Supersymmetric contribution to the electroweak ρ parameter

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Contributions to the electroweak ρ parameter, the ratio of the neutral- and charged-current strengths at zero-momentum transfer, are studied in the minimal extension of the standard model (SM) with softly broken supersymmetry. The effects of the extended Higgs sector, the gaugino-Higgsino sector, the light-squark-slepton sector and that of the stop-sbottom sector are studied separately, and the role of the custodial $SU(2)_V$ symmetry in each sector is clarified. The stop-sbottom sector is found to give potentially a most significant contribution to $\delta\rho$ which can double the standard-model contribution from the top-bottom sector, whereas all the remaining sectors contribute to $\delta\rho$ at the level of at most a few $\times 10^{-3}$. In the supergravity model with radiative electroweak gauge symmetry breaking there are no extra sources of the $SU(2)_V$ breaking at the grand unification scale other than those present already in the SM, and the resulting $\delta\rho$ is found to be significantly smaller than in the general case. Constraints on the allowed range of $\delta\rho$ in the supergravity models are given by taking account of existing and prospective experimental mass limits of additional particles at CERN LEP and $Spp\bar{S}$ and Fermilab Tevatron.

I. INTRODUCTION

The standard model (SM) of electroweak interactions as developed¹ in the 1960s by Glashow, Weinberg, and Salam has been known for some time to successfully describe various weak-interaction data on the level of 10% accuracy. However, only fairly recently² have these data become sufficiently precise to allow for the test of quantum effects. Very recently, the first data from the SLAC Linear Collider (SLC) and CERN LEP have allowed us to measure³ the mass of the Z boson with an error of less than 60 MeV, which already begins to constrain⁴ the values of the Higgs boson and especially the top-quark mass within the SM. Further precision measurement of electroweak parameters can be expected from experiments at the SLC and LEP in the very near future.

So far no deviation from the SM has been observed. We think nevertheless that the rapid accumulation of new electroweak data calls for a comprehensive analysis of one-loop effects that can be expected in extensions of the SM. We focus here on perhaps the best motivated of its extensions, supersymmetry⁵ (SUSY). Supersymmetric models do not suffer⁶ from the fine-tuning problem⁷ related to the smallness of the weak energy scale as compared to the Planck or grand-unified theory (GUT) scale. Furthermore, the currently most promising of our candidates for a "theory of everything," superstring theory,⁸ also implies the manifestation of supersymmetry at some energy scale (although not necessarily at the weak scale).

Among the large variety of supersymmetric radiative corrections, those from vertex and box diagrams are usually⁹ quite small, once existing bounds on the masses of superpartners are incorporated; they are also process

dependent. In contrast, corrections involving two-point functions, i.e., corrections to W and Z propagators, are universal and can in principle be sizable¹⁰⁻¹² even if all sparticles are heavy. In this paper we focus our attention on corrections to the electroweak ρ parameter, defined¹³ as the ratio of the strengths of neutral and charged currents at vanishing momentum transfer. These corrections are conceptually very simple, since they are uniquely determined in terms of the W and Z two-point functions at zero external momentum. Furthermore, the parameter ρ is known experimentally¹⁴ rather precisely,

$$\rho = 0.998 \pm 0.0086, \quad (1.1)$$

leading to the bound $m_t \lesssim 180$ GeV in the SM (Ref. 15). Some discussions of SUSY corrections to ρ already exist in the literature.^{11,12} However, in these papers only a few "typical" or extreme choices of parameters were discussed. While our results agree with these references in the region of overlap, we tried to cover the full parameter space of supersymmetric models with minimal particle content, respecting, of course, existing experimental bounds¹⁶ on sparticle masses.

The remainder of this paper is organized as follows. In Sec. II we set up our notation; we briefly discuss the role of the custodial global $SU(2)$ symmetry¹³ which, if unbroken, guarantees the validity of the SM tree-level result $\rho = 1$ to all orders. In Sec. III we discuss contributions to $\delta\rho (= \rho - 1)$ involving Higgs bosons (Sec. III A), gauginos and Higgsinos (Sec. III B), the superpartners of light quarks and leptons (Sec. III C), and the superpartners of t and b quarks (Sec. III D). For each case we give upper and, where they exist, lower bounds on $\delta\rho$ as a function of a relevant sparticle mass. In Sec. IV we discuss the to-

tal SUSY contribution to $\delta\rho$ ($\delta\rho_{\text{SUSY}}$) on supergravity models with radiative gauge symmetry breaking.¹⁷ In these models the complete sparticle spectrum is given in terms of a few parameters. We show how the resulting correlations reduce the maximal possible value of $\delta\rho_{\text{SUSY}}$ as compared to the general case. We also discuss the special case¹⁸ (so-called “no-scale” models) where a common gaugino mass at ultrahigh energies is the only seed for supersymmetry breaking in the visible sector. Finally, in Sec. V we summarize our results and draw some conclusions.

II. FORMALISM

We want to compute corrections to the ratio ρ of the overall strengths of neutral and charged currents, normalized to its tree-level value ($\rho=1$ in the SM), for momentum transfer small as compared to M_W , $|q^2|/M_W^2 \ll 1$; this last condition is satisfied in all deep-inelastic scattering experiments to date, as well as in measurements of parity violation in atoms. At the tree level, $\rho \neq 1$ would be a signal for a deviation from the relation $M_W = M_Z \cos\theta_W$, due to, e.g., a more complicated Higgs-boson structure. At the quantum level, the parameter ρ as defined in Ref. 13 is no longer directly related to gauge-boson masses, since it probes the gauge-boson two-point functions at small $|q^2|$, not at $q^2 = M_V^2$ ($V = W, Z$) where the physical masses are defined. In fact we can safely neglect¹³ the q^2 dependence of the radiative corrections in the region of $|q^2|/M_W^2 \ll 1$, and write

$$\begin{aligned} \delta\rho &= \frac{\Pi_{+/-}(0) - \Pi_{33}(0)}{M_W^2} \\ &= \frac{\Pi_{+/-}(0)}{M_W^2} - \frac{\Pi_{ZZ}(0)}{M_Z^2}, \end{aligned} \quad (2.1)$$

where $\Pi_{+/-}$, Π_{33} , and Π_{ZZ} are the coefficients of $-ig_{\mu\nu}$ in the vacuum-polarization amplitudes of charged and neutral W bosons and Z bosons, respectively. Hence all we need are various one-loop contributions to these vacuum-polarization amplitudes.

To this end we have to deal with three types of contributions: Fermion loops, loops of non-Higgs scalars [which do not have a nonvanishing vacuum expectation value (VEV)], and loops of Higgs scalars. The gauge-boson loops are the same as in the SM, and in fact they are treated as part of the SM renormalization and do not contribute to $\delta\rho$ in the framework of Ref. 14.

The contribution of two (possibly identical) fermions with masses m_1, m_2 and coupling

$$\bar{\psi}_1(a + b\gamma_5)\gamma_\mu\psi_2 V^\mu \quad (V = W, Z) \quad (2.2)$$

is

$$\begin{aligned} \Pi(\text{fermions}) &= \frac{1}{16\pi^2} [|a|^2 f_f(m_1, m_2) \\ &\quad + |b|^2 f_f(-m_1, m_2)] \end{aligned} \quad (2.3)$$

with

$$\begin{aligned} f_f(m_1, m_2) &= (m_1 - m_2)^2 \ln \frac{\Lambda^4}{m_1^2 m_2^2} - 2m_1 m_2 \\ &\quad + \frac{2m_1 m_2 (m_1^2 + m_2^2) - m_1^4 - m_2^4}{m_1^2 - m_2^2} \ln \frac{m_1^2}{m_2^2}, \end{aligned} \quad (2.4)$$

where Λ is an ultraviolet cutoff. Note that expression (2.1) is finite, all Λ -dependent terms thus have to cancel in the end, which provides us with a good check on masses and mixings. The contribution of non-Higgs scalars with masses m_1, m_2 and coupling

$$ic\phi_1^* \overleftrightarrow{\partial}_\mu \phi_2 V^\mu \quad (V = W, Z) \quad (2.5)$$

is

$$\Pi(\text{scalars}) = \frac{|c|^2}{16\pi^2} f_s(m_1^2, m_2^2), \quad (2.6)$$

with

$$f_s(m_1^2, m_2^2) = \frac{m_1^2 m_2^2}{m_1^2 - m_2^2} \ln \frac{m_2^2}{m_1^2} + \frac{1}{2} (m_1^2 + m_2^2). \quad (2.7)$$

Finally, the contribution of Higgs-gauge loops from a Higgs boson with mass m_H and coupling dg_{HVV} , i.e., a factor of d times the HVV coupling of the SM,

$$\mathcal{L}_{HVV} = gM_W \left[W_\mu^+ W^{-\mu} + \frac{1}{2\cos^2\theta_W} Z_\mu Z^\mu \right] H, \quad (2.8)$$

is

$$\delta\rho(\text{Higgs}) = \frac{3\alpha|d|^2}{16\pi\sin^2\theta_W} f_H \left[\frac{m_H^2}{M_Z^2}, \theta_W \right] \quad (2.9)$$

with

$$f_H(x, \theta_W) = x \left[\frac{\ln(\cos^2\theta_W/x)}{\cos^2\theta_W - x} + \frac{\ln x}{\cos^2\theta_W(1-x)} \right]. \quad (2.10)$$

We note here that

$$f_H(x, 0) = 0; \quad (2.11)$$

likewise,

$$f_f(m_f, m_f) = f_s(m_s^2, m_s^2) = 0. \quad (2.12)$$

These are manifestations of a global $\text{SU}(2)_V$ symmetry¹⁹ of the gauge-boson mass matrix which survives the spontaneous breaking of gauge symmetry. If this symmetry can be extended¹³ to the rest of the Lagrangian, it will protect the relation $\rho=1$ from quantum corrections. In the case of minimal supersymmetry with two Higgs doublets, $H, \bar{H}$, with hypercharge $Y(\bar{H}) = -Y(H) = \frac{1}{2}$, the gauge boson mass matrix can be written as

$$\mathcal{L}_{\text{mass}} = \frac{g^2}{8} \text{tr}(W_\mu W^\mu) \text{tr}(\Phi \Phi^\dagger), \quad (2.13)$$

where $W^\mu = \sum_{i=1}^3 W_i^\mu \sigma_i$ with σ_i ($i=1, 2, 3$) the Pauli matrices and the matrix Φ is given by

$$\Phi = \begin{pmatrix} H^0 & \bar{H}^+ \\ H^- & \bar{H}^0 \end{pmatrix}. \quad (2.14)$$

We thus see that $\mathcal{L}_{\text{mass}}$ is, in fact, invariant under *any* $\text{SU}(2)_V$ under which W^μ transforms like a triplet:

$$W_\mu \rightarrow G W_\mu G^\dagger. \quad (2.15)$$

As discussed in Refs. 11 and 12 the transformation properties of the matter fields in supersymmetric theories should be chosen as

$$\begin{aligned} \Phi &\rightarrow G \Phi G^\dagger, \\ Q &\equiv \begin{pmatrix} U_L \\ D_L \end{pmatrix} \rightarrow G Q, \quad \bar{Q} \equiv \begin{pmatrix} U_R^c \\ D_R^c \end{pmatrix} \rightarrow G^* \bar{Q}, \\ L &\equiv \begin{pmatrix} \nu_L \\ l_L \end{pmatrix} \rightarrow G L. \end{aligned} \quad (2.16)$$

Notice that Eqs. (2.16) are written for superfields; they are thus equally valid for scalars and fermions.

