v_{i\parallel}$ decreases. A less favourable situation for the electron cooling at LEAR resulted in a behaviour similar to that of the other data. The LEAR data are compared with theoretical predictions from the binary collision model [Eqs. (43c), (43d), and (54c)]. To some extent, this model can reproduce the rise of the frictional force (Fig. 56b), which is virtually all due to the magnetic force. The slightly higher data from the antiproton measurements at LEAR are, however, not considered as being due to the sign of the charge (see subsection 3.5) but rather to coincide with proton data within the experimental errors for the absolute normalization.

The dispersion in the cooler was included in the formula for the longitudinal cooling time given by the Novosibirsk Group [116]:

$$\tau_{\parallel \text{lab}} = \frac{\beta \gamma v_{i\parallel}}{24 \pi r_e r_i n_e \eta_c c^2} \sqrt{\left(\frac{\alpha_0 \beta_0 c}{2}\right)^2 + \Delta_e^2 + \left(1 - \frac{D_{\text{cooler}}}{\beta_0 c} \frac{dv_{e\parallel}}{dr}\right)^2} v_{i\parallel}^2. \quad (170)$$

At Novosibirsk the proton beam at injection was already rather cool ($\Delta p/p \approx 3 \times 10^{-4}$), and measurements of longitudinal cooling times for large initial momentum spreads were not possible. However, studies of longitudinally hot beams were carried out at ICE (CERN) and at FNAL. At FNAL the total cooling time for the reduction of an initial momentum spread of 3×10^{-3} to equilibrium was of the order of 4 s [4]; similar values were obtained at ICE [3], but no systematic investigations of the cooling-time dependence on momentum spreads were made until recently, when extensive studies were done at LEAR [109]. In these measurements the momentum distribution of a cooled beam was blown up in the longitudinal direction by stochastic heating and then cooled back to equilibrium (as shown in Fig. 47). The resulting total laboratory cooling times were around 100 ms for initial momentum spreads between 1×10^{-3} and 1.3×10^{-2} [109].

10.6.3 Equilibrium properties

The determination of the equilibrium emittances and of the momentum spread provides information about the cooling force for small ion velocities in the rest frame, provided the heating processes are well known. In any case, they are of practical interest. The flattened electron velocity distribution ($T_{e\parallel} \ll T_{e\perp}$) is imposed on the ion beam if there is no coupling between longitudinal and transverse degrees of freedom. Transverse thermal equilibrium leads ideally to the condition $\theta_{i\perp} = v_{i\perp}/\beta_0 c = 1/2 \sqrt{m_e/m_i} \theta_{e\perp}$, as discussed in subsection 9.3 [Eq. (156)]. For $\beta_0 = 0.3$ and $T_{e\perp} = 0.2$ eV, one expects, at best, $\theta_{i\perp} = 3 \times 10^{-5}$. The measured beam divergences in the cooler were usually above 10^{-4} . The momentum spread was frequently lower than this value. The experiments show that a flattened distribution was always reached for the proton beam. The amount of flattening depended on the contribution from intrabeam scattering and on practical limits from the electron beam (acceleration voltage ripple). The transverse equilibrium beam properties were determined from direct measurements of the beam size using the various techniques described before (subsections 7.13 and 10.5), or were deduced from the divergence of the atomic hydrogen beam measured downstream from the cooler. As mentioned before, the transverse equilibrium is governed by the relation between the transverse velocities (angles) of the electrons and protons in the cooling region [Eqs. (155) and (156)], so these values are to be used as references for the comparison of the results from the different laboratories. Therefore the lowest attainable beam sizes and emittances depend on the β -functions in the cooling region. The values obtained are summarized in Table 10.3.

Initially (in NAP-M) the equilibrium momentum spread was determined from the debunching time, but later it was extracted from the Schottky noise signal. The remarkable discovery made during the first Schottky noise measurements at Novosibirsk [117] was the big difference between the Schottky signal amplitude of an uncooled and a cooled beam (Fig. 57) and the signal levelling-off at low proton intensity (beam energy 65 MeV). A further investigation of the Schottky signal yielded the double-peak structure discussed in subsection 7.3. Recently, the Schottky signal of a cold beam was investigated in great detail at LEAR [109] and at the TSR [112] as a function of ion intensity (Fig. 58), which confirmed the findings at NAP-M. The splitting of the Schottky signal was also observed in the cooling experiments at the IUCF [113].

The behaviour of the ion beam leading to a distortion of the Schottky signal can be described in the framework of a dielectric model, and one can find a suitable dielectric function governing the beam-wall interaction. From this treatment one can derive a formula which describes the shape of the Schottky signal [109a, 118]

$$\langle A^2(f) \rangle = e^{-t^2} \left[\left(1/s^2 + 1 - 2t e^{-t^2} \int_0^t u^2 du \right)^2 + \pi t^2 e^{-2t^2} s^2 \right]^{-1} s^{-2}$$

with

$$s = \frac{f_c}{h_f \Delta f} \quad \text{and} \quad t = \frac{f - h_f f_0}{\sqrt{2} h_f \Delta f}.$$

The frequency splitting f_c [Eqs. (144)] depends on N_i and $Z_{\parallel h}$, and these quantities can in principle be measured separately. Therefore the only parameter in the fit is $s \propto 1/\Delta f$. The momentum spread $\Delta p/p \propto \Delta f/f$ can thus be extracted unambiguously. This procedure was developed for the evaluation of the LEAR data and was also applied to the TSR results. The above equation shows also that the total power of the Schottky signal around a harmonic $P_A = \int \langle A^2 \rangle df$ is decreasing with decreasing momentum spread since $P_A \propto s^{-2} \propto \Delta p^2$. The frequency splitting f_c scales with the square root of the number of stored ions [Eq. (144a)], if the longitudinal impedance $Z_{\parallel h}$ remains constant (constant beam size). The deviation from this scaling observed at the TSR for high ion intensity (Fig. 58c) could be an indication of a dynamical change of the longitudinal impedance [Eq. (129)]. The deformed Schottky signal has not always the form shown in Figs. 58a and b: a triple-peak structure was sometimes observed also in the LEAR measurement (Fig. 58d), where the central peak position corresponded to the position of the harmonic of the revolution frequency.

The evaluation of the Schottky signals yielded the equilibrium momentum spreads displayed in Fig. 59. The NAP-M points are identical with those from Fig. 57 and show the levelling-off at a momentum spread (FWHM) of about 1.7×10^{-6} . The increase with N_i seems to be more rapid than $N_i^{0.4}$, as expected from Eq. (161). The ICE data however scale very nicely with $N_i^{0.4}$ and can be fitted by Eq. (161) with reasonable parameters. Also the LEAR data for the optimized cooling seem to show a levelling-off, but at a higher momentum spread; this is however in agreement with the electron acceleration voltage ripple. For less optimal cooling conditions the scaling is different. The TSR data can also be reproduced reasonably well with the intrabeam scattering model developed in subsections 7.6 and 9.4.3. In this case even the scaling with Z , A , and β is reproduced.

Some data from the longitudinal equilibrium studies are also compiled in Table 10.3. They show that there is a remarkable difference between the divergence and the momentum spread obtained at Novosibirsk and those obtained elsewhere. The NAP-M values are nearly one order of magnitude lower than the others. The longitudinal plasma correlation factor $\Gamma_{i\parallel}$ is about 5 for the Novosibirsk data.

10.6.4 Beam lifetime

One immediate result of cooling is the increase in the beam lifetime, which is achieved by the reduction of the losses due to multiple scattering. Ideally, only single-scattering losses contribute to the beam lifetime, at least for protons and bare ions. For antiprotons and partly stripped ions, the annihilation and charge-changing reactions, respectively, become important at low energy. These losses cannot be avoided by using electron cooling. The measured beam lifetimes with and without cooling are summarized in Table 10.4.

10.6.5 Cooling of bunched beams, and accumulation

Longitudinal and transverse electron cooling measurements of bunched beams were performed at all facilities; the cooling times found were similar to those for coasting beams. However, the studies of the bunched beams were not carried out in such detail as those for the coasting beams. The purpose of the initial studies (NAP-M, ICE, and FNAL) was more to make a check in view of particle accumulation, which was to be tried out after the successful cooling of bunched beams.

The question, How strongly could the longitudinal phase space be compressed? was answered by the determination of the ion distribution within the bunch. This information was obtained from the time-of-flight (TOF) spectrum of the neutral hydrogen atoms. The particle distribution within the bunch is shown in Fig. 60 or different RF voltages. In LEAR the bunch length was reduced from 40 m to 2 m through cooling within a time compatible with the longitudinal cooling time for a radio frequency corresponding to the first harmonic of the revolution frequency. Shorter bunches are, of course, obtained when using a higher harmonic of the revolution frequency for the RF (Fig. 61).

The cooling of bunched beams permitted successive batches of protons to be stacked. Different techniques were used to accumulate protons [3, 4] and resulted in a roughly tenfold increase of the stored intensity. The limit was due either to the beam lifetime [4] or to instabilities [3]. Recently, stacking in the presence of electron cooling was also performed at the TSR [112].

10.6.6 *Instabilities*

Several kinds of instabilities were observed when cooling coasting and bunched proton beams. Longitudinal instabilities leading to a self-bunching of the coasting beam were reported from NAP-M and from ICE. In the latter case, the instability appeared only when the machine was operated above transition energy. The bunch was quite stable (many seconds), which allowed its detection and the measurement of its width ($3 \text{ ns} = 25 \text{ cm}$) through the TOF spectrum. The most likely origin of the instability might have been the negative mass effect. The two different Schottky spectra shown in Fig. 62 illustrate the longitudinal beam distribution for an unbunched and for a self-bunched beam. The corresponding signal for a bunched beam is shown in Fig. 53.

Another instability was encountered when the number of stored protons reached was about 3×10^9 during the accumulation tests in ICE (operation below transition). From the momentum spread measured for the coasting beam, it could be deduced that this number was close to the Keil-Schnell limit [Eq. (130a)].

Pronounced instabilities, which occurred when the transverse phase space shrank, were recently seen at LEAR. They were detected by observing the change in the amplitudes of the sidebands in the Schottky spectrum of transverse pick-ups (Fig. 33). After cooling, the sidebands disappeared and the proton beam assumed a low emittance. Suddenly, however, new peaks occurred at various positions in the spectrum, accompanied by a rapid emittance growth, which stopped when the emittance reached a certain value. Transverse cooling decreased the beam size again, and a new 'hiccup' followed. The period between hiccups was irregular and intensity-dependent (Fig. 63). A possible explanation of this effect might be the trapping of ions or electrons in the proton beam [119].

10.6.7 *Comparison of the frictional force exerted on positive and negative ions with MOSOL*

As mentioned in subsection 10.3, a linear arrangement was set up to verify the suspicion that magnetically confined electrons scatter differently from negative and positive ions, and yield different momentum transfers which result in a difference of the frictional force. This was confirmed [24c, 49] for both the longitudinal (Fig. 64) and the transverse force. The maximum of the cooling force is at about 10^6 cm/s , hence roughly a factor of 20 below $\Delta_{\perp}$. The dependence of the effect on the magnetic confinement of the electrons was studied (Fig. 65). The data show, in agreement with theory, that a difference in the maximal longitudinal cooling force arises only for large magnetic fields. While the maximum of the force for protons increases little and remains between 1 and 2 eV/m (for $n_e = 10^8 \text{ cm}^{-3}$) it rises to values above 6 eV/m for H^- ions because the contributions from small impact parameters become more and more important.

10.7 Discussion of results

Generally we can state the following. The frictional force was directly measured for the longitudinal direction over a wide range of velocities. Its dependence on various parameters was studied in detail. In the transverse direction the damping decrement and the cooling time were deduced. The ion velocity domain above and below the mean transverse electron velocity $\Delta_{\perp}$ was covered for both the transverse and longitudinal phase space. Clear indications of the magnetic term came from the continuously rising longitudinal force for decreasing ion velocity (Fig. 56). The maximum of the longitudinal force and the regime where it is proportional to the ion velocity ($v_i < \Delta_{\parallel}$) were investigated only in the LEAR, IUCF, and MOSOL experiments. The predicted scaling behaviour is in general reproduced.

The results of the measurements of the longitudinal frictional force in the different laboratories are in remarkable agreement for large ion velocities. For lower velocities, there is a discrepancy between the LEAR and IUCF data. The spread of the data at low velocity shows also that the attainment of high frictional force values depends strongly on the cooling conditions. Even different measuring techniques may lead to different results. For a precise comparison of the data sets with each other and with theory in this velocity regime all the relevant variables ($v_{i\parallel}$, $v_{i\perp}$, $\Delta_{\perp}$, $\Delta_{\parallel}$) have to be measured simultaneously. Only then can the magnetic force prediction be tested quantitatively. The same is true for a comparison between binary collision and dielectric model.

The transverse cooling was mainly studied in NAP-M; the measurements from the other laboratories for large transverse velocities fit to the scaling obtained in Novosibirsk.

The perturbation of the Schottky signal, originally weakly seen in Novosibirsk, then clearly observed and studied in detail in LEAR, and confirmed by TSR measurements, also for higher-charged ions, has lead to some excitement. However, the observed phenomena are in accordance with theoretical predictions and are well understood. They demonstrate that a strong cooling can efficiently counteract longitudinal instabilities, leading to a dynamical equilibrium and extending the domain of beam stability in a storage ring beyond classical limits.

The satisfactory understanding of the longitudinal cooling force allows us to investigate quantitatively the interplay between electron cooling and intrabeam scattering. With the help of Eqs. (160) and (161) we can reproduce, surprisingly well, the behaviour of the measured data when adjusting the variables correspondingly (Figs. 59). The scaling of the equilibrium momentum spread proportional to $N_i^{0.4}$ seems to be valid for most of the data. However, the NAP-M results have a steeper dependence on N_i . It should be noted that the equilibrium momentum spread should level off for a decreasing number of stored ions when the ion temperature reaches the longitudinal electron temperature. So far, in the NAP-M experiment only this has been observed. It is thus clear that the present equilibrium data are insufficient for a precise check of the model. The plateau has to be found, so as to verify that the longitudinal electron temperature is consistent with it. Also the beam size should be known for each momentum spread, in order to fix the intrabeam scattering blow-up rate.

The initial electron cooling experiments confirmed the general theoretical description. A more rigorous test of the cooling force now requires precise measurements, in particular in the low velocity range. This would provide more experimental information about the magnetic frictional force. Also needed is an experimental check of the binary model prediction that the transverse frictional force changes sign for $v_{i\perp} < \sqrt{2}v_{i\parallel}$, as well as an investigation of the region with $v_{i\parallel} \ll \Delta_{\parallel}$ (linear part of the magnetic cooling force).

The electron cooling experiments have moreover encountered transverse instabilities that deserve further detailed studies. This is also connected with the need for more precise measurements of the transverse equilibrium values and their dependence on beam intensity and machine parameters. An interesting domain to be investigated is the cooling at working points close to resonances, close to

transition and above transition, because, as in the case of the longitudinal instabilities, electron cooling might increase the region of stability.

The study of the cooling of bunched beams yielded cooling times similar to those for coasting beams. Further investigation of this operation is of practical interest but provides no new information.

The cooling theory is tested to the extent that it can be used to predict the evolution of beam emittance and momentum spread with sufficient precision for experiments with cooled beams. While the comparison of cooling force measurements with the appropriate formula can still be made directly, the details have to be evaluated with simulation programs containing the cooling force from the binary collision and the dielectric model. They allow us to simulate a particular measurement and to obtain a more accurate check. The quality of such a simulation is shown in Fig. 66, where the increase of the Schottky signal in a specific frequency window is computed for the case of electron cooling of a longitudinally blown up beam (cf. Fig. 47) and compared with the LEAR measurements.

Future cooling experiments with highly charged ions will also be susceptible to electron density effects (plasma polarization) and to the possible influence of electron-ion states around the ionization threshold. The difference in cooling for like-charged and unlike-charged partners should be studied in the context of the cooling of antiprotons by electrons, and of positive ions by positrons. Another interesting field is the cooling of partly stripped ions. Here the effects of the size of the ion and its inner degrees of freedom (excitation) have to be studied. Also, the cooling of molecules is awaiting investigation.

11. CONCEIVABLE FUTURE DEVELOPMENTS OF ELECTRON COOLING

11.1 New cooling-ring projects

As a result of the successful initial electron cooling experiments, new projects for low- and medium-energy storage rings were started [111, 112, 120-126]. Their design parameters are compiled in Table 11.1. The range of electron cooling and the ion species are shown in Fig. 67. The physics programme at the new cooling rings is divided into four areas:

- i) atomic physics with light and heavy ions (ASTRID, TSR, ESR, CRYRING, HISTRAP);
- ii) nuclear and intermediate-energy physics (LEAR, IUCF, CELSIUS, ESR, COSY);
- iii) accelerator physics (TARN II, ESR, TSR, IUCF);
- iv) particle physics (LEAR).

The forthcoming cooling experiments from these rings should provide new information about the scaling of the cooling force with the ion charge and mass, the influence of the atomic structure of the ion, plasma effects, and the behaviour of the Schottky signal under extreme situations. The cooling can be studied in new domains: the regimes of high charges, high intensity, and low temperature. We are awaiting, with much curiosity, the outcome of the attempts to crystallize beams. The future data will not overthrow the theory of electron cooling, but rather it will provide material to round off the picture. It is more the application of electron cooling for the improvement of the experimental situation that could pave the way to new discoveries.

The electron coolers for these rings have conventional designs. Less conventional designs are discussed below. They might find their applications in specific tasks.

11.2 Linear cooler

Electron coolers are S- or U-shaped in order to merge ion and electron beams and to keep the electron gun and the collector away from the interaction region. The thermocathode and the requirement of an ultra-high vacuum necessitate this. For special applications it might, however, be interesting to use a linear arrangement. Electrons from a small cathode installed directly in the vacuum chamber of the ion beam and mounted sufficiently off-axis could be accelerated parallel to

the ion beam and then merged with it. After a sufficiently long cooling region, the electrons are decelerated, spread out in a decreasing magnetic field, and captured in a collector with an opening for the ion beam (Fig. 68). The electron beam blow-up after deceleration could be amplified by a second solenoid, which provides an opposite field. The second solenoid could, in addition, be used to compensate the effect of the cooling solenoid on the ion beam (coupling of emittances, spin rotation). This configuration would take up less space in the storage ring and would also be much simpler. It requires a small electron beam size and a low current to keep the vacuum load within reasonable limits, and could be useful for maintaining the low temperature of the ion beam when working with internal targets. A similar configuration has been developed for use as an electron target in dielectronic recombination studies [47].

