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Note on QCD corrections to hadronic Higgs decay

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ABSTRACT

The $\mathcal{O}(\alpha_s)$ QCD correction to the hadronic decay of a scalar Higgs boson, for which contradicting results have been reported, is calculated. Our result is in agreement with that of Braaten and Leveille but disagrees with that of Janot. The possible origin of the discrepancy is discussed. The total hadronic decay rate and the differential rate to $q\bar{q}g$ of a pseudoscalar Higgs boson are also presented.

1. Introduction

The Higgs boson plays a fundamental role in the $SU(2) \times U(1)$ symmetry breaking and the generation of the masses of W , Z , quarks, and leptons. The search for the Higgs boson is thus of prime importance in clarifying the origin of the Fermi scale. If the Higgs boson mass is below the W pair threshold, it has a large decay branching fraction to hadrons. The lowest-order decay rate, $H \rightarrow q\bar{q}$, has been known for a long time.[†] Since quarks have color interactions, it is necessary to evaluate the QCD correction to the decay if one needs an accurate estimate of the rate and a realistic simulation of the decay. Although the $\mathcal{O}(\alpha_s)$ correction has been calculated a decade ago by Braaten and Leveille,² it has remained relatively unknown. Recently we made an independent calculation of the correction,³ which is in agreement with Ref. 2, and extended it to the case of nonminimal and pseudoscalar Higgs bosons. A new calculation by Janot⁴ also appeared recently, who apparently did not know the old result of Ref. 2. His result, however, does not agree with Ref. 2 and ours. Nevertheless, his results seem to have been used in obtaining recent experimental bounds⁵ on Higgs bosons at LEP.

In this note, we present our calculation of the correction in some detail and try to clarify the origin of the difference, because we have listed only the final results in Ref. 3. We will also show the differential rate for the decay into $q\bar{q}g$. The pseudoscalar case has not been discussed in the literature. Finally, we discuss the summation of leading logarithms, which is necessary to have a well-behaved corrections in the large Higgs mass limit. We thus provide all relevant formulas for the hadronic decay of a scalar and pseudoscalar Higgs particle, which are

[†] The formula for the leptonic decay $H \rightarrow \ell\bar{\ell}$ can be found in Ref. 1. Following the public recognition of QCD, the hadronic decay rate has been understood to be given by the same formula with the suitable replacement of the fermion mass and the inclusion of a color factor of 3, provided that m_H is large enough that perturbative QCD is applicable.

correct up to both $\mathcal{O}(\alpha_s)$ and leading logarithms. In this paper we denote a scalar and pseudoscalar Higgs boson by H and P , respectively.

2. Scalar Higgs decay

2.1 LOWEST ORDER

The lowest order amplitude $H \rightarrow q\bar{q}$ is given by the Feynman diagram of Fig. 1, in which our convention for the momenta is also shown:

$$\mathcal{M}_0 = f \bar{u}(p) v(\bar{p}) , \quad (2.1)$$

where f is the Yukawa coupling defined as

$$\mathcal{L} = f \bar{q} q H . \quad (2.2)$$

In the minimal standard model,

$$f = -\frac{gm}{2m_W} = -(\sqrt{2}G_F)^{1/2} m , \quad (2.3)$$

where g is the SU(2) gauge coupling, m is the quark mass, and m_W is the W mass, and G_F is the Fermi coupling constant. Squaring the amplitude (2.1) and summing over spins and color, we obtain

$$\sum |\mathcal{M}_0|^2 = 2N_C f^2 m_H^2 \beta_0^2 , \quad (2.4)$$

where

$$\beta_0 = \sqrt{1 - \frac{4m^2}{m_H^2}} , \quad (2.5)$$

and $N_C = 3$ is a color factor. The decay is isotropic and integrating over the

two-body phase space

$$d\Phi_2 = \frac{\beta_0}{8\pi} \quad (2.6)$$

gives the lowest-order decay rate

$$\begin{aligned} \Gamma_0(H \rightarrow q\bar{q}) &= \frac{1}{2m_H} \int d\Phi_2 \sum |\mathcal{M}_0|^2 \\ &= \frac{N_C f^2}{8\pi} m_H \beta_0^3 \\ &= \frac{N_C G_F m^2}{4\sqrt{2}\pi} m_H \beta_0^3. \end{aligned} \quad (2.7)$$

2.2 VIRTUAL ONE-LOOP CORRECTIONS

We now proceed to the QCD corrections. The contributing graphs are depicted in Fig. 2. The diagram (a) gives the vertex correction, (b) and (c) are quark self-energy corrections, and (d)–(f) are the counterterm contribution. We will work in the Feynman gauge.

