CERN-TH.5672/90
FTUAM/05-90

ONE-LOOP CORRECTIONS TO THE SKYRMION MASS

Antonio DOBADO¹
CERN, CH-1211 Geneva, Switzerland
and
Juan TERRON
Departamento de Física Teórica
Universidad Autónoma de Madrid
Cantoblanco 28049 Madrid, Spain

Abstract

The one-loop corrections to the Skyrmon mass are computed by the use of a non-local effective Lagrangian. A Skyrmon mass of 1.4GeV is obtained for the values of the parameters that fit the $\pi - \pi$ elastic scattering data in the context of ChPT. Strong indications of the stability of the Skyrmon when one-loop corrections are included are also presented.

¹On leave of absence from Departamento de Física Teórica de la Universidad Complutense de Madrid, 28040 Madrid, Spain.

In the last years, a lot of work has been done in the Phenomenological Lagrangian Approach or so-called Chiral Perturbation Theory (ChPT) [1,2] to describe meson physics under $1/\Lambda^2$. ChPT is a systematic expansion in the derivatives of the meson fields of the strong amplitudes. Working up to some given power of the momenta of the external particles requires only a finite number of parameters that eventually could be obtained from QCD. Nevertheless, as we are not able at the present moment to use QCD at low energies, these parameters have to be fixed to fit the experimental data. The model is in principle not renormalizable but working at a given order in the external momenta, one can neglect the new counterterms that appear when loops are computed. This is due to the fact that when they are not of the same kind as those existing in the original Lagrangian, then they are proportional to higher order powers in the external momenta. In the simplest version ChPT is formulated as a theory of pions based on the chiral group $SU(2)_L \times SU(2)_R$. The lowest order in the derivative expansion has two derivatives and reproduces the standard current algebra results. Going to the next order i.e. four derivatives, one has to add two more terms (see below) and two arbitrary constants. Consistency in power energy counting requires including in the computation of the amplitudes the one-loop effects coming from the two-derivative term [3], together with the tree-level contributions coming from the four-derivative terms and making some appropriate renormalization of the parameters.

Although ChPT has been shown to be useful to describe meson dynamics, it is incomplete since it does not take into account baryons. Nevertheless there is a very well-known way to introduce baryons into the kind of Lagrangians used in ChPT at least classically. The idea was proposed by Skyrme almost thirty years ago [3], well before the advent of concepts such as chiral symmetry and QCD. It consists of unifying the description of pions and nucleons (i.e. mesons and baryons) by considering the baryons as topological solitons (Skyrmions) that under some conditions are developed by the pion fields. This beautiful picture of baryons was largely forgotten for years until some work by Balachandran et al. [4], further work by Witten, who showed how the Skyrmions can carry half-integer spin [5], and the success of the qualitative descriptions of the baryonic phenomenology pioneered by Adkins, Nappi and Witten [6]. In the last years a lot of work has been done trying to describe the complexities of the hadronic world with the Skyrmin model (see [7] and [8] for recent reviews). The results are very promising.

Unfortunately there is one piece lacking in all these works, at least to the knowledge of the authors (see the end of this paper for an exception). Most of the work in the Skyrmin model has been done at the classical level, i.e. without including one or more loops corrections in the computations. From our point of view this is not consistent since, as it is clear in the ChPT context, one-loop corrections to the two-derivative Lagrangian are of the same order in energy as the four-derivative terms needed to stabilize the Skyrmin.

There is another important drawback in working only at the classical level in Skyrmin physics. As there are no loops there is no renormalization of the free parameters. Therefore, one cannot compare the parameters that fit the proton mass with, say, the ones that fit the $\pi - \pi$ scattering (the last ones depend on some energy scale since they are renormalized parameters). Attempts to unify the parameters describing the meson world with the ones describing the baryon world have been made in the literature [9] at the classical level. From ChPT we know that loops are important in $\pi - \pi$ scattering and therefore we have

to find some way to include them also in the Skyrmin physics context.