11.3 Circular cooler

For some applications it is imperative to have a circular electron cooler, namely for

- i) cooling at highly relativistic energies,
- ii) cooling with positrons.

The cooling of highly relativistic ions ($\gamma \gg 1$) with circulating electrons was considered for antiproton-proton colliders [2b, 127, 128] and for heavy-ion colliders [129]. In these schemes the electrons themselves are cooled by synchrotron radiation. Electron cooling at very high energy has to act against the factor γ^2 and the low η_c [Eqs. (154)]. Although the advantage of electron cooling is that it is not dependent on the intensity of the stored ions, much more effort is required than for stochastic cooling at present colliders. The time needed to cool a 1 TeV proton beam ($\gamma = 1000$) can be estimated from Eqs. (73b) and (154c), and is of the order of weeks. Such long cooling times are not of much interest for any application. The situation is much better for superconducting heavy-ion colliders, as γ is not much larger than 10 and η_c could be kept at the 1% level. For highly charged ions ($Z \approx 90$), the higher γ and lower η_c can be partly compensated by the Z^2/A dependence of the cooling time. Values of the order of minutes can be anticipated, provided the electron beam itself is cooled rapidly enough. This is not evident, because synchrotron radiation cooling is not fast enough for this regime of γ .

A recirculator, rather than a storage ring, would be sufficient if the cooling with a low-density beam of electrons or positrons is considered. In this case, the period during which the cooling medium is recirculated should be such that its temperature is not increased by much more than, say, a factor e . After this period, the cooling beam is dumped and the recirculator is filled with a fresh beam. Recirculation times of the order of a second could be practical. Then the electrostatic electron accelerator can be operated in a pulsed mode, and this technique could be applied to increase the energy range for electron cooling with a smaller, more compact device. Access to the MeV region ($\gamma \leq 5$) would be easier with pulsed acceleration than with a d.c. electron beam. Another application for a recirculator would be in the cooling of ions with positrons. This might turn out to be mandatory if the intention is to cool highly charged ions over long periods. The recombination of electrons with stored ions could prevent continuous cooling over a long time, but when electrons are replaced by positrons, this problem is removed. Two major cases for such applications can be foreseen:

- i) the high-resolution spectroscopy (ring-internal) of bare or few-electron heavy ions;
- ii) the investigation of rare, highly charged ions (rare isotopes, fragments, etc.).

In both cases it takes some time to prepare the beam with the desired properties, so one does not want to lose it after a few seconds. A suitable positron beam for cooling highly charged ions requires positron densities of about 10^6 cm^{-3} and temperatures below 1 eV. Positrons can be obtained from electroproduction or from radioactive β^+ -emitters. However, they are not useful for cooling. In positron physics, methods have been developed to thermalize positrons with an efficiency of about 10^{-3} [130], but this reduces their intensity to a level that is not acceptable for cooling. However, an

accumulation scheme for slow positrons has been proposed, which could deliver about 10^8 s^{-1} cold positrons [131]. Positrons from such a source could possibly provide the required density and temperature and could be used in a recirculator. Either the positrons are dumped after being heated and a new injection from the positron accumulator follows, or the positron beam is continuously stored and cooled by electrons. The cooling of positrons by electrons is favoured by their mass ratio of unity. A similar scheme was proposed for antihydrogen production [132].

11.4 Cooling with MeV electrons

As can be seen from Table 11.1, the highest energies at which electron cooling will be applied in the upcoming cooling rings is 300 keV. This is a practical limit dictated by the required technical effort. Projects exist for electron cooling with electrostatically accelerated electrons at still higher energies [59–61, 133–136]. The original goal for the application of these devices was the enabling or improvement of antiproton accumulation at the production maximum. Another goal was the cooling of antiprotons at about 2 GeV/c in proton–antiproton collision experiments [61]. A prototype MeV electron beam, developed by the FNAL–NEC Collaboration [59, 136], recently produced its first beam. The 1 MeV device developed at Novosibirsk [43, 135] has been operating for technical studies for some time. The major challenge in these devices—the electron collection—seems to have been mastered. Although the cooling times that are achievable with these machine are interesting for many kinds of applications, in particular for intense beams where stochastic cooling becomes less efficient, the interest in electron cooling in this energy range for (anti)proton rings decreases. High-energy electron cooling could be of much more relevance for heavy-ion rings because of the large number of production thresholds that can be studied with internal targets, and because of the shorter cooling times.

11.5 Cooling with sub-keV electrons

Another conceivable future development could be in the direction of cooling at very low velocities (below 1 keV electron energy) in small storage rings and at eV energies in traps. Nowadays, many laboratories are equipped with new ion sources delivering high-charge states. The high charge facilitates acceleration, but the low ion intensity could necessitate accumulation. In this case, electron cooling could be helpful for reducing the phase space and increasing the lifetime of the beam, and could pave the way for a simple installation that would be available to small institutes. Apart from cooling, the electron beam could serve also as a target for studying electron–ion interactions.

Experimentation with trapped ions has boomed during recent years, and research is going on at many institutes. For particular systems, partially stripped ions can be laser-cooled very efficiently, or cooled with ambient gas. In some cases (e.g. bare ions), cooling with electrons or positrons is an attractive or even necessary alternative. These aspects merit special discussion, since some specific features of electron cooling change at low energy.

11.5.1 *Electron cooling in small storage rings*

We have seen before that the time required for cooling a beam of a given initial momentum spread and divergence remains virtually unchanged at all the operational energies of a constant-perveance electron gun, provided $v_i < \Delta_\perp$. This is partly due to the contribution of the magnetic force. However, the importance of the magnetic force depends also on the flattening of the electron velocity distribution. The velocity spreads in the longitudinal and transverse directions differ less for low energy, as $\Delta_\perp/\Delta_\parallel \propto \beta_0$ [Eq. (95c)]. Therefore the influence of the magnetic force is attenuated and the equilibrium temperatures are higher. At very low energy, the times required for the damping of betatron oscillations and momentum spread will become longer than at intermediate energy. This can hardly be compensated by an increase of the electron density, because space-charge

effects would deteriorate the situation. On the other hand, η_c can be substantially increased for small rings. The ion-beam lifetimes in storage rings become very short at low energy [Eqs. (139)] and the instability thresholds more severe. The latter usually scale as $\beta_0^2 \gamma^3$ [Eqs. (122) and (130)]. These effects limit the applicability of electron cooling in storage rings at very low energy [137, 138], although technically this is very simple. Stabilization of the acceleration voltage is rather easy, as is also the electron re-collection, because the high-voltage could be provided by power supplies that are stable, such as batteries, and the electron current is very low (milliamperes or less). To overcome the obstacle of the high transverse electron temperature, a photocathode or a cold plasma electron source could be used.

So far, the lowest electron cooling energy reached in storage rings has been about 800 eV (1.5 MeV proton energy), corresponding to $\beta_0 = 5.6 \times 10^{-2}$ [2]. The beam lifetime was of the order of a few seconds and comparable to the cooling time of betatron oscillations. With ions that are twice as fast (5 MeV), a more favourable situation was recently brought about at LEAR, where beam lifetimes of a fraction of an hour and cooling times of a few seconds were obtained (vacuum below 10^{-11} Torr) for (anti)proton beams [110, 118].

11.5.2 *Electron cooling in traps*

When the ions are of such low energy that they can be stored in traps rather than in rings, we are in the position to obtain a long beam lifetime. In traps, the residual gas and intra-ion scattering does not usually lead to losses, since the confinement forces in all three spatial directions can be of the same order. It is, rather, the electron capture that leads to losses. The electron cooling of ions in traps is promising, but it has not yet been exploited. Physically, the situation is similar to that in plasma machines, except that here the electrons are used for cooling the plasma and not for heating it. The cooling times can be estimated using Spitzer's formula [Eq. (21)] directly. However, the heating of the electrons should be taken into account, unless they can also be continuously renewed in the trap or cooled by synchrotron radiation or by resistive cooling. Because of the capture process, trapped ions should preferentially be cooled by particles of like charge. An example of the application of electron cooling in traps has been considered for thermalizing trapped keV antiprotons to room temperature in order to carry out high-resolution mass measurements [139, 140]. This arrangement was realized very recently, and antiprotons confined in a Penning trap were cooled, in tens of seconds, from a few keV to a fraction of 1 eV by electrons (also confined in the trap) [141]. Similarly, the cooling with positrons of bare ions captured in traps can be contemplated. As an illustration, let us consider a static trap consisting of a longitudinal magnetic field and a static electric potential at either end of the solenoid (Fig. 69). Trapping of positive ions and positrons is considered first. The filling of the trap with the two species is done, for instance, as follows. One component is axially injected by grounding the entrance potential. We take the cold positrons first. When the pulse is reflected from the other end, a central electrode (previously grounded) is biased to prevent the positrons from leaking back out. Then the second component (hot ions) is injected, being reflected also from the end electrode. Before the ions reach the entrance again, the potential of this electrode is brought to a confining value. Ions traverse the positron cloud and are cooled. Then the central electrode can be grounded again. Cooling leads to an equalization of the temperatures.

For unlike charges (e.g. positive ions and electrons) we have to choose a different geometry. Here we can follow the suggestion of Ref. [139] to use 'nested traps', which means that the trap of the colder component is small and is inside a larger trap for the hotter component (Fig. 69). For this purpose, the region of the extended central electrode can be used to trap the electrons.

As long as the ions are hot, they can surpass the potential well confining the electrons. When their longitudinal temperature decreases below the value corresponding to this potential, they no longer overlap with the electrons. Therefore the central potential has to be lowered. This leads to an

escape of the hotter electrons, and the ions now pass through colder electrons until their energy is again lower than this potential. The procedure can be repeated until all electrons are evaporated from the central trap. The evaluation of the cooling times depends here on the exact procedure.

In these kinds of traps, usually rather high magnetic fields (a few teslas) are used. Therefore the magnetic force should play an important role and could modify Spitzer's formula considerably. If, in addition, the cooling gas has a flattened velocity distribution, the reduction of the axial ion velocity should be rather fast. On the other hand, the large difference in the oscillation frequencies for the light cooling component and the ions provides the possibility of reducing the temperature increase of the cooling gas by other methods, e.g. synchrotron radiation cooling, and thus obtaining cooling without the need for frequent loading.

11.6 Subthermic cooling beams

The development of subthermic electron beams is stimulated mainly by the intention to produce crystalline ion beams. As already mentioned, an electron beam colder than the standard beams is also needed for the cooling in storage rings at low ion energies. The main goal is the reduction of the transverse temperature (1300 K for a thermionic cathode). The currently considered sources for cold electrons are photocathodes or a cold electron plasma (see subsection 6.1). A cold electron cooling source based on a photocathode is at present under construction [38]. The design parameters of this device are given in Table 11.2.

The principal problem, besides the production of cold electrons, is the prevention of beam heating during acceleration. Particle trajectory calculations have been performed for cold electron sources in order to optimize the acceleration region [38]. For a few hundred volts acceleration, heating can certainly be kept below the meV level. At keV electron energies, this will be difficult to realize.

A possible reduction of the transverse electron temperature raises the question of the importance of other contributions. This mainly concerns alignment errors, properties of the magnetic field and of the vacuum chamber, and space-charge effects. The latter impose a limit on the electron-beam current. If, for instance, we require that the longitudinal velocity spread across the electron beam does not exceed the transverse temperature [$n_e \pi r_e^2 m_e c^2 \leq kT_{e\perp}$, Eq. (98)], we get the condition that the electron-beam current should fulfil:

$$I_e \leq \frac{kT_{e\perp}}{m_e c^2} \frac{ec\beta_0}{r_e}, \quad (171)$$

which results in 1 μ A for a beam radius of 1 mm, $kT_{e\perp} = 1$ meV, and $\beta_0 = 0.03$.

A low transverse electron-beam temperature leads to an increase of the cooling force at low ion velocities in the rest frame (v_i^{-2} dependence). This would help to achieve and maintain a low ion-beam temperature and to get cooling times that are comparable to or shorter than beam lifetimes at very low energy.

Certainly not all the possibilities of producing subthermic electron beams have yet been explored. Also, techniques have to be developed for measuring the temperature of such cold electron beams.

11.7 New techniques

The possibilities provided by electron cooling are still a long way from being fully exploited, and some new techniques might emerge after experience has been gained in the cooling of heavy-ion beams. It might be possible to profit from the resonance-like behaviour of dielectronic recombination in order to manipulate partly stripped ion beams [142]; because of narrow peaks in the recombination cross-section, narrow velocity bands in the ion beam could be projected out.

Recombination leads, however, to a down-charged ion in an auto-ionizing state. Spontaneous or induced auto-ionization in the cooler could restore the initial ion charge. Also, field ionization might be applicable.

Recently, it was proposed to enhance electron cooling through microwave radiation introduced into the cooler [143]. It is argued that microwave radiation stimulates Landau transitions of the magnetically confined electrons, and that such transitions will enhance the cross-section for binary collisions. This method will be used instead of electron cooling [144a] at the planned storage ring RCNP-MSR in Osaka [144b].

Electron cooling could be combined with other cooling techniques to obtain improved performances. Its combination with laser cooling might enable the latter to be carried out at a fixed laser frequency, and the ion to be swept through the resonance with the help of the electron beam [145]. The joint operation of electron and laser cooling could drive an ion beam more quickly into the region covered by laser cooling, and would counteract transverse heating as a result of longitudinal laser cooling. The combination of electron and stochastic cooling could be advantageous for the cooling of large-emittance beams. The simultaneous operation of both cooling systems is straightforward. Some time ago, a hybrid cooling technique was proposed [146] which consisted in the use of the electron beam as a pick-up, and possibly also as a kicker, for stochastic cooling. This would be an ideal combination if ways could be found to detect the stochastic noise of the ion beam through the electron beam. Using the electron beam as a kicker is probably not so difficult.

As already mentioned, electron cooling imposes its flattened velocity distribution on the ion beam. In some cases, intrabeam scattering leads to a blow-up of the momentum distribution. It would be important to study the conditions under which the energy that is necessary for the blow-up is taken from the transverse temperature and not from the coherent longitudinal motion. If this could be controlled, the transverse ion dimension could probably be reduced beyond the thermal equilibrium limit. The usefulness of the intrabeam scattering of partly stripped ions for beam cooling was pointed out some years ago [147]. The cooling action is based on the excitation of ions in intrabeam collisions, which extracts kinetic energy from the beam [147]. If de-excitation proceeds via photon emission, a part of the energy is radiated away. This cooling process has to compete with charge-changing reactions that lead to losses. Also the diffusion in this process is rather high. This method is not reserved exclusively for electron cooling; it can also be combined with other methods.

The electron beam can be used to excite partly stripped ions. The excitation cross-section reaches maximal values, of the order of the atomic radius squared, when the electron-ion velocity difference corresponds to approximately three times the excitation energy. This cross-section exceeds by far the Rutherford cross-section on which electron cooling is based. In order to achieve excitation, the electron beam has to be detuned in energy. The friction is based on the momentum transferred in the excitation process. The de-excitation proceeds via photon emission and leads to diffusion. This method can possibly be exploited for fast momentum cooling and for beam acceleration or deceleration.

11.3 Polarization of an ion beam through electron cooling

A question that has frequently been asked is, Would it be possible to polarize ions when cooling with polarized electrons? It turns out that, for the cooling of bare ions, the spin effects are so small that a polarization build-up can be excluded. The situation is different for partly stripped ions. In this case the exchange channel is open, and a bound unpolarized electron can be exchanged with a free polarized electron from the cooler without changing the charge state of the ion. The nucleus could subsequently be polarized through hyperfine interaction. Polarized electrons can be obtained from photocathodes. The magnetic field in the cooling region requires longitudinal electron polarization, which complicates the electron-beam transport from the cathode to the cooling region.

It would be interesting to study this process, although a similar effect can be achieved with resonant electron transfer when the ion beam is passed through a polarized target.

12. APPLICATIONS OF ELECTRON COOLING IN STORAGE DEVICES

The reasons for applying electron cooling in storage devices are rather evident from what has been discussed in the foregoing. They are summarized in this section, and complemented by other uses not yet discussed.

12.1 Increase of the beam lifetime

The increase of the beam lifetime is of importance at low energy, in particular for heavy ions [Eqs. (139)]. Electron cooling is advantageous when the multiple scattering can be reduced to single scattering losses, and when other losses (instabilities, charge-changing processes, nuclear reactions) are negligible. The gain in beam lifetime through electron cooling at low energy facilitates

- i) injection and stacking;
- ii) slow extraction (in particular after deceleration);
- iii) stored-beam experiments (accelerator studies, atomic physics).

For instance, antiprotons have to be decelerated from accumulation energy to the lower MeV range and be extracted over many minutes in order to supply stop-experiments. Because of the adiabatic blow-up, the antiproton beam would fill the acceptance of the storage device after deceleration, and scattering losses would immediately occur. Stochastic cooling at intermediate energy is possible, but owing to its slowness at low energies it increases the deceleration time. Electron cooling could reduce the beam size and the momentum spread within a few seconds, and would therefore limit the losses when slow extraction is planned; hence it would increase the net event rate for low-energy experiments [5]. Similar arguments hold for the deceleration of highly charged ions, which in some cases have first to be accelerated to high energies for stripping and then slowed down again for experimental investigation at low energy.

12.2 Increase in luminosity

Electron cooling provides the possibility of reducing the beam size to very small values (see Table 10.3). The luminosity L (event rate/cross-section) available for a reaction depends on the number density of reaction centres within the ion-beam diameter. Although it is rare that a decrease in beam size helps to increase the luminosity for fixed-target experiments, small beam sizes are crucial for beam-beam experiments. The luminosity for colliding beams is given by

$$L = N_1 N_2 k_b \ell_{ov} A_{ov} (C_1 C_2 A_1 A_2)^{-1} |\beta_1 \pm \beta_2| c, \quad (172)$$

where $N_{1,2}$ are the number of stored particles per bunch in each beam, circulating in k_b (the number of bunches) with the velocity $\beta_{1,2}c$, on orbits of length $C_{1,2}$, and ℓ_{ov} is the length of the overlap region. The quantities $A_{1,2}$ define the cross-sectional area of the particle beams, and A_{ov} is the total overlap area. The negative sign in Eq. (172) stands for parallel beams. The increase in luminosity is inversely proportional to the square of the average beam sizes. Consequently, electron-cooled beams provide much higher event rates for colliding (parallel, antiparallel, crossed) beam experiments. Examples of such experiments are: the formation of antihydrogen in merged parallel beams of antiprotons and positrons [132]; the formation of antiprotonic hydrogen ($\bar{p}p$ atoms) in merged parallel beams of antiprotons-protons [2] and of antiprotons- H^- -ions [148]; the breeding of antideuterons with merged parallel antiproton beams [149]; the study of the production of charmonium and other narrow resonances in intermediate-energy antiproton-proton collisions [150]; the investigation of super-heavy molecules [151] with crossed beams of heavy ions [123].