The diagram (a) gives the correction to the lowest-order vertex Eq. (2.3). Denoting the full vertex function (up to the coupling factor) as $\Lambda = \Lambda_0 + \Lambda_1$, where $\Lambda_0 = 1$ is the lowest order term, we find for the one-loop correction from the diagram (a)

$$\begin{aligned} \Lambda_{1a} &= \frac{C_F \alpha_s}{2\pi} \left\{ 2 \left(\frac{1}{\epsilon} - \log \frac{m^2}{\mu^2} \right) - \frac{1 + \beta_0^2}{2\beta_0} \log \frac{\beta_0 + 1}{\beta_0 - 1} \log \frac{m^2}{\lambda^2} \right. \\ &\quad + \frac{1 + \beta_0^2}{\beta_0} \left[\text{Li}_2 \left(\frac{1 - \beta_0}{1 + \beta_0} \right) - \log \frac{2\beta_0}{\beta_0 + 1} \log \frac{\beta_0 + 1}{\beta_0 - 1} - \frac{1}{4} \log^2 \frac{\beta_0 + 1}{\beta_0 - 1} + \frac{\pi^2}{12} \right] \\ &\quad \left. + \frac{1 - \beta_0^2}{\beta_0} \log \frac{\beta_0 + 1}{\beta_0 - 1} + 3 \right\} \end{aligned} \quad (2.8a)$$

for $q^2 < 0$, and

$$\begin{aligned} \Re e \Lambda_{1a} = & \frac{C_F \alpha_s}{2\pi} \left\{ 2 \left(\frac{1}{\epsilon} - \log \frac{m^2}{\mu^2} \right) - \frac{1 + \beta_0^2}{2\beta_0} \log \frac{1 + \beta_0}{1 - \beta_0} \log \frac{m^2}{\lambda^2} \right. \\ & + \frac{1 + \beta_0^2}{\beta_0} \left[\text{Li}_2 \left(\frac{1 - \beta_0}{1 + \beta_0} \right) + \log \frac{1 + \beta_0}{2\beta_0} \log \frac{1 + \beta_0}{1 - \beta_0} - \frac{1}{4} \log^2 \frac{1 + \beta_0}{1 - \beta_0} + \frac{\pi^2}{3} \right] \\ & \left. + \frac{1 - \beta_0^2}{\beta_0} \log \frac{1 + \beta_0}{1 - \beta_0} + 3 \right\} \end{aligned} \quad (2.8b)$$

for $q^2 > 4m^2$. Although the physical vertex is at $q^2 = m_H^2$, we have also shown the value for negative q^2 for the purpose of discussing the renormalization condition later. (In the definition of β_0 , the factor of m_H^2 should be replaced by q^2 in general.) In Eq. (2.8), $\text{Li}_2(x)$ is Spence function

$$\text{Li}_2(x) = - \int_0^x \frac{dt}{t} \log(1 - t), \quad (2.9)$$

$\alpha_s = g_s^2/4\pi$ is the QCD coupling constant, and $C_F = 4/3$ is a color factor. We have regularized the ultraviolet divergence by dimensional regularization with $D = 4 - 2\epsilon$, μ is the renormalization scale, and the infrared singularity is regularized by a gluon mass λ . No problem occurs from the use of a finite gluon mass, since the three-gluon coupling does not appear in this order.