In the SM this custodial $\text{SU}(2)_V$ is broken¹³ by hypercharge interactions and by unequal Yukawa couplings for up-type and down-type quarks or neutral and charged leptons. If SUSY were exact, this would also be the only source of breaking in the supersymmetric case. The introduction of soft SUSY-breaking terms can, however, introduce additional sources of explicit breaking of $\text{SU}(2)_V$ if the soft SUSY-breaking mass terms for $\tilde{u}_R^c$ and $\tilde{d}_R^c$ are different, or if the trilinear interactions for up-type and down-type squarks or sleptons (A terms) are different. [On the one-loop level, the former source of $\text{SU}(2)_V$ breaking is only relevant in the case of sizable mixing between left- and right-handed sfermions because right-handed sfermions do not couple directly to W bosons.] In addition, $\text{SU}(2)_V$ can be broken spontaneously if $|\omega| \equiv |\langle \bar{H}^0 \rangle / \langle H^0 \rangle| \neq 1$. It is worth mentioning that in minimal supergravity models with radiative gauge symmetry breaking,¹⁷ these additional sources of $\text{SU}(2)_V$ breaking are *absent* at the Planck scale (M_P), where all scalar masses and all trilinear soft breaking parameters are assumed to be equal. However, through the renormalization-group equations¹⁷ relating physics at M_P to that at the weak scale, the sources of $\text{SU}(2)_V$ breaking present in the SM (unequal Yukawa couplings and non-vanishing hypercharge interactions) lead to $m_{\tilde{u}_R}^2 \neq m_{\tilde{d}_R}^2$, $A_t \neq A_b$ and $|\omega| \neq 1$ at scale M_W . We will come back to this point in Sec. IV. Before discussing specific models

we first study the various supersymmetric contributions to $\delta\rho$ in a general softly broken SUSY model.

III. CONTRIBUTIONS TO $\delta\rho$ IN SOFTLY BROKEN SUPERSYMMETRY

As mentioned in the Introduction we will assume minimal particle content, i.e., two Higgs-doublet superfields and three generations of quark and lepton superfields. We then have to compute contributions to $\delta\rho$ from four different sectors: Higgs bosons; gauginos and Higgsinos; the superpartners of light (essentially massless) quarks and leptons; and the stop-sbottom sector. We devote one subsection to each of these four sectors.

A. Higgs bosons

As is well known,²⁰ the minimal supersymmetric Higgs-boson sector contains 5 physical degrees of freedom; one complex charged scalar ($H^\pm$), two real neutral scalars (H_1^0, H_2^0), and one real pseudoscalar (P^0). It can be described²⁰ in terms of just two parameters, which we take to be the mass of the physical pseudoscalar m_P and the ratio of the two Higgs VEV's:

$$\omega = \tan\beta = \langle \bar{H}^0 \rangle / \langle H^0 \rangle \quad (3.1)$$

as defined in Sec. II. The other masses are then given by

$$\begin{aligned} m_{H^\pm}^2 &= m_P^2 + M_W^2, \\ M_{H_{1,2}^0}^2 &= \frac{1}{2} \left\{ m_P^2 + M_Z^2 \pm \left[(m_P^2 + M_Z^2)^2 - 4 \left(\frac{\omega^2 - 1}{\omega^2 + 1} \right)^2 m_P^2 M_Z^2 \right]^{1/2} \right\}, \end{aligned} \quad (3.2)$$

with $m_{H_1^0} \leq m_{H_2^0}$. We also need the gauge couplings of these bosons. These are determined by a factor $\cos(\alpha - \beta)$, which describes the overlap between the heavier scalar mass eigenstate (H_2^0) and the combination of H and $\bar{H}$ which has a nonzero VEV, the “true Higgs boson.” It is given by

$$\cos^2(\alpha - \beta) = \frac{m_{H_1^0}^2}{m_P^2} \frac{M_Z^2 - m_{H_1^0}^2}{M_Z^2 + m_P^2 - 2m_{H_1^0}^2}. \quad (3.3)$$

The total Higgs contribution to $\delta\rho$ is then

$$\begin{aligned} \delta\rho_{\text{Higgs}} &= \frac{\alpha}{16\pi \sin^2\theta_W} \left\{ 3 \left[\cos^2(\beta - \alpha) f_H \left[\frac{m_{H_2^0}^2}{M_Z^2}; \theta_W \right] + \sin^2(\beta - \alpha) f_H \left[\frac{m_{H_1^0}^2}{M_Z^2}; \theta_W \right] \right] \right. \\ &\quad + \frac{1}{M_W^2} [\sin^2(\beta - \alpha) f_s(m_{H^\pm}^2, m_{H_2^0}^2) + \cos^2(\beta - \alpha) f_s(m_{H^\pm}^2, m_{H_1^0}^2)] \\ &\quad \left. + f_s(m_{H^\pm}^2, m_P^2) - \sin^2(\beta - \alpha) f_s(m_P^2, m_{H_2^0}^2) - \cos^2(\beta - \alpha) f_s(m_P^2, m_{H_1^0}^2) \right\}. \end{aligned} \quad (3.4)$$

Some limiting cases may be of interest. In the limit $m_P \rightarrow 0$ one finds

$$m_{H^+} \rightarrow M_W, \quad m_{H_2^0} \rightarrow M_Z, \quad m_{H_1^0} \rightarrow m_P |\cos 2\beta| \rightarrow 0, \quad \cos^2(\beta - \alpha) \rightarrow \cos^2 2\beta. \quad (3.5)$$

This gives

$$\begin{aligned} \delta\rho_{\text{Higgs}}(m_P \rightarrow 0) &\rightarrow \frac{\alpha}{16\pi \sin^2 \theta_W} \left[3 \cos^2 2\beta \left[\frac{\ln(\cos^2 \theta_W)}{\cos^2 \theta_W - 1} - \frac{1}{\cos^2 \theta_W} \right] - \sin^2 2\beta \frac{\ln(\cos^2 \theta_W)}{\cos^2 \theta_W - 1} + 1 \right] \\ &\simeq \frac{\alpha}{16\pi \sin^2 \theta_W} (0.54 \cos^2 2\beta - 0.13 \sin^2 2\beta), \end{aligned} \quad (3.6)$$

where in the last line we have taken $\sin^2 \theta_W = 0.22$. This limit is interesting, since it maximizes $\delta\rho_{\text{Higgs}}$, if we further take the limit $|\cos 2\beta| = 1$, i.e., either $\omega \rightarrow \infty$ or 0. Notice that $\delta\rho_{\text{Higgs}}$ is positive in this limit.

In the opposite extreme, $m_P \rightarrow \infty$, one has

$$\begin{aligned} m_{H^+} &\rightarrow m_P, \quad m_{H_2^0} \rightarrow m_P, \\ m_{H_1^0} &\rightarrow M_Z |\cos 2\beta|, \quad \cos^2(\beta - \alpha) \rightarrow 0, \end{aligned} \quad (3.7)$$

so that

$$\delta\rho_{\text{Higgs}}(m_P \rightarrow \infty) \rightarrow \frac{3\alpha}{16\pi \sin^2 \theta_W} f_H(\cos^2 2\beta; \theta_W), \quad (3.8)$$

which is negative semidefinite. Setting again $\cos^2 2\beta = 1$ we obtain the absolute minimum of $\delta\rho_{\text{Higgs}}$. We thus find, as absolute bounds,

$$-3.1 \times 10^{-4} \leq \delta\rho_{\text{Higgs}} \leq 3.6 \times 10^{-4}, \quad (3.9)$$

which is always uninterestingly small in itself. It is important, however, that in minimal SUSY, there cannot be a sizable negative contribution to $\delta\rho$ from the Higgs sector. In the SM for a 1-TeV Higgs boson $\delta\rho_{\text{Higgs}}$ could be as small as -2.2×10^{-3} , which is enough to cancel the positive contribution of a top quark with mass 85 GeV; and in a general nonsupersymmetric two-doublet model, $\delta\rho_{\text{Higgs}}$ can even be made to cancel the effect of a 200-GeV top quark.²¹

B. Gauginos and Higgsinos

In the minimal model this sector consists of two charged Dirac fermions (charginos $\tilde{W}_{1,2}$ with $|m_{\tilde{W}_1}| \leq |m_{\tilde{W}_2}|$) and four neutral Majorana fermions (neutralinos $\tilde{Z}_i$ with $|m_{\tilde{Z}_1}| \leq |m_{\tilde{Z}_2}| \leq |m_{\tilde{Z}_3}| \leq |m_{\tilde{Z}_4}|$). Without any assumptions this sector can be described by four parameters: the SUSY-breaking masses M_1 and M_2

of the $U(1)_Y$ and $SU(2)$ gauginos, the supersymmetric Higgsino mass μ , and the ratio ω of the two Higgs VEV's introduced in Sec. II. We will further assume that M_1 and M_2 are related to each other and to the gluino mass M_3 , as is the case in supergravity models with canonical kinetic terms for gauge superfields:

$$M_1 = \frac{5}{3} \tan^2 \theta_W M_2 = \frac{5}{3} \frac{\alpha}{\cos^2 \theta_W \alpha_s} M_3; \quad (3.10)$$

we use the second identity only to relate one of the parameters of the electroweak gaugino sector to the directly measurable gluino mass. (In general, the gaugino masses M_i can have either sign; the physical gluino mass is then $|M_3|$.)

