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Neutrino Counting at the Z-Peak and Right-Handed Neutrinos

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ABSTRACT

We consider the implications of extending the Minimal Standard Model, with n families of quarks and leptons, by introducing an arbitrary number of right-handed neutrinos, for neutrino-counting via the "invisible width" of the Z . It is shown that the effective number of neutrinos, $\langle n \rangle$, satisfies the inequality $\langle n \rangle \leq n$, where $\langle n \rangle$ is defined by $\Gamma(Z \rightarrow \text{neutrinos}) \equiv \langle n \rangle \Gamma_0$ and Γ_0 is the standard width for one massless neutrino. Thus, in the case of three families, the neutrino-counting can give a result which is less than three, if there are right-handed neutrinos.

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1. Introduction

Recently, after several years of patient waiting, we have had the great fortune of learning about the first results coming from the $e^+ - e^-$ colliders operating at the Z-pole. One of the most interesting questions, to be answered by these colliders, was "what is the number of families?". This question has now been answered, provided that one assumes the validity of the Minimal Standard Model [1] which is defined to have n families of quarks and leptons with the usual quantum numbers (left-handed doublets and right-handed singlets) and no right-handed neutrinos. The very first result came [2], in the summer of 1989, from the MARK II detector at SLAC, which reported $< n > = 3.8 \pm 1.4$. This number is defined by $\Gamma(Z \rightarrow \text{neutrinos}) \equiv < n > \Gamma_0$ where Γ_0 is the standard width for one massless neutrino. In the Minimal Standard Model this number equals the number of families n_{fam} . This number was much improved, after the coming into operation of the LEP accelerator. The results, as of the end of 1989, for the $< n >$, read

MARK II [2]	$= 3.8 \pm 1.4$
MARK II [3]	$= 2.8 \pm 0.6$
ALEPH [4]	$= 3.27 \pm 0.30$
ALEPH [5]	$= 3.01 \pm 0.15 \text{ (exp)} \pm 0.05 \text{ (theo)}$
DELPHI [6]	$= 2.4 \pm 0.4 \text{ (stat)} \pm 0.5 \text{ (syst)}$
L3 [7]	$= 3.42 \pm 0.48$
L3 [8]	$= 3.29 \pm 0.17$
OPAL [9]	$= 3.1 \pm 0.4$

The above results indicate very clearly that, within the Minimal Standard Model, the number of families is simply three. Indeed, at the present stage of our understanding, this result is marvelous. The general structure of the quark mixing matrix in the case of

four or more families is so unattractive that it seems as a "blessing" that Nature appears to be content with just the minimal structure needed to have both **family mixing and CP violation** [for a review see, for example [10]]. The higher families, again at the present stage of our understanding, look like irrelevant complications. However, to quote Dirac, "one should always guard against getting too attached to one particular line of thought". We certainly do not believe that the Minimal Standard Model is the final theory of elementary particles. There is most probably a "beyond", but we have no experimental clue as to how it looks like.

The simplest possible extensions of the Minimal Standard Model are obtained by adding right-handed neutrinos to the particle content of the theory. Doesn't the requirement of simplicity oblige us to exclude such extensions? We feel that the answer is no. In fact, there is a major difference between i) adding more families to what we have got and ii) introducing one or more right-handed neutrinos. In the former case we do not obtain any fundamentally new phenomenon, which was not there with only three families. The general structure is just as before but more complicated: there are many more terms, of the same kind as before, in our Lagrangian. The introduction of right-handed neutrinos, on the other hand, is very different. By introducing them we can get new structures such as massive neutrinos, lepton mixing and CP violation in the leptonic sector. Indeed, right-handed neutrinos are natural inhabitants of many theoretical models. We would like to know as much as possible about them, the first question being, of course, do they exist or not? In this letter we study the implication of right-handed neutrinos for neutrino-counting measurements via the "invisible width" of the Z. The major result obtained is a "theorem" which states that the measured value, $< n >$, is always less than or equal to the true number of families, if one adds right-handed neutrinos to the particle spectrum of the Minimal Standard Model.

The plan of this paper is as follows: in Section 2, we briefly discuss the simplest possible extension of the Minimal Standard Model in which only a single right-handed

neutrino is assumed to exist. This scheme was recently studied by the author [1]. Here, we shall only quote the results and then, in Section 3, present the generalization, concerning the neutrino-counting, for the case of arbitrary numbers of families and right-handed neutrinos. Finally, our conclusions are presented in Section 4.

