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THE MULTIPARTICLE CORRELATION STRUCTURE
OF HIGH ENERGY COLLISIONS

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ABSTRACT

On the basis of the extensive experimental evidence on multiplicity distributions (MDs) in restricted phase space domains, especially the wide occurrence of negative binomial MDs, we construct an approximate expression for all multiparticle correlations in terms of the single particle distribution and a two-particle "aggregation function". We propose to test it experimentally by measuring multiple factorial moments of the MDs in several small domains of phase space as functions of their separations.

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1. In the last five years, many high energy experiments have established that the multiplicity distributions (MDs) of charged and negative particles produced in hadronic and semileptonic collisions and in e^+e^- annihilation have to good approximation a negative binomial (NB) shape for full phase space and many restricted phase space domains (see [1,2] for reviews and references). Furthermore, motivated by the search for intermittency [3,4], numerous groups have measured the factorial moments of MDs

$$F_r = \langle n(n-1)\dots(n-r+1) \rangle \quad (1)$$

in very small rapidity windows [5], and it has recently been noticed [6,7] that the measured F_r 's approximately obey the relations characteristic of NB distributions. Also, various authors have described the measured F_r 's in small rapidity windows by postulating simple expressions for the higher correlation functions in rapidity, $C_r(y_1\dots y_r)$ with $r \geq 3$, in terms of the two-body correlation $C_2(y_1 y_2)$ [8,9]. These expressions, however, do not account for the observed NB properties of the MDs [8,9], or do so only in very small rapidity windows [7]. We show in this note that approximate NB behaviour is obtained by adopting a modified expression for the higher C_r 's in terms of C_2 and the single particle distribution $Q_1(y) = dn/dy$.

2. We denote by $Q_r(y_1\dots y_r)$ the inclusive r -particle rapidity distribution; it is defined as the probability density to find $r-1, 2 \dots$ particles at rapidities $y_1\dots y_r$ irrespective of the presence of further particles. The Q_r are related to the correlation functions C_r by well known equations [10] which are conveniently summarized in functional form:

$$G = \exp(g), \quad G = 1 + \sum_{r=1}^{\infty} \frac{(r!)^{-1}}{r!} \int Q_r(y_1\dots y_r) \prod_{i=1}^r u(y_i) dy_i \quad (2)$$

$$g = \int Q_1(y) u(y) dy + \sum_{r=2}^{\infty} \frac{(r!)^{-1}}{r!} \int C_r(y_1\dots y_r) \prod_{i=1}^r u(y_i) dy_i \quad (3)$$

with $u(y)$ an arbitrary function. The Q_r and C_r are fully symmetric in $y_1\dots y_r$. For any domain D , the factorial moments

(1) are given by

$$F_r^D = \langle n_D(n_D-1)\dots(n_D-r+1) \rangle = \int_D Q_r(y_1\dots y_r) \prod_{i=1}^r dy_i \quad (4)$$

where n_D denotes the multiplicity in D . If the distribution of n_D is NB with parameters $\bar{n}_D$ and k_D , the F_r^D are given by

$$F_r^D = (\bar{n}_D)^r (1+1/k_D) \dots (1+(r-1)/k_D) \quad (5)$$

so that from (4) the Q_r obey the recurrence relation

$$\int_D Q_{r+1}(y_1\dots y_{r+1}) \prod_{i=1}^{r+1} dy_i = \bar{n}_D (1+r/k_D) \int_D Q_r(y_1\dots y_r) \prod_{i=1}^r dy_i \quad (6)$$

Our aim is to propose a relation between the Q_r 's which is independent of D and implies eq.(6) at least approximately for such domains D where the NB gives a good description of the data.

The solution is obvious in the Poisson limit of infinite k_D for all D . Since

$$\bar{n}_D = \int_D Q_1(y) dy \quad (7)$$

eq.(6) with $r/k_D=0$ is immediately obtained if we take

$$Q_{r+1}(y_1\dots y_{r+1}) = \{Q_r(y_1\dots y_r) Q_1(y_{r+1})\}_s \quad (8)$$

where $\{\dots\}_s$ means symmetrization in all y_i (averaging over all permutations). By iteration eq.(8) gives

$$Q_r(y_1\dots y_r) = \prod_{i=1}^r Q_1(y_i) \quad (9)$$

and by eqs.(2,3) all correlation functions then vanish (independent emission of particles).

For finite k_D 's our proposal is to add terms to eq.(8) in order to take into account the basic feature of NB distributions, i.e., the aggregation effect by which the presence of r particles enhances the probability to find additional particles by an amount proportional to r , see [11] and appendix of [1]. It is natural to assume the aggregation effect to be controlled by

the position of the particles already present. This leads us to propose the following generalization of eq.(8):

$$Q_{r+1}(y_1 \dots y_{r+1}) = \left\{ Q_r(y_1 \dots y_r) Q_1(y_{r+1}) \left[1 + \sum_{i=1}^r a(y_{r+1}, y_i) \right] \right\}_S \\ = \left\{ Q_r(y_1 \dots y_r) Q_1(y_{r+1}) [1 + r a(y_{r+1}, y_1)] \right\}_S \quad (10)$$

The aggregation function $a(y_{r+1}, y_i)$ expresses the probability enhancement near the particle at rapidity y_i ; we take it to be independent of r so as to make the overall enhancement approximately proportional to r . Note that in general there is no reason for this function to be symmetric. As shown in the Appendix on the basis of eqs.(2,3), our ansatz (10) corresponds to the following recurrence relation for the correlation functions

$$C_{r+1}(y_1 \dots y_{r+1}) = r \left\{ C_r(y_1 \dots y_r) Q_1(y_{r+1}) a(y_{r+1}, y_1) \right\}_S \quad (11)$$

where for $r=1$ one replaces C_1 by Q_1 .