In this letter we report some early results of a research program intended to unify ChPT and Skyrmin physics. Our final goal is to extend the ChPT approach to the topological non-trivial sector of the configuration space, in order to describe the phenomenology of mesons and baryons at the same level.

Our starting point is the $SU(2)_L \times SU(2)_R$ effective chiral Lagrangian given by :

$$L = L_0 + L_1 \quad (1)$$

where

$$L_0 = \frac{f_\pi^2}{4} \text{tr} \partial_\mu U \partial^\mu U^\dagger \quad (2)$$

$$L_1 = M \text{tr} (\partial_\mu U \partial^\mu U^\dagger) \text{tr} (\partial_\nu U \partial^\nu U^\dagger) + N \text{tr} (\partial_\mu U \partial^\mu U^\dagger) \text{tr} (\partial_\nu U \partial^\nu U^\dagger) \quad (3)$$

Here the $U(x)$ field belongs to $SU(2)$ and can be parametrized in terms of the pion fields as $U(x) = \exp(i \pi^a(x) \sigma^a)$. From this Lagrangian one obtains the following low energy amplitudes for $\pi - \pi$ elastic scattering

$$T(\pi_a \pi_b \rightarrow \pi_c \pi_d) = A(s, t, u) \delta_{ab} \delta_{cd} + A(t, s, u) \delta_{ac} \delta_{bd} + A(u, t, s) \delta_{ad} \delta_{cb} \quad (4)$$

with

$$A(s, t, u) = \frac{s}{f_\pi^2} + \frac{4}{f_\pi^4} [2M(\nu)s^2 + N(\nu)(t^2 + u^2)] + \frac{1}{(4\pi)^2 f_\pi^4} \left[-\frac{1}{12}(3t^2 + u^2 - s^2) \log\left(\frac{-t}{\nu^2}\right) - \frac{1}{12}(3u^2 + t^2 - s^2) \log\left(\frac{-u}{\nu^2}\right) - \frac{1}{2}s^2 \log\left(\frac{-s}{\nu^2}\right) \right] \quad (5)$$

The above amplitudes include the tree-level contributions coming from L and the one-loop contribution coming from L_0 in the spirit of the effective Lagrangian approach of ChPT [1,2]. The renormalized parameters $M(\nu)$ and $N(\nu)$ depend on an arbitrary scale ν . By demanding the physical amplitudes to be independent on that scale it is easy to find the evolution equations:

$$M_R(\nu) = M_R(\nu_0) - \frac{1}{12(4\pi)^2} \log\left(\frac{\nu}{\nu_0}\right) \quad (6)$$

$$N_R(\nu) = N_R(\nu_0) - \frac{1}{6(4\pi)^2} \log\left(\frac{\nu}{\nu_0}\right)$$

The values of the M_R and N_R parameters at some scale can be chosen to fit the $\pi - \pi$ elastic scattering data. For instance, in [10] the values $M_R(1\text{GeV}) = -0.011$ and $N_R(1\text{GeV}) = 0.0046$ were obtained. A further fit of the data was made in [11] where the Padé approximant technique was used to unitarize the chiral series producing a great improvement of the fit. The values of the parameters obtained were $M_R(1\text{GeV}) = -0.0041$ and $N_R(1\text{GeV}) = 0.0062$. The amplitudes in eq.5 have been derived in many ways in the literature [1,2], [?], but from the point of view of Skyrmin physics the most relevant is the one by Cheyette and Gaillard (see [14] and previous work quoted there). In this reference the authors compute an effective Lagrangian that reproduces the contribution

of the one-loop corrections coming from L_0 to the $\pi - \pi$ scattering amplitudes, (in fact they work in another different physical context), but their results apply here). Adding the L_4 contribution and introducing the renormalized parameters $M_H(\nu)$ and $N_H(\nu)$ it is not difficult to obtain the effective Lagrangian that corresponds to the amplitudes in eq.5. This Lagrangian may be written as:

$$L_{eff} = \frac{1}{2} H_\mu^\alpha H^{\mu\alpha} + \frac{1}{f_\pi^2} (H_\mu^\alpha H^{\mu\alpha})(H_\nu^b H^{\nu b}) + \frac{1}{f_\pi^2} (H_\mu^\alpha H_\nu^\alpha)(H^{\mu\nu} H^{\mu\nu}) + \frac{1}{64\pi^2 f_\pi^2} \{ H_\mu^\alpha H^{\mu\alpha} \log \frac{\nu^2}{\partial^2} H_\nu^b H^{\nu b} + H_\mu^\alpha H^{\mu\alpha} \log \frac{\nu^2}{\partial^2} H_\nu^c H^{\nu c} + \frac{1}{3} \{ H_\mu^\alpha H^{\mu\alpha} \log \frac{\nu^2}{\partial^2} H_\nu^b H^{\nu b} - H_\mu^\alpha H_\nu^b \log \frac{\nu^2}{\partial^2} H_\mu^\alpha H_\nu^b \} \} \quad (7)$$

where $H_\mu^\alpha(x)$ is defined by the relation $i\sigma^\alpha B_\mu^\alpha = f_\pi \partial_\mu U U^\dagger$, i.e. $H_\mu^\alpha = f_\pi \text{Tr}(U^\dagger \sigma^\alpha \partial_\mu U)/2i$. One can check that this Lagrangian is ν -independent provided $M_H(\nu)$ and $N_H(\nu)$ scale according to eq.6. Another feature is its non-locality due to the $\log \partial^2$ operators appearing in it. This is not surprising since L_{eff} takes into account the effect of pion loops.

The Lagrangian in eq.7 is very appropriate for working with Skyrmions because it is explicitly lacking of any divergence (neither infra-red nor ultra-violet) and it is ν -invariant. The mass functional for time-independent fields may be written as

$$M[U(\vec{x})] = - \int d^3x L_{eff}(\vec{x}) \quad (8)$$

Using this formula requires the computation of expressions such as $\hat{\Omega}_\nu \phi_{\alpha\beta}^a(\vec{x})$ where $\hat{\Omega}_\nu \equiv \log(-\nabla^2/\nu^2)$ and $\phi_{\alpha\beta}^a(\vec{x}) \equiv B_\alpha^a(\vec{x}) B_\beta^a(\vec{x})$. This task can be done by the use of the Fourier transform of $\phi_{\alpha\beta}^a(\vec{x})$ i.e.

$$\hat{\Omega}_\nu \phi_{\alpha\beta}^a(\vec{x}) \equiv \int d\vec{k} \phi_{\alpha\beta}^a(\vec{k}) \log(k^2/\nu^2) \exp(i\vec{k} \cdot \vec{x})$$

$$\phi_{\alpha\beta}^a(\vec{k}) \equiv \frac{1}{(2\pi)^3} \int d\vec{x} \phi_{\alpha\beta}^a(\vec{x}) \exp(-i\vec{k} \cdot \vec{x}) \quad (9)$$

Let us concentrate now on the problem of the computation of the Skyrmon mass from eq.8. The standard ansatz for the Skyrmin field is $U_S(\vec{x}) = \exp(i f(r) \hat{x} \sigma^a)$ where $f(r)$ is the so-called chiral angle. As we are interested in the topological sector with winding number equal to one, the boundary conditions for $f(r)$ are $f(0) = \pi$ and $f(\infty) = 0$. Now, after some amount of algebra, the mass functional can be written as

$$M[U_S] = M_{tree}[U_S] + M_{1-loop}[U_S] \quad (10)$$

$$M_{tree}[U_S] = 4\pi \int r^2 dr \left\{ \frac{f^2}{2} (f'^2 + \frac{2s^2}{r^2}) - 4M(\nu)(f'^2 + \frac{2s^2}{r^2})^2 - 4N(\nu)(f'^4 + \frac{2s^4}{r^4}) \right\} \quad (11)$$

$$M_{1-loop}[U_S] = \frac{1}{8\pi^2} \int_0^\infty k^2 dk \log(k^2/\nu^2) \sum_{j=1}^n c_j [W_j(k)]^2 \quad (12)$$