The self-energy diagrams (b) and (c) are cancelled by the counterterm diagrams (e) and (f) if one uses the on-shell renormalization scheme as usually adopted in QED. The mass and wave-function renormalization counterterms in this scheme is given by

$$\begin{aligned} \frac{\delta m}{m} &= \frac{C_F \alpha_s}{4\pi} \left[3 \left(\frac{1}{\epsilon} - \log \frac{m^2}{\mu^2} \right) + 4 \right] \\ Z_2 - 1 &= \frac{C_F \alpha_s}{4\pi} \left[- \left(\frac{1}{\epsilon} - \log \frac{m^2}{\mu^2} \right) + 2 \log \frac{m^2}{\lambda^2} - 4 \right]. \end{aligned} \quad (2.10)$$

Finally, the counterterm for the vertex (d) should be included. In arbitrary Yukawa theories, this counterterm is fixed by the renormalization condition for the Yukawa coupling, which can be chosen at will. For the Higgs boson, however, there is no arbitrariness in the choice of the renormalization condition, since the quark mass and the Yukawa coupling are proportional to each other. In fact the mass comes from the vacuum expectation value of the Higgs field:

$$\mathcal{L} = f\bar{q}q(v + H), \quad (2.11)$$

with $m = -fv$. Since we are concerned with the QCD radiative corrections and the Higgs field carries no color, the combination $v + H$ has to be renormalized as a whole. Thus the counterterm for the Yukawa coupling is fixed in terms of the quark mass and wave-function renormalization constants:

$$\begin{aligned} \Lambda_{1d} &= (Z_2 - 1) - \frac{\delta m}{m} \\ &= \frac{C_F \alpha_s}{2\pi} \left[-2 \left(\frac{1}{\epsilon} - \log \frac{m^2}{\mu^2} \right) + \log \frac{m^2}{\lambda^2} - 4 \right]. \end{aligned} \quad (2.12)$$

It should be stressed that this counterterm is not equal to the “naive” one obtained by subtracting the vertex function at $q^2 = 0$:

$$\Lambda_{1d}^{(\text{naive})} = -\Lambda_{1a}(q^2 = 0) = \frac{C_F \alpha_s}{2\pi} \left[-2 \left(\frac{1}{\epsilon} - \log \frac{m^2}{\mu^2} \right) + \log \frac{m^2}{\lambda^2} - 1 \right]. \quad (2.13)$$

This is in sharp contrast with the case of the vector vertex, for which Ward identity and zero momentum subtraction give identical counterterms. We note that the same counterterm can be used for multi-Higgs cases and also for pseudoscalar Higgs bosons (up to the overall Yukawa coupling factor).

The total virtual correction is the sum of Eq. (2.8b) and Eq. (2.12), which is

free from ultraviolet singularities:

$$\begin{aligned}
\Re e \Lambda_1 = \frac{C_F \alpha_s}{2\pi} & \left\{ - \left(\frac{1 + \beta_0^2}{2\beta_0} \log \frac{1 + \beta_0}{1 - \beta_0} - 1 \right) \log \frac{m^2}{\lambda^2} \right. \\
& + \frac{1 + \beta_0^2}{\beta_0} \left[\text{Li}_2 \left(\frac{1 - \beta_0}{1 + \beta_0} \right) + \log \frac{1 + \beta_0}{2\beta_0} \log \frac{1 + \beta_0}{1 - \beta_0} - \frac{1}{4} \log^2 \frac{1 + \beta_0}{1 - \beta_0} + \frac{\pi^2}{3} \right] \\
& \left. + \frac{1 - \beta_0^2}{\beta_0} \log \frac{1 + \beta_0}{1 - \beta_0} - 1 \right\}, \tag{2.14}
\end{aligned}$$

and the lowest-order rate Eq. (2.7) is multiplied by a factor $1 + 2\Re e \Lambda_1$:

$$\Gamma_{1v}(H \rightarrow q\bar{q}) = \Gamma_0 \cdot 2\Re e \Lambda_1. \tag{2.15}$$

2.3 GLUON EMISSION CORRECTIONS

The virtual correction (2.14) still contains an infrared divergence. In order to obtain a finite result, one has to add corrections from the emission of a real gluon. In our treatment of the singularity, it is convenient to divide the gluon emission into soft and hard parts by a cutoff ω_0 ($\ll m$) in the gluon energy. The contributing diagrams are shown in Fig. 3.