2. The Simplest Scheme

The simplest possible extension of the Minimal Standard Model is obtained by adding just a single right-handed neutrino to the particle content of the Minimal Three-Family Standard Model. We assume, as usual, that there is a single Higgs doublet. In the most general case, however, there may exist n families of quarks and leptons and $N - n$ right-handed neutrinos whereby the leptonic content of the theory would be

$$\begin{pmatrix} \nu_{jL}^c \\ l_{jL}^c \end{pmatrix}, \quad l_{jR}^c, \quad \nu_{kR}^c, \quad j = 1, 2, \dots, n; \quad k = n, \dots, N \quad (1)$$

Here the primes on the matter fields denote that these fields are not yet the physical ones. Note that in the Standard Model there is no known reason why the number of the left-handed and right-handed neutrinos should be equal.

One class of popular models, in which right-handed neutrinos appear is the so-called "see-saw" models [2], where one argues that the reason for the known neutrinos being light is that there are as many right-handed neutrinos which are very heavy. These models have some attractive features but they are far from being compelling because of two reasons:

- i) Even if we set aside the neutrinos, the spectrum of fermion masses is very strange. Indeed, $m_e/v \simeq 10^{-6}$, where $v \simeq 250 \text{ GeV}$ is the scale of symmetry breaking, while the quantity m_t/v is of the order of unity. Yet both the electron and the top mass are produced by the same "mother Higgs", in the Standard Model.
- ii) The second problem with the "see-saw" mechanism is that as soon as one intro-

duces more than one right-handed neutrino the picture becomes much more complicated. In general, one must make further assumptions to obtain say 3 large mass scales, in order to get three light neutrinos. The point is that the elements of the mass matrix of the right-handed neutrinos can all have large magnitudes and yet its determinant can vanish, as happens in "democratic" scenarios, where all the matrix elements are equal. Therefore, we feel that "one should not get too attached" to the see-saw thinking. We shall not assume that the right-handed neutrinos are very heavy. We feel that the question of their mass, provided they exist at all, has to be settled by experiment.

The leptonic Lagrangian, containing the particles in Eq. (1), is of the form

$$L = L(W) + L(Z) + L(H, l) + L(H, \nu) + L(l - mass) + L(\nu - mass) \quad (2)$$

We shall assume, without loss of generality, that the Lagrangian in Eq. (2) is written in a basis where the lepton mass matrix, in $L(l - mass)$, has been diagonalized and the leptonic mixing matrix, in $L(W)$, is the unit matrix (for details see [1]). In the Minimal Standard Model the terms $L(\nu - mass)$ and $L(H, \nu)$ are missing. Introduction of right-handed neutrinos brings in more parameters into the theory. These new parameters all enter through the term $L(\nu - mass)$ which contains the neutrino mass matrix. In the simplest extension of the Three-Family Standard Model, where only a single right-handed neutrino is added, $L(\nu - mass)$ is of the form

$$L(\nu - mass) = - \sum_{j=1}^3 g_j \bar{\nu}_{jL}^c \nu_R^c - \frac{M}{2} \bar{\nu}_R^c \nu_R^c + h.c.$$

Written in matrix form this equation reads

$$L(\nu - mass) = - \frac{1}{2} \bar{\mathbf{N}} \begin{pmatrix} 0 & 0 & 0 & a_1 \\ 0 & 0 & 0 & a_2 \\ 0 & 0 & 0 & a_3 \\ a_1 & a_2 & a_3 & M \end{pmatrix} \mathbf{N}^c + h.c. \quad (3)$$

where

$$\mathbf{N} = \text{col}(\nu_{1L}^c, \nu_{2L}^c, \nu_{3L}^c, \nu_R^c)$$

In Eq. (3) we have used the identity $\overline{\nu_{jL}} \nu_R = \nu_R^C \nu_{jL}$. The a_j , $j = 1, 2, 3$, and M are four constants. Without loss of generality, one may take all four of them to be positive. M describes the mass parameter of the right-handed neutrino. It is there because we have no right to exclude it. Its nature is similar to that of the electron mass in pure QED or the quark mass in pure QCD. The a_j are different. They arise from the couplings of the lepton doublets to the Higgs and the right-handed neutrino. In [11], the diagonalization of the neutrino mass matrix was described and a convenient parametrization for the ensuing lepton mixing matrix was presented. In this simplest scheme, there are four physical neutrinos which we denote by ν_1 , ν_2 , ν_P , ν_F . The first two are massless while the last two are massive Majorana neutrinos with masses

