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LANDAU LEVELS: TRANSLATIONS AND ROTATIONS
IN A CONSTANT ELECTROMAGNETIC FIELD

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ABSTRACT

The spectrum of a charged particle in a constant magnetic field consists of equally-spaced Landau levels $E_n = \hbar\omega(n + \frac{1}{2})$ which are infinitely degenerate. Using the magnetic group we show that this structure is present for any charged system which is rotation-translation invariant. The result is extended to the case of a constant electric field. The separation between baricentric and relative coordinates is discussed.

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1. ~ INTRODUCTION

The behaviour of a charged particle in an external constant magnetic field B is rather interesting and reveals important features which are of a very general character. It is well known that the level structure of our system is that of a harmonic oscillator

$$E_n = \hbar \omega \left(n + \frac{1}{2} \right) \quad (1.1)$$

where

$$\omega = \frac{qB}{mc} \quad (1.2)$$

is the Larmor (or cyclotron) frequency, i.e., the angular velocity of the classical circular motion of the particle. We also know that each level is infinitely degenerate. This degeneracy has a very fundamental rôle in physics and is the main cause of the quantum Hall effect. The exact infinite degeneracy of each level is due to the special invariance properties of our problem.

Indeed, if one looks at the problem naively, one might be led to consider invariance under translation. This is obviously true classically, since the constant field B appears in the expression for the Lorentz force. In the quantum theory, the potential $A_1(x)$, which is linear in x, appears in the Hamiltonian

$$H = \frac{1}{2m} \left(\vec{p} - \frac{q}{c} \vec{A} \right)^2 \quad (1.3)$$

If we perform a translation, $A_1(x)$ is changed so that H is not immediately invariant under the transformation. The original Hamiltonian can still be recovered if we perform an extra gauge transformation which restores the A_i to their original form.

We thus have invariance under a magnetic translation group whose generators w_i (to be defined in the next section) are indeed constants of the motion [1]. The degeneracy of the Landau levels is due to the fact that in our case (in contrast to the free case) the generators do not commute, but

$$[w_\mu, w_\nu] = -ik \varepsilon_{\mu\nu} B \quad (1.4)$$

This non-commutativity is the fundamental new feature introduced by the magnetic fields. The degeneracy of the Landau level follows immediately from the algebra (4) of the constants of motion w_i :

$$[H, w_\mu] = 0 \quad (1.5)$$

The group theoretical origin of the Landau degeneracy suggests that the phenomenon has a much wider validity than the one-particle case, but that it applies to any translation-invariant system subject to a constant magnetic field. This has indeed been shown by Yip [2] who generalizes a partial result obtained by Kohn [3] almost thirty years ago.

In this paper we offer a completely general treatment of this topic in a formalism which is not bound to the choice of any particular gauge. This will allow us to obtain two interesting new results:

- 1) The existence beyond the magnetic translation group of a magnetic rotation whose generator λ automatically corrects for any change of the form of $A_\mu(x)$ due to the rotations.
- 2) The extension of the treatment to the case in which a constant electric field $\vec{E}$ is present. Indeed, Landau degeneracy is present in any translation-invariant system, but also the breaking of the multiplet is always the same since it follows only from invariance arguments.

In Section 2 we shall review the classical and quantum treatment of a particle in a constant magnetic field. In Section 3, we shall show how Landau degeneracy is present and can be treated in a translation-rotation invariant treatment in a constant magnetic and electric field. Finally, in Section 4 we shall deal with systems of identical particles and show that in this case one can completely separate baricentric and relative motion.

2. - THE ONE-PARTICLE CASE

In our treatment we shall orient the magnetic field B along the positive x_3 axis. We shall choose an electric field $\vec{E}$ orthogonal to $\vec{B}$. The electric field will therefore lie in the (1,2) plane. The problem will thus be two-dimensional and we shall work consistently in the (1,2) plane.

- a) A brief classical discussion will be helpful as an introduction to the subject. The classical equations of motion are

$$m \frac{dx_L}{dt} = \pi_L \quad (2.1a)$$

$$\frac{d\pi_L}{dt} = \omega \varepsilon_{Lr} \pi_r + q E_L \quad (2.1b)$$

where

$$\omega = \frac{qB}{mC}$$

and the antisymmetric tensor ε_{ij} is given by $\varepsilon_{11} = \varepsilon_{22} = 0$, $\varepsilon_{12} = -\varepsilon_{21} = 1$. It will be useful to define the vector

$$W_L = \pi_L - m \omega \varepsilon_{Lr} x_r \quad (2.2)$$

which obeys the equation

$$\frac{dW_L}{dt} = q E_L \quad (2.3)$$

so that in the absence of an electric field w_1 and w_2 will be constants of the motion. Those quantities will play a fundamental rôle in all our treatment.

