THE PHASE STRUCTURE OF THE U(1) HIGGS-FERMION LATTICE THEORY

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Abstract

We study the phase structure of the U(1) lattice scalar-fermion theory with Yukawa interaction in the mean field approximation. Four phases are found: spontaneously broken phase ($\langle\phi\rangle\neq 0$), symmetric phase ($\langle\phi\rangle=0$) with massless fermions, symmetric phase without them and antiferromagnetically ordered phase. The phase transition lines are in a good agreement with recent Monte Carlo simulations.

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1. Introduction

Nonperturbative study of the Higgs sector of the standard model of electroweak interaction has attracted much attention recently. In the case of heavy Higgs particle and heavy fermion (e.g. fourth generation) there are two large couplings: Higgs selfcoupling and Yukawa. Neglecting small couplings (gauge and Yukawa for light fermions) one obtains the scalar-fermion theory with Yukawa interaction, that should be studied nonperturbatively, e.g. on a lattice.

The lattice models of this type were investigated recently [1-13]. In this paper we extend our previous work [13] where 1-component model was considered and study in the mean field approximation (MFA) the lattice theory of complex scalar field with bare selfcoupling $\lambda=\infty$ and even number D of staggered fermion species. The action of the theory is:

$$\begin{aligned}
 S &= S_B + S_F + S_Y ; \quad S_B = -2k \sum_{x,\mu} \text{Re} \phi_x^* \phi_{x+\hat{\mu}} ; \\
 S_F &= \sum_{\alpha=1}^D \sum_{x,\mu} (\eta_{x\mu}/2) \bar{\chi}_x^\alpha (\chi_{x+\hat{\mu}}^\alpha - \chi_{x-\hat{\mu}}^\alpha) ; \\
 S_Y &= \sum_{\alpha=1}^{D/2} \sum_x Y \exp(i\varphi_x \theta_x) \bar{\chi}_x^\alpha \chi_x^\alpha + \sum_{\alpha=D/2+1}^D \sum_x Y \exp(-i\varphi_x \theta_x) \bar{\chi}_x^\alpha \chi_x^\alpha \\
 \eta_{x\mu} &= (-1)^{x_1+\dots+x_{\mu-1}} \text{ for } \mu > 1, \eta_{x1} = 1 ; \varphi_x = (-1)^{\sum_{\mu} x_\mu} ; \\
 \mu &= 1, 2, \dots, d ; \quad \phi_x = \exp(i\theta_x).
 \end{aligned} \tag{1}$$

The fields are defined in the sites of d -dimensional ($d=4$) hypercubic lattice. The action (1) is invariant under $U(1)$ chiral transformation that changes the phase of ϕ_x .

2. The mean field approximation for the scalar-fermion theory.

Integrating over fermion degrees of freedom in the partition function of the theory (1) we obtain the pure bosonic theory with the fermion determinant contribution:

$$\begin{aligned} Z &\equiv \exp(-NW) = \int (\prod_x d\theta_x) \int (\prod_{x,\alpha} d\chi_x^\alpha d\bar{\chi}_x^\alpha) \exp(-S) = \\ &= \int (\prod_x d\theta_x) \exp(-S_B + D \operatorname{Re} \ln \det M), \end{aligned} \quad (2)$$

where

$$\begin{aligned} M_{xy} &= \sum_{\mu} (\eta_{x\mu}/2) (\delta_{y, x+\hat{\mu}} - \delta_{y, x-\hat{\mu}}) + Y \exp(i\varphi_x \theta_x) \delta_{xy} \equiv \\ &\equiv K_{xy} + Y \exp(i\varphi_x \theta_x) \delta_{xy}. \end{aligned} \quad (3)$$

Following the ideas of [14,15] we rewrite the right hand side of (2) in the form of the sum over configurations of uncorrelated random field ϕ_x with the probability distribution $Z_H^{-1} \exp(\sum_x \operatorname{Re} H^* \phi_x)$, where H is the complex field (mean field) acting on each ϕ_x , apply the inequality $\langle \exp A \rangle \geq \exp \langle A \rangle$ and obtain the estimate for the free energy:

$$W \leq W(H) = N^{-1} \langle S_B - D \operatorname{Re} \ln \det M + \sum_x \operatorname{Re} H^* \phi_x \rangle_H - N^{-1} \ln Z_H, \quad (4)$$

where

$$\begin{aligned} Z_H &= \int (\prod_x d\theta_x) \exp(\sum_x \operatorname{Re} H^* \phi_x) = (2\pi I_0(|H|))^N, \\ \langle A \rangle_H &= Z_H^{-1} \int (\prod_x d\theta_x) \exp(\sum_x \operatorname{Re} H^* \phi_x) \cdot A. \end{aligned} \quad (5)$$