This sector has been discussed extensively in the literature; see Refs. 5 and 22 for reviews and further references. We follow closely the notation of Tata *et al.*,^{23–25} where all signs and phases are given explicitly. The small notational changes that we adopt in this paper are as follows: the supersymmetric Higgsino mass is denoted by μ rather than $2m_1$, and the mass eigenstates are numbered in the order of increasing masses ($\tilde{W}_{1,2}$ for charginos, $\tilde{Z}_{1,2,3,4}$ for neutralinos, $H_{1,2}^0$ for neutral Higgs scalars, and $\tilde{t}_{1,2}$ for stop quarks). Without going into further details we note here that the chargino mass matrix can easily be diagonalized analytically, while the 4×4 neutralino mass matrix is more easily treated numerically. Apart from the eigenvalues of the mass matrices we need the various gauge couplings of the eigenstates. The $W\tilde{W}_i\tilde{Z}_j$ vector and axial-vector couplings X_i^j and Y_i^j , as well as the $Z\tilde{W}_1\tilde{W}_2$ vector and axial-vector couplings x and y can be found in Ref. 23, the corresponding $Z\tilde{W}_i\tilde{W}_i$ couplings x_i and y_i are given in Ref. 24; and the $Z\tilde{Z}_i\tilde{Z}_j$ couplings W_{ij} , which are either purely vector or purely axial vector, are given in Ref. 25. Altogether one has

$$\begin{aligned} \delta\rho(\text{gaugino}) &= \frac{\alpha}{4\pi M_W^2 \sin^2 \theta_W} \left\{ \sum_{i=1,2} \sum_{j=1}^4 [|X_i^j|^2 f_f(|m_{\tilde{Z}_j}|, |m_{\tilde{W}_i}|) + |Y_i^j|^2 f_f(-|m_{\tilde{Z}_j}|, |m_{\tilde{W}_i}|)] \right. \\ &\quad - |y_1|^2 f_f(-|m_{\tilde{W}_1}|, |m_{\tilde{W}_1}|) - |y_2|^2 f_f(-|m_{\tilde{W}_2}|, |m_{\tilde{W}_2}|) \\ &\quad \left. - \frac{1}{2} |x|^2 f_f(-m_{\tilde{W}_1}, m_{\tilde{W}_2}) - \frac{1}{2} |y|^2 f_f(m_{\tilde{W}_1}, m_{\tilde{W}_2}) - 2 \sum_{i,j=1}^4 |\tilde{W}_{ij}|^2 f_f(-m_{\tilde{Z}_i}, m_{\tilde{Z}_j}) \right\}, \end{aligned} \quad (3.11)$$

where $\tilde{W}_{ij} = \sin\theta_W \cos\theta_W W_{ij}/e$. Notice that by letting i and j in the last term in Eq. (3.11) run over the full range the difference between the cases of identical and nonidentical Majorana fermions is automatically taken into account, since the latter contribute twice. Note also that we have kept the signs of the fermion masses in Eq. (3.11), which allows for a more compact notation; the physical masses are, of course, again given by the absolute values.

In Figs. 1(a) and 1(b) we show lines of constant gaugino-Higgsino contribution to $\delta\rho$ in the $m_{\tilde{g}} - \mu$ plane, for $\omega=1$ and $\omega=5$, respectively. In principle, μ , M_i , and ω could all have either sign; however, only the sign of the product $\mu M_3 \omega$ matters physically, so that we can choose ω and M_3 to be positive here and identify the latter with the physical gluino mass $m_{\tilde{g}}$. The region between the inner dotted curves is excluded experimentally, since it violates either the bound¹⁶ $m_{\tilde{g}} > 50$ GeV or the KEK TRISTAN bound $m_{\tilde{W}_1} > 25$ GeV (Refs. 26 and 27). The absence of conspicuous monojet events at CERN $Sp\bar{p}S$ and the Fermilab Tevatron probably allows²⁸ us to exclude additional regions of the parameter space, but since these “bounds” so far are only based on theorists’ analyses, we have not implemented them here. The outer dotted curves show the region that is excluded²⁷ by the recent measurements of the total decay width of the Z . Note that these new bounds exclude some of the regions where $\delta\rho(\text{gaugino})$ is relatively large. We have extended our curves into the newly excluded region in order to demonstrate this effect.

The comparison of Figs. 1(a) and 1(b) shows that moving ω away from unity increases $\delta\rho(\text{gaugino})$ quite dramatically, by a factor of 2 or more. This is due to the custodial $SU(2)_V$ symmetry discussed in Sec. II, which is broken spontaneously if $\omega \neq 1$. For $\omega=1$, the only source of $SU(2)_V$ breaking in the gaugino-Higgsino sector is due to hypercharge interactions. In fact, we find that in this case the expression inside the large parentheses in Eq. (3.11) vanishes in the limit $\sin^2\theta_W \rightarrow 0$, which means that $\delta\rho(\text{gaugino})$ is suppressed by an additional factor of $\sin^2\theta_W$.

Figure 1 also indicates that $\delta\rho(\text{gaugino})$ is maximal if $m_{\tilde{g}}$ and $|\mu|$ are both small. This can again be understood in terms of the custodial $SU(2)_V$. These mass terms do not break the custodial $SU(2)_V$ symmetry, but they do tend to increase the overall mass scale in the gaugino-Higgsino sector, eventually leading to a decoupling of the sector from low-energy phenomena. Indeed, when we perform a scan of ω , μ , and $m_{\tilde{g}}$ within the experimentally allowed range, keeping $m_{\tilde{W}_1}$ fixed, we find that $\delta\rho(\text{gaugino})$

FIG. 1. Lines of constant Higgsino-gaugino contribution to $\delta\rho$ for $\omega=1$ (a) and $\omega=5$ (b). The region between the inner dotted lines is excluded experimentally by the pre-LEP constraints $m_{\tilde{W}_1} > 25$ GeV and $m_{\tilde{g}} > 50$ GeV. Between the outer dotted lines the LEP-constraint $\Gamma(Z \rightarrow \text{gauginos}) < 120$ MeV is violated.

no) is always maximized for $m_{\tilde{g}} = 50$ GeV, the lowest allowed value, and μ very close or equal to zero. The result of this scan is shown as the full curve in Fig. 2 where the maximal gaugino-Higgsino contribution to $\delta\rho$ is shown as a function of the mass of the lighter chargino ($\tilde{W}_1$).

Note that $m_{\tilde{g}} = 50$ GeV implies $M_2 \simeq 15$ GeV, see Eq. (3.10), which is quite small compared to M_W . We have therefore also computed $\delta\rho(\text{gaugino})$ in the limit $M_2 = M_1 = \mu = 0$. In this case all masses and couplings can be computed analytically. In particular,

$$m_{\tilde{W}_1} = \sqrt{2} M_W \min(\sin\beta, \cos\beta), \quad (3.12)$$

where $\tan\beta \equiv \omega$, and one finds¹²

$$\delta\rho(\text{gaugino}; \mu = M_1 = M_2 = 0)$$

$$= \frac{\alpha}{16\pi \sin^2\theta_W} \left[-6 + \frac{\cos^2 2\beta}{\cos^2\theta_W} + \frac{8 \cos^2\beta \cos^2\theta_W - 1 + 4 \sin^2\theta_W - \cos 2\beta}{\cos^2\theta_W - 1/(2 \cos^2\beta)} \ln(2 \cos^2\beta \cos^2\theta_W) \right. \\ \left. + \frac{8 \sin^2\beta \cos^2\theta_W - 1 + 4 \sin^2\theta_W + \cos 2\beta}{\cos^2\theta_W - 1/(2 \sin^2\beta)} \ln(2 \sin^2\beta \cos^2\theta_W) \right]. \quad (3.13)$$

FIG. 2. The maximal possible Higgsino-gaugino contribution to $\delta\rho$ as a function of the mass of the lighter chargino. The solid curve results from a numerical scan of the parameter space under the constraint $m_{\tilde{g}} > 50$ GeV. For the dashed curve, $m_{\tilde{g}}$ and μ are fixed to zero, in which case the chargino masses and $\delta\rho$ are uniquely determined by ω , see Eqs. (3.12) and (3.13).

As one can see from Fig. 2, Eq. (3.13) describes the upper bound on $\delta\rho(\text{gaugino})$ very well. Since in the presence of light charginos it is not easy²⁹ to obtain improved lower bounds on $m_{\tilde{g}}$, a significant effect due to the lower bound on $m_{\tilde{g}}$ is not to be expected soon. On the other hand, nonobservation of the $\tilde{W}_1$ in Z decays at LEP has recently led to an increase of the lower bound on $m_{\tilde{W}_1}$ to 45 GeV or so, which reduces the maximal gaugino-Higgsino contribution to $\delta\rho$ from its pre-LEP value of 2×10^{-3} to about 1.4×10^{-3} . Notice that Eq. (3.13) is no longer useful if the lower bound on $m_{\tilde{W}_1}$ surpasses M_W , since this necessitates μ , $m_{\tilde{g}} \neq 0$, see Eq. (3.12). The region $m_{\tilde{W}_1} \geq M_W$ can, however, only be probed³⁰ at LEP 200 or beyond.

Finally, it should be mentioned that $\delta\rho(\text{gaugino})$ is always positive; however, even for small $m_{\tilde{W}_1}$, $\delta\rho(\text{gaugino})$ can be brought arbitrarily close to 0 by increasing $m_{\tilde{g}}$ or μ , see Fig. 1. The reason is that in this limit, the chargino and neutralino mass eigenstates are actually equal to the current states, i.e., the off-diagonal elements in the mass matrices, which originate from the breaking of $SU(2)_L \times U(1)_Y$ and also contribute to the breaking of the custodial $SU(2)_V$ through $M_Z \neq M_W$ and possibly $\omega \neq 1$, become irrelevant. The only source of breaking in this sector is then the $U(1)_Y$ gaugino, which, however, couples to W bosons only at the two-loop level. At the one-loop level, the custodial $SU(2)_V$ is thus effectively restored if $m_{\tilde{g}}$ or μ becomes very large.

C. Superpartners of light quarks and leptons

In the limit of zero lepton or quark mass the superpartners of right-handed SM fermions do not contribute to $\delta\rho$ since they couple neither to $W^\pm$ nor to W^3 , see Eq. (2.1). On the other hand, gauge symmetry ensures that

the explicitly SUSY-breaking contribution to the masses of members of an $SU(2)$ doublet are equal. Differences between these masses, and hence nonzero contributions from this sector to $\delta\rho$ [recall that $f_s(m_s^2, m_s^2) = 0$ in Eq. (2.7)], can only be induced by the so-called D -term contributions⁵ to sfermion masses, which originate from gauge symmetry breaking. This gives

$$m_{\tilde{e}_L}^2 = m_{\tilde{\nu}_L}^2 - M_W^2 \cos 2\beta, \quad m_{\tilde{d}_L}^2 = m_{\tilde{u}_L}^2 - M_W^2 \cos 2\beta, \quad (3.14)$$

with $\cos 2\beta = (1 - \omega^2)/(1 + \omega^2)$. Obviously there is no mass splitting and no contribution to $\delta\rho$ in the case of exact custodial $SU(2)_V$, $\omega = 1$, whereas $\delta\rho$ is maximized by choosing $|\cos 2\beta| = 1$ ($\omega = 0$, or ∞). Furthermore, one finds that $\delta\rho$ is maximal if the explicitly SUSY-breaking contributions to the sfermion masses are chosen as small as possible. One can thus maximize $\delta\rho$ by choosing $m_{\tilde{\nu}} = 0$ for all neutrino species, and $\omega \rightarrow \infty$ ($\cos 2\beta \rightarrow 1$), which leads to the bound (for N_f generations)

$$\delta\rho(\text{sleptons}) \leq \frac{\alpha N_f}{16\pi \sin^2 \theta_W} \simeq 1.9 \times 10^{-3}, \quad (3.15)$$

where in the last step $N_f = 3$ has been assumed. Notice that in this situation, the left-handed charged sleptons have mass M_W , well beyond any present or soon achievable bound. On the other hand, LEP should soon be able^{31,27} to detect sneutrinos with masses up to 40 GeV or so. A negative outcome of this search would tighten the bound (3.15) substantially:

$$\delta\rho(\text{sleptons}) \leq 1.0 \times 10^{-3} \quad (N_f = 3, m_{\tilde{\nu}} \geq 40 \text{ GeV}). \quad (3.16)$$