Here $s \equiv \sin f(r)$ and $c \equiv \cos f(r)$. The functions $W_j(k)$ are defined as

$$W_j(k) \equiv \int_0^\infty r^2 dr J_j(r) \quad (13)$$

The values of the constants c_j and the functions $J_j(r)$ may be found in table 1. Since we have been able to write the mass of the Skyrmin in terms of the chiral angle $f(r)$ it is possible in principle to compute numerically the mass corresponding to any given $f(r)$. Nevertheless, the convergence of the numerical integration process involved in the computation of eq.12 is very slow because of the $k^2 \log(k^2/\nu^2)$ factors. To overcome this problem we make an integration by parts in the equation giving M_{1-loop}^ν . Thus we obtain

$$M_{1-loop}^\nu[U_S] = \frac{1}{8\pi^2} \sum_{j=1}^n c_j \left(\int_0^\infty k^2 dk \log(k^2/\nu^2) [W_j(k)]^2 + \int_\nu^\infty \frac{dk}{k} \log(k^2/\nu^2) [W_j(k)]^2 \right) \quad (14)$$

where

$$W_j(k) \equiv \int_0^\infty dr J_j(k, r) \quad (15)$$

The $J_j(k, r)$ functions are displayed in table 2. In this new form, the numerical convergence of the integrals involved in the computation of the Skyrmin mass is greatly improved. The next thing to do at this stage is to find the function $f(r)$ that minimizes the mass functional and has the right boundary conditions. This turns out to be a very hard task. Note that one has to explore the behaviour of a highly complex functional in the corresponding functional space. Obviously, before doing that, it is useful to have some feeling of what could be the effect of the one-loop corrections to the Skyrmin mass in a simpler way. One possibility is restricting ourselves to some reasonable parametrized family of functions $f_\lambda(r)$ in the search for the minimum. Assuming that the one-loop corrections do not change drastically the pattern of $f(r)$, one could use as $f_\lambda(r)$ any parametrization that works in the classical case (i.e. when one uses L instead of L_{eff} to compute the Skyrmin mass). In order to get some numerical results we have used the parametrization of the chiral angle recently proposed by Atiyah and Manton [16]:

$$f_\lambda(r) = \pi [1 - (1 + \lambda^2/r^2)^{-1/2}] \quad (16)$$

This parametrization fits very well the numerical results in the classical case. Using the numerical values $f_\pi = 93 \text{ MeV}$, $1M_H(1 \text{ GeV}) = -0.011$, $1N_H(1 \text{ GeV}) = 0.0016$ of [9] as indicative of the values relevant for hadron physics, we have obtained the minimum of the mass functional in eq.8 for the Atiyah and Manton parametrization. The minimum is found for $\lambda^2 = 15.1 \text{ GeV}^{-2}$ corresponding to a Skyrmin mass of

$$M = 1.424 \text{ GeV} \quad (17)$$

which splits into

$$M_{tree}^\nu = 1.611 \text{ GeV}$$

$$M_{1-loop}^\nu = -0.2177 \text{ GeV}$$

for $\nu = 16\epsilon V$.