Using

$$\langle \phi_x \phi_{x+\hat{\mu}} \rangle_H = \langle \phi_x \rangle_H \langle \phi_{x+\hat{\mu}} \rangle_H, \quad \langle \phi_x \rangle_H \equiv \sigma = u'(|H|) \exp(i \arg H), \quad (6)$$

where $u(|H|) = N^{-1} \ln Z_H$, we find:

$$W(H) = -2kd(u'(|H|))^2 - N^{-1} \langle D \operatorname{Re} \ln \det M \rangle_H + |H| u'(|H|) - u(|H|). \quad (7)$$

The best estimate for W is $\min_H W(H)$ and the corresponding H is the selfconsistent mean field. The symmetric phase $H=0$ (i.e. $\langle \phi \rangle = 0$) can be stable if:

$$\left. \partial^2 W(H) / \partial |H|^2 \right|_{H=0} > 0 \quad (8)$$

otherwise the spontaneous symmetry breaking occurs.

For $Y=0$ and $Y=\infty$ the value of $\det M$ is independent of ϕ_x , the theory reduces to the XY-model with action S_θ and (7), (8) give that the symmetric phase becomes unstable at $k > 1/2d$.

To compute $\langle \ln \det M \rangle_H$ we perform the expansion in powers of Y :

$$\begin{aligned} \ln \det M &= \ln \det K + \ln \det (\delta_{xy} + (K^{-1})_{xy} Y \exp(i\zeta_y \theta_y)) = \\ &= \ln \det K - (Y^2/2) \sum_{xy} (K^{-1})_{xy} (K^{-1})_{yx} \exp(i\zeta_x \theta_x) \exp(i\zeta_y \theta_y) - \dots \end{aligned} \quad (9)$$

One can easily see, using (5), (6) that the Y^n term in (9) contributes $O(|H|^n)$ to $W(H)$. Thus only the Y^2 term affects the local stability of the symmetric phase (8):

$$\left. \partial^2 W(H) / \partial |H|^2 \right|_{H=0} = (1/2)(1 - 2kd - DY^2 c/2), \quad (10)$$

where

$$c \equiv -(K^{-2})_{xx} = \int d^d p (2\pi)^{-d} \left(\sum_{\mu} \sin^2 p_{\mu} \right)^{-1} = (2/d)(1 + O(1/d)). \quad (11)$$

From (8) we find that the symmetry breaking occurs when

$$k > (1/2d)(1 - DY^2 c/2). \quad (12)$$

This formula is correct at $Y=0$ but is obviously wrong at $Y=\infty$.

One can also expand $\ln \det M$ in powers of $1/Y$:

$$\ln \det M = \ln \det (Y \exp(i\zeta_x \theta_x) \delta_{xy}) + \ln \det (\delta_{xy} + K_{xy} Y^{-1} \exp(-i\zeta_y \theta_y)) =$$

$$= N \ln Y + i \sum_x \gamma_x \theta_x - (1/2 Y^2) \sum_{xy} K_{xy} K_{yx} \exp(-i \gamma_x \theta_x) \exp(-i \gamma_y \theta_y) - \dots \quad (13)$$

and obtain

$$\left. \frac{\partial^2 W(H)}{\partial |H|^2} \right|_{H=0} = (1/2)(1 - 2kd - Dd/4Y^2). \quad (14)$$

The condition for the symmetry breaking becomes:

$$k > (1/2d)(1 - Dd/4Y^2), \quad (15)$$

which is correct at $Y=\infty$ but not at $Y=0$. We suppose that the regions of applicability of (12) and (15) merge at some intermediate value $Y=Y_*$. Note that the MFA is correct in the leading order in $1/d$. Thus we get the selfconsistent large d picture if we neglect $O(1/d)$ terms in (11) and let $Y_* = \sqrt{d/2}$, where (12) and (15) coincide¹.

As a result we find approximate phase transition lines separating the symmetric ($\langle \phi \rangle = 0$) and broken ($\langle \phi \rangle \neq 0$) phases (fig.1, lines AA' and BB'):

$$k = \begin{cases} (1/2d)(1 - DY^2/d) , & Y < \sqrt{d/2} \\ (1/2d)(1 - Dd/4Y^2) , & Y > \sqrt{d/2} \end{cases} \quad (16)$$

3. The fermion propagator.

The fermion propagator in the MFA is given by $\langle (M^{-1})_{xy} \rangle_H$. (This result can be obtained by adding the sources $\sum_x \bar{\chi}_x^\alpha \eta_x^\alpha$, $\sum_x \bar{\eta}_x^\alpha \chi_x^\alpha$ to the action (1), applying the MFA described in sect.2 and

¹The value $Y = \sqrt{d/2}$ is remarkable also because $\det M$ vanishes for some configurations (e.g. when $\phi_x = 1$ in all sites except one where $\phi_x = -1$) at $Y^2 = (d/2)(1 + O(1/d))$. The same phenomenon occurs in the theory with 1-component real ϕ_x [13].

differentiating the approximate free energy over $\overline{\eta_x^\alpha}$ and η_x^α .)