12.3 Increase of spatial and momentum resolution

The decrease in the momentum spread and in the size of the ion beams is required for measurements aiming at high spatial and energy resolution. This concerns

- i) particle (π , K , η , $\bar{p}$, $\bar{n}$, etc.) production threshold studies [152–154];
- ii) spectroscopy of nuclear states;
- iii) mass spectroscopy of circulating ions [155–157];
- iv) atomic spectroscopy experiments (reduction of Doppler effect etc.);
- v) separation of different beam components, e.g. spin states [87b,c];
- vi) separation of the decay point of short-lived particles from their production point.

12.4 Increase of the duty cycle

Pulsed machines have duty cycles (ratio of 'beam-on' to 'beam-off' time for experiments) substantially below unity because of the time needed for beam handling. In particular, the duty cycle becomes bad for a decelerated beam, since adiabatic blow-up requires countermeasures. This is illustrated in Fig. 70, where part I shows the machine cycle without cooling; part II, the cycle with slow cooling; part III, the cycle with fast cooling. Let us consider accumulation at high energy and slow extraction at low energy. When no cooling is available, the machine acceptance is rapidly filled. This requires frequent deceleration, and often results in small batches of extracted beam (Fig. 70, part I). Slow cooling already allows more particles to be accumulated. The conservation of this stack during deceleration necessitates intermediate cooling. The extracted beam intensity is higher, but the batches arrive less frequently (Fig. 70, part II). Short cooling times provide fast accumulation and require only a brief intermediate stop (Fig. 70, part III). Eventually, cooling can be done simultaneously with deceleration. This illustrates that electron cooling can pave the way for a high duty cycle and a high average beam intensity. This kind of operation would provide slow antiprotons ($\beta \leq 0.1$) with a higher efficiency [158] than is available at present, and it would also be desirable for the delivery of accumulated nuclear fragments or bare heavy ions at low energy.

12.5 Accumulation of ions

Originally, electron cooling was proposed for the accumulation of antiprotons. Stochastic cooling turned out to be more suitable at the high energy of the antiproton production maximum. Accumulation is required when rare species from weak sources are to be used as projectiles. For bare ions, nuclear fragments, and polarized ions, the accumulation energy is much lower than for antiprotons, and the region is very suitable for electron cooling. The following applications can be foreseen:

- i) the stacking of polarized ions, i.e. light and heavy ions from cyclotrons and linear injectors;
- ii) accumulation of nuclear fragments and radioactive ions produced in secondary reactions;
- iii) accumulation of heavy ions after the last stripping stage (e.g. Pb^{80+} , U^{92+}).

The stacking of beams from linear injectors or cyclotrons can be facilitated by electron cooling. Multiturn injection and momentum stacking fills the machine acceptance within less than a second. Fast cooling by electrons compresses the phase space, moves the beam to the stack orbit, and provides space for another injection so that the space-charge limit for the injected beam can be rapidly reached, even for weak sources [159]. Stacking tests were successfully performed in the initial cooling experiments at CERN and FNAL (subsection 10.6.5) and also recently in the TSR [122]. In combination with charge-exchange injection for H^- beams, it could possibly prevent the stripped ions from passing again through the stripping foil by dragging them away and thus preventing the circulating beam from blowing up [159].

Sometimes it is argued that electron cooling would give too long cooling times in the case of large initial emittances and momentum spreads. However, this can be counteracted by a large

β -function in the cooler (see subsection 9.5.2), and by sweeping techniques (see below). Electron cooling could even be advantageous for high-energy heavy-ion machines. Used in a small booster/accumulator for sampling and condensing the ions after stripping off the last electrons, it would provide denser and more intense beams for high-energy colliders and fixed-target machines.

12.6 Ion-beam deceleration with electrons

A rapid energy change of the ion beam can be provoked by detuning the electron velocity to such an extent that the frictional force is at its maximum. A simultaneous magnetic-field change of the ring keeps the beam on the nominal orbit while it is being accelerated or decelerated. This procedure was successfully applied in NAP-M and in ICE. The maximal drag rate can be calculated from the longitudinal magnetic force, which reaches its maximum at $v_{i\parallel} = \Delta_{\parallel}$ [see Eq. (53a) and Fig. 15]. We have (assuming $v_{i\parallel} \approx v_{i\perp}$):

$$|F_{\parallel \max}| \approx |F_{\parallel}^{\text{ad}}(v_{i\parallel} = v_{i\perp} = \Delta_{\parallel})| = \frac{3}{2} F_0 \left(\frac{c}{\Delta_{\parallel}} \right)^2 L_C^{\text{ad}} = 6\pi Z^2 n_e r_e^2 m_e c^2 \left(\frac{c}{\Delta_{\parallel}} \right)^2 L_C^{\text{ad}}. \quad (173a)$$

The value of $|F_{\parallel \max}|$ can be maximized by increasing n_e or decreasing $\Delta_{\parallel}$. However, both quantities are correlated. The electron velocity spread $\Delta_{\parallel}$ depends on the interelectron potential V_{ee} , on the electron velocity distribution caused by space-charge effects, on the ripple of the acceleration voltage, and on the intrabeam scattering within the electron beam. The last two effects will be neglected for this consideration. The electron velocity spread given by V_{ee} is [Eq. (95c)]:

$$(\Delta_{\parallel}/c)^2 = 4r_e (4/3 \pi n_e)^{1/3}.$$

The velocity spread seen by an ion beam passing the electron beam is obtained from Eqs. (100b) and (95b):

$$(\Delta_{\parallel}/c)_{sc}^2 = 1/4 (n_e \pi r_{oi}^2 r_e)^2 (\gamma + 1)^2 \beta_0^{-2} \gamma^{-4} \approx (n_e \pi r_{oi}^2 r_e)^2 \beta_0^{-2}.$$

Therefore, we have

$$|F_{\parallel \max}| = \begin{cases} 3/2 \pi (4/3 \pi)^{-1/3} r_e m_e c^2 L_C^{\text{ad}} Z^2 n_e^{2/3} & \text{for } n_e \leq n_{\text{crit}}, \\ 6\pi m_e c^2 L_C^{\text{ad}} Z^2 \beta_0^2 (\pi r_{oi}^2)^{-2} n_e^{-1} & \text{for } n_e \geq n_{\text{crit}}, \end{cases} \quad (173b)$$

where n_{crit} is the critical electron density above which $|F_{\parallel \max}|$ decreases as $1/n_e$ (Fig. 71):

$$n_{\text{crit}} = (2^8 \pi/3)^{1/5} (\pi r_{oi}^2)^{-6/5} \beta_0^{6/5} r_e^{-3/5}. \quad (173c)$$

Hence $|F_{\parallel \max}|$ reaches its maximum at $n_e = n_{\text{crit}}$ and one obtains, for an ion beam radius of 1 mm,

$$|F_{\parallel \max}| \leq Z^2 100 \text{ eV/m}.$$

In this case $n_{\text{crit}} = 7 \times 10^8 \text{ cm}^{-3}$ and $\Delta_{\parallel} \approx 10^6 \text{ cm/s}$. The value of $|F_{\parallel \max}|$ at $n_e = 10^8 \text{ cm}^{-3}$ is close to the experimentally determined maximum of the longitudinal frictional force (Fig. 56). The deceleration rate achievable in the laboratory system is

$$|F_{\parallel \max}| \ell f_{\text{rev}} = |F_{\parallel \max}| \eta_c \beta_0 c. \quad (173d)$$

For our standard case we estimate a deceleration rate of

$$|F_{\parallel \max}| \eta_c \beta_0 c = 33 \text{ MeV/s } Z^2 .$$

Whilst in general this value is too small to compete with RF acceleration, it might be useful for the deceleration of beams at low energy. It would have the advantage of providing continuous beam cooling during deceleration, and would deliver a cold beam when the extraction energy is reached. The application of this technique was proposed for the deceleration of antiprotons from a few MeV to trapping energies [160]. The arrangement consists of a fixed-magnetic-field machine of the racetrack Microtron type. Racetrack Microtrons usually work only with electrons, because phase stability during RF acceleration requires that the particle velocity should not change, which is the case for highly relativistic particles. In the proposed decelerator [160], this obstacle is overcome for non-relativistic particles by replacing the RF cavity with an electron beam, which provides deceleration through friction. Such a device might also be interesting for the deceleration of highly charged ions to trapping energies (a few hundred keV).

12.7 Provision for internal target operation

The short electron cooling times allow the use of internal targets even at low energy, where the Rutherford cross-section [Eq. (132)] becomes very large. The combination of both techniques is foreseen in LEAR, IUCF, TSR, ESR, CELSIUS, TARN II, and COSY. By dragging the ion beam with the electrons, narrow resonances or thresholds can be scanned without the ion beam having to be bunched.

The potential of a combination of internal target and electron cooling is illustrated when studying, for instance, the reaction $\bar{p}p \rightarrow \bar{n}n$ around its threshold (98 MeV/c, $\beta_0 = 0.1$). High reaction rates and resolution are achievable, although Rutherford scattering limits the hydrogen target density to about 10^{-10} g/cm^2 [161]. With an external beam and target, a high-resolution study is excluded at this low energy. Another application is being prepared for the TSR and LEAR to try out a method (spin filter) of polarizing stored ions [85].

12.8 Beam extraction and diagnostics

Cooling electrons are picked up by ions during the cooling process (Section 5). The down-charged ions are separated from the primary beam and can be extracted [162], thus providing an external beam even when the machine is working in storage-ring mode (e.g. internal target, atomic physics experiments). The extraction rate can be adjusted through the electron current. Also, for partly stripped ions, dielectronic recombination [Eq. (76b)] can be used [162, 163]. Even if the external-beam intensity is too low for experiments, this type of extracted beam provides an excellent means of doing diagnostics and monitoring during an internal target and cooling operation.

The electron beam can be used to extract the ion beam in a way that is similar to the ultraslow extraction of antiprotons [164]. By detuning the electron relative to the ion energy, the frictional force can be precisely adjusted to a desired value. Faster electrons accelerate the ions (Fig. 72). The ion momentum distribution remains unchanged for a large velocity difference (slow variation of the frictional force). The acceleration enables the beam to be moved slowly and precisely into an extraction resonance. Prior to this—when the velocities are still matched—the ion beam can be blown up longitudinally by a fast modulation of the electron high voltage [109]. By adjusting the momentum spread and the drag rate correspondingly, the beam could be extracted over many minutes with a constant spill rate. Longer extraction times could be achieved by reshaping the momentum distribution and repeating the procedure.

12.9 Production of short bunches

Electron cooling experiments with bunched beams have shown that short bunches can be produced, even when bunching on the first harmonic of the revolution frequency and with a low RF voltage. Short bunches are of interest for experiments having the following requirements:

- i) TOF information for particle spectroscopy (e.g. neutron spectroscopy);
- ii) a low duty cycle in order to avoid misidentification of particles that follow one another within short times but have their origin in different reactions;
- iii) improvement of the peak-to-background ratio;
- iv) production of dense beam pulses for inertial fusion experiments.

The production of short bunches with RF and electron cooling is straightforward. However, the use of longitudinal instabilities to compress the beam in a controlled way has not yet been considered, although it might be feasible. In ICE, longitudinal beam instabilities led to a rather stable self-bunching of the beam, as witnessed by the stability of time distribution of the neutral hydrogen atoms [11].

12.10 Cooling of molecules

This application has not yet been considered. With the commissioning of low-energy ion storage rings, the spectroscopy of relativistic molecules could open up new experimental possibilities (use of the Doppler effect). The kinetics of the cooling of charged molecules is the same as that of ions, apart from possible contributions due to the rich spectrum of molecular excitation. However, the lower molecular binding energies render their storage more difficult because they are easily broken up.

The simplest kind of molecule is the rather strongly bound H_2^+ ion. For more weakly bound ions, the situation is comparable to that of the cooling of H^- ions (binding energy 0.75 eV). The merging of an electron beam with such ion beams may therefore lead to an initial break-up, until the relative electron-ion velocity is below the break-up threshold.

The electron cooling of molecules or even larger clusters stored in traps could also be considered. In these cases, electron cooling has to compete with other methods (laser cooling, buffer, gas cooling), and the method to be used depends on the particular application.

12.11 Axial loading of traps

In this application it is the friction, rather than the cooling with an electron beam, that is used for the external filling of a trap with ions. This method is needed for experiments with highly charged ions or antiprotons (formed at high energies) at thermal energy or below, and is also of relevance for the accumulation of positrons.

Let us consider, for instance, incoming positrons of energy $E_+ \gg e\Phi_{\text{trap}}$, where Φ_{trap} is the confining electric potential of the trap (Fig. 73). The e^+ beam is merged with a beam of a little slower electrons over a trap length ℓ_{trap} . The velocity difference u is adjusted to get the maximum longitudinal friction ($u \approx \Delta_{\parallel}$). The positrons are slowed in the electron beam by more than their energy spread. The positrons, radially confined by a longitudinal magnetic field, come up against the potential well of the end electrode and are reflected back. When they again reach the entrance of the trap they cannot cross this well, because their energy is lowered by $F_{\text{max}} \ell_{\text{trap}}$. The arrangement of Fig. 73 is very suitable for the trapping and accumulation of positrons [131] from a radioactive source after the first moderation. The random arrival of decay positrons renders other schemes more difficult and less efficient. Positron trapping through electron friction could become a very interesting possibility for feeding storage rings (synchrotron light sources, high-energy colliders).

12.12 Beam dynamics studies, ordering

The ability to reach very low beam temperatures makes electron cooling a powerful tool for the study of beam dynamics. Here, the major considerations are the testing of intrabeam scattering theories and the investigation of beam instabilities and beam ordering. Electron cooling of intense beams has produced transverse and longitudinal instabilities that are not entirely understood. Their investigation under the same machine conditions but for different charge states of ions [$A^{q+}/A^{(q+n)+}$, $\bar{p}/p$] could reveal new information. Longitudinal instabilities merit a thorough investigation. In particular, the region from the start of the splitting of the Schottky signal to the onset of the instability has to be studied. A check should also be made to find out whether the electron beam is stabilizing in the region close to transition energy.

The effects of intrabeam scattering observed in electron cooling experiments are in agreement with theory, at least for standard beam intensities and temperatures and for standard machine parameters. The situation may change for higher intensities and lower temperatures, and may produce surprises. In particular, the role of intrabeam scattering has not yet been investigated in sufficient detail in the region of transition from disorder to order. The degree of order is characterized by the plasma correlation factor. In the longitudinal direction it is inversely proportional to the beam momentum spread and to the third root of N_i [Eq. (141b)]:

$$\Gamma_{\parallel} \propto (\sigma_p/p)^{-2} N_i^{1/3}.$$

The cooling experiments have shown that intrabeam scattering limits in most cases the momentum spread and we have found that $(\sigma_p/p)_{\text{IBS}}$ increases proportionally to $N_i^{2/5}$ [Eq. (161), Figs. 42 and 59]. If our intrabeam scattering model holds, then $\Gamma_{\parallel}$ increases proportionally to $N_i^{-1/2}$ for decreasing N_i until thermal equilibrium is reached (Fig. 74). Calculations done on electron cooling simulations resulted in a scaling of $(\sigma_p/p)_{\text{IBS}}$ proportional to $N_i^{1/3}$ [109c, 118]. In this case we get $\Gamma_{\parallel} \propto N_i^{-1/3}$. The scaling with the charge of the ion and its mass is $\Gamma_{\parallel} \propto (Z/A^2)^{2/5} A^{-1}$; it favours light ions, in contrast to an earlier opinion [82a]. Therefore light ion beams of low intensity facilitate the achievement of longitudinal beam ordering. Also the ratio of cooling time to electron capture rate is more favourable for light ions. Electron capture could prevent us from reaching sufficiently long beam lifetimes because, if strong cooling (large n_e) is needed to obtain ordering, also the capture rate becomes high, which in turn could disturb the ordering and reduce the ion beam lifetime to values that would render its detection impossible.

The attempt to produce ordered beams will be an important application of electron cooling. Its success requires special machine settings [165] and depends very much on the realization of low-temperature electron beams. Beam ordering is somewhat contrary to beam stability. The theoretical models that predict structured beams of high order are still far away from reality, and refined calculations are very computer-time-consuming; an experimental investigation might be faster and more conclusive, provided sensitive diagnostic means exist.

12.13 Special techniques

For completeness, we will mention a few special techniques, some of which have already been tested in previous cooling experiments. It is possible to decrease the cooling time for a large initial momentum spread by sweeping the electron beam in momentum space across the ion beam. This has been investigated qualitatively in previous experiments. Simulations show that cooling times could probably be reduced considerably for large initial momentum spreads.

It is not always desirable to reduce the momentum spread to low values. The equilibrium momentum spread can be adjusted to a certain value by modulating the electron acceleration voltage accordingly (longitudinal blow-up) [110]. Similarly, the transverse cooling can be worsened by

inclining the electron beam with respect to the ion beam. When working with intense ion beams, these techniques could be of importance in order to avoid instabilities.

It is often unnecessary to apply the cooling continuously (e.g. for stretcher-ring operation). The cooling action can be almost stopped by stepping down the electron-beam energy by a few keV (parking position). For dense electron beams, this procedure is safer than a full turning-off because the tune change [Eq. (123)] is smaller. The electron beam has to be on for only the length of time the ion pulse takes to pass through the cooling region, when a bunched ion beam is cooled and the duty cycle lies around 1%. By pulsing the electron beam in the same way as the ion beam, the instantaneous electron current could possibly be increased without greatly increasing the problems related to the transport and collection of a high-current d.c. electron beam. Pulsing may, however, deteriorate the stability of the high-voltage power supply, and it is in general a more delicate operation; therefore it is probably more suitable for achieving short initial cooling times (e.g. for accumulation). On the other hand, high electron currents amplify the space-charge effects, and it is necessary to find ways of suppressing them. Although electron space-charge compensation by trapped ions is frequently discussed, there is as yet no convincing experimental proof that this is a viable means of stable operation.