At this point some comments are in order. First of all, the Skyrmon mass in the formalism considered here does not depend on the scale ν . We have checked this fact directly in our numerical results. Note that, because of the structure of eq.11 and eq.12, this is a non-trivial test of our computations. The results, shown in table 3, are highly independent of the renormalization scale ν . On the other hand, it is interesting to stress that although M is ν -independent, M_{tree}^ν and M_{1-loop}^ν are both ν -dependent. Only the addition of these two magnitudes is ν -independent. This is a very nice property of the formalism considered here. It tells us that it is meaningless to compute only the contribution of the one-loop corrections to the mass of the Skyrmon. One has to work at the same time with the terms coming from L_0 and L_1 and the one-loop corrections from L_0 just in the spirit of the effective Lagrangian approach or ChPT.

Another important point we have to deal with in studying Skyrmions is the problem of their stability. In the classical case it is customary to split L_1 in eq.1 in a different way (the so-called Skyrme and non-Skyrme terms) using the parameters $e^2 \equiv 1/16N$ and $\gamma \equiv (1 + M/N)/2$. The Skyrme-term contribution to the Skyrmon mass is positive provided e^2 is positive (phenomenologically $e^2 \sim 31$ [10]). On the other hand, it has been shown in the literature [8] that the Skyrmon is unstable for $\gamma > 0.12$. For $\gamma < 0$ there is no problem and the classical Skyrmon is stable. What happens in the region $\gamma \in (0, 0.12)$? We have studied this point with the aid of the parametrization $f_\lambda(r) = \pi(1 - (r/\lambda)^\gamma)$ if $r < \lambda$ and $f_\lambda(r) = 0$ if $r > \lambda$. With this kind of chiral angle function it is possible to show analytically that the classical mass-functional is not bounded from below for any $\gamma > 0$. We have also designed the following parametrization

$$f_\lambda(r) = \pi \exp(-(r/\lambda)^\gamma) \quad (18)$$

that allows us to find numerically the instabilities. In the classical case, we find that for any $\gamma > 0$, if we increase α then λ_{min} (the λ value that minimizes the mass-functional) goes to zero and that there is always a value of α (α_c) such that the minimum does not exist (i.e. $M \rightarrow -\infty$) for $\alpha > \alpha_c$. This result suggests that it does not make much sense to consider positive values of γ because, even if there were some local minimum in that case, it would decay by tunnelling in some configuration of arbitrary large negative energy.

Motivated by the fact that the ansatz introduced above seems to be useful in discovering instabilities of the classical Skyrmon we have applied it in the one-loop approach considered in this paper. By introducing this ansatz in the mass-functional of eq.8 we have observed that when α increases, the value of λ_{min} increases. This is the same kind of behaviour that we found in the classical case in the region of stability ($\gamma < 0$). This result strongly suggests that, for the numerical values of the renormalized parameters $M_R(\nu)$ and $N_R(\nu)$ considered here, the one-loop Skyrmon is very likely to be stable.

Finally we would like to conclude by stressing that the results presented here are only the beginning of a more ambitious program. As mentioned in the introduction, the goal of this program is to incorporate baryons in ChPT through the Skyrmon model in a consistent way. This will allow us to make a global fit of the ChPT parameters both from meson and baryon physics. From the early results presented here this program seems to be hard but feasible. Work is in progress in this direction [17].

Note added: In [18] the authors obtain a one-loop effective Lagrangian to compute the soft pion corrections to the Skyrmon. Nevertheless, due to the local nature of their effective Lagrangian, they have both ultra-violet and infra-red divergences in their results. From the discussion above we consider the non-local effective Lagrangian of [14] more appropriate for the reconciliation of Skyrmon physics with ChPT.

Table captions

- Table 1: Values of the constants c_j and the functions $I_j(r)$ appearing in eq.12 and eq.13 ($v \equiv \sin(kr)/kr$ and $v' \equiv d\psi/d(kr)$).
- Table 2: Values of the functions $J_j(k, r)$ appearing in eq.15 ($\sigma'_j(r) \equiv d\sigma_j(r)/dr$).
- Table 3: Values of M_{free}^ν , M_{l-loop}^ν and M for different values of ν (all of these magnitudes are given in $(GeV)^{-2}$) and for $\Lambda^2 = (5.5GeV)^{-2}$.