$$m(\nu_P) = \frac{1}{2}(\sqrt{M^2 + 4a^2} - M), \quad m(\nu_F) = \frac{1}{2}(\sqrt{M^2 + 4a^2} + M) \quad (4)$$

where

$$a^2 = a_1^2 + a_2^2 + a_3^2 \quad (5)$$

In [11], we also showed that, when we rewrite the charged currents, $L(W)$, in terms of the physical fields we get a leptonic mixing, V , of the form

$$V = \Phi R \quad (6)$$

where Φ is a diagonal phase matrix and R is an orthogonal matrix. The matrix Φ can be taken to be

$$\Phi = \text{diag}(1, 1, i, 1)$$

and the orthogonal matrix R is displayed below, viz.,

$$L(W) \sim (\overline{\nu_{1L}}, \overline{\nu_{2L}}, \overline{\nu_{PL}}, \overline{\nu_{FL}}) \gamma^\lambda \Phi \begin{pmatrix} c_\beta & -s_\beta s_\gamma & -s_\beta c_\gamma & 0 \\ 0 & c_\gamma & -s_\gamma & 0 \\ c_\alpha s_\beta & c_\alpha c_\beta s_\gamma & c_\alpha c_\beta c_\gamma & -s_\alpha \\ s_\alpha s_\beta & s_\alpha c_\beta s_\gamma & s_\alpha c_\beta c_\gamma & c_\alpha \end{pmatrix} \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \\ 0 \end{pmatrix} W_\lambda \quad (7)$$

to which we have to add its hermitian conjugate. Here c and s denote the cosine and sine of their respective arguments. The angles α , β and γ , lie in the first quadrant and are related to the parameters of the mass matrix through

$$\begin{aligned} s_\alpha &= \sqrt{m(\nu_P)/(m(\nu_P) + m(\nu_F))} \\ s_\beta &= a_1/a \\ c_\beta s_\gamma &= a_2/a \\ c_\beta c_\gamma &= a_3/a \end{aligned} \quad (8)$$

Note that we have four neutrinos but only three charged leptons. Therefore, we have extended the charged lepton column matrix (by putting a zero on the fourth row) in order to be able to manifestly exhibit the unitarity property of the four by four lepton mixing matrix. Thus the four parameters of this minimal scheme are three angles and one scale parameter, with the dimension of a mass, e.g., one of the two Majorana masses. The reason for the presence of the phase matrix Φ was given in [11]. The origin of such phases, which have nothing to do with CP violation, is well understood [13]. When computing the invisible width of the Z , the phase matrix is irrelevant because we only require the moduli of the elements of the lepton mixing matrix. Another essential point about our parametrization of the lepton mixing matrix is that we have, without loss of generality, introduced a convention where the electron does not couple to the second massless neutrino (see [11] for a discussion of this point).

The coupling matrix of the neutrinos to the Z , written in the physical basis, reads

where x_{pp} is the phase space suppression factor due to the non-zero mass of ν_F , and so on. These suppression factors are given by

$$x_{ij} = \sqrt{\lambda(M_{2i}^2, m_i^2, m_j^2)/M_Z^2} \quad (14)$$

Here λ is the triangular function defined by

$$\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + bc + ca)$$

Thus x_{ij} are bounded by unity whereby $\Gamma(Z \rightarrow \nu's) \leq 3\Gamma_0$. Note that since the ν_P and ν_F are Majorana neutrinos their diagonal couplings to the Z is purely axial and their nondiagonal interaction, $Z \rightarrow \nu_P + \nu_F$, is purely vector. These facts are irrelevant, insofar as our arguments concerning the neutrino-counting are concerned.