The experimental situation for left-handed squarks is less clear. The Collider Detector at Fermilab (CDF) Collaboration has derived³² the bound $m_{\tilde{q}} > 74$ GeV; however, this assumes five flavors of degenerate left- and right-handed squarks, as well as 100% branching ratio for the direct $\tilde{q} \rightarrow q + \tilde{Z}_1$ decay, which leads to the missing- p_T signature looked for in the experiment. Both assumptions can be wrong. In fact, Eq. (3.14) shows that in the interesting case of $|\cos 2\beta| = 1$, where we can expect large effects on $\delta\rho$, sizable mass splittings even among left-handed squarks are possible, while the masses of right-handed squarks could in principle be totally unrelated to left-handed squark masses. Furthermore, as discussed in Refs. 29 and 33 the missing- p_T signature especially for left-handed squarks would be substantially reduced in the presence of a light chargino $\tilde{W}_1$ or $\tilde{Z}_2$. We therefore adopt somewhat arbitrarily 50 GeV as a “reasonable” lower bound on any squark mass (other than the light stop $\tilde{t}_1$; see the next subsection). In this case the maximal contribution to $\delta\rho$ from the first two generations of squarks is given by

$$\delta\rho(\tilde{u}, \tilde{d}, \tilde{s}, \tilde{c}) \leq 1.7 \times 10^{-3}. \quad (3.17)$$

We would now like to discuss how the bounds (3.15)–(3.17) are altered in minimal supergravity (SUGRA) models⁵ with radiative gauge symmetry breaking. We discuss this point here, rather than in Sec. IV, since the renormalization-group equation (RGE) relevant

for this particular sector can easily be solved analytically. The assumption of SUGRA models relevant to our discussion here is that all squark and slepton masses are assumed to be equal at the grand unification scale M_X , which also holds for the gaugino masses. This leads to the relation

$$m_{\tilde{u}_L, \tilde{c}_L}^2 = m_{\tilde{v}}^2 - \frac{2}{3} M_Z^2 \sin^2 \theta_W \cos 2\beta + 0.79 m_g^2. \quad (3.18)$$

Note that in these models $m_t > m_b$ implies $\omega > 1$, i.e., $\cos 2\beta < 0$. Assuming²⁹ $m_g \geq 50$ GeV then implies $m_{\tilde{u}_L} \geq 58$ GeV in the relevant situation of $\cos 2\beta = -1$, which is only slightly above our “experimental” limit of 50 GeV. If, however, LEP raises the lower bound on $m_{\tilde{v}}$ to 40 GeV, Eq. (3.18) gives $m_{\tilde{u}_L} \geq 70$ GeV and hence

$$\delta\rho_{\text{SUGRA}}(\tilde{u}, \tilde{d}, \tilde{s}, \tilde{c}) \leq 1.1 \times 10^{-3} \quad (m_g \geq 50 \text{ GeV}, m_{\tilde{v}} \geq 40 \text{ GeV}), \quad (3.19)$$

which is significantly stronger than the general bound (3.17).

Finally, in so-called no-scale models,¹⁸ squarks and sleptons are actually assumed to be massless at the

Planck scale M_P . This class of models is attractive since it allows the description of realistic sparticle spectra in terms of only one SUSY-breaking parameter, the common gaugino mass at M_P , or, equivalently, the gluino mass $\tilde{m}_g$. We find that the total slepton-squark contribution to $\delta\rho$ is still maximized for $\omega \rightarrow \infty$ and $m_{\tilde{v}} = 0$, which implies¹⁷ $m_g \simeq 265$ GeV at the weak scale, this in turn gives via Eq. (3.18) $m_{\tilde{u}_L} \simeq 240$ GeV, or

$$\delta\rho_{\text{no scale}}(\tilde{u}, \tilde{d}, \tilde{s}, \tilde{c}) < 1.5 \times 10^{-4}, \quad (3.20)$$

which is entirely negligible. Note, however, that even in this very restrictive class of models the bounds (3.15) and (3.16) for the slepton contribution cannot be improved.

D. Superpartners of b and t quarks

In this sector, quark masses can clearly no longer be neglected. In this subsection we will keep only the top-quark mass finite; the bottom-quark mass will be introduced in the fully numerical treatment of the next section. In this case the only mixing between left- and right-handed squarks occurs in the stop sector. The corresponding mass matrix can in general be written as

$$(\tilde{t}_L^*, \tilde{t}_R^c) M_{\tilde{t}}^2 \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R^* \end{pmatrix} = (\tilde{t}_L^*, \tilde{t}_R^c) \begin{pmatrix} m_{\tilde{b}_L}^2 + m_t^2 + M_W^2 \cos 2\beta & m_t m_{LR} \\ m_t m_{LR} & m_{\tilde{t}_R}^2 + m_t^2 + \frac{2}{3} M_Z^2 \sin^2 \theta_W \cos 2\beta \end{pmatrix} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R^* \end{pmatrix}, \quad (3.21)$$

where $m_{\tilde{b}_L}$ is the mass of the physical left-handed sbottom (which does not mix with the $\tilde{b}_R$ in the limit of $m_b = 0$ considered here), and $m_{\tilde{t}_R}^2$ is the soft SUSY-breaking contribution to the right-handed stop mass. Finally, the off-diagonal mass m_{LR} is⁵ a combination of the relevant soft SUSY-breaking A parameter and the supersymmetric Higgsino mass μ . Allowing for future reference for nonzero mixing also in the sbottom sector, the $\tilde{t} - \tilde{b}$ contribution to $\delta\rho$ is given by¹²

$$\delta\rho(\tilde{t} - \tilde{b}) = \frac{3\alpha}{8\pi M_W^2 \sin^2 \theta_W} \{ \cos^2 \phi_t [\cos^2 \phi_b f_s(m_{\tilde{t}_1}^2, m_{\tilde{b}_1}^2) + \sin^2 \phi_b f_s(m_{\tilde{t}_1}^2, m_{\tilde{b}_2}^2)] \\ + \sin^2 \phi_t [\cos^2 \phi_b f_s(m_{\tilde{t}_2}^2, m_{\tilde{b}_1}^2) + \sin^2 \phi_b f_s(m_{\tilde{t}_1}^2, m_{\tilde{b}_2}^2)] \\ - \cos^2 \phi_t \sin^2 \phi_t f_s(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) - \cos^2 \phi_b \sin^2 \phi_b f_s(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2) \}, \quad (3.22)$$

where the physical masses are given by

$$m_{\tilde{t}_{1,2}}^2 = \frac{1}{2} (m_{\tilde{b}_L}^2 + m_{\tilde{t}_R}^2 + 2m_t^2 + M_W^2 \cos 2\beta (1 + \frac{2}{3} \tan^2 \theta_W)) \pm \{ [m_{\tilde{b}_L}^2 - m_{\tilde{t}_R}^2 + M_W^2 \cos 2\beta (1 - \frac{2}{3} \tan^2 \theta_W)]^2 + 4m_t^2 m_{LR}^2 \}^{1/2}. \quad (3.23)$$

Here $\tilde{t}_1$ denotes the lighter mass eigenstate ($m_{\tilde{t}_1} \leq m_{\tilde{t}_2}$), while the mixing angle ϕ_t is given by

$$\tan \phi_t = \frac{m_{\tilde{t}_1}^2 - m_{\tilde{b}_L}^2 - m_t^2 - M_W^2 \cos 2\beta}{m_t m_{LR}}. \quad (3.24)$$

Similar equations hold for the sbottom sector. In particular, Eqs. (3.23) and (3.24) show once again that L - R squark mixing vanishes in the limit of vanishing quark masses, which we assume here for b quarks.

It is important to note that the LL element of the mass matrix (3.22) is determined by the sbottom mass $m_{\tilde{b}_L}$, m_t , and ω . However, the LR and RR elements are arbitrary in a general softly broken SUSY model. It is then easy to see that the stop-sbottom contribution to $\delta\rho$ can be made

arbitrarily large, at least in principal, even for a finite top mass. Choosing, e.g., $m_{\tilde{t}_R}^2 = m_{\tilde{b}_L}^2$, and m_{LR} such that $m_{\tilde{t}_1}^2 \ll m_{\tilde{b}_L}^2$, one finds, in the limit of $m_{\tilde{b}_L}^2 \gg M_W^2, m_t^2$,

$$m_{\tilde{t}_2}^2 = 2m_{\tilde{b}_L}^2, \quad \cos^2 \phi_t = \frac{1}{2}, \quad (3.25)$$

$$\delta\rho(\tilde{t} - \tilde{b}) = \frac{3\alpha m_{\tilde{b}_L}^2}{8\pi \sin^2 \theta_W M_W^2} (\frac{3}{4} - \ln 2),$$

which can obviously be arbitrarily large. For fixed $m_{\tilde{b}_L}$, even larger values of $\delta\rho(\tilde{t} - \tilde{b})$ are possible if $m_{\tilde{t}_R}^2 > m_{\tilde{b}_L}^2$.