Introducing

$$\hat{\pi}_L = \pi_L - m v_L^D \quad (2.4)$$

where $\vec{v}^D$ is the drift velocity defined by

$$v_L^D = \frac{c}{B} \varepsilon_{Lr} E_r \quad (2.5)$$

Eq. (2.1b) becomes

$$\frac{d\hat{\pi}_L}{dt} = \omega \varepsilon_{Lr} \hat{\pi}_r \quad (2.6)$$

so that

$$X_L = \frac{\varepsilon_{Lr}}{m\omega} (\hat{w}_r - \hat{\pi}_r) \quad (2.7)$$

where

$$\begin{aligned} \hat{W}_L &= W_L - m v_L^D \\ \frac{d\hat{W}_L}{dt} &= q E_L \end{aligned} \quad (2.8)$$

We introduce the two important quantities

$$\begin{aligned} \hat{E} &= \frac{1}{2m} \hat{\pi}_L^2 \\ \hat{\lambda} &= \frac{1}{2m\omega} (\hat{\pi}^2 - \hat{W}^2) \\ &= \frac{(\hat{\pi}_L + \hat{W}_L) \varepsilon_{Lr} x_r}{2} \end{aligned} \quad (2.9)$$

which obey the equations

$$\begin{aligned} \frac{d\hat{E}}{dt} &= 0 \\ \frac{d\hat{\lambda}}{dt} &= \varepsilon_{Lr} \hat{W}_L v_r^D \end{aligned} \quad (2.10)$$

The classical trajectories can be easily obtained by integrating Eqs. (2.6) and (2.8) and using Eq. (2.7):

$$\hat{\pi}_L = \sqrt{\frac{E}{2m}} c_L(t) \quad (2.11a)$$

$$\hat{W}_L = \hat{W}_L^0 + q E_L t \quad (2.11b)$$

which leads to

$$X_L = \xi_L^i + v_L^D t - \frac{1}{\omega} \sqrt{\frac{E}{2m}} c_L'(t) \quad (2.11c)$$

where

$$c_L(t) = \begin{pmatrix} \sin \omega t \\ \cos \omega t \end{pmatrix} ; \quad c_L'(t) = \omega \begin{pmatrix} \cos \omega t \\ -\sin \omega t \end{pmatrix}$$

Let us consider in some detail the situation in the absence of an electric field. In that case we have four constants of motion ϵ , w_1 , w_2 , λ which in the quantum case will correspond to the Hamiltonian and to the magnetic translations and rotation.

The constants of motion

$$\dot{z}_c = \frac{1}{2m\omega} \epsilon_{cr} w_r \quad (2.12)$$

represent the co-ordinates of the centre of the circular motion of the charged particle.

b) Let us now move on to the quantum treatment. Let us first consider $\vec{E} = 0$. The Hamiltonian is

$$H = \frac{1}{2m} (\pi_r^2 + \pi_c^2) \quad (2.13)$$

where

$$\pi_c = p_c - \frac{q}{c} A_c(x) \quad (2.14)$$

The vector potential A_i is related to B by

$$\epsilon_{cr} \partial_r A_r = B \quad (2.15)$$

whose general solution is

$$A_c = -\frac{B}{2} (\epsilon_{cr} x_r - \frac{\partial \phi}{\partial x_c}) \quad (2.16)$$

where the scalar function ϕ determines the gauge in which we are working. Most of the results of this paper are actually independent of the choice of ϕ . We have the commutators

$$[\pi_c, \pi_r] = i\hbar m\omega \epsilon_{cr} \quad (2.17)$$

As in the classical case, the constants of motion are

$$w_c = \pi_c - m\omega \epsilon_{cr} x_r \quad (2.18)$$

$$\lambda = \frac{\vec{\pi}^2 - \vec{w}^2}{2m\omega} \quad (2.19)$$

Indeed, since

$$[w_c, \pi_r] = 0 \quad (2.20)$$

$$[\lambda, \pi_c] = -i\hbar \epsilon_{cr} \pi_r \quad (2.21)$$

w_1 and λ commute with H :

$$[H, w_c] = 0 \quad (2.22)$$

$$[H, \lambda] = 0 \quad (2.23)$$

The "angular momentum" constant of motion λ looks trivial since in Eq. (2.18) it is a linear combination of other known constants of motion. As will be seen better in the next section, this is in general not true. In our case, we notice that if we add to the Hamiltonian a rotation-invariant external potential $V(r)$, w_1 and w_2 are no longer constants of motion, whereas Eq. (2.23) is still true.