In analogy with the expansion of $\ln \det M$ (9), (13) we expand the $(M^{-1})_{xy}$ in powers of Y for $Y^2 < d/2$ and in powers of $1/Y$ for $Y^2 > d/2$. In the symmetric phase ($\sigma=0$) one can easily compute the average of these expansions over uncorrelated random field ϕ_x and find that the fermion propagator is $(K^{-1})_{xy}$ for $Y^2 < d/2$ and is zero for $Y^2 > d/2$. Thus the region $\sigma=0$ is divided by the phase transition line $Y^2 = d/2$, characterized by the discontinuous behavior of the fermion propagator, into two different phases p and p' (see fig.1). The phase p contains massless fermions. In the phase p' the fermions do not propagate (i.e. they are infinitely heavy in the MFA). Such symmetric phase without massless fermions was predicted recently for real ϕ theory in [10,13]. The line of the phase transition should continue into the broken phase, because the fermion propagator is continuous at the second order phase transition line.

Computing the fermion propagator in coinciding points we get the MFA result for fermion condensate:

$$\langle \chi \bar{\chi} \rangle = \langle (M^{-1})_{xx} \rangle_H = \begin{cases} (2Y/d)\sigma + O(\sigma^3), & Y < \sqrt{d/2} \\ \sigma/Y + O(\sigma^3), & Y > \sqrt{d/2} \end{cases} \quad (17)$$

4. Antiferromagnetic (AFM) phase.

The XY-model possesses also the AFM phase. In our case one can easily check the invariance of the action (1) under the transformation [13]:

$$\begin{aligned} k &\rightarrow -k, \quad \phi_x \rightarrow \zeta_x \phi_x, \\ Y &\rightarrow -iY, \quad \chi_x \rightarrow \chi_x \exp(i\zeta_x \pi/4), \quad \bar{\chi}_x \rightarrow \bar{\chi}_x \exp(i\zeta_x \pi/4), \end{aligned} \quad (18)$$

which interchanges ferro- and antiferromagnetic ordering.

Combining this with the MFA result (16) we find the paramagnetic (symmetric) - AFM phase transition lines (fig.1, lines CC', EE'):

$$k = \begin{cases} -(1/2d)(1 + DY^2/d), & Y < \sqrt{d/2} \\ -(1/2d)(1 + Dd/4Y^2), & Y > \sqrt{d/2} \end{cases} \quad (19)$$

5. Comparison with Monte Carlo (MC) data

We compared our results with the MC simulations of the 4-dimensional lattice theory of complex scalar field with fixed length ($\lambda=\infty$) and two species of naive Dirac fermions carried by Stephenson and Thornton [3]. This theory is equivalent to (1) with $d=4$, $D=8$.

The phase diagram [3] that was computed in the region $0 < k < 0.19$, $0 < Y < 4$ agrees qualitatively with the MFA (fig.1). Table 1 presents the coordinates of the points where the second order phase transition lines obtained in [3] cross the $Y=0$ (at $k=k_c$) and $k=0$ (at $Y=Y_1, Y_2$) axes in comparison with the MFA. From (16) we find that

$$k_c = 1/2d, \quad Y_1 = \sqrt{d/D}, \quad Y_2 = \sqrt{Dd/4} \quad (20)$$

6. Conclusion

We applied the MFA to investigate the phase structure of the d -dimensional $U(1)$ scalar-fermion lattice theory with Yukawa interaction. We studied the local stability of the symmetric phase ($\langle \phi \rangle = 0$) and found four phases (fig.1). The lines of the second order phase transition are in a good agreement with the MC results [3]. As we studied only local stability of the symmetric phase, additional first order phase transition lines are possible. The approximate formula (17) for $\langle \bar{\chi}\chi \rangle$ were derived.

Our investigation shows that the most interesting phenomena

take place in the region $k < 0$ near $Y \approx \sqrt{d/2} \approx 1.4$. Therefore this region deserves careful MC study.

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Table 1. The coordinates of the points where the phase transition lines obtained by MC simulation in [3] and by the MFA (eqs.(16),(20)) cross $Y=0$ and $k=0$ axes.

Figure captions

Fig.1. Approximate phase diagram of the model (1). AA' , BB' , CC' , EE' are the second order phase transition lines. GG' is the line of discontinuous phase transition. The position of its ends is not clear. Letters f , af , p and p' denote ferro-, antiferro-, and two different paramagnetic phases.

Table 1

	k_c	Y_1	Y_2
4^4 lattice [3]	0.15	0.64	2.44
6^4 lattice [3]	0.16	0.68	2.48
MFA (eq.(20))	0.13	0.71	2.83

$D=8$