Soft gluon corrections

The emission of a soft gluon is described by a universal factor multiplying the tree amplitude (up to a color factor):

$$\mathcal{M}_s = f g_s \bar{u}(p) \left(\frac{p^\alpha}{k \cdot p} - \frac{\bar{p}^\alpha}{k \cdot \bar{p}} \right) T^a v(\bar{p}) \varepsilon_\alpha^*. \tag{2.16}$$

Here ε is the polarization vector of the gluon, and T^a is the SU(3) generator in the fundamental representation. The soft-gluon emission probability integrated

over the soft-gluon phase space (defined as $E_g < \omega_0$) is

$$\begin{aligned}
\Delta_{1s} &= \sum_{\substack{\text{gluon} \\ \text{pol., color}}} \int_{E_g < \omega_0} d\Phi_g \left| g_s \left(\frac{\varepsilon^* \cdot p}{k \cdot p} - \frac{\varepsilon^* \cdot \bar{p}}{k \cdot \bar{p}} \right) \right|^2 \\
&= \frac{C_F \alpha_s}{\pi} \left\{ \left(\frac{1 + \beta_0^2}{2\beta_0} \log \frac{1 + \beta_0}{1 - \beta_0} - 1 \right) \log \frac{4\omega_0^2}{\lambda^2} \right. \\
&\quad + \frac{1 + \beta_0^2}{\beta_0} \left[\text{Li}_2 \left(\frac{1 - \beta_0}{1 + \beta_0} \right) + \log \frac{1 + \beta_0}{2\beta_0} \log \frac{1 + \beta_0}{1 - \beta_0} - \frac{1}{4} \log^2 \frac{1 + \beta_0}{1 - \beta_0} - \frac{\pi^2}{6} \right] \\
&\quad \left. + \frac{1}{\beta_0} \log \frac{1 + \beta_0}{1 - \beta_0} \right\}.
\end{aligned} \tag{2.17}$$

(The color factor C_F is effective in the sense that it is meaningful only in the rate.) The soft gluon correction to the decay rate is given by this factor multiplying the lowest-order rate:

$$\Gamma_{1s}(H \rightarrow q\bar{q}g) = \Gamma_0 \cdot \Delta_{1s}. \tag{2.18}$$

Adding this and virtual correction, we find that the λ dependence disappears, which shows the cancellation of the infrared singularity explicitly.

Hard gluon corrections

The amplitude for hard gluon emission is calculated from the diagrams in Fig. 3:

$$\mathcal{M}_h = f g_s \bar{u}(p) \left(\frac{\gamma^\alpha \not{k} + 2p^\alpha}{2k \cdot p} - \frac{\not{k} \gamma^\alpha + 2\bar{p}^\alpha}{2k \cdot \bar{p}} \right) T^a v(\bar{p}) \varepsilon_\alpha^*. \tag{2.19}$$

In general, there are only two independent kinematic variables in spin-summed three-body decays. (Although the three-body phase space has five variables, three of them reduce to trivial angular variables, on which the squared matrix element does not depend.) As the two independent variables, we may choose two

of the following three invariants:

$$\begin{aligned}
s &= (p + \bar{p})^2 = 2p \cdot \bar{p} + 2m^2, \\
t' &= 2p \cdot k = t - m^2, \\
u' &= 2\bar{p} \cdot k = u - m^2,
\end{aligned} \tag{2.20}$$

with the constraint $s + t' + u' = m_H^2$. Another choice is β and θ , defined as follows:

$$\beta = \sqrt{1 - \frac{4m^2}{s}}, \tag{2.21a}$$

$$p \cdot k = \frac{1}{4}(m_H^2 - s)(1 - \beta \cos \theta). \tag{2.21b}$$

In the c.m. frame of the $q\bar{q}$ system, β is the velocity of the quark, and θ is the angle between the quark and gluon directions.

The three-body phase space in terms of these variables is

$$\begin{aligned}
d\Phi_3 &= \frac{1}{128\pi^3 m_H^2} ds dt' \\
&= \frac{m^2}{32\pi^3} \frac{(\beta_0^2 - \beta^2)\beta^2}{(1 - \beta^2)^3} d\beta d\cos\theta.
\end{aligned} \tag{2.22}$$