The infinite degeneracy of all Landau levels follows from the existence of the three constants of motion λ , w_1 , w_2 obeying the algebra

$$[w_c, w_r] = -i\hbar m\omega \epsilon_{cr} \quad (2.24)$$

$$[\lambda, w_c] = -i\hbar \epsilon_{cr} w_r \quad (2.25)$$

In order to study the levels of the system, one needs to diagonalize, besides the Hamiltonian, another constant of motion (for example, w_2 or λ). The other two

constants of motion, which will not be diagonal, will connect different Landau sublevels with the same energy.

It is well known that the different Landau levels correspond to energies $E_n = \hbar\omega(n + \frac{1}{2})$, just the harmonic oscillator spectrum. This can be seen by enlarging the algebra to include the "kinetic operator" π_i . We have

$$[\pi_z, \pi_y] = i\hbar m\omega \epsilon_{zy} \quad (2.26)$$

$$[H, \pi_z] = i\hbar\omega \epsilon_{zy} \pi_y \quad (2.27)$$

$$[\lambda, \pi_z] = -i\hbar \epsilon_{zy} \pi_y \quad (2.28)$$

which, together with Eq. (2.17), fix the form of the spectra of H and of λ . One can express H and λ in the standard form

$$H = \omega f \quad (2.29)$$

$$\lambda = f - g \quad (2.30)$$

where

$$f = \frac{1}{2m\omega} \vec{\pi}^2 \quad (2.31)$$

$$g = \frac{1}{2m\omega} \vec{w}^2 \quad (2.32)$$

In the Appendix we shall see that, because of the algebra (2.24)-(2.26), f and g have the spectra

$$f = (n + \frac{1}{2})\hbar \quad (2.33)$$

$$g = (m + \frac{1}{2})\hbar \quad (2.34)$$

where m and n are integer numbers.

The Landau degeneracy is exhibited by the fact that the eigenvalues of H do not depend on the quantum number m. On the other hand, the level is fully defined if one diagonalizes simultaneously H and λ .

c) Let us now discuss briefly the gauge dependence of our results. The magnetic constants of motion w_i and λ differ from the standard Euclidean operators p_i and λ by x dependent functions. The explicit forms for the two most frequently used gauges (symmetric and Landau) are:

a) Symmetric Gauge ($\phi = 0$)

$$\pi_z = p_z + \frac{m\omega}{2} \epsilon_{zy} x_y$$

$$w_z = p_z - \frac{m\omega}{2} \epsilon_{zy} x_y$$

$$\lambda = \epsilon_{zy} p_z x_y = \ell \quad (2.35)$$

b) Landau Gauge ($\phi = x_1 x_2$)

$$\begin{aligned} A_1 &= 0 & \pi_1 &= p_1 & w_1 &= p_1 - m\omega x_2 \\ A_z &= B x_1 & \pi_z &= p_z - m\omega x_1 & w_z &= p_z \\ \lambda &= \epsilon_{zy} p_z x_y + \frac{m\omega}{2} (x_1^2 - x_2^2) \end{aligned} \quad (2.36)$$

Equations (2.35) and (2.36) give a clear indication of the physical meaning of the magnetic translation rotation group. The "deformation" due to A_i depends on the gauge. It is possible that in some gauges a subgroup of the Euclidean group is presented. For example, in the symmetric case the Hamiltonian is rotation-invariant; this implies, as shown in Table I, that in that gauge $\lambda = \ell$. In the Landau gauge the Hamiltonian is invariant with respect to rotations along the z axis and thus $w_z = p_z$.

We thus see that, although a gauge-invariant treatment is in general preferable, the choice of one particular gauge is mainly dictated by the physical features we want to emphasize.

3. - EXTENSION TO EUCLIDEAN-INVARIANT SYSTEMS

The results of the last section were obtained for the elementary case of a single particle in a constant magnetic field. However, since they follow from general invariance arguments, they can easily be extended to more complex situations.