Note that (3.25) is independent of m_t . In this limit the relevant source of $SU(2)_V$ breaking is the LR element in the mass, i.e., the trilinear soft SUSY-breaking parameter

A_t describing the strength of $\tilde{t}_L \tilde{t}_R^c \bar{H}^0$ interactions. Indeed one finds that in the limit considered here, m_{LR} has to be very large:

$$m_{LR} \simeq \sqrt{m_{\tilde{b}_L}^2 m_{\tilde{t}_R}^2} / m_t. \quad (3.26)$$

Such a large m_{LR} can only be achieved³⁴ by choosing A_t very large; this, however, will in general lead to the occurrence³⁵ of false vacua, i.e., minima of the scalar potential that have $\langle \tilde{t}_{L,R} \rangle \neq 0$ and thus break charge and color symmetry. For sufficiently large A_t , these minima lie below the desired minimum in which only $\langle H^0 \rangle$ and $\langle \bar{H}^0 \rangle$ are nonzero. On the other hand, it is possible³⁵ that the tunneling rate between the physical vacuum and these false vacua is small even on cosmological scales, so that their occurrence cannot strictly be ruled out. Furthermore, the false vacua also necessitate³⁵ nonvanishing VEV's for other fields than $\tilde{t}_L$ and $\tilde{t}_R$ typically $\bar{H}$; in principle the occurrence of such false vacua can thus be avoided by choosing the squared masses of these other scalars to be very large, of order A_t^2 . However, we find both large hierarchies between scalar masses and the idea of the Universe somewhat being trapped in a very shallow physical minimum, ignoring much deeper false minima, very unappealing. We thus impose the bound

$$m_{LR} < 3m_{\tilde{b}_L}, \quad (3.27)$$

which roughly corresponds to the well-known³⁵ constraint $A < 3m_0$, where A and m_0 are the common trilinear soft SUSY-breaking parameter and the common scalar mass at scale M_P in minimal supergravity models. It is quite possible that the constraint (3.27) is not strong enough to rule out false vacua; in particular, in minimal supergravity models one always has $m_{\tilde{t}_R}^2 < m_{\tilde{b}_L}^2$, and usually also $m_{\tilde{H}}^2 < m_{\tilde{b}_L}^2$ holds, so that false vacua with $\langle \tilde{t}_L \rangle = \langle \tilde{t}_R \rangle = \langle \bar{H}^0 \rangle \neq 0$ may still exist. The bound (3.27) would be sufficient to rule them out only if

$$m_{\tilde{b}_L}^2 = m_{\tilde{t}_1}^2 (\min) + \frac{1}{2} m_t^2 + \frac{1}{2} M_W^2 (1 + \frac{2}{3} \tan^2 \theta_W) + \left[\frac{45}{4} m_t^4 + 9 m_t^2 [m_{\tilde{t}_1}^2 (\min) + \frac{1}{2} M_W^2 (1 + \frac{2}{3} \tan^2 \theta_W)] + \frac{M_W^4}{4} (1 - \frac{2}{3} \tan^2 \theta_W)^2 \right]^{1/2}. \quad (3.30)$$

Note that a light stop, being lighter than its superpartner, can lead to a quite different signature³⁶ than the other squarks do. Consequently, the usual bounds on squark masses from $p\bar{p}$ collider experiments might not apply. We take a conservative bound $m_{\tilde{t}_1} > 25$ GeV, since an even lighter stop³⁷ should have been seen at TRISTAN. In this case the constraint $\omega > 1$ does not strengthen the bound (3.28) much, especially at not too big values of m_t , $m_t < 150$ GeV. Since the new SLC and LEP experiments fail to detect a stop,²⁶ the stop-quark mass limit would soon become $m_{\tilde{t}_1} > 45$ GeV. In this case the bound (3.28) changes to

$m_{\tilde{H}}^2, m_{\tilde{t}_R}^2 \geq m_{\tilde{b}_L}^2$. Nevertheless (3.27) is strong enough to shift the choice of parameters that maximize $\delta\rho(\tilde{t}-\tilde{b})$ to the point $\omega \rightarrow 0$, $m_{LR} = 0$, with $m_{\tilde{b}_L}$ at its minimal allowed value, which we again take to be 50 GeV. In this case only the first term in Eq. (3.22) contributes, since $\phi_t = \phi_b = 0$. Numerically we find, for $75 \text{ GeV} < m_t < 200$ GeV,

$$\delta\rho(\tilde{t}-\tilde{b}) \leq \left[0.89 + \left[\frac{53 \text{ GeV}}{m_t} \right]^4 \right] \delta\rho(t-b) \quad (m_{\tilde{b}_L} > 50 \text{ GeV}), \quad (3.28)$$

where the SM top-bottom contribution is given by

$$\delta\rho(t-b) = \frac{3\alpha m_t^2}{16\pi \sin^2 \theta_W M_W^2}. \quad (3.29)$$

We note here that in the case of stops, the sign of $\cos 2\beta$ does matter, unlike all the other cases discussed so far. The reason is that even without the $\tilde{t}_L$ - $\tilde{t}_R$ mixing, two different sources of $SU(2)_V$ breaking contribute; the explicit breaking due to $m_t \neq m_b$, and the spontaneous breaking due to $\omega \neq 1$. Only for $\omega < 1$ do the two contributions add up constructively, as can be seen from the LL element of the $\tilde{t}$ mass matrix (3.21). In the absence of the L - R mixing, the RR element decouples and is thus unimportant. As it happens, in minimal supergravity models the inequality $m_t > m_b$ implies via RGE scaling¹⁷ $m_{\tilde{H}}^2 < m_{\tilde{H}}^2$ and hence $\omega > 1$. In this case, however, $\cos 2\beta < 0$ and the two sources of breaking tend to cancel. In this case, $\delta\rho(\tilde{t}-\tilde{b})$ is no longer maximized for $m_{LR} = 0$. Rather we find that it becomes maximal when the bound (3.27) is saturated ($m_{LR} = 3m_{\tilde{b}_L}$), $m_{\tilde{t}_L}^2$, $\omega \rightarrow \infty$, and $m_{\tilde{b}_L}$ is chosen such that $m_{\tilde{t}_1}$ is as small as experimentally allowed:

$$\delta\rho(\tilde{t}-\tilde{b}) \leq \left[0.66 + \left[\frac{41 \text{ GeV}}{m_t} \right]^4 \right] \delta\rho(t-b) \quad (m_{\tilde{t}_1} > 45 \text{ GeV}, \omega > 1). \quad (3.31)$$

Just like the parametrization (3.28), that of (3.31) describes the actual maximum of $\delta\rho(\tilde{t}-\tilde{b})$ in the region $75 \text{ GeV} < m_t < 200 \text{ GeV}$ to better than 5%.

Recall that the bound (3.31) is saturated if the sbottom mass is given by Eq. (3.30). Even for the weaker constraint $m_{\tilde{t}_1} > 25$ GeV this corresponds to quite heavy sbottoms of about 230 GeV for $m_t = 75$ GeV, or 400 GeV

for $m_t = 150$ GeV. Notice that within minimal SUGRA models, squarks of the first two generations would be even heavier, since $m_{\tilde{b}_L}$ is reduced via RG scaling¹⁷ due to top-Higgsino and stop-Higgs-boson loops. It is thus interesting to explore the region of smaller sbottom mass. This region is also interesting since here *lower* bounds on $\delta\rho(\tilde{t}-\tilde{b})$ can be derived, at least if $m_t > m_W$. In this case the only possibility to effectively restore $SU(2)_V$ invariance is the introduction of a big $SU(2)_V$ -invariance mass, i.e., $m_{\tilde{b}_L} \gg m_t^2 - M_W^2$. For given $m_{\tilde{b}_L}$ and m_t we will vary ω , m_{LR} , and $m_{\tilde{t}_R}^2$ within the boundaries defined by the constraint (3.27) as well as the inequalities $\omega > 1$ and $m_{\tilde{t}_R}^2 < m_{\tilde{b}_L}^2$ valid in supergravity models.

Under these circumstances, $\delta\rho(\tilde{t}-\tilde{b})$ is maximized either by the choice $\omega=1$ and $m_{LR}=0$, or for $\omega \rightarrow \infty$, $m_{\tilde{t}_R}^2$ and m_{LR} chosen such that $m_{\tilde{t}_1}$ is at its experimental minimum. In the second case, which becomes relevant if $m_{\tilde{b}_L} > m_t$, the favored choice is $m_{\tilde{t}_R}^2 = m_{\tilde{b}_L}^2$; this might, however, lead via Eq. (3.26) to too large a value of m_{LR} if $m_{\tilde{t}_1}$ is to be kept fixed. In this case, $\delta\rho(\tilde{t}-\tilde{b})$ is maximized by setting m_{LR} to its maximal value (3.27) and by using Eq. (3.23) to choose $m_{\tilde{t}_R}^2$ such that $m_{\tilde{t}_1}$ is at its experimental lower bound.

The minimum of $\delta\rho(\tilde{t}-\tilde{b})$ is always reached for $\omega \rightarrow \infty$, which minimizes $SU(2)_V$ breaking in the LL element of the $\tilde{t}$ mass matrix (3.21). In the unlikely case of $m_{\tilde{b}_L}^2 \ll m_t^2$, the minimum is reached for $m_{\tilde{t}_R}^2 \rightarrow 0$, $m_{LR} \rightarrow 3m_{\tilde{b}_L}$. For larger $m_{\tilde{b}_L}$, $\delta\rho(\tilde{t}-\tilde{b})$ is minimal if $m_{\tilde{t}_R}^2 = m_{\tilde{b}_L}^2$, which maximizes a mass term which does not contribute to the $SU(2)_V$ breaking in the absence of $\tilde{b}_L - \tilde{b}_R$ mixing ($m_b = 0$). However, the damping effect of this mass can only be felt in $\delta\rho$ if there exists some amount of L - R mixing in the stop sector. If $m_{\tilde{b}_L}$ is rather small, the minimum of $\delta\rho(\tilde{t}-\tilde{b})$ is actually reached for maximal $\tilde{t}_L - \tilde{t}_R$ mixing, i.e., when m_{LR} is chosen in (3.23) such that $m_{\tilde{t}_1}$ is at its experimental minimum. For bigger $m_{\tilde{b}_L}$, however, the minimum is reached at some intermediate value of m_{LR} , which is most easily determined by just numerically scanning m_{LR} within the limits $0 < m_{LR} < 3m_{\tilde{b}_L}$.

The resulting upper and lower bounds on $\delta\rho(\tilde{t}-\tilde{b})$ for values of $m_{\tilde{b}_L}$ that can be probed in the near future at the $Sp\bar{p}S$ and Tevatron are shown in Fig. 3 for three different values of m_t . For $m_t = 100$ GeV we only show the upper bound, since the lower bound is always below 10^{-4} and thus not of interest to us. The kink in the upper bound for $m_t = 100$ GeV originates from switching between the two choices of parameters discussed above. The kink in the lower bound for $m_t = 190$ GeV has a similar origin.

Obviously the lower bound increases much more rapidly with the top mass than the upper bound does, this is not surprising since the lower bound is zero for $m_t < M_W$, while the upper bound approaches a finite, m_t independent value in the unrealistic limit of $m_t^2 \ll M_W^2$. Further-

FIG. 3. Maximal and minimal contributions of the $\tilde{t}-\tilde{b}$ sector to $\delta\rho$. Mixing between the superpartner of left- and right-handed b -quarks has been neglected, so that $m_{\tilde{b}_L}$ is the physical mass of the $\tilde{b}_L$ -squark. The condition (3.21) on the magnitude of L - R mixing in the stop sector has been imposed in order to avoid false vacua, as discussed in the text. Furthermore, it has been assumed that $\omega \geq 1$ and $m_{\tilde{b}_L} > m_{\tilde{t}_R}$ as is the case in minimal supergravity models; this last condition is numerically not very significant. For $m_t = 100$ GeV only the upper bound is shown, because the lower bound lies below 10^{-4} .

more, in the opposite extreme, $m_t^2 \gg M_W^2, m_{\tilde{b}_L}^2$, all nonsupersymmetric contributions to the $\tilde{t}$ mass matrix (3.21) can be neglected, in which case one finds^{11,12} that $\delta\rho(\tilde{t}-\tilde{b}) \rightarrow \delta\rho(t-b)$. Figure 3 shows, however, that even in the most extreme realistic situation, $m_t = 190$ GeV and $m_{\tilde{b}_L} = 50$ GeV, there is still a difference of a factor of 2 between the lower and upper bounds, which means that realistically the nonsupersymmetric entries in the mass matrix can never be neglected. ($m_t > 190$ GeV is not possible,¹⁷ unless there is an intermediate scale between M_W and M_X , since in this case the running top quark Yukawa coupling has a Landau pole at some scale Q below M_X .)