3. Eqs.(10) and (11) represent our proposal for the approximate description of the correlation structure of multiparticle production in high energy collisions. They express all correlations in terms of the aggregation function $a(y', y)$ and the single particle distribution $Q_1(y)$, usually denoted by dn/dy . In a given domain D , our ansatz leads to an exact NB distribution if the following ratio R_r is independent of r :

$$R_r = \frac{\int_D Q_r(y_1 \dots y_r) w_D(y_1) \prod_{i=1}^r dy_i}{\int_D Q_r(y_1 \dots y_r) \prod_{i=1}^r dy_i} \quad (12)$$

$$w_D(y) = \int_D Q_1(y') a(y', y) dy \quad (13)$$

The common value of the R_r 's is then $\bar{n}_D/k_D$.

Barring singularities, constancy of R_r with respect to r will always hold when D is a small interval where the variations of the functions a and Q_1 with rapidity can be neglected. For

general D , exact NB behaviour obtains when $w_D(y)$ is independent of y in D . This occurs in particular if $a(y', y)$ is independent of y in full phase space, the case studied in [12] where it was shown to be the only situation in which NB behaviour holds for all D 's composed of arbitrary sets of disconnected intervals. In this case the ratio $\bar{n}_D/k_D$ is additive in D , a property which does not hold in experiment [13] nor in the QCD parton shower model [14].

In many cases one should expect some variation of the ratio (12) with r . When these variations are small the corresponding MD will be close to NB shape, as occurs frequently in experiment. Thus, approximate NB behaviour can be expected when the function $w_D(y)$ varies little over the dominant part of the integration range in eq.(12).

4. Our ansatz (11) gives the following expressions for the first correlation functions [we introduce the reduced correlation functions c_r and write $a(1, j)$ as abbreviation for $a(y_1, y_j)$]:

$$C_r(y_1 \dots y_r) = c_r(y_1 \dots y_r) \prod_{i=1}^r Q_1(y_i) \quad (14)$$

$$c_2(y_1, y_2) = \{a(1, 2)\}_S \quad (15)$$

$$c_3(y_1, y_2, y_3) = \{a(1, 2)a(2, 3) + a(1, 3)a(2, 3)\}_S \quad (16)$$

$$c_4(y_1 \dots y_4) = \{a(1, 2)a(2, 3)a(3, 4) + a(1, 4)a(2, 4)a(3, 4) + a(1, 3)a(2, 3)a(3, 4) + 3a(1, 2)a(2, 4)a(3, 4)\}_S \quad (17)$$

These expressions simplify when the aggregation function $a(y, y')$ is symmetric, as is expected to be the case in the central region where $Q_1(y)$ is constant. Eqs.(15-17) then reduce to

$$c_2 = a(1, 2); \quad c_3 = 2\{a(1, 2)a(2, 3)\}_S \quad (18)$$

$$c_4 = 2\{2a(1, 2)a(2, 3)a(3, 4) + a(1, 4)a(2, 4)a(3, 4)\}_S \quad (19)$$

5. The above correlation functions differ from those proposed by other authors, e.g. eqs.(17) and (28-30) of [7] or eq.(25) of [9]. All such "Ansätze" including ours are no more than educated guesses, and it will be for experiment to decide to

what degree they are able to describe the data. To this end we propose to study first the double factorial moments $F_{rr'}$ [4] for two small (pseudo)rapidity windows d, d' with centers y, y' separated by a variable distance $y-y'$ ranging, say, from 0.2 upward. The $F_{rr'}$ are defined by

$$F_{rr'} = \int \prod_d \prod_{d'} \int_{y_1}^{y_2} \int_{y'_1}^{y'_2} dy_j Q_{rr'}(y_1 \dots y_{r+r'}) \\ = \langle n(n-1) \dots (n-r+1) n'(n'-1) \dots (n'-r'+1) \rangle \quad (20)$$

with n, n' the multiplicities in d, d' respectively. In absence of singularities, the $F_{rr'}$ reduce for small d and d' to

$$F_{rr'} \approx d^r d'^{r'} Q_{r+r'}(y \dots y, y' \dots y') \quad (21)$$

where in $Q_{r+r'}$ one has r values y and r' values y' . In our description of correlations, these $Q_{r+r'}$ values are then expressed in terms of the $Q_1(y)$, $a(y, y')$, $Q_1(y')$, $a(y, y')$ deduced from ordinary factorial moments in d and d' , plus the new quantity $a(y, y')$ which controls the correlation between d and d' (and similarly for the proposals of other authors). In the central region where $Q_1(y)$ and $a(y, y')$ vary little with (pseudo)rapidity y , one can augment statistics by averaging over y and y' with constant separation $y-y'$ and constant bin sizes d, d' . Given enough statistics, this type of analysis can be extended to multiple factorial moments by considering 3, 4 ... small bins instead of two. Regarding the existing data for factorial moments in single small bins, their description in our scheme is by construction essentially the same as in [7].