Table 1:

j	1	2	3	4	5	6	7	8
c_j	1	2	-2/3	-1/3	-1/3	1	-10/3	-2/3
$I_j(r)$	$\alpha_1 \psi''$	$\alpha_1 \psi'/kr$	$\alpha_2 \psi''$	$\alpha_2 \psi'/kr$	$\alpha_3 \psi'$	$\alpha_1 \psi$	$\alpha_5 \psi$	$\alpha_6 \psi$
$\alpha_j(r)$	$r'^2 - \frac{r^2}{r'^2}$	$\frac{\alpha_1}{r^2} f' - \frac{r^2}{r'^2}$	$\frac{r^2}{r'^2}$	$f'^2 + \frac{r^2}{r'^2}$	$\frac{r^2}{r'^2}$	$\frac{\alpha_1}{r^2} f' + \frac{r^2}{r'^2}$		

Table 2:

j	$J_j(k, r)$
1	$((r\alpha_1)'' + 2(\alpha_1' + \alpha_1/r)) \sin kr$
2	$(\alpha_1' + \alpha_1/r) \sin kr$
3	$((r\alpha_2)'' + 2(\alpha_2' + \alpha_2/r)) \sin kr$
4	$(\alpha_2' + \alpha_2/r) \sin kr$
5	$(r\alpha_3'' + 3\alpha_3') \cos kr$
6,7,8	$(r\alpha_{6,7,8})'' \sin kr$

Table 3:

ν	M_{free}^ν	M_{l-loop}^ν	M
0.1	0.961276	0.459899	1.421175
1.0	1.642202	-2.18027	1.421175
10.	2.320128	-895952	1.421176

References

- [1] S. Weinberg, *Physica* **96A** (1979) 327
- [2] J. Gasser and H. Leutwyler, *Ann. of Phys.* **158** (1984) 142
- [3] T.H.R. Skyrme, *Proc. Roy. Soc. London* **260** (1961) 127; *ibid* **262** (1961) 237.
T.H.R. Skyrme, *Nucl. Phys.* **31** (1962) 556
- [4] A.P. Balachandran et al., *Phys. Rev. Lett.* **49** (1982) 1124; *Phys. Rev.* **D27** (1983) 1153
- [5] E. Witten, *Nucl. Phys.* **B223** (1983) 122; *ibid* 133
- [6] G.S. Adkins, C.R. Nappi and E. Witten, *Nucl. Phys.* **B228** (1983) 552
- [7] I. Zahed and G.E. Brown, *Phys. Rep.* **142** (1986) 1
- [8] Chiral Solitons, Ed. Keh-Fei Lin, (World Scientific, Singapore 1987).
- [9] J.F. Donoghue, E. Golowich and B.R. Holstein, *Phys. Rev. Lett.* **53** (1984) 717
- [10] J.F. Donoghue, C. Ramirez and G. Valencia, *Phys. Rev.* **D38** (1988) 2195
- [11] A. Dobado, M.J. Herrero and J.N. Truong, *Phys. Lett.* **B235** (1990) 131
- [12] H. Lehmaun, *Phys. Lett.* **41B** (1972) 529
- [13] D. Bessin and J. Zinn-Justin, *Phys. Rev.* **D5** (1972) 1313
- [14] O. Cheyette and M.K. Gaillard, *Phys. Lett.* **B197** (1987) 205
- [15] A. Dobado and M.J. Herrero, *Phys. Lett.* **B228** (1989) 195
- [16] M.F. Atiyah and N.S. Manton, *Phys. Lett.* **B222** (1989) 138
- [17] A. Dobado and J. Terron, in preparation.
- [18] I. Zahed, A. Wirzba and U.G. Meissner, *Phys. Rev.* **D33** (1986) 830.