3. Arbitrary Numbers of Families and Right-Handed Neutrinos

From the structure of Eqs. (10) and (12), we see that the above result may easily be generalized to arbitrary numbers of families and of right-handed neutrinos. Hence we have the following theorem:

Theorem:

In the Standard Model, with n left-handed lepton doublets and $N - n$ right-handed neutrinos, the effective number of neutrinos, $\langle n \rangle$, defined by

$$\Gamma(Z \rightarrow \text{neutrinos}) \equiv \langle n \rangle \Gamma_0$$

where Γ_0 is the standard width for one massless neutrino, satisfies the inequality

$$\langle n \rangle \leq n \quad (15)$$

Proof: With n doublets and $N - n$ right-handed neutrinos the matrix exhibited in Eq. (9) has n ones and $N - n$ zeros along its diagonal and the matrix $\mathbf{V}$ is a N by N unitary matrix. The flavour coefficients, for $Z \rightarrow \nu's$, are then given by

$$N_{\rho\sigma} = \sum_{j,k=1}^n \mathbf{V}_{\rho j} \delta_{jk} \mathbf{V}_{\sigma k}^* = \sum_{j=1}^n \mathbf{V}_{\rho j} \mathbf{V}_{\sigma j}^* \quad (16)$$

$$L(Z, \nu) \sim (\overline{\nu_L}, \overline{\nu_{2L}}, \overline{\nu_{FL}}) \gamma^\lambda \mathbf{V} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_{2L} \\ \nu_F \\ \nu_{FL} \end{pmatrix} Z_\lambda \quad (9)$$

The flavour structure of Eq. (9) is of the form

$$\overline{\nu_\rho} N_{\rho\sigma} \nu_\sigma$$

where the indices ρ and σ , running from one to four, denote our four physical neutrino states. Using the unitarity of the lepton mixing matrix, we find

$$N_{\rho\sigma} = \sum_{j,k=1}^3 \mathbf{V}_{\rho j} \delta_{jk} \mathbf{V}_{\sigma k}^* = \delta_{\rho\sigma} - \mathbf{V}_{\rho 4} \mathbf{V}_{\sigma 4}^* \quad (10)$$

Thus we obtain

$$L(Z, \nu) \sim (\overline{\nu_L}, \overline{\nu_{2L}}, \overline{\nu_{FL}}) \gamma^\lambda \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c_\alpha^2 & i c_\alpha s_\alpha \\ 0 & 0 & -i c_\alpha s_\alpha & s_\alpha^2 \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_{2L} \\ \nu_F \\ \nu_{FL} \end{pmatrix} Z_\lambda \quad (11)$$

A very important property of the neutral current coupling coefficients is that

$$\sum_{\rho\sigma} |N_{\rho\sigma}|^2 = 3 \quad (12)$$

The 3, on the right hand side of Eq. (12), is simply the number of leptonic doublets, which equals the number of families. This means that, due to unitarity of the lepton mixing matrix, the invisible width of the Z cannot exceed $3\Gamma_0$, where Γ_0 is the width for each massless neutrino pair. Therefore, the invisible width of the Z , in this minimal scheme, is always smaller than predicted in the Minimal Standard Model. The reason is as follows. From Eq. (11), we have that

$$\Gamma(Z \rightarrow \nu's) = \Gamma_0 \{ 2 + c_\alpha^4 x_{FF} + 2c_\alpha^2 s_\alpha^2 x_{FF} + s_\alpha^4 x_{FF} \} \quad (13)$$

where now ρ and σ run from 1 to N . The width of Z decaying to neutrino pairs is then given by $\langle n \rangle > \Gamma_0$ where

$$\langle n \rangle = \sum_{\rho, \sigma=1}^N |N_{\rho\sigma}|^2 x_{\rho\sigma} \leq \sum_{\rho, \sigma=1}^N |N_{\rho\sigma}|^2 \quad (17)$$

where we have used the fact that the phase space suppression factors satisfy $x_{\rho\sigma} \leq 1$ [see Eq. (14)]. Substituting Eq. (16) into Eq. (17) and performing the sums, by using the unitarity of V , we find $\langle n \rangle \leq n$. This result is somewhat discomforting. It means that, strictly speaking, one can not draw a definite conclusion on the number of families from neutrino-counting at the Z -peak, if there are right-handed neutrinos. Mathematically speaking, we could have, say, four families and a few right-handed neutrinos and yet get three from neutrino-counting. Cosmological and other arguments are needed to examine such unusual possibilities. The most exciting possibility would be to find a number slightly less than three.

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