Only the total $\delta\rho$ can be directly measured, of course. In Fig. 4 we thus show lines of constant maximal [Fig. 4(a)] and minimal [Fig. 4(b)] $\delta\rho(\tilde{t}-\tilde{b}) + \delta\rho(t-b)$, see Eq. (3.29). Figure 4(a) shows that the present experimental bound (1.1) starts to constrain parameters of the $\tilde{t}$ mass matrix (3.21) if $m_t > 130$ GeV; from Fig. 4(b) one can read off that the region of small $m_{\tilde{b}_L}$, $m_{\tilde{b}_L} < 50$ GeV, can be excluded from the present measurement of ρ if $m_t > 150$ GeV or, conversely, if a light left-handed sbottom is found, the upper bound on m_t would be reduced to about 150 GeV.

IV. $\rho - 1$ IN MINIMAL SUPERGRAVITY MODELS

So far we have discussed the various supersymmetric contributions to $\delta\rho$ in quite general softly broken supersymmetric models with minimal particle content, al-

FIG. 4. Lines of constant maximal (a) and minimal (b) $\delta\rho(t-b)+\delta\rho(\bar{t}-\bar{b})$. The parameters ω , m_{LR} , and $m_{\tilde{t}_R}$ are varied within the limits $m_{LR} < 3m_{\tilde{b}_L}$, $m_{\tilde{t}_R} < m_{\tilde{b}_L}$, and $\omega > 1$.

though we have mentioned the case of the more restrictive minimal supergravity (SUGRA) models a few times. In this section we will discuss this class of models in more detail; in particular, the correlations between the various contributions to $\delta\rho$ that have been discussed separately in Sec. III will here be fully taken into account.

We start by reminding the reader of some of the basics of minimal SUGRA models. The main assumption is that supersymmetry is broken spontaneously in a “hidden sector,” which couples to the fields of the visible sector only gravitationally. After supersymmetry is broken, this interaction induces^{5,38} a common nonsupersymmetric contribution m_0^2 to the squared masses of all scalar fields, a common mass M for all gauginos, and trilinear and bilinear nonsupersymmetric interactions among the scalars of the model described by the dimensional parameters A and B , which are of order m_0 , with $B = A - m_0$. However, this simple picture is valid only at the scale where supersymmetry is broken in the hidden sector, which usually happens at or very close to the Planck scale $M_P = 1.2 \times 10^{19}$ GeV. At lower scales radiative corrections due to the interactions between the fields of the visible sector have to be taken into account. We will here

neglect effects³⁹ due to a possible grand unified theory (GUT) sector; effectively this amounts to the assumption that SUSY is broken not at scale M_P , but at the scale $M_X \approx 2 \times 10^{16}$ GeV, where the three running gauge couplings of the SM meet. Finally, we assume the validity of the “grand desert” scenario, which means that there is no additional physics scale between M_W and M_X . (The scale of sparticle masses is by assumption close to M_W , since otherwise fine-tuning of the parameters is reintroduced.^{5,40})

Because M_X/M_W is large even on a logarithmic scale, the summation of the leading logarithms of the radiative corrections is crucial. This is most easily done by using renormalization-group equations (RGE’s). Since we have assumed that there is no new physics (except for SUSY, of course) below M_X , the relevant set of RGE can be directly derived from the supersymmetric version of the SM. We do not reproduce these well-known¹⁷ equations here, but only make a few general remarks. First, we have taken into account all Yukawa couplings of the third generation of quarks and leptons; this is important, since we will sometimes encounter situations where $\omega \approx m_t/m_b$, so that the top and bottom Yukawa couplings are of the same size. We have not, however, tried to include threshold effects at the GUT or weak scale. It has been argued^{41,42} that the latter can be quite important, and a systematic procedure for their treatment has recently been suggested.⁴² However, as shown in Ref. 23, relations between physical parameters (which is what we are interested in) are usually rather insensitive to variations of the scale where the RG scaling is terminated, if one requires M_W to have the correct value at the weak scale Q_0 . In view of the many assumptions made in minimal SUGRA calculations, incorporation of a rather involved scheme to compute the residual Q_0 dependence therefore seems something of an overkill to us, at least for our purpose, which is the investigation of how correlations between various sparticle masses predicted in SUGRA models affect the total SUSY contribution to $\delta\rho$. We thus fix $Q_0 = 100$ GeV henceforth.

This leaves us with six free parameters, which we take to be m_0 , M , A , μ , and the top and bottom Yukawa couplings h_t and h_b , respectively, the latter three quantities taken at the scale M_X . We also have two constraints, i.e., that the W mass has its correct value of 80 GeV at scale Q_0 and so does the b quark, $m_b(Q_0) = 4.5$ GeV. We assume $h_t(M_X) = h_b(M_X)$, which is exactly true in minimal SU(5) models, and has always to be at least approximately true phenomenologically. In addition we will always assume that the mass of the top quark is known; i.e., we present our results for a given m_t rather than for given h_t . For given m_0 , M , and A these constraints fix (up to a certain possible discrete ambiguity, see below) the values of μ , h_t , and h_b .

In practice one first has to make sure that the Higgs-boson potential along the neutral direction

$$V = \frac{1}{8}(g^2 + g'^2)(|H^0|^2 - |\bar{H}^0|^2)^2 + m_1^2 |H^0|^2 + m_2^2 |\bar{H}^0|^2 - m_3^2 (H^0 \bar{H}^0 + H^{0*} \bar{H}^{0*}) \quad (4.1)$$

satisfies the following two conditions:

$$m_1^2 m_2^2 < m_3^4, \quad (4.2)$$

which guarantees that gauge symmetry breaking is taking place; and

$$m_1^2 + m_2^2 \geq 2|m_3^2|, \quad (4.3)$$

since otherwise the Higgs-boson potential is unbounded from below in the direction $\langle H^0 \rangle = \langle \bar{H}^0 \rangle$. The m_i^2 in Eqs. (4.2) and (4.3) are the diagonal ($i=1,2$) and off-diagonal ($i=3$) elements of the Higgs-boson mass matrix for $\langle H^0 \rangle = \langle \bar{H}^0 \rangle = 0$, i.e., in the unbroken phase. At the scale M_X they are given by

$$m_1^2(M_X) = m_2^2(M_X) = m_0^2 + \mu^2(M_X), \quad (4.4)$$

$$m_3^2(M_X) = B\mu(M_X) = (A - m_0)\mu(M_X). \quad (4.5)$$

Note that Eq. (4.4) is incompatible with the conditions (4.2) and (4.3). However, the equality of m_1^2 and m_2^2 is destroyed at the loop level by the imbalance between the top and bottom quark masses, so that at scales $Q < M_X$ one always has $m_2^2(Q) < m_1^2(Q)$. Only if conditions (4.2) and (4.3) are satisfied is the meaningful definition of gauge-boson and quark masses possible.

Finally, after h_t , h_b , and μ are fixed for given m_0 , M , and A , one has to check for the occurrence of lower-lying minima of the scalar potential that break charge and color, as discussed in Sec. III D. The most dangerous of the various simple vacua discussed in the literature^{17,35} occurs in the direction $\langle \tilde{t}_L \rangle = \langle \tilde{t}_R \rangle = \langle \bar{H}^0 \rangle$; it leads to a minimum of the scalar potential V with $\langle V \rangle < 0$ unless

$$A_t^2 \leq 3(m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2 + m_{\tilde{t}}^2). \quad (4.6)$$

In practice the requirement that this vacuum should not lie below the physical one, rather than below zero, leads to a constraint very similar to (4.6). We require this to hold also at the GUT scale, since otherwise the Universe would have probably been trapped in this false vacuum.

In Ref. 42 it has recently been argued that the conventional^{17,35} treatment of false vacua is problematic, since one has used RGE valid in the unbroken phase down to a value which often is much smaller than a typical VEV in the false vacuum; such a treatment is inconsistent since gluon and photon masses in the false broken phase are of order of this typical VEV, and hence obviously not negligible compared to Q_0 . In fact, it has been demonstrated that some classes of false vacua do not survive in a consistent treatment. However, the false vacuum related to the condition (4.6) has VEV's of order m_0/h_t , which is of the order Q_0 (Ref. 44), since $h_t(Q_0) > 0.4$ for $m_t > 75$ GeV. We therefore use the constraint (4.6), but not the well-known^{17,35} similar constraints valid for false vacua with $\langle \tilde{\tau} \rangle$ or $\langle \tilde{b} \rangle \neq 0$.

We are now in a position to discuss some numerical results. In Figs. 5(a) and 5(b) we present some “typical” results for a general minimal SUGRA model, where m_0 , M , A , and B can all be nonzero. We have chosen a rather small value $M = 20$ GeV, corresponding to $m_g \simeq 60$ GeV, in order to be as different as possible from the no-scale model case $m_0 = A = B = 0$ discussed below. We see that under these circumstances $\delta\rho_{\text{SUSY}}$ always decreases with

increasing m_0 and decreasing m_t . Note that for the various curves of Fig. 5(a) the ratio $A/m_0 (= \hat{A})$ is fixed, not A itself. Increasing m_0 therefore also increases A . Since we have earlier identified the various A parameters at the weak scale as a potential source of $\text{SU}(2)_V$ breaking, one might think that increasing m_0 would increase the amount of $\text{SU}(2)_V$ breaking and hence lead to an increase of $\delta\rho$, contrary to our observation. We remark here that the A terms do not constitute a genuinely new source of $\text{SU}(2)_V$ breaking, since in the limit of equal Yukawa couplings, $h_b = h_t$, which restores the $\text{SU}(2)_V$ invariance of

FIG. 5. The total contribution to $\delta\rho$ from all superpartners but excluding $\delta\rho(t-b)$ in minimal supergravity models with gaugino mass $M = 20$ GeV at M_X and $B = A - m_0$, for $m_t = 150$ GeV as a function of the common scalar mass m_0 at scale M_X (a), and for $m_0 = 100$ GeV as a function of the top mass m_t (b). Curves with different texture differ in the choice of $\hat{A} = A/m_0$. Whenever two curves with the same texture are shown, the lower curve corresponds to the solution with larger μ , as discussed in the text. The curves end when one of the following constraints is violated: $m_{\tilde{W}_1} > 25$ GeV, denoted by $\tilde{W}_1$; $m_{\tilde{t}_1} > 25$ GeV, denoted by $\tilde{t}_1$; and $m_{\tilde{\nu}}^2 \geq 0$, denoted by $\tilde{\nu}$. Except for the curve with $A = 3m_0$ (solid), which extends to the largest possible value of m_t , all curves in (b) are terminated at the high- m_t end by the constraint on the chargino mass.

the Yukawa couplings, the A terms also automatically respect the symmetry. At the same time an increase of m_0 leads to a larger overall mass scale for scalars and thus, via decoupling of heavy particles, leads to a decrease of $\delta\rho$, as discussed in Sec. III. We find that in minimal SUGRA models this second effect always wins, i.e., an increase of the overall sparticle mass scale always leads to a decrease of $\delta\rho$.