The spin- and color-summed matrix element squared is

$$\begin{aligned}
\sum |\mathcal{M}_h|^2 &= 4f^2 N_C C_F g_s^2 \left[\frac{u'}{t'} + \frac{t'}{u'} + \frac{2(q^2 - 2m^2)(s - 4m^2)}{t'u'} + 2 \right. \\
&\quad \left. + 4m^2 \left(\frac{1}{t'} + \frac{1}{u'} \right) - 2m^2(q^2 - 4m^2) \left(\frac{1}{t'^2} + \frac{1}{u'^2} \right) \right] \\
&= 16f^2 N_C C_F g_s^2 \left\{ \left[1 + \frac{2sq^2\beta_0^2}{(q^2 - s)^2} \right] \frac{1}{1 - \beta^2 \cos^2\theta} \right. \\
&\quad \left. - \frac{8m^2 q^2 \beta_0^2}{(q^2 - s)^2} \frac{1}{(1 - \beta^2 \cos^2\theta)^2} \right\}.
\end{aligned} \tag{2.23}$$

Integrating over the three-body phase space with the cutoff $E_g > \omega_0$ gives^{*}

$$\begin{aligned}
\Gamma_{1h}(H \rightarrow q\bar{q}g) &= \Gamma_0 \\
&\times \frac{C_F\alpha_s}{\pi} \left\{ \left(\frac{1+\beta_0^2}{2\beta_0} \log \frac{1+\beta_0}{1-\beta_0} - 1 \right) \log \frac{m^2}{4\omega_0^2} \right. \\
&\quad + \frac{1+\beta_0^2}{\beta_0} \left[2\text{Li}_2\left(\frac{1-\beta_0}{1+\beta_0}\right) + 2\text{Li}_2\left(-\frac{1-\beta_0}{1+\beta_0}\right) \right. \\
&\quad \left. \left. - \log \frac{2}{1+\beta_0} \log \frac{1+\beta_0}{1-\beta_0} + \frac{1}{2} \log^2 \frac{1+\beta_0}{1-\beta_0} - \frac{\pi^2}{6} \right] \right. \\
&\quad \left. - 3 \log \frac{4}{1-\beta_0^2} - 4 \log \beta_0 \right. \\
&\quad \left. + \frac{1}{16\beta_0^3} (3 + 2\beta_0^2 + 3\beta_0^4) \log \frac{1+\beta_0}{1-\beta_0} + \frac{1}{8\beta_0^2} (-3 + 29\beta_0^2) \right\}. \tag{2.24}
\end{aligned}$$

Summing all virtual, soft, and hard corrections, it is readily seen that all singularities cancel and no ω_0 dependence remains:

$$\begin{aligned}
\Gamma_1(H \rightarrow q\bar{q} + q\bar{q}g) &= \Gamma_{1v} + \Gamma_{1s} + \Gamma_{1h} \\
&= \Gamma_0 \cdot \frac{C_F\alpha_s}{\pi} \Delta_H, \tag{2.25}
\end{aligned}$$

where

$$\Delta_H = \frac{1}{\beta_0} A(\beta_0) + \frac{1}{16\beta_0^3} (3 + 34\beta_0^2 - 13\beta_0^4) \log \frac{1+\beta_0}{1-\beta_0} + \frac{3}{8\beta_0^2} (-1 + 7\beta_0^2), \tag{2.26}$$

* The gluon energy can be defined either in the Higgs rest frame or in the $q\bar{q}$ e.m. frame. In the limit $\omega_0 \ll m$, the result does not depend on this choice. Needless to say, one has to employ larger cutoffs to circumvent the soft and collinear singularities in realistic simulations of the three-jet rate.

with

$$\begin{aligned}
A(\beta_0) = (1 + \beta_0^2) & \left[4\text{Li}_2\left(\frac{1 - \beta_0}{1 + \beta_0}\right) + 2\text{Li}_2\left(-\frac{1 - \beta_0}{1 + \beta_0}\right) - 3\log\frac{2}{1 + \beta_0}\log\frac{1 + \beta_0}{1 - \beta_0} \right. \\
& \left. - 2\log\beta_0\log\frac{1 + \beta_0}{1 - \beta_0} \right] - 3\beta_0\log\frac{4}{1 - \beta_0^2} - 4\beta_0\log\beta_0.
\end{aligned}
\tag{2.27}$$

This result agrees with Braaten and Leveille,² who employed dimensional regularization to treat both ultraviolet and infrared divergences. The final result thus does not depend on how the singularities are regularized, as it should. Since we have used the on-shell renormalization scheme, the quark mass parameter m should be regarded as the position of the pole of the quark propagator, which may be estimated using QCD sum rules.⁶

We remark that the results shown above hold also for scalar bosons in multi-Higgs models, provided that the proper Yukawa coupling f is used. (Only Eq. (2.3) and the last equality of (2.7) should be modified.)