12.14 Limits

The limits of electron cooling have not yet been reached in experiments, as far as the following properties are concerned:

- i) electron density,
- ii) electron temperature,
- iii) frictional force.

The electron density can certainly be increased many times above the present-day values of electron coolers, in particular when working with small electron beams. For an uncompensated beam, limits arise from the velocity profile across the electron beam [Eq. (98)], from the azimuthal twist of the electron beam in the solenoidal field [Eq. (101)], and from the rise of the longitudinal temperature [Eq. (95c)] with increasing electron density. It is not possible to give exact general boundaries for cooling, since they depend on the specific case (cooling of hot/cold ions, long/short storage times, etc.).

The present-day values for the transverse electron temperature can certainly be reduced by using colder electron sources. The price one has to pay for this reduction is the lower electron density. Photocathodes could possibly deliver electrons at the temperature of liquid-nitrogen and cold-plasma sources, and even at liquid-helium temperature. At these low initial temperatures there will be imperfections in the acceleration, which will affect the electron temperature at cooling energy. The limits on the longitudinal temperature are determined by the electron source, by the high-voltage ripple, and by the intrabeam scattering in the cooling beam.

High cooling rates are of interest for beam crystallization, for a rapid beam-stacking, and for internal target operation. It is therefore necessary to work close to the maximum of the cooling force, which, for the longitudinal and the transverse component of the frictional force, is at different values of the ion velocities. The contribution of the magnetic force can be maximized by increasing the magnetic field. The maximum value of the cooling force is increased by reducing the relevant electron temperatures, which shifts this maximum to lower ion velocities. This helps mainly to increase the cooling rate for *cold* ion beams. The increase of the magnetic field opens up the possibility to extend the region of adiabatic collisions to smaller impact parameters, provided no other limits enter (see Fig. 10). The effects of an increase of the solenoidal (and toroidal) field on the ion beam have to be watched (Section 8). Basically, a stronger cooling force for *hot* beams is obtained by raising the electron density, but there is the danger of encountering the limits discussed above.

12.15 Open questions

Despite the well-understood kinetics of electron cooling, there remain a number of open questions to be answered by future experiments. These concern

- i) the behaviour of the cooling force at low ion velocity (in the rest frame) [Eqs. (53) and (54)] and a sensitive comparison of binary collisions and the dielectric model (clarify the impact parameter problem);
- ii) the effects of the inner degrees of freedom of ions (e.g. excitation of partly stripped ions);
- iii) the scaling of the cooling force with n_e for highly charged ions;
- iv) the coherent contribution of the finite electron plasma;
- v) the effect of polarization of the plasma by highly charged ions;
- vi) the influence of possible bound states around the ionization threshold.

The study of points (iii) to (vi) should be executed with highly charged ions, because new phenomena would be more easily detectable.

13. ATOMIC PHYSICS APPLICATIONS OF ELECTRON COOLING

Many applications use the cooling beam directly as an electron target, which is a unique feature of this cooling method. These uses are mainly directed towards atomic physics experiments that are investigating the reactions listed in Section 5. The interplay between these studies and electron cooling is discussed below, and are briefly mentioned where these applications provide information that is relevant to this cooling. More details can be found in Refs. [166] and [167].

13.1 Spontaneous radiative electron capture

The atomic physics motivation for the study of this process [Eq. (76a)] is to be able to check the recombination models for low electron-ion energy and highly excited ionic states (initial population). The reaction is studied by measuring the spectrum of the emitted photons (X-rays). The measurement of the intensity, energy, and width of these electromagnetic transitions provides information about the initial population of the states, the atomic cascade, the lifetime, and the binding energies of atomic states. Interest is focused on the precise measurement of the binding energy of few-electron ions, and the aim is to test QED predictions (e.g. Lamb shift, hyperfine splitting) and to disentangle the various contributions.

Depending on the angle θ of emission with respect to the ion trajectory, the energy ($E_\theta = h\nu$) of the photons is Doppler-shifted in the laboratory [Eq. (83)]. The ratio of sum to difference energy between a forward ($\theta = 0$) and backward ($\theta = \pi$) emitted photon yields the absolute beam velocity independently of the photon energy:

$$\frac{E_0 + E_\pi}{E_0 - E_\pi} = \beta_0. \quad (174)$$

The width of a transition is composed of the natural level width of the final state, the width of the electron continuum (electron temperature), the momentum spread of the ion beam, and the Doppler spread due to a finite solid angle of observation $\Delta\Omega$. For the last two contributions, we have

$$\frac{\Delta E}{E_{\theta, \text{lab}}} = \frac{\beta_0 \cos \theta}{1 - \beta_0 \cos \theta} \left[\frac{\Delta\beta_0}{\beta_0} + \frac{\Delta\Omega}{2\pi} \cos \theta \right], \quad (175)$$

which means that the momentum spread enters at the same level as the contribution from a finite solid angle for forward and backward emission. In the case of X-ray transitions, these contributions dominate the contribution from the electron temperature. The level width is usually negligible.

The population probability of the hydrogen-like atomic levels is described by Eq. (77). If the electron binding energy is not high enough, the electrons are again stripped off by the motional electric field experienced by the ions when moving through a transverse magnetic field $B_{\perp}$ (e.g. toroid, bending dipole of ring). The corresponding stripping-energy threshold is given by

$$E_{\text{str}} = 2m_e c^2 \sqrt{\frac{Z\alpha\beta_0\mu_B B_{\perp}}{(m_e c^2)}} = 1.3 \text{ eV } \sqrt{Z\beta_0 B_{\perp} [\text{T}]}, \quad (176)$$

where μ_B is the Bohr magneton. The dipole field is given by the beam rigidity $B\rho$ and the bending radius ρ [Eq. (112b)]; the transverse component in the toroid is given by the solenoidal field, and by the effective angle of the toroid. The corresponding energy thresholds are

$$E_{\text{dip}} = 2.3 \text{ eV } \beta_0 \sqrt{\frac{A\gamma}{\rho[\text{m}]}} \quad (177a)$$

$$E_{\text{tor}} = 2.3 \text{ eV } \sqrt{Z\beta_0 \sin \phi_0 B_{\text{sol}}}. \quad (177b)$$

Electrons that are captured into levels with binding energies $E_a < E_{\text{tor}}$ are stripped in the toroid and return to the original beam component. The others survive the passage through the toroid and, for a cooled beam on axis, they pass also through the quadrupoles without being stripped. On the way to the bending magnet, the high- n states will be de-excited through radiative transitions. If the electrons have not reached levels with sufficient binding energy, i.e. if $E_a < E_{\text{dip}}$, they will be stripped when they arrive in the ring dipole magnet. The principal quantum numbers corresponding to the limits given in Eqs. (177) are, for hydrogen-like ions,

$$n_{\text{dip}} = 3.4 Z \beta_0^{-1/2} \gamma^{-1/4} (\rho[\text{m}]/A)^{1/4}, \quad (178a)$$

$$n_{\text{tor}} = 4.6 Z^{3/4} (\beta_0 \sin \phi_0 B_{\text{sol}} [\text{T}])^{-1/4}. \quad (178b)$$

Ions that are stripped in the dipole and return to the original charge state perform large betatron oscillations. The down-charged ions with $E_a > E_{\text{dip}}$ will be separated from the primary beam because of their higher beam rigidity. They will survive only if the original and the lower charge state can be accommodated simultaneously in the ring (momentum acceptance larger than the percentage of q/A change). Otherwise they will exit from the ring and can be measured with the appropriate detectors. The recombination coefficient [Eq. (79)] and the average temperature of the electron beam at the position through which the ions pass can be determined from the measurement of the down-charged ion rate [Eqs. (81) and (82)] if the cut-off in Eq. (79) and de-excitations are taken into account correctly. It should be noted that the recombination models used so far do not take into account the presence of both the solenoidal magnetic field and the space-charge field of the electron beam in the cooling region.

The study of spontaneous electron capture therefore provides—besides the atomic physics data—valuable overall information about the cooling beam, the absolute ion velocity, and the status of the cooling. The recombination rate should increase continuously for the cooling of an initially hot ion beam ($v_i \geq \Delta_{\perp}$) until a plateau is reached when approaching equilibrium ($v_i \ll \Delta_{\perp}$). Transverse instabilities are nicely seen in fluctuations of this rate (Fig. 75).

Some time ago it was proposed to make direct use of the radiation emitted in spontaneous electron capture [168] to provide a tuneable X-ray line source. Interest in this idea could be rekindled with the advent of the new heavy-ion rings and in view of the cold electron beam developments.

13.2 Induced radiative electron capture

If the electron cooling region is illuminated by a radiation field that matches a transition from the continuum to an atomic level, then electron capture is stimulated:

In this reaction, a specific transition is filtered out through a corresponding wavelength selection of the radiation field. High spectral density and resolution can be achieved with lasers. The ratio of induced to spontaneous capture probabilities $W_n^{\text{ind}}/W_n^{\text{spon}}$ (gain factor g) is given by [27]

$$g = W_n^{\text{ind}}(\Delta\nu)/W_n^{\text{spon}}(\Delta\nu) = u(\nu) B_\nu/A_\nu, \quad (180)$$

where $u(\nu) = P_{\text{laser}}/(cA_{\text{laser}}\Delta\nu)$ is the spectral photon density (P_{laser} is the laser power, A_{laser} is the cross-sectional area of the laser beam, and $\Delta\nu$ is the spectral width). The subscript n denotes captures into atomic levels with principal quantum number n . The Einstein coefficients A_ν and B_ν are related by $A_\nu = 8\pi h\nu^3/c^3 B_\nu$. The indication $(\Delta\nu)$ in Eq. (180) means that a ratio is formed between transitions within the spectral width essentially defined by the laser resolution. Inserting the photon density and the relation between the Einstein coefficients, we get

$$g = P_{\text{laser}}c^2/(8\pi h\nu^3 A_{\text{laser}}\Delta\nu). \quad (181)$$

The transition probabilities determine the recombination coefficients α_n^{ind} and α_n^{spon} ; therefore, it holds that

$$g = \alpha_n^{\text{ind}}(\Delta\nu)/\alpha_n^{\text{spon}}(\Delta\nu). \quad (182)$$

The recombination coefficients include the electron velocity distribution [Eqs. (78)]. Evaluating this for a spherical distribution yields

$$g_{\text{sph}} = \pi^2 \frac{P_{\text{laser}}}{A_{\text{laser}}} \left(\frac{m_e c^2}{h\nu} \right)^3 \frac{\lambda_e^3}{cT_{e\perp}}, \quad (183)$$

where $\lambda_e = r_e/\alpha (= 3.86 \times 10^{-11} \text{ cm})$ is the Compton wavelength of the electron. By comparing a spherical distribution with a flattened one, we find [27]

$$g_{\text{flat}} = 1/2 g_{\text{sph}} \Delta_\perp/\Delta_\parallel. \quad (184)$$

Depending on the laser power, the wavelength, and the laser beam diameter, g can assume values up to a thousand. The study of laser-induced capture refines the possibilities given by spontaneous capture, in the sense that the region around the ionization threshold can be scanned with a resolution given mainly by the laser. This is done by measuring the number of down-charged ions as a function of the laser frequency. Since spontaneous capture to other levels not populated by stimulated capture produces also down-charged ions and background, we have to profit from the electromagnetic stripping in magnets in order to reduce these contributions. Whilst for atomic physics the induced capture gives access to the region around the ionization threshold (Rydberg states, influence of magnetic and electric fields), for the cooling process it provides direct information on the electron distribution—in particular on the flattening—and yields a precise measurement of the ion velocity. With this method it might also be possible to detect effects due to the electron plasma polarization, or the formation of Debye spheres around highly charged ions passing through the cooler.

13.3 Antihydrogen production

The direct application of the electron cooling arrangement and of spontaneous and induced radiative capture provides the possibility of producing antihydrogen according to Eqs. (76a) and (179) [132]. The investigation of these antimatter atoms aims at using the high-resolution spectroscopy of atomic levels for a precise test of the CPT theorem [169]. For such experiments, capture rates comparable to those in the electron cooling of protons are required. Consequently, the positron beam will also act as a cooling medium for antiprotons. Owing to these requirements, the technology for the production of antihydrogen is related to the technique of cooling with a recirculated beam of positrons or electrons.

13.4 Three-body recombination

The perspective of producing subthermal electron beams of high density opens up the possibility of reaching a domain for the electron plasma where multibody capture processes come into play. Recent calculations [34, 35] show that the contributions from collisional three-body recombination [Eq. (76e)] become detectable for electron coolers planned for heavy-ion rings. Such contributions should show up through their different scaling with electron temperature and density [Eq. (86)]. Three-body collisional recombination has not yet been investigated experimentally for highly charged ions, and its study at storage rings could reveal new information. For electron cooling, the detection of many-body effects would indicate a deviation from the general theory.

13.5 Dielectronic recombination

One important application of the cooling electron beam would be in the study of dielectronic recombination [Eq. (76b)]. For these measurements the electron beam has to be detuned in energy. Depending on the amount of detuning, the cooling will almost cease, or it will be greatly attenuated. The resonance condition results in a narrow peak (depending on the level lifetime and on the electron temperature) in the down-charge rate as a function of detuning energy, which is similar to the case of induced radiative capture. (Remember that the electron energy has to match the excitation energy of a core electron minus the binding energy of the captured electron.) Detuning can be done in either of two ways: by increasing the electron energy, or by decreasing it. It is the cross-section and the width of these states that are of interest for atomic physics. On the other hand, the effective electron temperature can be determined from the width of the peak. The ion velocity can be deduced from the difference between the positions of the high-energy and low-energy peaks, without knowledge of the absolute peak position [Eq. (84a)].

Once the ion storage rings are in full operation, these applications and their combinations will certainly be refined, and they will be exploited to provide new insights into atomic physics problems that have hitherto been unexplorable. The usefulness of electron cooling as a beam-cooling method for nuclear physics and intermediate-energy physics is uncontested. Its use in accelerators might pave the way for their operation in new regimes of stored beams.

14. SUMMARY

It has been shown that the mechanism of electron cooling is nothing else but the temperature equalization through collisions in a mixture of hot (ions) and cold (electrons) gas. More precisely, it is a two-component plasma that approaches thermal equilibrium. The heat is exchanged through Rutherford scattering. Plasma physics has for a long time provided the theoretical framework for the description of the underlying processes. The electron cooling of ions stored in traps comes closest to the standard situation found in the physics of dilute plasmas. In the electron cooling of ions circulating in rings, the common relativistic motion modifies the picture. The standard situation is restored by observing the processes in the co-moving frame. Of practical interest is the prediction of

the time it takes to approach equilibrium, and the equilibrium temperature that can be reached. Already the temperature relaxation time of a two-component plasma, derived by Spitzer 50 years ago, gives the right order for the cooling times. Also, from this theory, one can estimate the equilibrium temperature, even taking into account the diffusion process in the ion beam (intrabeam scattering), which prevents us from reaching thermal equilibrium for intense ion beams.

Budker and his co-workers have admirably realized the potential of this configuration and have consequently applied it to proton beams. The pioneering work performed in Novosibirsk was based, both in theory and in practice, on the implements of plasma physics. The early theory developed by Derbenev and Skrinsky made use of this close connection. Initially the omission of the magnetic confinement of the electrons resulted in predictions for the cooling time that were longer than the experimental values. A more comprehensive approach, taking into account the magnetic field, produced results that were in agreement with experiments, but it ran into the long-standing plasma physics problem of binary collision models — namely, the impact parameter cut-off for the scattering cross-section, which was even more problematic for the magnetically confined electrons. Sørensen and Bonderup, being aware of this problem of binary collisions, applied a plasma continuum model — the dielectric description — to electron cooling, which enabled the obstacle of cut-off for magnetized electron beams to be overcome. This model takes into account the continuum properties of the electron plasma, and is probably also more suitable for the cooling of highly charged ions. In this case we expect that polarization of the electron plasma by the ions could produce a situation that would be significantly different from the one with light ions and protons. It may even turn out that a linear theory does not then suffice. The binary collision model, however, provides us with an analytic description of the cooling force, which is very helpful for the understanding of the underlying processes.

The fundamental work of the Novosibirsk group on the theory was complemented by their equally pioneering experimental study. Although, as far back as the late forties, Pierce had already made up the recipe for the production of cold electrons, it was the Novosibirsk team that developed the basis for the generation of a cooling electron beam, its adiabatic acceleration and transport, its merging with the proton beam, and the re-collection of the electrons with high efficiency. After the first suitable electron beam was obtained in 1974, electron cooling experiments followed immediately in the NAP-M ring in Novosibirsk and yielded spectacular results. There then followed a period of intensive experimental study, which provided the data for checking the theory. This activity was complemented by the experimental work performed in the ICE ring at CERN and in the cooling ring at FNAL. These experiments showed that electron cooling was also suitable for the cooling of very hot beams. Originally, Budker and his co-workers proposed electron cooling as a means of accumulating antiprotons for high-energy colliders. The comparison of electron and stochastic cooling on the basis of practicability and performance, which was made at CERN with ICE, showed that the latter was more suitable in view of the high energy of the antiproton production maximum. When the first electron cooling results were obtained with ICE, it soon became clear that this method could considerably extend the experimental possibilities in the low-energy antiproton ring LEAR. The LEAR cooler became the first device developed for routine operation. It also soon became clear that a cooling beam represents an excellent free electron target for atomic physics experiments. The interest in this technique was increased enormously by the promise of obtaining superb stored ion beams of small size, divergence, and momentum spread, by the prospect of using internal targets in conjunction with electron cooling, and by the electron target application. At the First Topical Workshop on Electron Cooling held in Karlsruhe in 1984, ten cooling projects were presented. There are now three installations working, and the major part of the remainder will come into operation in 1990. Probably all the originally proposed cooling rings will be realized. At the Karlsruhe Workshop, two further items of news from Novosibirsk — which previously had not been accorded the attention

they deserved — amplified the interest in electron cooling. The first was the difference in frictional force for the cooling of negative and positive ions by magnetized electrons, and the second was the claim to have observed ordering in the proton beam as a consequence of cooling to a very low temperature. The first item has been confirmed during recent experimental studies at Novosibirsk with a single-pass arrangement. The second triggered a lot of theoretical activity and speculation, and we are eagerly awaiting the outcome from the new storage rings. Electron cooling has now passed into a consolidation phase. The focus has moved from the mechanism of electron cooling to its results and its exploitation, which will provide new information and hopefully some surprises.