Figure 5(b) shows that for $M^2 \ll m_0^2$, the purely supersymmetric contribution to $\delta\rho$ tends to increase even *faster* with m_t than the top-bottom contribution itself. The reason is that a larger m_t does not only mean a larger amount of $SU(2)_V$ breaking in the stop mass matrix, but also increases via RG scaling the discrepancy between m_2^2 and m_1^2 , and thus the difference between the VEV's of the two Higgs-boson fields, which we have identified as another source of $SU(2)_V$ breaking which affects all the four sectors discussed in Sec. III.

The dependence on the A parameter is more complicated. We have already seen in Sec. III D that depending essentially on the ratio $m_{\tilde{b}_L}/m_t$, increasing A can increase or decrease $\delta\rho(\tilde{t}-\tilde{b})$. The situation is further complicated by the assumed relation between A and B , $B = A - m_0$, which gives A a direct role in gauge symmetry breaking.

For a given value of B , there are often two values¹⁷ of μ leading to the correct W mass. This can be seen from Eqs. (4.4) and (4.5). Often the condition (4.2) cannot be satisfied if $\mu=0$, i.e., $m_3^2=0$. (Note that μ renormalizes multiplicatively: if $\mu=0$ at some scale, it is zero everywhere, which also implies $m_3^2=0$.) Increasing μ will at first increase $|m_3^2|$ faster than m_1^2 and m_2^2 , since the latter depend on μ only quadratically. At some value of μ , the correct amount of gauge symmetry breaking will be reached. However, for larger values of μ , $\mu=O(m_0)$, a further increase of μ will decrease gauge symmetry breaking, since now m_1^2 and m_2^2 grow faster than m_3^2 does. In general there can then be a second value of μ where M_W again has the correct value. This explains the occurrence of two curves with $\hat{A} (= A/m_0) = -1$ and $\hat{A} = -2$ in Fig. 5(a), and for $\hat{A} = -1$ and $\hat{A} = 0$ in Fig. 5(b). The curves for $\hat{A} = 0$ are actually connected at a point where the two solutions of μ coincide. In all these cases the solution with the larger value of μ leads to a smaller $\delta\rho$, because large μ tend to decrease the gaugino-Higgsino contribution (see Sec. III B). Furthermore, since μ enters m_1^2 and m_2^2 in the same way, large values of μ tend to decrease the difference between m_1^2 and m_2^2 , and hence lead to smaller ω . For the other values of $\hat{A}$ used in Fig. 5 only one solution is shown; this happens when the constraint (4.3) is violated even for $\mu=0$ and hence only the solution with large μ exists, which is indeed the case for the curves with $\hat{A}=3$; or the solution with large μ violates the condition (4.3) at the GUT scale, which would mean $M_W \simeq M_X$; or both solutions exist, but one of them leads to a phenomenologically unacceptable sparticle spectrum.

So far we have discussed the situation $M^2 \ll m_0^2$. Note that existing lower bounds^{29,32} on the gluino mass imply $M \neq 0$, since gaugino masses renormalize multiplicatively.

However, bounds on squark and slepton masses do not directly imply bounds on m_0 ; in fact, they can be created purely radiatively, so that $m_0=0$ is possible. One can go even beyond that and also set $A=B=0$, which is the case for so-called “no-scale” models.¹⁸ Apart from their possible theoretical advantages, they are attractive since they allow to describe the entire sparticle spectrum in terms of one SUSY-breaking parameter M (μ , h_t , and h_b are again fixed by M_W , m_t , and m_b).

In Fig. 6 we show some results for this class of models. We see that $\delta\rho_{\text{SUSY}}$ is again a steeply falling function of the overall sparticle mass scale, which is here characterized by the gaugino mass M . However, the results depend much less on m_t than they do in the previous case, especially in the region of large m_t ; just after the break in the curves, $\delta\rho_{\text{SUSY}}$ even *decreases* with m_t . The breaks signal a flip in the sign of $\omega = \langle \bar{H}^0 \rangle / \langle H^0 \rangle$. In our convention, $\text{sgn}(\omega) = -\text{sgn}(m_3^2)$. In the given case of $B=0$, $m_3^2 > 0$ at scale M_X , but it is created radiatively. (Recall that $\mu \neq 0$.) The RGE for m_3^2 contains two groups of terms. The first one contains electroweak gaugino masses M_1 and M_2 and coupling g^2, g'^2 and tends to increase m_3^2 (for our choice $M, \mu > 0$), the second group contains products of A parameters and Yukawa couplings and decreases m_3^2 if these A parameters are positive, which is always true here, since their relevant A parameters are themselves only created radiatively, essentially from terms containing the gluino mass. We thus have a one-loop electroweak effect battling a two-loop effect proportional to h_t^2 . For small m_t , below 100–120 GeV, the first effects wins, and the resulting ω is negative. In this case, increasing m_t means decreasing $|m_3^2|$ and hence increasing $|\omega|$, which tends to increase $\delta\rho$; this explains the positive slope of the curves in Fig. 6 to the left of the break. Eventually, the second effect wins, and m_3^2 and ω flip sign. Increasing m_t even more leads to larger $|m_3^2|$, and

FIG. 6. The total supersymmetric contribution to $\delta\rho$ for the “no-scale” ansatz $m_0 = A = B = 0$. The break in the curves for $M = 100$ and 150 GeV is due to a change of $\text{sgn}(\omega)$, as discussed in the text; to the left of the break, $\omega < 0$, and to the right, $\omega > 0$. The curve for $M = 75$ GeV is terminated by the requirement $m_v^2 \geq 0$.

thus to smaller $|\omega|$ and $\delta\rho$. Finally, at very large m_t , this decrease is overcome by an increase of $\delta\rho(\tilde{t}-\tilde{b})$, causing the curves to again bend up somewhat.

Note that there is a finite region of m_t where solutions with either sign of ω are possible. In this region $|m_3^2|$ is quite small, so that effects due to the bottom Yukawa coupling become important. There are then two possible solutions: either $|\omega|$ is only moderately large, around 5, leading to a fairly small h_b and hence a positive m_3^2 and a negative ω (for $\omega^2 \gg 1$, the requirement $m_b = 4.5$ GeV essentially fixes h_b/ω) or $|\omega|$ is very large, close to m_t/m_b , in which case h_b is almost as big as h_t , and m_3^2 is negative, i.e., ω is positive. Since this latter solution has a larger absolute value of ω , it leads to a large SUSY contribution to $\delta\rho$.

We now turn to the question how large $\delta\rho_{\text{SUSY}}$ can possibly be in minimal SUGRA models. To answer this question we have scanned the space of the parameters m_0 , M , and A , still assuming $B = A - m_0$ and keeping track of possible double solutions differing in μ or ω as discussed above. In addition to the “theoretical” constraints (4.3) and (4.6) we also have implemented the bounds $m_{\tilde{q},\tilde{g}} > 50$ GeV, where $m_{\tilde{q}}$ is an average squark mass and $m_{\tilde{e}}^2, m_{\tilde{\nu}_1}^2, m_{\tilde{t}_1}^2 > 25$ GeV; finally we have also studied the effect of raising these last bounds to $m_{\tilde{e}}^2, m_{\tilde{\nu}_1}^2, m_{\tilde{t}_1}^2 > 40$ GeV, which would roughly describe the situation of not finding any SUSY signal in the first year of LEP (Refs. 26 and 27).

Our results are summarized by the solid lines in Fig. 7(a) and 7(b) for the case of general minimal SUGRA models and by Fig. 8 for the “no-scale” ansatz $m_0 = A = B = 0$. The solid lines in Fig. 7 are only indicative. In this general situation a considerable numerical effort is necessary to determine the maximal $\delta\rho_{\text{SUSY}}$ for given m_t . Each curve thus consists of only 9 points, which have been connected by straight lines. In reality, these curves should not probably be continuous, since it is possible that certain solutions “die out” as m_t is changed, i.e., that a region of parameter space where all constraints are satisfied shrinks to a point and vanishes when m_t is changed. This explains, e.g., why the point for $m_t = 105$ GeV in Fig. 7(a) is below the one for 90 GeV.

Nevertheless the overall tendency is still one of increasing maximal $\delta\rho_{\text{SUSY}}$ with increasing m_t , at least in the general case of Fig. 7. Note that in this case, two different constraints are just saturated on the solid lines. In Fig. 7(a) they are the requirement $m_{\tilde{\nu}}^2 > 0$ and the requirement of correct gauge symmetry breaking with small μ , while for the more stringent constraints of Fig. 7(b) the bound on $\delta\rho$ is actually set by the bounds on sneutrino and chargino masses, for the highest m_t values and the stricter constraints, the false-vacuum constraint (4.6) and the gluino mass bound also become important, which explains the downturn of this curve at $m_t > 180$ GeV.

For comparison, Fig. 7 also shows bounds on $\delta\rho_{\text{SUSY}}$ in various general softly broken SUSY models. In all these curves the m_t dependence is purely due to the direct

effect of m_t in the $\tilde{t}$ mass matrix (3.21). All curves include m_t -independent contributions of 3.6×10^{-4} from the Higgs-boson sector, see Eq. (3.9), 2.0×10^{-3} (1.4×10^{-3}) from gauginos and Higgsinos, Eq. (3.13), and 1.9×10^{-3} (1.0×10^{-3}) from sleptons, see Eqs. (3.15) and (3.16), in Figs. 7(a) and [7(b)]. The dotted and short-dashed curves also contain a contribution of 1.7×10^{-3} from the first two generations of squarks, see Eq. (3.17). For the curves labeled “general,” the only constraint included for the stop sector (besides the bound on $m_{\tilde{t}_1}$) is the requirement (3.27) meant to forbid false vacua. Recall that in this case $\delta\rho$ is maximal for $\omega \rightarrow 0$. This is

FIG. 7. The maximal total supersymmetric contribution to $\delta\rho$ as a function of m_t . The constraints $m_{\tilde{q},\tilde{g}} > 50$ GeV have been imposed, where $\tilde{q}$ denotes any squark other than the stop. In addition, the pre-LEP constraints $m_{\tilde{e}}, m_{\tilde{\nu}_1}, m_{\tilde{t}_1} > 25$ GeV have been imposed in (a), while for (b) $m_{\tilde{e}}, m_{\tilde{\nu}_1}, m_{\tilde{t}_1} > 45$ GeV and $m_{\tilde{\nu}} > 40$ GeV have been required, which roughly correspond to the limits obtainable by one year of LEP physics without any SUSY signal. The solid curves result from a numerical scan of the parameter space of minimal supergravity models. The other curves correspond to more general softly broken supersymmetric models, as discussed in the text.