So far we have discussed the QCD correction to $H \rightarrow q\bar{q}$, but our result is directly applicable to the $\mathcal{O}(\alpha)$ QED correction to $H \rightarrow f\bar{f}$ (f being a quark or lepton) with the replacement $C_F\alpha_s \rightarrow \alpha e_f^2$ (and $N_C \rightarrow 1$ for a lepton).

2.4 COMPARISON WITH JANOT'S RESULT

In Ref. 4, Janot provides some detail of his calculation, which makes it possible to pin down the origin of the difference to a certain degree. (Hereafter we quote the equations in Ref. 4 as (J*m.n.*.) First, his result of real emission (J5.8) is in agreement with ours, if we correct a misprint; the factor β_0^2 in the denominator of the infrared-singular term in (J5.8) (on the first line) should be β_0 . In addition, the expression for the second Spence function is somewhat ambiguous; the argument should be squared, not the function itself.

Hence the difference should have stemmed from the virtual corrections. His result appears in Eq. (J4.7). Besides the same misprint in the infrared singular

term (β_0^2 should be β_0), we find two disagreements. In the notation of Ref. 4, we obtained the constant factor of -1 instead of -2 , and for the coefficient of $\ln((1 - \beta_0)/(1 + \beta_0))$ (the last term), we found $-(1 - \beta_0^2)/\beta_0$ instead of $-(1 + 2\beta_0^2)/2\beta_0$. We cannot track down the reason of discrepancy further, except that Janot seems to have used the four-dimensional γ matrix algebra instead of the D -dimensional one (see the sentence above (J4.5)), which can lead to inconsistent results in dimensional regularization. Moreover, there is no remark how the renormalization was actually done.

2.5 COMPARISON WITH QCD SUM RULE CALCULATIONS

Closely related calculations with ours have been made in the context of QCD sum rules. Reinders *et al.*⁷ calculated the $\mathcal{O}(\alpha_s)$ QCD correction to the two-point function of the scalar current, which is similar to our calculation. In their calculation, however, there is no principle to fix the renormalization condition, because their scalar current is just an external current which has nothing to do with the quark mass. Therefore, they subtract the vertex function at zero momentum, giving a result which is slightly different from ours.

Similar calculations were made for the pseudoscalar current also. The result in Ref. 7 was derived with zero momentum subtraction and differs from our result for P decay. However, the result in a review⁶ was derived from the divergence of the axial vector current and is in agreement with ours.

A related calculation of the QED corrections to the decay of a pseudoscalar boson into a lepton pair has been done by Bergström.⁸ He restricted his calculation to the case $m_P \gg m$ and obtained a result which is identical to Eq. (4.2b), using the correct renormalization condition.

3. Pseudoscalar Higgs decay

Let us now turn to the hadronic decay of a pseudoscalar Higgs boson P . Because the steps are parallel to that for a scalar H , we will only mention the main differences. The Yukawa interaction is taken as

$$\mathcal{L} = f \bar{q} i \gamma_5 q P, \quad (3.1)$$

and the lowest-order rate is

$$\Gamma_0(P \rightarrow q\bar{q}) = \frac{N_C f^2}{8\pi} m_P \beta_0. \quad (3.2)$$

Note that the threshold behavior is S wave, in contrast to the P wave behavior of $H \rightarrow q\bar{q}$. The virtual correction factor differs slightly:

$$\Re e \Lambda_{1a}(P \rightarrow q\bar{q}) = \Re e \Lambda_{1a}(H \rightarrow q\bar{q}) - \frac{1 - \beta_0^2}{\beta_0} \log \frac{1 + \beta_0}{1 - \beta_0} \quad (q^2 > 4m^2), \quad (3.3)$$

i.e., the single logarithm term is absent. The renormalization counterterm is exactly the same as for the scalar case, as is discussed in the last section. The soft-gluon factor is also the same. The hard-gluon emission matrix element squared is