In this section we shall discuss their generalization to any Euclidean-invariant system and to the case in which we add a constant electric field.

a) Many-Particle System

We shall schematize the problem by considering a system of n particles with co-ordinates x^n and p^n , charges q^n and masses m^n , bound together by a many-body Euclidean-invariant potential $U(x)$ which obeys the equations

$$\sum_i \partial_i^2 U = 0 \quad (3.1)$$

$$\sum_i \epsilon_{ij} \partial_i^2 x_j^2 U = 0 \quad (3.2)$$

The Hamiltonian H_0 is

$$H_0 = \sum p^n + U \quad (3.3)$$

where

$$\epsilon^n = \frac{1}{2m^n} (\vec{p}^n)^2 \quad (3.4)$$

$$\pi_\nu^n = p_\nu^n - \frac{q^n}{c} A_\nu(x) \quad (3.5)$$

In analogy with Section 2, we define

$$W_\nu^n = \pi_\nu^n - \frac{q^n B}{c} \epsilon_{ij} x_j \quad (3.6)$$

$$\lambda^n = \frac{(\vec{p}^n)^2 - (\vec{W}^n)^2}{2 q^n B/c} \quad (3.7)$$

We have the commutators

$$[\epsilon^n, W_\nu^n] = 0$$

$$[\epsilon^n, \lambda^n] = 0$$

We see that any w^n of λ^n commutes with the kinetic part of the Hamiltonian. In order to have a vanishing commutator with the potential, we define

$$W_\nu = \sum_n W_\nu^n$$

$$\Lambda = \sum_n \lambda^n \quad (3.8)$$

We immediately see that [see Eqs. (3.1) and (3.2)]

$$[U, W_\nu] = 0$$

$$[U, \Lambda] = 0$$

In conclusion, we see that

$$[H_0, W_\nu] = 0 \quad (3.9)$$

$$[H_0, \Lambda] = 0 \quad (3.10)$$

Unlike the one-particle case, here W_ν and Λ are independent operators whose conservation stems from different physical situations. Indeed, Eq. (3.9) follows from Eq. (3.1), whereas (3.10) follows from (3.2). The algebra between the constants of motion is

$$[W_\nu, W_\delta] = -i \hbar \frac{qB}{c} \epsilon_{\nu\delta} \quad (3.11)$$

$$[\Lambda, W_L] = -i\hbar \varepsilon_{\alpha\beta} W_\beta \quad (3.12)$$

where $Q = \sum q^n$. Equations (3.9) and (3.10), together with the algebra (3.11) and (3.12), show that each eigenstate of the Hamiltonian is (like in the one-particle case) infinitely degenerate.

In order to analyze better this degeneracy, it is useful to introduce the operator

$$G = \frac{c}{Q_B} (W_1^2 + W_L^2) \quad (3.13)$$

which obeys the algebra

$$[G, \Lambda] = 0 \quad (3.14)$$

$$[G, W_L] = -i\hbar \varepsilon_{\alpha\beta} W_\beta \quad (3.15)$$

As a consequence, the combination $\Lambda - G$ commutes with Λ and with W_1 and is constant along the degenerate level. Since the eigenvalues of G are $(m+\frac{1}{2})\hbar$, we have

$$\Lambda = \Lambda - G + (m + \frac{1}{2})\hbar \quad (3.16)$$

Therefore, as in the free case, each substate belonging to the same degenerate Landau level corresponds to a different angular momentum labelled by the integer m .

b) Let us now introduce a constant external electric field $\vec{E}$ lying in the (1,2) plane. The Hamiltonian will be

$$H = H_0 - E_\alpha \sum_n q^n x_\alpha \quad (3.17)$$

Using Eq. (3.6)

$$q^n x_\alpha = \frac{c}{B} \varepsilon_{\alpha\beta} (W_\beta - \pi_\beta)$$

we get

$$H = H_0 + v_L^2 (Z\pi^2 - W_L) \quad (3.18)$$

where the drift velocity $\vec{v}^D$ is given by Eq. (2.5). Recalling Eq. (3.3), we can write

$$H = \sum \frac{1}{2m} (\hat{\pi}^a)^2 + U - v_L^2 \hat{W}_L + \frac{1}{2M} (\vec{\nabla}^0)^2 \quad (3.19)$$

$$(M = \sum m^a)$$

where

$$\hat{\pi}_L^a = \pi_L^a - m^a v_L^a = T^{-1} \pi_L^a T$$

$$\hat{W}_L = W_L - M v_L^0 = T^{-1} W_L T \quad (3.20)$$

and T is the generator of the Galilei transformation

$$T = \exp \left[-i \frac{v_L^0 \sum m^a x_a}{\hbar} \right] \quad (3.21)$$

We can use the canonical transformation T to rewrite H in a more convenient form. In the new basis we shall have ($H = T^{-1} H_{\text{new}} T$)

