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A Comment on the t-Quark Mass

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ABSTRACT

Phenomenological considerations lead us to conjecture a relation linking lepton and quark masses: $\sqrt{m_c/m_t} + \sqrt{m_s/m_b} = \sqrt{m_\mu/m_\tau}$. Consequences for the t mass are discussed.

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The observed hierarchy in the mass spectrum of the quarks and the charged leptons clearly singles out the third family: the masses of the τ -lepton and of the b and t quarks dominate the spectrum in their respective channels to such an extent that in a first approximation the masses of the first and second family members appear negligible. This certainly suggests a "cascade" scenario for the mass generation dynamics: in the first stage only members of the third family would acquire a mass while in a second stage the masses of the second family would be generated through "radiative corrections" involving the masses of the third family, and finally in a last stage the masses of the first family would be obtained as "second order effects". In this note we shall deal exclusively with the generation of mass for the second family.

A particular implementation of the first stage of this scenario is provided by the so-called democratic mass matrices¹⁾. The subsequent stages have been parametrized recently²⁾.

Although one has no clues on the dynamical mechanism of mass generation — perhaps related to a substructure of the quarks and the leptons — one would certainly expect that the masses of the members of the first and second generations reflect the dynamical mixing between these families and the third one: if there is no mass, there is no mixing.

One particular way to describe the relations between mass eigenvalues and the observable electroweak mixing angles was discussed some time ago³⁾. There the strength of the mixing between the third and the second family is given by

$$\text{the ratio } \sqrt{\frac{m_\mu}{m_\tau}}, \sqrt{\frac{m_s}{m_b}}, \sqrt{\frac{m_c}{m_t}}, \text{ respectively.}$$

More generally one may expect the ratio $\sqrt{\frac{m_\mu}{m_\tau}}, \sqrt{\frac{m_s}{m_b}}, \sqrt{\frac{m_c}{m_t}}$, respectively) to describe the amount of "leakage" of the third family into the second one, giving rise to the mass of the corresponding member of the second family. For this reason it is interesting to consider these ratios more in detail. We use the values (see, for example, ref. (4)):

$$m_s[1 \text{ GeV}] = 175 \pm 55 \text{ MeV} \quad m_c[1 \text{ GeV}] = 1350 \pm 50 \text{ MeV.}$$

$$m_b[1 \text{ GeV}] = 5300 \pm 100 \text{ MeV}$$

Furthermore we shall assume $80 \text{ GeV} < m_t(m_t) < 250 \text{ GeV}$. The lower limit is given by the experimental searches for the t-quark effects in the collider experiments. The upper limit follows from the measurements of the ρ -parameter. In fact these measurements give⁵⁾ $m_t \leq 200 \text{ GeV}$, but in view of the uncertainties involved we allow a slightly larger range (see also ref. (6)). One obtains:

$$\frac{m_\mu}{m_\tau} = 0.0592 \pm 0.0001 \quad (= \alpha \cdot 8.113) \quad \sqrt{\frac{m_\mu}{m_\tau}} = 0.2433 \pm 0.0002 \quad (1)$$

$$\frac{m_s}{m_b} = 0.033 \pm 0.011 \quad (= \alpha \cdot 4.5 \pm 1.5) \quad \sqrt{\frac{m_s}{m_b}} = 0.18 \pm 0.03$$

$$\frac{m_c}{m_t} = 0.003 \dots 0.010 \quad (= \alpha \cdot (0.41 \dots 1.37)) \quad \sqrt{\frac{m_c}{m_t}} = 0.055 \dots 0.10.$$

Nothing is known about the neutrino mass ratio $m_{\nu_\mu} / m_{\nu_\tau}$. We have indicated that the mass ratios can be written as the finestructure constant α multiplied with a coefficient of the order of one, which suggests to interpret the masses of the members of the second generation as radiative corrections.

We find from (1):

$$\sqrt{\frac{m_\mu}{m_\tau}} \sqrt{\frac{m_s}{m_b}} = 0.06 \pm 0.03 \quad \sqrt{\frac{m_c}{m_t}} = 0.055 \dots 0.10, \quad (2)$$

i.e. a large part of both allowed regions overlap. This observation leads us to propose for phenomenological reasons the relation:

$$\sqrt{\frac{m_s}{m_b}} + \sqrt{\frac{m_c}{m_t}} = \sqrt{\frac{m_\mu}{m_\tau}} \quad (3)$$

We suppose this is an exact relation between the lepton and quark masses. In this case we obtain from (2) and (3):

$$\sqrt{\frac{m_c}{m_t}} = 0.055 \dots 0.090,$$

which gives:

$$100 \text{ GeV} < m_t < 250 \text{ GeV.} \quad (4)$$

(We use a QCD rescaling factor $m_t(m_t)/m_t(1 \text{ GeV}) = 0.6$, appropriate for the mass region considered here.)

In case of the central values of the quark masses, in particular for $m_s = 175 \text{ MeV}$, we obtain from (3):

$$\begin{aligned} m_t(1 \text{ GeV}) &\approx 200 \text{ GeV.} \\ m_t(m_t) &\approx 120 \text{ GeV.} \end{aligned} \quad (5)$$

The range of t-quark mass values given in eq. (4) is in good agreement with the range one obtains from considering higher order radiative corrections of the Z-parameters⁷⁾.

The relation (3) brings to mind two analogies: one is with the universality of the weak force which implies that the sum of the squares of the β -decay and Λ -decay constants equals the square of the μ -decay constant. The second analogy is with anomaly cancellation, e.g. the one which implies that the sum of the charges in each generation vanishes.

Whether a universality or anomaly cancellation concept could be useful for the understanding of the yet unknown mechanism of mass generation remains of course to be seen.

The relation (3) leaves no room for a contribution from the neutrinos on the lepton side. If neutrinos are massless, this poses no problem. If they are massive, we would conclude that only the τ -neutrino can acquire a sizeable mass, while ν_μ and ν_e are nearly or exactly massless. Furthermore no neutrino mixing is present either.

Numerically the square root ratios could be consistent with

$$\sqrt{\frac{m_c}{m_t}} : \sqrt{\frac{m_s}{m_b}} : \sqrt{\frac{m_\mu}{m_\tau}} = 1 : 2 : 3. \quad (6)$$

In this case we have

$$\sqrt{\frac{m_s}{m_b}} = 0.162 \quad \sqrt{\frac{m_c}{m_t}} = 0.081, \quad (7)$$

which leads to $m_s(1 \text{ GeV}) = 140 \text{ MeV}$ and $m_t(m_t) = 125 \text{ GeV}$, i.e. mass values quite consistent with the experimental constraints.

If our conjectured mass relation turns out to be correct, it would considerably strengthen the cascade scenario we have mentioned. It would be difficult to understand such a relation without invoking a substructure of the quarks and the leptons. The connection between the charges of the leptons and quarks — which follows from an anomaly cancellation — requires the sum of the charges for each generation to vanish. Our mass relation implies a further connection between the mass ratios of the second and third families which could be a starting point of a further understanding of the dynamics leading to the mass spectrum.

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