$$\begin{aligned} \sum |\mathcal{M}_h|^2 &= 4f^2 N_C C_F g_s^2 \left[\frac{u'}{t'} + \frac{t'}{u'} + \frac{2q^2(s - 2m^2)}{t'u'} + 2 \right. \\ &\quad \left. - 2m^2 q^2 \left(\frac{1}{t'^2} + \frac{1}{u'^2} \right) \right] \\ &= 16f^2 N_C C_F g_s^2 \left\{ \left[1 + \frac{2sq^2}{(q^2 - s)^2} \right] \frac{1}{1 - \beta^2 \cos^2 \theta} \right. \\ &\quad \left. - \frac{8m^2 q^2}{(q^2 - s)^2} \frac{1}{(1 - \beta^2 \cos^2 \theta)^2} \right\}, \end{aligned} \quad (3.4)$$

and the integrated rate is in the form of (2.24), with the last line changed to

$$+\frac{1}{16\beta_0}(3+2\beta_0^2+3\beta_0^4)\log\frac{1+\beta_0}{1-\beta_0}+\frac{1}{8}(29-3\beta_0^2). \quad (3.5)$$

Summing all the corrections, we find

$$\Gamma_1(P \rightarrow q\bar{q} + q\bar{q}g) = \Gamma_0(P \rightarrow q\bar{q}) \cdot \frac{C_F\alpha_s}{\pi}\Delta_P, \quad (3.6)$$

where

$$\Delta_P = \frac{1}{\beta_0}A(\beta_0) + \frac{1}{16\beta_0}(19+2\beta_0^2+3\beta_0^4)\log\frac{1+\beta_0}{1-\beta_0} + \frac{3}{8}(7-\beta_0^2). \quad (3.7)$$

4. Limiting behaviors

The correction factors Δ_H and Δ_P show interesting behaviors in the limiting cases.

Near the threshold $m_H \rightarrow 4m$ ($\beta_0 \rightarrow 0$), the corrections are positive and develop a $1/\beta_0$ singularity:

$$\Delta_H \simeq \frac{\pi^2}{2\beta_0} - 1, \quad (4.1a)$$

$$\Delta_P \simeq \frac{\pi^2}{2\beta_0} - 3. \quad (4.1b)$$

It can be seen that these terms solely come from virtual corrections; both soft and hard gluon corrections vanish at the threshold. The $1/\beta_0$ singularity can be understood as coming from the color-Coulomb interaction between the quark and antiquark, and signals the formation of bound states.** A realistic treatment of this threshold region requires the inclusion of the mixing of the Higgs boson with the quarkonium states, both below and above the open flavor threshold. This phenomenon has been fully discussed in Ref. 3.

** For further discussion, see Ref. 7.

In the opposite limit $m_H \gg m$ ($\beta_0 \rightarrow 1$), the corrections change the sign. Moreover, they contain a logarithmic singularity

$$\Delta_H \simeq -\frac{3}{2} \log \frac{m_H^2}{m^2} + \frac{9}{4}, \quad (4.2a)$$

$$\Delta_P \simeq -\frac{3}{2} \log \frac{m_H^2}{m^2} + \frac{9}{4}. \quad (4.2b)$$

The asymptotic equality of the scalar and pseudoscalar corrections indicates the restoration of chiral symmetry. The appearance of the logarithmic term is related to the ultraviolet behavior of the mass operator, which we further discuss in the next section.

Because of the log factor, the correction becomes large if the quark mass is much smaller than the Higgs mass, rendering the one-loop result inappropriate (the corrected rate eventually becomes negative). In this case, one can sum the leading logarithmic terms as discussed in Ref. 2. As long as the leading term is concerned, this is equivalent to replacing the quark mass parameter in the coupling with the running quark mass:

$$\Gamma(H \rightarrow q\bar{q}) = \frac{N_C G_F \tilde{m}(m_H)^2}{4\sqrt{2}\pi} m_H \beta_0^3, \quad (4.3)$$

where $\tilde{m}(m_H)$ is the running quark mass evaluated at m_H :

$$\begin{aligned} \tilde{m}(m_H) &= \tilde{m}(2m) \left[\frac{\alpha_s(m_H)}{\alpha_s(2m)} \right]^{12/(33-2N_f)} \\ &= \tilde{m}(2m) \left[\frac{\log(4m^2/\Lambda^2)}{\log(m_H^2/\Lambda^2)} \right]^{12/(33-2N_f)}, \end{aligned} \quad (4.4)$$