FIG. 8. The maximal supersymmetric contribution to $\delta\rho$ for the “no-scale” ansatz $m_0 = A = B = 0$. For the solid curves only the pre-LEP constraints $m_{\tilde{q}}, m_{\tilde{g}} > 50$ GeV, $m_{\tilde{e}}, m_{\tilde{\tau}_1}, m_{\tilde{\mu}_1} > 25$ GeV have been imposed, while for the dashed curves stricter lower bounds have been assumed, as indicated. The origin of the break in the curves is again the flip of $\text{sgn}(\omega)$ as explained in Fig. 6.

quite unrealistic in view of the fact that $\omega < 1$ implies that Yukawa couplings are even more different than quark masses; in fact, for $m_t > 75$ GeV, h_t will have a Landau pole below M_X if $\omega < 0.4$. Also, all models with radiative electroweak symmetry breaking, including superstring-inspired ones, predict $\omega \geq 1$. This constraint is implemented in the short-dashed curves. As discussed in Sec. III D, $\delta\rho_{\text{SUSY}}$ is now maximal for large ω and $m_{\tilde{t}_1}$ as small as allowed experimentally; the discrepancy between the dotted and the short-dashed curve thus grows with the lower bound on $m_{\tilde{t}_1}$.

As discussed in Sec. III D, this choice requires the mass of the left-handed sbottom to be quite large. At the same time, we assumed some other squark masses to be at their experimental lower limit of 50 GeV. Such a large hierarchy among squark masses is not natural; it could also induce dangerously large contributions⁴⁵ in $K^0\bar{K}^0$ and $B^0\bar{B}^0$ mixing, since it partially destroys the super Glashow-Iliopoulos-Maiani (GIM) cancellation. Finally, if squark masses are equal at some super-high-energy scale, as in SUGRA models, the big top Yukawa coupling will actually decrease $m_{\tilde{b}_L}$ as compared to $m_{\tilde{d}_L}$ and $m_{\tilde{s}_L}$, unless there are as yet unknown large superpotential couplings involving d and s quarks, but not b squarks. If one implements this inequality one finds that the total squark contribution to $\delta\rho$ is still maximized for the same choice of parameters that maximizes $\delta\rho(\tilde{t}-\tilde{b})$, even though the squarks of the first two generations are now so heavy that their contribution is negligible. The long- and short-dashed curves are thus parallel to each other.

Comparing the long-dashed and solid curves in Fig. 7 we see that the various nontrivial relation between parameters inherent in minimal SUGRA models reduce the maximal $\delta\rho_{\text{SUSY}}$ by about 30–40 % (40–55 %) for the

weaker (stronger) constraints on sparticle masses. The effects of allowing B to be independent of A , or of some possible³⁹ sizable Yukawa couplings on a GUT sector, can be at most of this size.

Finally, Fig. 8 shows that the no-scale ansatz $m_0 = A = B = 0$ reduces the maximal SUSY contribution by another factor of 2 or even 3, except in the region around $m_t = 120$ GeV where very large values of ω are possible. In this case, the bound is always set by the sneutrino mass constraint, i.e., $m_{\tilde{\nu}}^2 \geq 0$ for the solid curves and $m_{\tilde{\nu}}^2 > 40$ GeV for the dashed curves.

V. SUMMARY AND CONCLUSIONS

In this paper we have studied supersymmetric contributions to the electroweak ρ parameter, defined via the relative strengths of neutral- and charge-current interactions at zero momentum transfer. Since we have required minimal particle content, all Higgs-boson fields reside in $\text{SU}(2)_V$ doublets, and thus the custodial global $\text{SU}(2)_V$ symmetry^{13,19} of the gauge-boson mass matrix in the SM is also present in our case. The supersymmetric contributions to $\delta\rho$ via one-loop corrections to gauge boson propagators can hence be classified according to the various sources of $\text{SU}(2)_V$ breaking present in each sector.

The simplest case is that of the superpartners of massless SM fermions, in practice all sleptons and the first two generations of squarks. The only source of $\text{SU}(2)_V$ breaking here is given by a possible inequality between the VEV’s of the two neutral Higgs-boson fields of the model, $|\omega| = |\langle \tilde{H}^0 \rangle / \langle H^0 \rangle| \neq 1$. However, since the absolute scale of these VEV’s is given by the W mass, even the extreme cases $\omega \rightarrow 0$ or $\omega \rightarrow \infty$ do not constitute strong $\text{SU}(2)_V$ breaking; as a result, the maximal contribution from this sector amounts to (a few) $\times 10^{-3}$, as shown in Sec. III C.

The next complicated cases are the Higgs and Higgsino-gaugino sectors. Here the sources of $\text{SU}(2)_V$ breaking are $\omega \neq 1$ and hypercharge interactions. However, since the hypercharge gauge coupling is small, both sources are rather weak, so that these sectors again only contribute a few $\times 10^{-3}$, or in case of the Higgs-boson sector, at the level of 10^{-4} .

Only in the $\tilde{t}-\tilde{b}$ sector do all sources of $\text{SU}(2)_V$ breaking that can be present¹² in SUSY models contribute; in particular, here inequalities between Yukawa couplings as well as trilinear scalar interaction strengths are important. In principle the latter allow for arbitrarily large contributions to $\delta\rho$ even for a finite top quark mass, as shown in Sec. III D; in practice, however, this is only possible if the off-diagonal element in the stop mass-squared matrix is of the same order as the diagonal elements even when these diagonal elements are very large. Since the off-diagonal element contains a factor of m_t , this is only possible if the trilinear interaction strength becomes very large as compared to the overall sparticle mass scale. This, however, is known³⁵ to lead to lower-lying false vacua which break charge and color symmetries. Requiring these vacua to be absent then reduces the $\tilde{t}-\tilde{b}$ contribution to $\delta\rho$ to at most the same order as the t - b contribution. Nevertheless it remains the dominant supersym-

metric contribution in the case of heavy tops, $m_t > 140$ GeV.

In Sec. IV we discussed the important special case of minimal SUGRA models with radiative gauge symmetry breaking. In these models, supersymmetry breaking is described by only three parameters, leading to numerous relationships between sparticle masses and mixings at the physically relevant weak scale. However, we found that a good part of the effect of these relations can be mimicked by two simple inequalities: Requiring $\omega > 1$, which in models with radiative breaking is a consequence of $m_t > m_b$, and $m_{\tilde{b}_L} < m_{\tilde{d}_L}, m_{\tilde{s}_L}$ which results from the inequality $m_t > m_c, m_u$. The maximal SUSY contribution to $\delta\rho$ obtainable in a general softly broken SUSY model that respects these inequalities is only 30–50 % bigger than the corresponding result of minimal SUGRA models. We are therefore confident that our results will not change dramatically even if we allow for an arbitrary B parameter,³⁸ GUT-sector effects on Higgs-boson masses³⁹ at scale M_X , or even gaugino masses being different from each other⁴⁶ at M_X . On the other hand, we found that imposing the “no-scale” ansatz¹⁸ of vanishing scalar masses as well as nonsupersymmetric bilinear and trilinear scalar interactions at scale M_X does reduce the maximal SUSY contribution to $\delta\rho$ by as much as a factor 2 or even 3. Finally, we always found that $\delta\rho_{\text{SUSY}}$ decreases rapidly if the overall sparticle mass scale is increased. This is not surprising, because this increase gradually decouples the various sources of $\text{SU}(2)_V$ breaking from low-energy physics. As a result the maximal $\delta\rho_{\text{SUSY}}$ is reduced by 30–50 % if the lower bound on masses of all charged sparticles is raised from 25 to 45 GeV and the additional bound $m_{\tilde{\nu}} > 40$ GeV is imposed, conditions that roughly describe the situation after one year of LEP running without SUSY signals.

We have performed our analysis for models with minimal particle content, but we believe that the results of Sec. III would be basically unaltered in many more general models. In particular, since the introduction of a Higgs-boson singlet does not introduce any new source of $\text{SU}(2)_V$ breaking, we do not expect it to change our analyses of Secs. III A and III B much; probably the most dramatic effect is that the upper bound on the mass of the lightest neutral Higgs scalar increases from M_Z to about 150 GeV, which reduces the most negative $\delta\rho_{\text{Higgs}}$ to about -5×10^{-4} , which is still very small. Similarly, new vectorlike quarks and lepton superfields as present in

E_6 modes to not constitute new sources of $\text{SU}(2)_V$ breaking either; their direct effect will thus be small. In both cases the RGE analysis which is relevant if gauge symmetry is to be broken radiatively can change the situation quite dramatically;⁴⁷ in particular E_6 models can contain new Yukawa interactions that could conceivably even destroy the inequality $m_{\tilde{b}_L} < m_{\tilde{d}_L}$. Also, E_6 models can produce⁴⁸ a nonvanishing $\delta\rho$ even at a tree level via the mixing of a SM Z boson with an additional neutral gauge boson Z' . Note however that this $\delta\rho$ is also always bigger than zero, so that it cannot cancel contributions from the sources discussed in our paper.

We should also mention that although an analysis of supersymmetric contributions to physical-gauge-boson mass shifts in general and the ratio of W and Z masses in particular could proceed¹⁰ along lines similar to our analysis, the numerical results could be quite different. It is true that the custodial $\text{SU}(2)_V$ symmetry protects the ratio M_W/M_Z just as well as it does for the ratio of neutral-to-charged-current interaction strengths; a zero result for deviations in one case will therefore also mean that the second relation is preserved. However, our analysis is sensitive only to two-point functions at zero momentum transfer, whereas mass shifts have to be computed on the weak-boson mass shell, i.e., at $q^2 = m_V^2$ ($V = W, Z$). In this context one should recall that our results were biggest when some sparticle masses are well below M_W, M_Z , in which case there is clearly some structure in the two-point functions, which makes extrapolation from $q^2 = 0$ to $q^2 = M_V^2$ difficult without further analysis.¹⁵

In summary we have shown that even for the quite restrictive case of minimal supergravity models, the supersymmetric contributions to $\delta\rho$ can be twice as big as the standard-model contribution if the top mass is close to its present lower bound of about 75 GeV. Since we seem to be witnessing a steady increase of the lower bound on m_t and a decrease of the upper bound on $\delta\rho$, it seems fair to conclude that in the near future experimental constraints on $\delta\rho$ can lead to significant constraints on supersymmetric models.

Note added. After having completed the manuscript, we received a paper from R. Barbieri *et al.*⁴⁹ in which the radiative corrections to M_W , the ρ parameter and the lepton asymmetries at the Z poles are discussed in the minimal supersymmetric standard model. Where there is an overlap, our results agree with theirs.

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