The reader of this review should have learnt the following. Electron cooling is a way of compressing the phase space occupied by ions stored in rings or traps; its application is useful wherever this compression is needed. Its practical range of application lies below $\beta\gamma = 1$. In this energy range, e-folding cooling times τ below 1 s can be achieved for stored protons, with standard electron beams at any energy. The cooling time decreases even more for heavy ions ($\propto Z^2/A$). In most cases, equilibrium between cooling and heating is reached after a few e-folding times. For low ion intensities, thermal equilibrium can be attained. In standard electron coolers, the transverse electron temperature $T_{e\perp}$ is determined by the cathode temperature, and the longitudinal one by the acceleration voltage ripple $\Delta U/U_{\text{cath}}$ or the internal electron-electron potential. Therefore we have $T_{e\parallel} \ll T_{e\perp}$. In practice, $kT_{e\perp} \approx 0.2$ eV and $\Delta U/U_{\text{cath}} \approx 10^{-5}$. The flattened electron velocity distribution is imposed on the ion beam, which means that also $T_{i\perp} \gg T_{i\parallel}$. Thermal equilibrium in the transverse plane can often be realized, resulting in an ion beam emittance $\epsilon [\approx T_{i\perp} \langle \beta \rangle / (m_i c^2 \times \beta^2 \gamma^2)]$ that can be as low as $\epsilon \approx 10^{-3} \text{ A}^{-1} (\beta\gamma)^{-2} \pi \text{ mm} \cdot \text{mrad}$. In the longitudinal direction, we obtain, for the momentum spread of the ion beam, $\Delta p/p = 1/2 \Delta U/U_{\text{cath}}$ for low beam intensity and low charge states. Under special conditions (highly stabilized power supply, low electron density, weak heating processes), longitudinal temperatures below 1 K can be reached. Then it might be possible to arrive at some degree of order in the ion beam. Higher beam intensities and charge states of the ions lead to higher values of $\Delta p/p$ owing to intrabeam scattering. The evolution of the ion beam properties and of the attainable equilibrium in the presence of electron cooling can be predicted quite precisely by computer programs that contain the relevant frictional forces and the machine parameters. They make it possible to anticipate the experimental conditions and to precalculate the luminosity, the beam lifetime, and the resolution.

The major technical problems for the generation of a standard cold electron beam can be considered as being solved, and its production is state of the art. There still remain a few technical challenges for special applications, which include the development of a cold electron source, of a recirculator, and of a positron cooler. Also, the cooling with protons of highly charged ions stored in traps could be a future development. Concerning cooling at high energy, I can foresee but one possible application, namely in superconducting heavy-ion colliders. It is possibly the only high-energy case where γ is not too large and where η_e is still in the 10^{-2} range, so that electron cooling could be more advantageous than the simpler (hardware-wise) stochastic cooling, because its efficiency does not depend on the stored beam intensity.

Electron cooling works for a given installation at any energy within its specified range, and it is not restricted to a few discrete energies; the matching of the velocities merely requires the adjustment of the electron acceleration voltage.

Electron cooling is based on the friction of ions deviating in velocity from the average electron velocity. This friction increases approximately linearly with small velocity deviations, reaches a maximum, and falls off inversely as the square of the velocity deviation. The position of the maximum friction is determined by the apparent velocity spread in the electron beam. The electrons are confined by a magnetic field parallel to their trajectory; therefore the thermal electron motion transverse to this field is frozen in gyrations. This results in a shift of the maximum friction to lower

velocity values (corresponding to the lower longitudinal electron velocity spread), and in a higher maximum value for the frictional force. The magnetization of the electrons therefore enhances the cooling effect for small velocity deviations; furthermore, another consequence of the magnetization is that in this regime the electron cooling of negative ions is faster than that of positive ions.

Inherent in the cooling is the diffusion caused by the scattering of ions from electrons. Diffusion acts against cooling, and both effects cancel out in thermal equilibrium. The effect of diffusion should not be forgotten when discussing beam ordering.

Positive ions can pick up electrons in the cooler. The charge of the ions is then reduced by one unit and they usually exit from the ring (however, they can still be stored in traps). In general, this does not limit the application of cooling, since the pick-up rates are always more than two orders of magnitude smaller than the inverse cooling times. The cooling times can be easily adjusted by varying the electron density. On the other hand, electron pick-up can be employed for beam extraction, and the extracted fraction of the beam can be used either for experiments or for diagnostics. Furthermore, electron-ion recombination will play an important role in future atomic physics applications of merged electron and ion beams. It is this special feature of the electron cooling arrangement that makes it possible to study atomic and plasma physics phenomena in dilute fully ionized gases with unprecedented precision. For a long time too little attention has been paid to this aspect, but the situation will change with the advent of the new heavy-ion cooling rings.

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Table 10.1
Parameters of the first cooler rings and electron coolers

	NAP-M [2]	ICE [3]	FNAL [4]
Circumference (m)	47	74	135
Operation energy (MeV)	1.5-85	46	114, 200
Number of stored protons	10^6 - 10^8	10^6 - 10^9	5×10^6
Average ring vacuum (Torr)	5×10^{-10}	2×10^{-9}	1×10^{-10}
Horiz. and vert. acceptance (π mm · mrad)	400, 200	80, 40	40, 20
Longitudinal acceptance (%)	± 1	± 0.25	± 1
Ring fraction occupied by cooler	0.02	0.04	0.037
Working point Q_h, Q_v	1.24, 1.34	1.71, 1.16	3.57, 5.57
Average horiz. β -function (m)	6	18	6
β_h and β_v in cooling section (m)	5.2, 5.2	3, 11	25, 40
Dispersion in cooling section (m)	6	5.7	0.1
Transition γ_{tr}	1.2	1.3	3.6
Cathode diameter (cm)	1, 2	5	10
Electron beam diameter (cm)	1, 2	5	5
Electron energy (keV)	0.7-46	26	62, 111
Electron current (A)	0.1-0.8	0.6, 1.3, 2.2	0.5-3
Electron density ($\times 10^8$ cm $^{-3}$)	0.09-3.7	0.2, 0.4, 0.8	0.1-0.6
Cooler magnetic field (kG)	1	0.5	0.7, 0.93
Toroidal angle (°)	45	36	90
Length of cooling section (m)	1	3	5
Gun-collector voltage (kV)	1	3	5
Electron current losses	$\leq 10^{-4}$	2.5%	$\leq 10^{-4}$
T_{e1} (eV)	0.12	0.24	0.6-1
T_{e2} (meV)	0.1	0.1	0.1

Table 10.2
Parameters of the LEAR electron cooling [109]

Circumference (m)	78.6
Operation energy for electron cooling (MeV)	5-76
Number of stored (anti)protons	10^6 -(3×10^9)
Average ring vacuum (Torr)	2×10^{-12}
Horiz. and vert. acceptance (π mm · mrad)	100, 48
Longitudinal acceptance (%)	± 0.5
Ring fraction occupied by cooler	0.02
Working point Q_h, Q_v	2.305, 2.730
β_h and β_v in cooling section (m)	1.9, 5.3
Dispersion in cooling section (m)	3.6
Transition γ_{tr}^2	-22
Electron beam diameter (cm)	5
Electron energy (keV)	0-40
Electron current (A)	0-4
Electron density (cm^{-3})	$0-10^8$
Cooler magnetic field (kG)	0.1-2
Toroidal angle ($^\circ$)	36
Length of cooling section (m)	1.5
Gun-collector voltage (keV)	0.7-3
Electron current losses	$\leq 5 \times 10^{-3}$

Table 10.3
Lowest equilibrium proton-beam values obtained in cooling experiments

	NAP-M [2]	ICE [3]	FNAL [4]	LEAR [109]	IUCF [113]
Beam velocity $\beta_0 c$ (c)	0.35	0.3	0.566, 0.45	0.31	0.3, 0.5
Beam divergence σ_{iy} (mrad)	0.03	0.12		0.3	≤ 0.15
$2kT_{i\perp} = m_i c^2 \beta_0^2 \gamma^2 \theta_{i\perp}^2$ (eV)	0.12	1.4	0.6 ^{*)} , 0.9 ^{*)}	8	≤ 2
Beam radius r_{0i} (mm)	0.25	1.3	0.8, 0.9	0.8	≤ 0.35
Emittance $= \pi r_{0i} \theta_{i\perp}$ (π mm · mrad)	0.0075	0.16	0.4 ^{*)}	0.24	0.05 ^{*)}
Momentum spread (FWHM)	1.7×10^{-6}	4×10^{-5}	3.8×10^{-5}	1.8×10^{-5}	$\leq 2 \times 10^{-5}$
for N_p stored protons	$\leq 2 \times 10^7$	8×10^6	$2 \times 10^6, 5 \times 10^5$	3×10^8	$\leq 10^7$
$2kT_{i\parallel} = m_i c^2 \beta_0^2 (\sigma_p/p)^2$ (meV)	0.1	70	100	10	12
$2T_{i\parallel} = m_i c^2 \beta_0^2 (\sigma_p/p)^2$ (K)	1	820	1100	110	140
$\Gamma_{i\perp}$ [Eq. (141a)]	4.7×10^{-4}	8.6×10^{-6}	3.2×10^{-7}	1.5×10^{-5}	4.2×10^{-5}
$\Gamma_{i\parallel}$ [Eq. (141b)]	5	1.2×10^{-3}	2.4×10^{-4}	2.9×10^{-2}	2×10^{-2}

*) Value measured independently.

Table 10.4
Proton beam lifetimes

| | NAP-M
[2] | ICE
[5] | FNAL
[4] | LEAR
[109] | IUCF
[113] | TSR
[112] |
|--------------------------------------|---------------------|--------------------|---------------------|---------------------|--------------------|---------------------|
| Beam velocity $\beta_0 c$ (c) | 0.3 | 0.3 | 0.566 | 0.31 | 0.3 | 0.21 |
| Average ring vacuum P_{N_2} (Torr) | 5×10^{-10} | 2×10^{-9} | 4×10^{-10} | 2×10^{-12} | 3×10^{-9} | 6×10^{-11} |
| Lifetime without cooling (s) | | 120 | 5-10 | > 10000 | 4 | 11000 |
| Lifetime with cooling (s) | 36000 | 8400 | 4000 | ≥ 10000 | 250 | 130000 |

Table 11.1

Cooling ring characteristics

| Name of ring
Reference | IUCF COOLER
[111] | TSR
[112] | CELSIUS
[120] | ASTRID
[121] | TARN II
[122] | ESR
[123] | CRYRING
[124] | COSY
[125] | HISTRAP
[126] |
|---------------------------------------|------------------------------------|---------------------|----------------------|-----------------------|-------------------|------------------------|-------------------|-------------------------|-----------------------|
| Circumference (m) | 86.8 | 55.4 | 81.8 | 40 | 77.8 | 108.4 | 51.6 | 184 | 47 |
| Magnetic rigidity (T · m) | 3.6 | 1.5 | 7 | 2 | 3.2 | 10 | 1.44 | 11 | 2.67 |
| Energy range (MeV/A) | 500 | 5-30 | 1360 (for p) | 0.1-190 | 100; 400 (for p) | 3-830 | 0.3-24 | 40-2500 | 0.02-80 |
| Type of ions: | | | | | | | | | |
| highest charge | 3 | 47 | 18 | | 10 | 92 | | 1 | 60 |
| highest A | 7 | 127 | 40 | | 20 | 238 | | 1 | 238 |
| lowest Q/M | 0.5 | 0.5 | | 0.5 | 0.5 | 0.5 | 0.5 | 1 | 0.5 |
| Expected ion intensity | 10 mA | 10 mA | 10 ¹¹ (p) | 5 × 10 ⁷ | 10 ⁸ | up to 10 ¹¹ | 10 ⁹ | ≤ 1 × 10 ¹⁰ | 2 × 10 ⁻¹² |
| Average ring vacuum (Torr) | < 3 × 10 ⁻⁹ | < 10 ⁻¹¹ | 10 ⁻¹¹ | 5 × 10 ⁻¹² | 10 ⁻¹¹ | 10 ⁻¹¹ | 10 ⁻¹¹ | ≤ 1 × 10 ⁻¹⁰ | |
| Horiz. acceptance (π mm · mrad) | 25 | 500 | 120 | 350 | 260 | 450 | 200 | 180 | 200 |
| Vert. acceptance (π mm · mrad) | 25 | 120 | 120 | 40 | 15 | 150 | 100 | 36 | 60 |
| Long. acceptance for ε = 0 (‰) | ± 0.2 | ± 3 | ± 0.3 | ± 2.1 | ± 0.1 | ± 2 | ± 1.7 | ± 0.5 | ± 4 |
| Length of cooling section (m) | 2.8 | 1.5 | 2.5 | 0.8 | 1.5 | 2.5 | 1.1 | 2 | 1 |
| Typical working point | | | | | | | | | |
| Q _h | 4.15 | 2.75 | 1.68 | 2.29 | 1.75 | 2.1-2.45 | 2.3 | 3.86 | 2.309 |
| Q _v | 5.15 | 2.825 | 1.9 | 2.73 | 1.8 | 2.1-2.45 | 2.27 | 4.12 | 2.274 |
| Average horiz. β-function (m) | 2.7 | 3.15 | 7.75 | 5.4 | 8 | 8-10 | 4 | 12 | 10 |
| β _h in cooling section (m) | 2.3 | 5.5 | 8.14 | 9.0 | 10 | 8 | 2.3 | 10 | 10 |
| β _v in cooling section (m) | 4 | 5.7 | 5.04 | 3.2 | 4 | 8-10 | 2.9 | 10 | 5 |
| Dispersion in cooling section (m) | 0 | 0-1 | -1.48 | 2.7 | 4.7 | 0-6 | 1.6 | 0 | 0 |
| Transition γ _t | 4.85 | 2.96 | 2.605 | 4.58 | 1.88 | 2.7 | 2.23 | 1-4 | 6.69 |
| Internal target type: | Jet/dust fibre | Storage cell | Cluster/C fibre | | | Jet | | Cluster | Jet |
| density (atoms/cm ²) | 1-100 ng/cm ² | 10 ¹⁴ | | | | 10 ¹⁴ | | 10 ¹⁴ | |
| nuclear mass | | | | | | | | | |
| First circulating beam | Jan. 1988 | May 1988 | Nov. 1988 | Summer '89 | 1989 | 1-129 | July 1990 | 1992 | 1994 |
| Cathode/beam diameter (cm) | 2.54 | 5 | 2 | 1 | 5 | 5 | 4 | 2.54 | 5 |
| Electron energy range (keV) | 10-270 | 3-20 | 10-300 | 0.1-2.5 | ≤ 120 | 10-310 | 2-20 | 0-30 (100) | 1-60 |
| Electron current (A) | 0-4.8 | 1 | 0-2.8 | 0.01-0.06 | ≤ 10 | 0.5-10 | 0.001-3 | 0-4 | 0.01-5 |
| Nom. gun perveance (μP) | 0.7 | 1.7 | 0.36 (40 kV) | 0.5 | 1 | 2 | 0.1-5 | 0.1-0.6 | 0.34 |
| Magnetic field (kG) | 1-1.5 | 0.55 | 0.5-2 | 0.27 | 1.2 | 0.1-2.5 | 2.5 | 0.8 | 0.5 |
| at electron energy (keV) | | | | | | | | | |
| Toroidal angle (°) | 60 | 45 | 90 | 2.5 | 120 | 90 | 13 | 22 | 48 |
| Gun-collector voltage (kV) | 2-7 | 1.2-2 | | 45 | 45 | 5 | 50 | 53 | 2 |
| at electron-energy (keV) | | | | 0.4 | | | 1.5 | 3 | 20 |
| and current (A) | | | | 2.5 | | 310 | 13 | 22 | 1 |
| Vacuum in cooler (Torr) | 1 × 10 ⁻⁹ | < 10 ⁻¹¹ | | 0.06 | 10 ⁻¹¹ | ≤ 10 ⁻¹⁰ | 10 ⁻¹¹ | 1 × 10 ⁻¹⁰ | < 10 ⁻¹¹ |
| Electron losses (collector) | 10 ⁻⁶ -10 ⁻³ | < 10 ⁻⁴ | | 10 ⁻¹¹ | | ≤ 10 ⁻³ | 10 ⁻⁴ | 1 × 10 ⁻⁴ | < 10 ⁻⁴ |
| First cooling experiment | March 1988 | End 88 | 1990 | 1989 | Oct. 1989 | Spring '90 | Spring '91 | 1992 | 1994 |

Table 11.2
Envisaged characteristics of a subthermic electron beam [38]

Current: continuous (mA) [pulsed (A)]	1-3 [15]
Electron energy spread	
at cathode ($T_{\text{cath}} = 77 \text{ K}$):	
$\Delta E_{\perp}$ (meV)	≤ 1
$\Delta E_{\parallel}$ (meV)	30-60
after acceleration:	
$\Delta E_{\perp}$ (meV)	≤ 1
$\Delta E_{\parallel}$ (μeV)	≤ 4
Final electron energy (keV)	≤ 50
Magnetic field (T)	≈ 0.1
Beam diameter (mm)	2

APPENDIX I: VALUES AND PARAMETERS FOR THE STANDARD ELECTRON COOLING CASE

This Appendix serves two purposes: first, it should provide the reader with a practical example and give him the possibility to cross-check his calculation with the standard case; secondly, it allows the author to control the most important formulae given in the main text, for possible errors. In Table AI.1 we list the standard parameters used as input for the calculations whose results are found in Table AI.2.

Table AI.1
Input parameters for the standard case

Electron energy	E_0	(keV)	100
Beam velocity	β_0	(c)	0.548
Electron beam radius	r_0	(cm)	2.5
Electron density	n_e	(cm ⁻³)	10 ⁸
Solenoid field	B_{sol}	(T)	0.1
Toroid radius	r_{tor}	(m)	1
Toroid angle	ϕ_0	(degree)	40
Cathode temperature	T_{cath}	(K)	1163
Length of cooling section	ℓ	(m)	2
Acceleration voltage ripple	σ_U	(V)	1
Radius of vacuum tube	r_{tube}	(cm)	11.2
Ring circumference	C_0	(m)	100
Betatron tune	$Q_x = Q_z = Q_y$		2.65
Transition	γ_{tr}		3
Longitudinal coupling impedance	$Z_{ } h/h_f$	(Ω)	10 ³
Horizontal ring acceptance	A_x	(π mm · mrad)	100
Vertical ring acceptance	A_z	(π mm · mrad)	50
Average ring vacuum	P_{gas}	(Torr)	10 ⁻¹⁰
Nuclear charge of residual gas	Z_{gas}		7
β -function in cooling region	$\beta_x = \beta_z$	(m)	4
Dispersion in cooling region	D	(m)	5
Magnetic radius	ϱ	(m)	6
Number of stored ions	N_i		10 ⁹
Charge of ions	Z		1
Mass number of ions	A		1
Initial ion beam emittance	ϵ_γ	(π mm · mrad)	20
Initial momentum spread	σ_p/p		10 ⁻³
Coulomb logarithm for residual gas	L_{res}		8

In the following table we list the values of some quantities calculated for the standard case, using the formulae summarized in Appendix III. The numbers in brackets are the equation numbers used in the text.