Λ is the QCD scale parameter, and N_f is the effective number of flavors. The running mass at the threshold $\tilde{m}(2m)$ may be replaced by m . The expression (4.3), however, only holds at $m_H \gg m$ and cannot be used closer to the threshold,

because the discarded finite $\mathcal{O}(\alpha_s)$ piece becomes comparable to the retained logarithmic term. We propose to use the following interpolation formula connecting the full one-loop expression near threshold and the leading-log result at large m_H :

$$\Gamma = \Gamma_0 \left[\frac{\log(4m^2/\Lambda^2)}{\log(m_H^2/\Lambda^2)} \right]^{24/(33-2N_f)} \left[1 + \frac{C_F \alpha_s}{\pi} \left(\Delta_H + \frac{3}{2} \log \frac{m_H^2}{m^2} \right) \right]. \quad (4.5)$$

How does the leading-log summation affect the $q\bar{q}g$ rate? The answer lies in the origin of the final log factor. The general theorem of the cancellation of mass singularities ensures that all soft and collinear singularities cancel in the rate. The remaining logarithm thus should come from the renormalization,² which suggests that the logarithm belongs to the overall Yukawa coupling factor. Therefore, the quark mass parameter in the coupling f in the rate for $q\bar{q}g$ should also be replaced by the running mass.

5. Conclusion

In this note we described the computation of the $\mathcal{O}(\alpha_s)$ QCD correction to the hadronic decay of a scalar and pseudoscalar Higgs boson. We gave the virtual and real corrections separately and the differential rate for the real emission $H, P \rightarrow q\bar{q}g$ is also given. The distribution for $P \rightarrow q\bar{q}g$ has not appeared elsewhere. A subtlety in the renormalization of the Yukawa coupling is discussed in detail. The difference between our result and that of Janot is shown to arise from the virtual correction which crucially depends on the renormalization condition.

The difference of the two results becomes prominent in the large Higgs mass limit $m_H \gg m$. If we denote the rate by $\Gamma = \Gamma_0 + \Gamma_1$, where Γ_0 is the lowest order result and Γ_1 is the $\mathcal{O}(\alpha_s)$ correction, Janot⁴ obtains a constant correction factor in this limit:

$$\frac{\Gamma_1^{(\text{Janot})}}{\Gamma_0} = \frac{5}{4} \frac{C_F \alpha_s}{\pi}. \quad (5.1)$$

On the other hand, Braaten and Leveille² and we found

$$\frac{\Gamma_1}{\Gamma_0} = \left(-\frac{3}{2} \log \frac{m_H^2}{m^2} + \frac{9}{4} \right) \frac{C_F \alpha_s}{\pi} . \quad (5.2)$$

The appearance of the logarithm may seem strange at first glance: Indeed, the QCD correction for a vector current (*e.g.* for the R ratio in $e^+e^- \rightarrow \text{hadrons}$) is a constant $((4/3)(C_F \alpha_s/\pi))$, as is well known. This difference, however, can be understood as follows. The logarithmic term is absent for the R ratio since the vector current is conserved and its anomalous dimension vanishes. The scalar current, or the mass operator, however, has nonvanishing anomalous dimension, which manifest itself in the logarithmic correction. As is mentioned in the previous section, summing over leading logarithm turns the quark mass parameter in the coupling factor into the running quark mass.

This high-mass region $m_H \gg m$ has been subsequently studied by Sakai⁹ and Inami and Kubota¹⁰ in a more rigorous framework, and a Russian group even derived the $\mathcal{O}(\alpha_s^2)$ correction in this limit.¹¹ Since the nature of the logarithmic factor is so well understood, we believe that the result of Ref. 4 which does not have the logarithmic term is incorrect.

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FIGURE CAPTIONS

- Fig. 1. Feynman diagram for the lowest-order hadronic Higgs decay, $H \rightarrow q\bar{q}$. The momenta of the particles are defined in the parentheses.
- Fig. 2. Diagrams for the one-loop corrections for $H \rightarrow q\bar{q}$. (a): vertex correction; (b), (c): self-energy corrections; (d)–(f): counterterms.
- Fig. 3. Diagrams for real gluon emission, $H \rightarrow q\bar{q}g$. The momenta of the particles are defined in the parentheses.

Fig. 1

(a)

(d)

(b)

(e)

(c)

(f)

Fig. 2

Fig. 3