$$H_{\text{new}} = H_0 + \frac{1}{2} M (\vec{v}_0)^2 - v_L^2 W_L \quad (3.22)$$

which shows that, also in the presence of an electric field, the Hamiltonian can be written in a simple algebraic form^{*}). The degeneracy in H is broken by the term $v_L^2 W_L$. Comparing with the expression

$$\Lambda = (\Lambda - G) + G \quad (3.23)$$

for the angular momentum, we see that the structure of the Landau level is determined by the operators G and W_L . Since those operators do not commute, we cannot diagonalize simultaneously H and Λ .

^{*}) From now on we shall work in the new basis and we shall drop for simplicity the subscript "new".

Let us orient $\vec{E}$ along the n axis so that $\vec{v}^D$ is oriented along the y axis. Equation (3.22) is then written as

$$H = H_0 + \frac{1}{2} M v^D{}^2 - v^D W_2 \quad (3.24)$$

with the expressions

$$[W_1, W_2] = -i k \frac{QB}{c} \quad (3.25)$$

$$G = \frac{c}{QB} (W_1^2 + W_2^2) \quad (3.26)$$

It is convenient to diagonalize W_2 :

$$W_2 = \hbar k \quad (3.27)$$

in order to get energy levels

$$E_k = H_0 + \frac{1}{2} M v_0^2 - \hbar k v^D \quad (3.28)$$

In this case we shall have

$$W_1 = -i k \frac{QB}{c} \frac{\partial}{\partial k} \quad (3.29)$$

$$G = \frac{c k^2}{QB} \left[k^2 + \left(\frac{QB}{\hbar k} \right)^2 \frac{\partial^2}{\partial k^2} \right] \quad (3.30)$$

One may, of course, prefer to diagonalize the angular momentum

$$G = k(m + \frac{1}{2}) \quad (3.31)$$

In this case W_2 , i.e., H , will be non-diagonal, with the following non-vanishing matrix elements:

$$\langle n+1 | W_2 | n \rangle = \frac{\sqrt{n+1}}{2i \sqrt{k QB/c}} \quad (3.32)$$

4. - SEPARATION OF THE BARICENTRIC CO-ORDINATES

In order to proceed further we specialize our choice of the system and deal with N identical particles of charges $q^n = q$ and masses $m^n = m^*$. The Hamiltonian will be

$$H_0 = \frac{1}{2m} \sum (\vec{p}^n)^2 + U \quad (4.1)$$

where

$$\pi_c^n = p_c^n - \frac{q}{c} A_c(x) \quad (4.2)$$

In this case we can recover the old Kohn [3] result and write

$$\pi_c = \sum \pi_c^n \quad (4.3)$$

Since

$$[\pi_c, U] = 0 \quad (4.4)$$

we obtain

$$[H, \pi_c] = i k \frac{QB}{c} \varepsilon_y \pi_y \quad (4.5)$$

together with the commutator

*) It would be enough if all "Larmor frequencies" $\omega^n = Bq^n/Cm^n = \omega$ were equal.

$$[\pi_c, \pi_r] = i\kappa \frac{N_1 B}{c} \varepsilon_{ij} \quad (4.6)$$

We also have.

$$[\Lambda, \pi_c] = -i\kappa \varepsilon_{ij} \pi_r \quad (4.7)$$

Equations (4.5) and (4.6) show that π_i act as raising and lowering operators for H_0 , so that each level belongs to a full Landau family labelled by the two quantum numbers (n, m) . In order to see this more precisely, we define [see Eq. (3.13)]:

$$G = \frac{c}{2N_1 B} (W_1^2 + W_2^2) \quad (4.8)$$

$$F = \frac{c}{2N_1 B} (\pi_1^2 + \pi_2^2) \quad (4.9)$$

using Eqs. (3.14), (3.15) and

$$[\Lambda, F] = 0 \quad (4.10)$$

$$[H, F] = 0 \quad (4.11)$$

$$[F, W_c] = 0 \quad (4.12)$$

$$[F, \pi_c] = -i\kappa \varepsilon_{ij} \pi_r \quad (4.13)$$

Recalling Eqs. (3.9), (3.12), (4.5) and (4.7), we see that if we define

$$\begin{aligned} H &= H_R + W F \\ \Lambda &= \Lambda_R + F - G \end{aligned} \quad (4.14)$$

the operators H_R and Λ_R commute between themselves and with all W_i and π_i . They are thus constant for a full Landau family

$$\begin{aligned} H &= H_R + (n + \frac{1}{2}) \kappa \omega \\ \Lambda &= \Lambda_R + (n - m) \kappa \end{aligned} \quad (4.15)$$