Table A I.2
Calculated values

<i>Electron beam properties</i>				
Lorentz factor	γ		1.196	
Electron current	I_e	(A)	5.17	(97)
Electron current density	j_e	(A/cm)	0.263	(97)
Gun perveance	P_{gun}	(μP)	0.163	(87)
Transverse electron temperature	$kT_{e\perp}$	(eV)	0.1	$= kT_{cath}$
Transverse velocity spread	$\Delta_{\perp}$	($cm \cdot s^{-1}$)	1.88×10^7	(95d)
Corresponding transverse diverg.	$\theta_{e\perp}$	(mrad)	1.1	(95d)
Electron plasma frequency	$\omega_{e,pl}$	(GHz)	0.563	(9)
Debye radius	λ_D	(μm)	235	(7)
Electron cyclotron frequency	$\omega_{e,c}$	(GHz)	14.7	(20)
Gyration radius	$r_{e,c}$	(μm)	12.8	(20)
Average electron distance	r_{av}	(μm)	13.4	(8)
Longitudinal electron temperature	$kT_{e\parallel}$	(eV)	7.5×10^{-5}	(94)
Longitudinal velocity spread	$\Delta_{\parallel}$	($cm \cdot s^{-1}$)	5.15×10^5	$\sqrt{2kT_{e\parallel}/m_e c^2}$
Ratio of temperatures (flattening)	Q_v^2		1333	(96)
Plasma correlation factor	$\Gamma_{e\perp}$		1×10^{-3}	(11)
	$\Gamma_{e\parallel}$		1.4	(11)
Pitch of gyration spiral	λ_s	(cm)	7	(89)
Azimuthal rotation velocity for $r = r_0$	v_{az}	($cm \cdot s^{-1}$)	2.25×10^7	(102)
Electron energy on axis	E_{ax}	(keV)	98.876	(99)
Energy diff. axial-peripheral electron
($r = r_0$)		(eV)	112	(98)
<i>Ion beam properties</i>				
Ion beam rigidity	$B\rho$	(T · m)	2.05	(112b)
Azimuthal rotation in solenoid	θ_{iaz}	(mrad)	0.097	(146a)
Horizontal deflection in toroid	θ_x	(mrad)	0.63	(148b)
Revolution frequency	f_0	(MHz)	1.6447	
Ion current	I_i	(μA)	26.3	(142b)
Average transverse ion temperature	$\langle kT_{i\perp} \rangle$	(eV)	666	(121d)
Longitudinal ion temperature	$kT_{i\parallel}$	(eV)	280	(121b)
Multiple scattering emittance	ϵ_{ms}	($\pi mm \cdot$	2×10^{-4}	(135b)
growth rate		mrad/s)		
Energy loss in residual gas	dE/dx	(eV/s)	11	(137a)
Single-scattering lifetime (for initial
emittance)	τ_{ss}	(s)	1.4×10^6	(139a)
Off-momentum function	η		0.58	(124)
Keil-Schnell momentum spread	$(\sigma_p/p_0)_{Keil-Schnell}$		2.9×10^{-4}	(130a)
Electron-beam-induced tune shift	ΔQ_e		1.8×10^{-3}	(123)

(cont.)

Table A I.2 (cont.)

<i>Electron capture</i>				
Radiative recombination coefficient	α_r	($\text{cm}^3 \text{s}^{-1}$)	7.4×10^{-13}	(81)
Radiative recombination rate	R_a	(s^{-1})	1046	(82)
Three-body recombination rate	R_{r3}	(s^{-1})	8.5	(82) + (86)
Stripping quantum number for dipole	n_{dip}		6	(178a)
Doppler shift for forward emission			2.65	(83)
Doppler shift for backward emission			0.54	(83)
<i>Cooling quantities</i>				
Maximal collision time	t_{cooler}	(ns)	12	(44a)
Cyclotron period	t_c	(ns)	0.4	(44b)
Langmuir period	t_{pl}	(ns)	11	(44c)
Impact parameters	$b_{\text{cooler}}(\Delta_{\perp})$	(μm)	2280	(46a)
	$b_{\text{mag}}(\Delta_{\perp})$	(μm)	80	(46b)
	$b_{\text{ad}}(\Delta_{\perp})$	(μm)	2100	(46c)
	$b_{\text{min}}(\Delta_{\perp})$	(μm)	0.007	(47)
	$b_{\text{min}}(\Delta_{\parallel})$	(μm)	9.5	(47)
	$b_{\text{min}}(\Delta_{\perp} \cdot \Delta_{\parallel})$	(μm)	0.26	(48)
Coulomb logarithm	$L_C(\lambda_D, \Delta_{\perp})$		10.4	(30)
	$L_C(r_{e,c}, \Delta_{\perp})$		7.5	(28)
	$L_C[r_{e,c},$		3.9	(28)
	$b(\Delta_{\perp} \cdot \Delta_{\parallel})]$			
	$L_C^{\text{ad}}(b_{\text{ad}}, b_{\text{mag}})$		3.3	(28)
Scaling factor for frictional force	F_0	(eV/m)	5×10^{-9}	(31c)
Approximate frictional force	$F(\Delta_{\perp})$	(eV/m)	0.067	(41)
Approximate frictional force	$F(\Delta_{\parallel})$	(eV/m)	89	(41)
Approximate cooling time for $\sigma_{v_i} = \Delta_{\perp}$	τ	(s)	0.03	(73b)
Spitzer's equilibration time for $\sigma_{v_i} = \Delta_{\perp}$	τ_{eq}	(s)	0.027	(21)
Laboratory cooling time for $\sigma_{v_i} = \Delta_{\perp}$	τ_{lab}	(s)	2.1	(154c)
Momentum spread for thermal equilibrium	σ_p/p_0		7.3×10^{-7}	(121b)

APPENDIX II: QUASIPOTENTIALS AND FRICTIONAL FORCE

We investigate the solution of the integral for particular distributions $f(v_e)$:

$$\Psi(v_i) = \int f(v_e) H(u) d^3 v_e, \quad u = v_i - v_e,$$

$$G(u) = \int H(u) u du.$$

II.1 SPHERICAL SYMMETRIC DISTRIBUTION $f(v_e) = f(v_e)$

$$\Psi(v_i) = \Psi(v_i) = \int f(v_e) H(u) d^3 v_e$$

$$u^2 = v_i^2 - v_e^2 - 2v_i v_e \cos \theta_{v_e}, \quad 2u du = 2v_i v_e \sin \theta_{v_e} d\theta_{v_e}$$

$$\Psi(v_i) = \frac{2\pi}{v_i} \int_0^\infty [G(u_{\max}) - G(u_{\min})] f(v_e) v_e dv_e$$

$$u_{\min} = \begin{cases} v_i - v_e & \text{for } v_i \geq v_e \\ v_e - v_i & \text{for } v_i \leq v_e \end{cases} \quad u_{\max} = v_i + v_e$$

II.1.1 Spherical Maxwellian distribution $f(v_e) = 1/(\Delta_e^3 \pi \sqrt{\pi}) e^{-v_e^2/\Delta_e^2}$

$$\Psi(v_i) = \frac{2}{\sqrt{\pi} \Delta_e^3 v_i} \int_0^\infty [G(u_{\max}) - G(u_{\min})] e^{-v_e^2/\Delta_e^2} v_e dv_e$$

with $v_e = \Delta_e t$

$$\Psi(v_i) = \frac{2}{\sqrt{\pi} \Delta_e v_i} \int_0^\infty [G(u_{\max}) - G(u_{\min})] e^{-t^2} t dt.$$

For $H(u) = 1/u$ we get

$$\Phi(v_i) = \frac{2}{\sqrt{\pi} \Delta_e v_i} \left(\int_0^{v_i/\Delta_e} 2\Delta_e t^2 e^{-t^2} dt + \int_{v_i/\Delta_e}^0 2v_i e^{-t^2} dt \right),$$

since

$$\int_{v_i/\Delta_e}^\infty t e^{-t^2} dt = \frac{1}{2} \int_{(v_i/\Delta_e)^2}^\infty e^{-z} dz = -\frac{1}{2} e^{-v_i^2/\Delta_e^2}.$$

This results in

$$\Phi(v_i) = \frac{2}{\sqrt{\pi} \Delta_e v_i} \left(2\Delta_e \int_0^{v_i/\Delta_e} t^2 e^{-t^2} dt + v_i e^{-v_i^2/\Delta_e^2} \right).$$

With the help of error function $\text{erf}(x)$

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt = \left(x e^{-x^2} + 2 \int_0^x x^2 e^{-t^2} dt \right) \frac{2}{\sqrt{\pi}}$$

we obtain

$$\Phi(v_i) = \frac{\text{erf}(v_i/\Delta_e)}{v_i}.$$

Similarly, we can solve the problem for $H(u) = u$.

The derivation of ϕ with respect to v_i yields

$$\Phi'(v_i) = \frac{d\Phi}{dv_i} = \left[\text{erf}\left(\frac{v_i}{\Delta_e}\right) - \frac{2}{\sqrt{\pi}} e^{-v_i^2/\Delta_e^2} \frac{v_i}{\Delta_e} \right] \frac{1}{v_i^2}.$$

Here we made use of

$$\frac{d \text{erf}(v_i/\Delta_e)}{d v_i} = \frac{e^{-v_i^2/\Delta_e^2}}{\Delta_e} \frac{2}{\sqrt{\pi}}.$$

For the frictional force we then get

$$F = -F_0 c^2 L_C \Phi(v_i) = -F_0 L_C \frac{c^2}{v_i^2} \left[\text{erf}\left(\frac{v_i}{\Delta_e}\right) - \frac{2}{\sqrt{\pi}} \frac{v_i}{\Delta_e} e^{-v_i^2/\Delta_e^2} \right].$$

II.1.2 Adiabatic frictional force for $f(v_{e\parallel}, v_{e\perp}) = \delta(v_{e\parallel}) f(v_{e\perp})$

From Ref. [19] we have

$$F_{\perp}^{\text{ad}} = -\frac{1}{2} F_0 c^2 L_C^{\text{ad}} \frac{2 v_{i\parallel}^2 - v_{i\perp}^2}{v_i^5} v_{i\perp} = -\frac{1}{2} F_0 c^2 L_C^{\text{ad}} \left(3 \frac{v_{i\parallel}^2}{v_i^2} - 1 \right) \frac{v_{i\perp}}{v_i^3},$$

$$F = -\frac{1}{2} F_0 c^2 L_C^{\text{ad}} \frac{3 v_{i\perp}^2 v_{i\parallel}}{v_i^5}.$$

With

$$\frac{v_{i\perp}}{v_{i\parallel}} = \text{tg } \phi_v, \quad \tilde{F} = -\frac{1}{2} F_0 \frac{c^2}{v_i^2} L_C,$$

we get

$$F_{\perp} = \tilde{F} (3 \cos^2 \phi_v - 1) \frac{v_{i\perp}}{v_i} = \tilde{F} (2 \cos^2 \phi_v - \sin^2 \phi_v) \frac{v_{i\perp}}{v_i}$$

and

$$F_{\parallel} = \tilde{F} 3 \sin \phi_v \cos \phi_v \frac{v_{i\parallel}}{v_i},$$

where

$$\operatorname{tg} \phi_F = \frac{F_{\perp}}{F_{\parallel}} = \frac{2 \cos^2 \phi_v - \sin^2 \phi_v}{3 \sin \phi_v \cos \phi_v}, \quad \phi_F \approx \pi/2 \left(1 - \frac{\phi_v}{\arccos 1/\sqrt{3}} \right),$$

and

$$\begin{aligned} |F| &= \sqrt{F_{\perp}^2 + F_{\parallel}^2} = \tilde{F} \frac{v_{i\perp}}{v_i} \sqrt{(2v_{i\parallel}^2 - v_{i\perp}^2)^2 + 9v_{i\perp}^2 v_{i\parallel}^2} \\ &= \tilde{F} \frac{v_{i\perp}}{v_i} \sqrt{\left(2 \frac{v_{i\parallel}^2}{v_i^2} + \frac{v_{i\perp}^2}{v_i^2} \right)^2 + 9 \frac{v_{i\parallel}^2}{v_i^2} \cdot \frac{v_{i\perp}^2}{v_i^2}}, \end{aligned}$$

$$|F| = \tilde{F} \sin \phi_v \sqrt{(\cos^3 \phi_v + 1)^2 + 9 \cos^2 \phi_v \sin^2 \phi_v}.$$

APPENDIX III: FREQUENTLY USED RELATIONS

In this Appendix we summarize the relation between quantities most frequently used in this review.

III.1 RELATIONS FOR PLASMA PROPERTIES

$$kT = \frac{1}{2} m \sigma_v^2, \quad kT_{\parallel} = \frac{1}{2} m \sigma_{\parallel}^2, \quad kT_{\perp} = \frac{1}{2} m \sigma_{\perp}^2 = \frac{1}{2} k(T_x + T_y) \quad (6c)$$

$$f_{\text{sph}}(v) = \left(\frac{mc^2}{2kT} \right)^{3/2} \frac{e^{-mv^2/2kT}}{c^3 \pi \sqrt{\pi}} \quad (6a)$$

$$f_{\text{flat}}(v) = \frac{mc^2}{2kT_{\perp}} \left(\frac{mc^2}{2kT_{\parallel}} \right)^{1/2} \frac{1}{c^3 \pi \sqrt{\pi}} e^{-mv_{\perp}^2/2kT_{\perp}} e^{-mv_{\parallel}^2/2kT_{\parallel}} \quad (6b)$$

$$r_e = \frac{e^2}{4\pi\epsilon_0 m_e c^2}$$

$$\omega_{e,pl} = \sqrt{4\pi n_e r_e c^2} \quad (9)$$

$$\lambda_D = \sqrt{\frac{\epsilon_0 k T_e}{e^2 n_e}} = \frac{c}{\omega_{e,pl}} \sqrt{\frac{k T_e}{m_e c^2}} \quad (7)$$

$$\omega_{e,c} = \frac{eB}{m_e \gamma} = \frac{176}{\gamma} \text{ GHz } B[T] \quad (20a)$$

$$r_{e,c} = \frac{v_{e\perp}}{\omega_{e,c}}, \quad \langle r_{e,c} \rangle = \frac{c}{\omega_{e,c}} \sqrt{\frac{2kT_{e\perp}}{m_e c^2}} \quad (20b)$$

$$r_{av} = (4/3 \pi n_e)^{-1/3} \quad (8)$$

$$\Gamma = \frac{(Ze)^2}{4\pi\epsilon_0 kT r_{av}} \quad (11)$$

$$\frac{\partial f}{\partial t} = \sum_{m,n} \frac{\partial \{ -f(F/m) + [\partial(fD_{mn}) / \partial v_n] \}}{\partial v_m} \quad (17)$$

$$\Delta x = 1.667 \sigma_x \quad (120d)$$

III.2 ELECTRON-BEAM AND ELECTRON-COOLER PROPERTIES

$$I_e = P_{\text{gun}} U_{\text{cath}}^{3/2} \quad (87)$$

$$n_e = I_e / (\pi r_0^2 e \beta_0 c) = j_e / (e \beta_0 c) \quad (97)$$

$$E_{\text{ax}} = e U_{\text{cath}} - n_e \pi r_0^2 r_e m_e c^2 [1 + 2 \ln (r_{\text{tube}} / r_0)] \quad (99)$$

$$E(r) = E_{\text{ax}} + n_e \pi r^2 r_e m_e c^2 \quad (98)$$

$$P_{\text{coll,max}} = I_e U_{\text{coll,min}}^{-3/2} = \frac{9\sqrt{2} P_0}{(1 - \zeta)[1 + 2 \ln (r_{\text{tube}} / r_0)]} \quad (106)$$

$$\frac{\Delta E(r)}{E_0} = \frac{\gamma}{9\sqrt{\gamma+1}} \frac{P_{\text{gun}}}{P_0} \left(\frac{r}{r_0}\right)^2 \quad (100a)$$

$$\frac{\Delta E(r)}{E_0} = \frac{I_e r_e}{ec} \frac{\gamma+1}{\beta_0^3 \gamma^2} \left(\frac{r}{r_0}\right)^2 \approx 1.2 \times 10^{-4} \frac{I_e [\text{A}]}{\beta_0^3} \left(\frac{r}{r_0}\right)^2 \quad (100b)$$

$$\omega_{\text{az}} = \frac{2\pi r_e n_e c^2}{\gamma \omega_{e,c}} \quad (101)$$

$$kT_{e\perp} = \frac{1}{2} m_e \Delta_{\perp}^2 = \frac{1}{2} m_e \beta_0^2 c^2 \Theta_{e\perp}^2 \geq kT_{\text{cath}} \quad (95d)$$

$$kT_{e\parallel} = \left(\frac{\sigma_{\bar{U}}}{U_{\text{cath}}} + \frac{kT_{\text{cath}}}{E_0} \right)^2 \frac{m_e \beta_0^2 c^2}{8} + 2m_e c^2 \frac{r_e}{r_{\text{av}}} \quad (94)$$

$$\frac{T_{e\parallel}}{T_{e\perp}} = \begin{cases} \frac{kT_{e\perp}}{8E_0} \frac{\gamma+1}{\gamma^2} & \text{for } kT_{\text{cath}} \gg e\sigma_{\bar{U}}, V_{ee} \quad (95a) \\ \frac{1}{4} \left(\frac{\sigma_{\bar{U}}}{U_{\text{cath}}} \right)^2 \theta_{e\perp}^{-2} & \text{for } e\sigma_{\bar{U}} \gg kT_{\text{cath}}, V_{ee} \quad (95b) \\ \left(\frac{2}{\beta_0 \theta_{e\perp}} \right)^2 \left(\frac{4\pi}{3} n_e \right)^{1/3} r_e & \text{for } V_{ee} \gg e\sigma_{\bar{U}}, kT_{\text{cath}} \quad (95c) \end{cases}$$

$$\lambda_s = 2\pi n_c \frac{\langle \beta_0 \rangle c}{\omega_{e,c}} \quad n_c = 1, 2, 3, \dots \quad (89)$$

$$B [\text{kG}] = 10.7 n_c \frac{\langle \beta_0 \rangle \gamma}{\lambda_s [\text{cm}]} \quad (90)$$