6. In section 2 we introduced our ansatz (10,11) on empirical grounds. It also finds a theoretical justification in the QCD parton shower model. As shown in [14], this model with Lund-type hadronization, which describes e^+e^- annihilation experiments very well, obeys a generalized form of local parton-hadron duality (LPHD) that links the inclusive distributions Q_r at partonic and hadronic levels. The link is given by eq.(5) of [14], which implies that the recurrence relations (10,11) we

have adopted above for the final hadrons also hold for the final partons of the shower if the aggregation function $a(y, y')$ is divided by the constant ρ defined in [14], section 3. This provides us with a QCD-based dynamical justification for our ansatz. The aggregation effect expressed in eq.(10) is natural for the final partons of a QCD shower model; any final parton of the shower is likely to be surrounded by others produced in nearby branches of the shower, an effect which produces the probability enhancement mentioned before eq.(10).

It should be noted, however, that LPHD has only been verified so far for the final metastable hadrons (mostly pions), not for hadron resonances. The effects of the latter should also be contained in the hadronic-level correlations, and our aggregation function $a(y, y')$ can incorporate them partially, presumably at the cost of some violation of LPHD. We nevertheless expect that our ansatz is too simple for a complete description of all resonance effects.

7. We end with a few remarks:

- i) As noted above, the aggregation function need not be symmetric. Only its symmetric part can be extracted from Q_1 and Q_2 . To study the antisymmetric part one must go to Q_3 .
- ii) Our description of multiparticle correlations may be also applicable in the fragmentation regions where Q_1 varies rapidly and NB properties are known to disappear. This should be tested experimentally.
- iii) In our discussion we have assumed $a(y, y')$ to be finite. This need not be the case; $a(y, y')$ could become infinite at $y=y'$ with an integrable singularity. This would imply intermittency, but new restrictions would be necessary to obtain NB behaviour in small rapidity windows [15].
- iv) Beyond its importance for QCD, multiparticle production is of considerable interest for statistical physics. It is one of the rare cases in physics where, given enough statistics, the correlation functions C_r are directly measurable for all

r , so that dynamical theories and models can be fully tested. In condensed matter physics, X-ray and neutron scattering experiments give only access to C_2 and there are no direct experimental tests for assumptions on higher correlations, like Kirkwood's famous superposition approximation of liquid theory (see [16], and e.g. [17] for a theoretical test).

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APPENDIX

From eq.(3) we calculate the functional derivative

$$\delta g/\delta u(y) = Q_1(y) + \sum_{r=1}^{\infty} (r!)^{-1} C_{r+1}(y, y_1, \dots, y_r) \prod_{i=1}^r u(y_i) dy_i \quad (22)$$

We multiply by $u(y)$, integrate over y and apply eq.(11):

$$\int [\delta g/\delta u(y)] u(y) dy = \int Q_1(y) u(y) dy + \int a(y_1, y_2) \prod_{i=1}^2 Q_i(y_i) u(y_i) dy_i + \sum_{r=1}^{\infty} [(r-1)!]^{-1} \int Q_{r+1}(y, y_1, \dots, y_r) C_r(y_1, \dots, y_r) \prod_{i=1}^r u(y_i) dy_i \quad (23)$$

In the last sum we replace r by $r+1$ and apply eq.(22), with the result

$$\int [\delta g/\delta u(y)] [1 - \int a(y', y) Q_1(y') u(y') dy'] u(y) dy = - \int Q_1(y) u(y) dy \quad (24)$$

This is the functional form of the recurrence relation (11). A similar calculation gives the functional form of eq.(10), the recurrence relation for the Q_r :

$$\int [\delta g/\delta u(y)] [1 - \int a(y', y) Q_1(y') u(y') dy'] u(y) dy = - G \int Q_1(y) u(y) dy \quad (25)$$

But the first eq.(2) gives $\delta G/\delta u(y) = G \delta g/\delta u(y)$, which establishes the equivalence of (24) and (25), hence also of the recurrence relations (10) and (11).

In the special case treated in [12] where $a(y', y)$ is independent of y , we were able to solve the functional equation. While the C_r and hence the Q_r turned out to have simple expressions, the corresponding exclusive distribution functions P_r were already quite complicated; it was nevertheless possible to give in [12] the condition under which they are everywhere positive, as they should be since they are probability densities. In our case, even the C_r get complicated for general r , the determination of the P_r is quite involved, and the question of their positivity is not yet resolved in the general case. But by continuity we know that the positivity result established in [12] must extend to the present case if the variation of $a(y', y)$ with y is not too strong.

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