Finally, we want to show that the expressions (4.14) for H_0 and Λ separate the contributions from the baricentric and relative co-ordinates. Indeed, F and G depend only on the baricentric variables π_i and W_i . On the other hand, it is easy to see that

$$H_R = \frac{1}{2mN} \sum_{rs} (\vec{\pi}^s - \vec{\pi}^r)^2 + \Lambda \quad (4.16)$$

$$\Lambda_R = \frac{c}{2N_1 B} \sum_{rs} [(\vec{\pi}^s - \vec{\pi}^r)^2 - (\vec{W}^s - \vec{W}^r)^2]$$

5. - CONCLUSIONS

The problem of a charged system subject to a constant magnetic and electric field appears frequently in condensed matter physics, superconductivity and the Hall effect. It is important to abstract those properties which are independent of the details but follow only from the general invariance principle. In this paper it has been shown that translation-rotation invariance implies the presence of infinitely degenerate (Landau) levels. In the case of identical particles, each level belongs to an equally-spaced Landau family. It is amusing that those arguments not only tell us about the degeneracy, but also predict how it will be broken in the presence of an electric field.

The general method can also be extended to the case in which a harmonic potential is present. This is being investigated in collaboration with W. Alberico and A. Molinari.

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A P P E N D I X

In this Appendix we wish to recall some elementary arguments leading to the Landau spectrum in Sections 2, 3 and 4. Let us define

$$f = \frac{1}{2\alpha} (\pi_1^2 + \pi_2^2) \quad (A1)$$

where the commutation relation is

$$[\pi_1, \pi_2] = i\hbar\alpha \quad (A2)$$

We define

$$a = \frac{1}{\sqrt{2\hbar\alpha}} (\pi_1 + i\pi_2) \quad (A3)$$

$$a^\dagger = \frac{1}{\sqrt{2\hbar\alpha}} (\pi_1 - i\pi_2)$$

We shall have

$$f = \hbar(a^\dagger a + \frac{1}{2}) \quad (A4)$$

$$[a, a^\dagger] = 1 \quad (A5)$$

This means that the spectrum of the operator f is

$$f = \hbar(n + \frac{1}{2})$$

$$|n\rangle = \frac{(a^\dagger)^n}{\sqrt{n!}} |0\rangle$$

where the lowest state obeys the equation

$$a|0\rangle = 0 \quad (A7)$$

Let us obtain the expressions of a and $a^\dagger$ in the Landau gauge. In that gauge we shall have

$$\pi_2 = p_2 + \frac{\alpha}{2} \epsilon_2 \times r \quad (A8)$$

leading to

$$a = \frac{1}{\sqrt{2\hbar\alpha}} \left[(p_1 + i p_2) - \frac{i\alpha}{2} (x_1 + i x_2) \right] \quad (A9)$$

$$a^\dagger = \frac{1}{\sqrt{2\hbar\alpha}} \left[(p_1 - i p_2) + \frac{i\alpha}{2} (x_1 - i x_2) \right]$$

Introducing

$$z = x_1 + i x_2$$

$$z^* = x_1 - i x_2 \quad (A10)$$

we finally get

$$a = -i\sqrt{\frac{\hbar}{2\alpha}} \left(\frac{\partial}{\partial z} + \frac{\alpha}{2\hbar} z \right) \quad (A11)$$

$$a^\dagger = -i\sqrt{\frac{\hbar}{2\alpha}} \left(\frac{\partial}{\partial z} - \frac{\alpha}{2\hbar} z^* \right)$$

In this representation, the equation $a|0\rangle = 0$ reads

$$\left(\frac{\partial}{\partial z} + \frac{\alpha}{2\hbar} z \right) \psi_0(z, z^*) = 0 \quad (A12)$$

of which the general solution is

$$\psi(z, z^*) = \chi(z) e^{-\frac{\alpha}{2\hbar} z z^*} \quad (A13)$$

where $\chi(z)$ can be any analytic function of z . The freedom in the choice of $\chi(z)$ is another expression of the degeneracy of the Landau levels.