III.3 ION-BEAM AND RING PROPERTIES

$$B_Q [\text{T} \cdot \text{m}] = 3.13 \beta \gamma \frac{\text{A}}{Z} \quad (112b)$$

$$\omega_{\text{spin}} = \omega_{e,c} \begin{cases} 1+G & \text{for } \mathbf{v}_i \parallel \mathbf{B}, \mathbf{B} \perp \sigma_i \\ \gamma G & \text{for } \mathbf{v}_i \perp \mathbf{B}, \mathbf{B} \perp \sigma_i \end{cases} \quad (142a)$$

$$\theta_{\text{iaz}} = \frac{B_{\text{sol}} \ell}{B_Q} \quad (146a)$$

$$\theta_x = \frac{B_{\text{sol}} r_{\text{tor}}}{B_Q} \left(\frac{1}{\cos \phi_0} - 1 \right) \quad (148b)$$

$$\theta_{\text{spin}} = (1 + G) \theta_{\text{iaz}} \quad (147)$$

$$\alpha_p = \frac{dC/C_0}{dp/p_0} = \gamma_{\text{tr}}^{-2} \quad (113a)$$

$$\eta = \frac{df/f_0}{dp/p_0} = 1 - \beta_0^2 - \frac{1}{\gamma_{\text{tr}}^2} \quad (125)$$

$$\xi_y = \frac{dQ_y/Q_y}{dp/p_0} \quad (117)$$

$$d_i(s) = D(s) (p_i - p_0)/p_0 \quad (113b)$$

$$\langle \beta_y \rangle \approx R_0/Q_y \quad (116)$$

$$\sigma_y = \sqrt{\frac{\epsilon_y}{\pi}} \beta_y \quad (119a)$$

$$\sigma_{y'} \approx \sqrt{\frac{\epsilon_y}{\pi \beta_y}} \quad (119b)$$

$$y' = v_{iy}/\beta_0 c \quad (120a)$$

$$V_{\text{beam}} \approx \frac{1}{2} \frac{C_0^2}{\bar{Q}} \frac{\bar{\epsilon}}{\pi} \approx C_0 \bar{\epsilon} \bar{\beta}_y \quad (119e)$$

$$I_i = \frac{N_i \beta_0 c Z e}{C_0} = Z e N_i f_{\text{rev}} \quad (142b)$$

$$kT_{iy}(s) = 1/2 m_i c^2 \beta_0^2 \gamma^2 \sigma_y^2(s) \quad (121a)$$

$$kT_{i\parallel} = 1/2 m_i c^2 \beta_0^2 \left(\frac{\sigma_p}{p_0}\right)^2 = 0.18 m_i c^2 \beta_0^2 \left(\frac{\Delta p}{p_0}\right)^2 \quad (120d) + (121b)$$

$$\langle kT_{i\perp} \rangle = \frac{1}{2} m_i c^2 \beta_0^2 \gamma^2 (\epsilon_x Q_x + \epsilon_z Q_z) / (2\pi R_0) \approx \frac{1}{4} m_i c^2 \beta_0^2 \gamma^2 \frac{\bar{\epsilon}/\pi}{\beta} \quad (121d)$$

$$\Gamma_{i\perp} = 2Z^2 r_i \left(\frac{4\pi \langle \beta_y \rangle^2}{3 C_0}\right)^{1/3} N_i^{1/3} \left(\frac{\epsilon}{\pi}\right)^{-4/3} \beta_0^{-2} \gamma^{-2} \quad (141a)$$

$$\Gamma_{i\parallel} = 2Z^2 r_i \left(\frac{4\pi}{3C_0 \langle \beta_y \rangle}\right)^{1/3} N_i^{1/3} \left(\frac{\epsilon}{\pi}\right)^{-1/3} \left(\frac{\sigma_p}{p_0}\right)^{-2} \beta_0^{-2} \quad (141b)$$

$$\frac{\dot{\epsilon}_{ms}}{\pi} = 8Z_{gas}^2 Z^2 n_{gas} r_i^2 \langle \beta_y \rangle \beta_0^{-3} \gamma^{-2} L_{res} \quad (135b)$$

$$-\frac{dE/dt}{E_i} = \frac{\dot{\epsilon}_{ms}}{\langle \beta_y \rangle} \frac{m_i}{m_e} \quad (137b)$$

$$\lambda_{IBS\parallel} = \frac{32}{3\sqrt{\pi}} \frac{r_i^2 c Z^4 L_C N_i}{C_0 \beta_0^3 \gamma^{1/2}} \left(\frac{\epsilon}{\pi}\right)^{-5/4} \left(\frac{\sigma_p}{p_0}\right)^{-5/2} \langle \beta_y \rangle^{-3/4} = \frac{32}{3\sqrt{\pi}} \frac{r_i^2 c Z^4 L_C N_i}{C_0 \beta_0^3 \gamma^{1/2}} \langle r_{0i} \rangle^{-5/2} \left(\frac{\sigma_p}{p_0}\right)^{-5/2} \langle \beta_y \rangle^{1/2} \quad (131f)$$

$$\tau_{ss} = \frac{\beta_0^3 \gamma^2}{4\pi Z_{gas}^2 Z^2 r_i^2 c n_{gas}} \theta_{acc}^2 \left(1 - \frac{\epsilon_y}{A_\epsilon}\right) \quad (139a) + (134c)$$

$$\tau_{ms} = k_0 \frac{A_\epsilon}{\dot{\epsilon}_{ms}} \quad k_0 = \begin{cases} 0.85 \text{ rectangular} \\ 2.16 \text{ round} \end{cases} \text{ acceptance} \quad (139b)$$

$$\left(\frac{\sigma_p}{p_0}\right)_{\text{Keil-Schnell}} \geq \sqrt{\frac{4\pi Z^2 r_i N_i}{\beta_0 \gamma |\eta| C_0} \left|\frac{Z_{\parallel} h_f}{\mu_0 c}\right|} \quad (130a)$$

$$\left(\frac{\sigma_p}{p}\right)_{IBS} = \left\{ \frac{32}{3\sqrt{\pi}} \frac{r_i^2 c Z^4 L_C N_i \tau_{\parallel}}{C_0 \beta^3 \gamma^{1/2}} \right\}^{2/5} \left\{ \frac{\epsilon}{\pi} \langle \beta_y \rangle \right\}^{-1/2} \langle \beta_y \rangle^{1/5} \quad (160)$$

$$L = \frac{N_1 N_2 k_b \ell_{ov} A_{ov}}{C_1 C_2 A_1 A_2} |\beta_1 \pm \beta_2| c \quad (172)$$

$$\Delta Q_{sp.ch.} = \frac{Z N_i}{2\pi \epsilon_y \beta_0^2 \gamma^3} r_i B_f \quad (122)$$

$$\Delta Q_e = \frac{Z}{2\beta_0^3 \gamma^3} n_e r_i \ell \langle \beta_y \rangle \quad (123)$$

$$f_{\text{syn}} = \frac{c}{C_0} \sqrt{\frac{Ze U_{\text{RF}} h_f |\eta|}{2\pi m_i c^2 \gamma}} \quad (126)$$

$$A_b = \frac{\pi}{4} \Delta E_b \Delta t_b = \frac{\pi}{4} \beta_0 \gamma m_i c \ell_b \frac{\Delta p}{p} \quad (127c)$$

$$\frac{Ze U_{\text{RF}}}{m_i c^2} = \frac{1}{2\pi} h_f |\eta| \beta_0^2 \gamma \left(\frac{\Delta p}{p}\right)^2 \left(\frac{C_0}{\ell_b}\right)^2 \quad (128a)$$

$$E_{\text{mot}} [\text{MV/m}] = 300 \beta_0 \gamma B_{\perp} [\text{T}] \quad (80)$$

$$E_{\text{str}} = 2m_e c^2 \sqrt{\frac{\mu_B B_{\perp}}{m_e c^2}} Z \alpha \beta_0 = 1.3 \text{ eV} \sqrt{Z \beta_0 B_{\perp} [\text{T}]} \quad (176)$$

$$E_{\text{dip}} = 2.3 \text{ eV} \beta_0 \sqrt{\frac{A \gamma}{\varrho [\text{m}]}} \quad (177)$$

III.4 RELATION FOR ELECTRON-ION INTERACTION

III.4.1 Non-cooling interactions

$$\alpha_r = 2\pi r_e^2 \alpha c^2 Z^2 \frac{\sqrt{\pi}}{\Delta_e} \sum_{n=1}^{n_{\text{max}}} \frac{1}{n} \quad (79)$$

$$R_a = N_i \eta_c \alpha_r n_e \gamma^{-2} \quad (82)$$

$$h\nu_{\text{lab}} = \frac{h\nu}{(1 - \beta_0 \cos \theta) \gamma} \quad (83)$$

$$n_{\text{dip}} = 3.4 Z \beta_0^{-1/2} \gamma^{-1/4} (\varrho [\text{m}]/A)^{1/4}, \quad (178a)$$

III.4.2 Cooling interactions

$$F_0 = \frac{4\pi Z^2 e^4 n_e}{(4\pi \epsilon_0)^2 m_e c^2} = \frac{\omega_{e,pl}^2 Z^2 e^2}{4\pi \epsilon_0 c^2} = \frac{\omega_{e,pl}^2 r_e m_e c^2 Z^2}{c^2} = 4\pi Z^2 n_e r_e^2 m_e c^2 \quad (31c)$$

$$\mathbf{F}^{\text{NM}}(\mathbf{v}_i) = -F_0 c^2 \int L_C(u) f(\mathbf{v}_e) \frac{\mathbf{u}}{u^3} d^3 \mathbf{v}_e \quad (33a)$$

$$F(v_i) = -F_0 L_C \left\{ \text{erf}(v_i/\Delta_e) - \frac{2v_i}{\Delta_e \sqrt{\pi}} e^{-v_i^2/\Delta_e^2} \right\} \frac{c^2}{v_i^2} \frac{v_i}{v_i} \quad (38)$$

$$F^{NM}(v_i) \approx -3/2 F_0 c^2 L_C \frac{v_i}{|v_i|^3 + 2\Delta_\perp^3} \frac{v_i}{v_i} \quad \text{for spherical } f(v_e) \quad (41)$$

$$F_\#^{NM}(v_i) = -2 \frac{F_0 c^2}{\sqrt{2\pi}} L_C^{NM}(\Delta_\#) \frac{v_{i\#}}{\Delta_\# \Delta_\perp^2} \left\{ \begin{array}{ll} \frac{\pi}{4} & \text{for } \# = \perp \\ 1 & \text{for } \# = \parallel \end{array} \right\} v_{i\#} < \Delta_\parallel \quad (43c)$$

$$F_\#^{NM}(v_i) \approx -\frac{F_0 c^2}{\Delta_\perp^2} \left\{ \begin{array}{ll} \sqrt{\frac{\pi}{8}} L_C^{NM}(\Delta_\perp) \frac{v_{i\perp}}{\Delta_\perp} & \text{for } \# = \perp \\ L_C^{NM}(v_{i\parallel}) - \sqrt{\frac{\pi}{2}} \frac{v_{i\parallel}}{\Delta_\perp} L_C^{NM}(\Delta_\perp) & \text{for } \# = \parallel \end{array} \right\} v_{i\#} > \Delta_\parallel \quad (43c)$$

$$F^{NM}(v_i) \approx -F_0 c^2 L_C^{NM}(v_i) \frac{v_i}{v_i^3} \quad v_i > \Delta_\perp$$

$$F_\#^{ad} = -\frac{F_0 c^2}{\sqrt{2\pi}} L_C^{ad}(\Delta_\#) \frac{v_{i\#}}{\Delta_\parallel^3} \left\{ \begin{array}{ll} \ln \frac{\Delta_\parallel}{v_{i\perp}} & \text{for } \# = \perp \\ 1 & \text{for } \# = \parallel \end{array} \right\} v_{i\#} < \Delta_\parallel \quad (54)$$

$$F_\#^{ad} = -\frac{1}{2} F_0 c^2 L_C^{ad}(U_{ad}) \left\{ \begin{array}{ll} \frac{v_{i\perp} - 2v_{i\parallel}^2}{v_i^2} \frac{v_{i\perp}}{v_i^3} & \text{for } \# = \perp \\ 3 \frac{v_{i\perp}^2}{v_i^2} \frac{v_{i\parallel}}{v_i^3} & \text{for } \# = \parallel \end{array} \right\} v_{i\#} > \Delta_\parallel \quad (53)$$

$$b_{\max} = \min(b_{\text{cooler}}, b_{\text{ad}}, r_0) \quad (46)$$

$$b_{\min}(v_i) = Z r_e c^2 \left\{ \begin{array}{ll} \frac{1}{v_i^2} & \Delta_\perp \ll v_i \\ \frac{1}{\Delta_\perp \Delta_\parallel} & \Delta_\parallel < v_i < \Delta_\perp \\ \frac{1}{\Delta_\parallel^2} & v_i < \Delta_\parallel \end{array} \right. \quad (48)$$

$$L_C(v_i) = \ln \left[\frac{\lambda_{ad}}{Z r_e} \left(\frac{v_i}{c} \right)^2 \right] = \ln \frac{\lambda_{ad}}{Z r_e} + 2 \ln \frac{v_i}{c} \quad (30)$$

$$L_C^{NM}(v_i) \approx \ln \frac{b_{mag}}{b_{min}} = \ln \frac{b_{mag}}{Z r_e} + \ln \left\{ \begin{array}{ll} \left(\frac{v_i}{c} \right)^2 & \Delta_{\perp} \ll v_i \\ \frac{\Delta_{\perp} \Delta_{\parallel}}{c^2} & \Delta_{\parallel} < v_i < \Delta_{\perp} \\ \left(\frac{\Delta_{\parallel}}{c} \right)^2 & v_i < \Delta_{\parallel} \end{array} \right\} \quad \begin{array}{l} \text{for exact limits} \\ \text{see Fig. 10} \end{array}$$

$$L_C^{ad}(v_i) \approx \ln \frac{b_{max}}{b_{mag}}, \text{ for exact limits see Fig. 10.}$$

$$F(v_i) = -F_0 \frac{c^2}{v_i^2} \int I(k) \frac{dk}{k} \quad (59b)$$

$$I = \frac{1}{\pi \omega_{e,pl}^2} \int \text{Im} \left(\frac{1}{\epsilon(\mathbf{k}, -\mathbf{k}v_i)} \right) \omega d\omega \quad (60)$$

$$\epsilon(k, \omega) = 1 - \lim_{\nu \rightarrow 0} \frac{\omega_{e,pl}^2}{\omega^2 - i\omega\nu} \quad \text{for electrons at rest} \quad (57)$$

$$\epsilon(k, \omega) = 1 - \lim_{\nu \rightarrow 0} \frac{\omega_{e,pl}^2}{k^2} \left(\frac{k_{\parallel}^2}{\omega^2 + i\omega\nu} + \frac{k_{\perp}^2}{\omega^2 - \omega_{e,c}^2 + i\omega\nu} \right) \quad (63)$$

$$\sigma_{v_m}^2 = \sigma_{v_{mo}}^2 e^{-2F_{mo}/m_i} + 2 \frac{D_{mm}}{F_{mo}} \quad \text{for } F = -F_{mo} v_i \quad (70) + (71)$$

$$T_{i\parallel} = T_e \lambda_{\perp} / \lambda_{\parallel}, \quad T_{i\perp} = 1/2 T_e \quad (72)$$

$$\tau = (6\pi Z^2 r_i r_e c^4 n_e L_C)^{-1} (\sigma_{v_i}^3 + 2\Delta_e^3) \quad (73b)$$

$$F_{\perp lab} = \eta_c F_{\perp}, \quad F_{\parallel lab} = \eta_c / \gamma F_{\parallel}, \quad \tau_{lab} = \gamma^2 / \eta_c \tau \quad (154a-c)$$

$$\tau_{\perp lab}^{emp} = \frac{(\alpha_0 \beta_0 c)^2 + \sigma_{v_{i\perp}}^2 + 11 \sigma_{v_{i\parallel}}^2}{12 \pi \sqrt{\pi} r_e r_p \gamma n_e c^4 \eta_c} \sqrt{\Delta_e^2 + 2 \sigma_{v_i}^2} \quad (168)$$

$$F_{\parallel \text{lab}}^{\text{emp}} = - \frac{3F_0 \gamma c^2 \eta_c}{\sqrt{\left[\left(\frac{\alpha_0 \beta_0 c}{2} \right)^2 + v_i^2 \right] \left[\left(\frac{\Delta_e}{2} \right)^2 + v_i^2 \right]}} \quad (169)$$

$$\tau_{\parallel \text{lab}} = \frac{\beta \gamma v_{i\parallel}}{24 \pi r_e r_i n_e \eta_c c^2} \sqrt{\left(\frac{\alpha_0 \beta_0 c}{2} \right)^2 + \Delta_e^2 + \left(1 - \frac{D_{\text{cooler}}}{\beta_0 c} \frac{dv_{e\parallel}}{dr} \right)^2} v_{i\parallel}^2 \quad (170)$$

$$\left(\frac{\sigma_p}{p} \right)_{\text{IBS}} = \left(\frac{4}{3\pi\sqrt{\pi}} \frac{m_e}{m_i} \frac{\Delta_{\parallel}^3}{c^3} \frac{Z^2 \gamma^{1/2}}{\beta_0^3 n_e \ell} \right)^{2/5} \frac{\langle \beta_y \rangle^{1/5}}{\langle r_{0i} \rangle} N_i^{2/5} \quad (161)$$

Figure captions

- Fig. 1 Illustration of electron cooling of an ion beam: a) schematic of the electron cooling arrangement in the cooling ring (the dashed arrows are electrons); b) the ion beam as seen from the coordinate system where the electrons (dots) are at rest; c) illustration of cooling as an energy loss of ions in a foil.
- Fig. 2 Schematic of the effect of beam cooling: a) on the beam size, and b) on the momentum spread.
- Fig. 3 Different types of plasmas shown in a double logarithmic temperature-density (T - n) plot. The extension of the electron beam area to low temperature concerns the longitudinal compression of the velocity spread due to acceleration. Straight lines indicate the degree of order characterized by the plasma correlation factor Γ . The ideal plasma is found for $\Gamma \ll 1$.
- Fig. 4 Kinematics of the collision between two charges and illustration of the equivalence of the scattering of a positive ion from a negative or positive charge.
- Fig. 5 Illustration of the integration over impact parameters b when ions pass through an electron gas of length dx .
- Fig. 6 The quasipotential Φ and the frictional force F as a function of ion velocity v_i for a Maxwellian electron velocity distribution of r.m.s. width Δ_e : a) linear plot, b) double logarithmic plot of the absolute value of force.
- Fig. 7 The diffusion coefficients D for the longitudinal ($D_{\parallel}$) and transverse ($D_{\perp}$) directions as a function of ion velocity v_i for a Maxwellian electron velocity distribution.
- Fig. 8 Three-dimensional plot of the cooling force [Eqs. (38) and (41)] as a function of $v_{i\parallel}$ and $v_{i\perp}$.
- Fig. 9 a) Equipotential lines (ellipses) for a prolate electron velocity distribution ($\Delta_{\parallel} \ll \Delta_{\perp}$). The force vector is normal to the ellipse and points to the inside. It halves the angle between the radius vectors of the ellipse. b) Three-dimensional plot of the quasipotential.
- Fig. 10 Impact parameter ranges that are relevant to electron cooling.
- Fig. 11 Scattering of a negative charge fixed on a string (infinitely strong magnetic field), from a positive charge (a) and a negative charge (b).
- Fig. 12 Three-dimensional plot of the adiabatic force as a function of the ion velocity components [Eq. (53a,b)]: a) transverse force showing the transverse heating effect for $v_{i\perp} < \sqrt{2} v_{i\parallel}$; b) longitudinal force.
- Fig. 13 Frictional force integrand $I(\zeta)$ versus ζ . Plots (a) and (b) are for electrons at rest and finite magnetic field: a) longitudinal component for protons moving parallel to the magnetic-field axis with the velocity $v_i = v_{i\parallel}$, $\zeta = \ln(k_{\perp} v_i / \omega_{e,c})$; b) transverse component for protons moving in the transverse plane with velocity $v_i = v_{i\perp}$, $\zeta = \ln(k_{\parallel} v_i / \omega_{e,c})$ [23]. Plots (c) and (d) are for electrons with finite velocity spread in the transverse direction and vanishing spread in the longitudinal one: c) longitudinal component for protons moving parallel to the magnetic-field axis with $v_{i\parallel} = 0.2\Delta_{\perp}$ and $\zeta' = \ln(k_{\perp} r_{e,c})$; d) transverse component for protons moving in the transverse plane with $v_{i\perp} = 0.5\Delta_{\perp}$, $0.2\Delta_{\perp}$, and $\zeta' = \ln(k r_{e,c})$ [23]. For all cases, $B = 700$ G and $n_e = 3 \times 10^8 \text{ cm}^{-3}$.
- Fig. 14 Modulus of the frictional force $F_{\text{emp}}^{\text{NM}}$ and angle $\phi_{F_{\text{emp}}}$ between the force components for electron cooling of negative ($\kappa = 1$) and positive ($\kappa = -1$) ions plotted versus the angle between ion velocity and magnetic-field direction [Eqs. (67)].
- Fig. 15 Longitudinal frictional force from the binary collision model plotted versus the ion velocity in a double logarithmic plot (different straight lines [Eqs. (43), (53), and (54)] are connected by the hand-drawn curve).

- Fig. 16 Atomic-level schemes illustrating the situation in the cooling region. a) Hydrogen-like atoms: cooling electrons populate the continuum above the ionization threshold; electron capture leads to the emission of characteristic photons. b) Doubly excited multielectron ions: cooling electrons are detuned in energy, the continuum band which they populate coincides with an auto-ionizing state; photon emission leads to dielectronic recombination.
- Fig. 17 The electron gun for the LEAR electron cooler: a) construction design: cathode (1), Pierce shield (2), fixation (3), cooled copper feedthrough (4), ceramic (5), high-voltage feedthrough (6), bellows (7), anodes (8) and (9), ceramic holder (10), solenoid (11); b) output from trajectory calculations (full lines: electron trajectories, dashed lines: potential lines); c) transverse electron energy after acceleration as a function of distance from the beam axis (Ref. [5]).
- Fig. 18 Separated-function electron gun (CELSIUS) with resonant first stage (extraction) and adiabatic second stage (acceleration): a) construction design: solenoid (1), correction coil (2), water cooling (3), cathode holder (4), cathode (5), guard electrodes (6), gun anode (7), corona rings and electrodes (8), acceleration tube (9), SF₆ chamber (10), transition piece (11); b) output from trajectory calculations [42].
- Fig. 19 Separated-function electron gun (ESR) with diode gun as first stage and single-gap acceleration as second stage [44]: a) output from trajectory calculations, b) transverse electron energy as a function of the distance from the cathode.
- Fig. 20 Illustration of the reason for the lower limit in longitudinal electron temperature after acceleration, showing the sum of the potentials from neighbouring electrons.
- Fig. 21 Faraday cup collector (CELSIUS) [42, 57]: cooling water connections (1), SF₆ chamber (2), Cu collector (3), collector anode (4), transition piece (5), second collector solenoid coil (6), corona rings and electrodes (7), retardation tube (8), collector solenoid (9).
- Fig. 22 LEAR/ICE collector [5]: a) construction design: water-cooled collector (1), pumping port (2), spike-electrode (3), mesh-electrode (4), repeller-electrode (5), ceramic (6), magnetic iron shunt (7) and (8), flange (9), collector coils (10); b) output from trajectory calculations.
- Fig. 23 Hollow collector (Novosibirsk) [53]: main solenoid (1), counter solenoid (2), collector (3).
- Fig. 24 High-voltage system for coolers: a) electric potential along the cooler, b) electric circuitry.
- Fig. 25 The LEAR electron cooler [5, 64] and its vacuum system.
- Fig. 26 Vacuum pressure measured close to the cooling region in the LEAR electron cooler as a function of the electron-beam energy. The inset shows the spectrum of gas composition for an electron beam of 27 keV and 2.5 A [5b, 64].
- Fig. 27 Microwave radiation spectrum received with an antenna located in the cooling section of the LEAR electron cooler (electron beam: 26.5 keV, 2.2 A; magnetic field: 472 G). Also shown is the calculated spectrum, and some cavity modes are indicated [70].
- Fig. 28 Thomson scattering of laser light from an electron beam (LEAR cooler): a) experimental set-up (PM: photomultiplier, PD: photodiode), b) rate of back-scattered photons as a function of laser wavelength for three values of the acceleration voltage U [72].
- Fig. 29 Universal curve for the growth rate due to intrabeam scattering with respect to reduced variables [79].
- Fig. 30 Longitudinal Schottky noise spectrum of a coasting 'warm' beam: a) signal from a single harmonic (35th) of the revolution frequency; b) spectrum from 1 MHz to 350 MHz, showing the relative height of the harmonics.
- Fig. 31 Calculated longitudinal Schottky line of cold coasting beams of different numbers of stored ions showing the effect of collective motion of a part of the beam.

- Fig. 32 Signal strength of Schottky noise as a function of the number of stored particles for positive and negative N_{crit} [95].
- Fig. 33 Transverse Schottky noise spectrum; the central peak corresponds to a harmonic of the revolution frequency, and the two satellites reflect the betatron motion, separated from the revolution peak by $h_r q_y$. Dotted line: before cooling; full line: after some cooling.
- Fig. 34 Geometry of the toroid section.
- Fig. 35 Effect of the toroids on the ion beam for S- and U-shaped coolers without and with correction dipoles.
- Fig. 36 The electron velocity profile and the ion-momentum dispersion in the cooling section: a) horizontal direction (finite dispersion), b) vertical direction (vanishing dispersion). The dashed parabola corresponds to the electron velocity profile after a voltage step down by ΔU . The dots represent the ions.
- Fig. 37 The shape of the longitudinal cooling force taking into account the electron-velocity profile and the ion-momentum dispersion for a zero emittance beam.
- Fig. 38 Illustration of angular mismatch of electron and ion beams (a), and its effect on the equilibrium point A for finite dispersion (b) and zero dispersion (c). It leads to a smearing of the ion velocity distribution.
- Fig. 39 Emittance change rates through cooling plotted versus momentum spread and emittance, taking into account the different contributions and effects in the cooling region [100].
- Fig. 40 Simulation of the electron cooling process; distribution of an ensemble of ions in the cooler: a) at $t = 0$; b) after 1 s; c) after 10 s. The lower row of frames shows the spatial distribution of the ion ensemble in the cooling region [100].
- Fig. 41 Time dependence of beam emittance (a) and momentum spread (b) (1σ values) for the electron cooler and lattice of ICE (10^9 protons at 300 MeV/c) [100].
- Fig. 42 Plot of $\text{Max}[(\sigma_p/p)_{\text{IBS}}, (\sigma_p/p)_{\text{therm.equit.}}]$ versus the number of stored ions [Eq. (161)] for the following conditions: $\Delta_{\perp} = 1.9 \times 10^7$ cm/s, $n_e = 1 \times 10^8$ cm $^{-3}$, $\langle\beta_y\rangle = 5$ m, $r_{oi} = 1$ mm, $\sigma_{\text{U}} = 1$ V and $\Delta_{\parallel}$ given by Eq. (94); protons: $\beta_0 = 0.2$, $^{12}\text{C}^{6+}$: $\beta_0 = 0.1$, and $^{238}\text{U}^{92+}$: $\beta_0 = 0.3$.
- Fig. 43 The first three cooling rings: a) the NAP-M ring in Novosibirsk [2]; b) the ICE ring at CERN [3]; c) the cooling ring at FNAL [4].
- Fig. 44 The electron coolers for the initial cooling experiments: cathode (1), gun solenoid (2), toroids (3), cooling solenoid (4), collector solenoid (5), collector (6), sputter ion pumps (7), correction dipoles (8), Ti-ball pumps (9), microwave antenna (10).
- Fig. 45 The MOSOL (Model Solenoid, Novosibirsk) arrangement to measure the frictional force for protons and H^- ions in a single pass: H^- source (1), electrostatic accelerator (2), magnesium vapour target (3), electron gun (4), solenoid (5), electron collector (6), beam spectrometer (7), plates of transverse ion spread (8), two-dimensional position detector (9) [49].
- Fig. 46 Longitudinal Schottky spectra showing the momentum cooling of a freshly injected beam; a scan is done every second [109].
- Fig. 47 Schottky spectrum of longitudinally heated ion beam (rectangular shape) and its cooling by the electron beam (scans for different times after the start of cooling, ICE).
- Fig. 48 Longitudinal Schottky spectrum showing the distribution of a stochastically blown-up beam without cooling (as in Fig. 47), the distribution of a cold beam without heating (narrow spike), and the distribution for the equilibrium between longitudinal heating and cooling (LEAR electron cooling).

- Fig. 49 Longitudinal Schottky spectrum after a voltage step (a) and time evolution of Schottky signal strength (b) measured at position 1 or 2 and with a bandwidth (W_1 , W_2) indicated in (a) [109a].
- Fig. 50 Horizontal beam profiles (scan every 800 ms) after a step up and down in the electron acceleration voltage [3]. The profiles are resolution limited.
- Fig. 51 Horizontal beam profiles at four different times after the beginning of cooling. The final width is about 0.3 mm (FWHM) corresponding to an emittance of less than 0.1 π mm mrad [113b].
- Fig. 52 Signal rise of an electrostatic position pick-up for a bunched beam as a result of cooling [109a]. The insets show the form of the pick-up signal before and after cooling.
- Fig. 53 Longitudinal Schottky spectrum of a cooled bunched beam (ICE), $U_{RF} = 10$ V [9] from 0–68 MHz.
- Fig. 54 Longitudinal Schottky spectrum for a cooled bunched beam around a harmonic showing the synchrotron oscillation peaks ($U_{RF} = 1$ kV, LEAR).
- Fig. 55 Compilation of transverse-cooling-time measurements from NAP-M (1.5, 35, 65 MeV) [115], ICE (46 MeV) [3], FNAL (114 MeV) [4], and LEAR (49 MeV) [109]. All data are scaled to $n_e = 10^8$ and $\eta_c = 0.02$. The lines show results from empirical formulae: full line from Eq. (167) and dashed line from Eq. (168), with $\alpha_0 = 0$, $\Delta_c = 1.9 \times 10^7$ cm/s and $\sigma_{v_{\parallel}} \ll \sigma_{v_{\perp}}$; the dash-dotted curve is as the dashed one but with $\alpha_0 = 1 \times 10^{-3}$ and $\beta_0 = 0.35$.
- Fig. 56 Compilation of longitudinal frictional force measurements from NAP-M (65 MeV) [115a], ICE (46 MeV) [3], FNAL (114 MeV) [4], LEAR (49 MeV and 10 MeV) [109, 110, 118], IUCF (45 MeV) [113c], and TSR (73 MeV, C^{6+}) [112]. All data are scaled to $n_e = 10^8$ and $\eta_c = 1$. The TSR data are further divided by $Z^2 = 36$. a) all data; the dashed line shows the prediction of the semiempirical formula [Eq. (169)] for $\alpha_0 = 5 \times 10^{-4}$, $\beta_0 = 0.35$. b) The LEAR data for an optimized cooler compared with the prediction of Eqs. (43c), (43d), and (54c), for $L_C^{NM} = 10$, $L_C^{ad} = 6$, $\Delta_{\parallel} = \Delta_{\perp}/30$, and $v_{i\perp} = \Delta_{\perp}/10$.
- Fig. 57 Power of longitudinal Schottky signal versus number of particles stored in NAP-M for uncooled and cooled proton beams [117].
- Fig. 58 Longitudinal Schottky spectrum of cooled ion beams for different beam intensities: a) proton beam stored in LEAR [110], full lines are fits to the data as described in Ref. [109a]; b) C^{6+} stored in TSR [112]; c) integrated power of Schottky signal versus square root of circulating C^{6+} ion current [112]; d) distorted Schottky line with central peak (LEAR).
- Fig. 59 Equilibrium momentum spread versus number of stored ions: a) NAP-M [2, 24] and ICE [3]; b) LEAR [109, 118]; c) TSR [112]. The straight lines show predictions according to Eq. (161) and Fig. 42.
- Fig. 60 Distribution of arrival times of neutral hydrogen atoms impinging on a scintillator for a bunched beam showing the particle distribution in the bunch for different radio-frequency voltages. The 1 kV distribution corresponds to Fig. 54 [109].
- Fig. 61 Signals from an electrostatic pick-up for a C^{6+} beam bunched on the 11th harmonic of the revolution frequency (scan every 150 ms) [112].
- Fig. 62 Wide-range (0–100 MHz) longitudinal Schottky spectrum for a coasting cooled beam (a) and for a self-bunched cooled (b) beam in ICE [9]. Note the change of the reference level: 230 μ V in (a) and 30 mV in (b).
- Fig. 63 Time evolution of a sideband amplitude from a transverse Schottky pick-up, showing the repeated occurrence of transverse blow-up (hiccup) due to an instability, and subsequent cooling for a proton beam stored in LEAR [109] (5 s per division).

- Fig. 64 Measurement of frictional force exerted on protons and on H^- ions in the single-pass experiment MOSOL at Novosibirsk [24c, 49].
- Fig. 65 The maximum of the frictional force (see Fig. 64) as a function of the magnetic field for protons and H^- ions [24c, 49].
- Fig. 66 Simulation of longitudinal electron cooling (lines) and comparison with experiment (dots) [118, 119]; dash-dotted line: 25 V HT ripple, dashed line: $\epsilon = 2.5 \pi \text{ mm mrad}$, full line: optimized cooling.
- Fig. 67 Operation range of electron coolers in existing and planned cooling rings.
- Fig. 68 Schematic arrangement of a linear electron cooler for storage rings.
- Fig. 69 Configuration for the simultaneous confinement of ions and electrons or positrons for cooling in a trap (see also Refs. [139] to [141]).
- Fig. 70 Illustration of the improvement of the duty cycle for a machine that has to accumulate ions at high energy (e.g. bare ions, antiprotons) and to deliver them at low energy to experimenters. Row a) beam energy, b) stored beam intensity, c) beam intensity delivered to user versus operation time. Part I: without cooling. Part II: with slow cooling at accumulation and two intermediate energies. Part III: with fast cooling at accumulation energy and during deceleration.
- Fig. 71 Illustration of the dependence of the longitudinal electron velocity spread and of the maximal adiabatic force on the electron density.
- Fig. 72 Illustration of slow beam extraction with the help of the electron beam: a) velocity (momentum) distribution of ions produced by an appropriate energy modulation of the electrons; b) electron velocity distributions for different settings of the acceleration voltage; c) modulation cycle of electron energy to smear out the ion velocity distribution as shown in (a).
- Fig. 73 Scheme for continuous loading of a trap with positive ions (e.g. positrons) via their deceleration in a single passage through an electron beam.
- Fig. 74 Illustration of the dependence of the longitudinal plasma correlation factor $\Gamma_{i\parallel}$ on the number of stored ions in the presence of electron cooling and intrabeam scattering.
- Fig. 75 Variation of the electron capture rate with time as a consequence of proton beam instabilities [109, 110].

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 9

Fig. 10

a)

b)

Fig. 11

Fig. 12

Fig. 13

Fig. 14

Fig. 15

Fig. 16

a)

b)

c)

Fig. 17

Fig. 18

Fig. 19

Fig. 20

Fig. 21

Fig. 22

Fig. 23

Fig. 24

Fig. 25

Fig. 26

Fig. 27

Fig. 28

Fig. 29

REF: 71.5 μ V

LINEAR

a)

CENTER FREQ: 41.589 MHz

SPAN: 150 kHz

RES BW: 3 kHz

VBW: 10 Hz

SWP: 10 s

b)

Fig. 30

Fig. 31

Fig. 32

Fig. 33

Fig. 34

Fig. 35

Fig. 36

Fig. 37

Fig. 38

Radial variation of electron
beam properties included

Dispersion (9.6 m) in
cooling region included

Fig. 39

Fig. 40

Fig. 41

Fig. 42

Fig. 43

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Fig. 48

Fig. 49

Fig. 51

Fig. 50

Fig. 52

Fig. 53

Fig. 54

Fig. 56

Fig. 57

a)

b)

Fig. 58

c)

d)

Fig. 58

a)

b)

c)

Fig. 59

Fig. 60

Fig. 61

Fig. 62

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