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HIGHER LEVEL KAC-MOODY STRING MODELS AND THEIR PHENOMENOLOGICAL IMPLICATIONS

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ABSTRACT

We present a systematic approach to string compactifications in which some of the gauge interactions are realized at higher Kac-Moody level. This can be realized in the current constructions of $4-d$ strings including symmetric and asymmetric compactifications. It is shown that there exist higher level models which can be obtained from exactly marginal deformations of a level-one model and a simple field theory interpretation is provided in terms of flat scalar field directions in the original level-one model. We then consider the phenomenological possibilities opened to $4-d$ higher-level string models. In particular we show how the levels associated to standard model interactions as well as the massless content of the theory are constrained by unitarity and the renormalization group equations. We also consider the prospects for string GUT models. Mechanisms like "missing partner" in $SU(5)$ or "see-saw" neutrino masses in $SO(10)$ are only possible for $k \geq 5$. For E_6 at any level only adjoints or fundamentals can be light. A generic feature of the unification models seem to be the existence of relatively light chiral multiplets transforming like $(8, 1, 0) + (1, 3, 0) + (1, 1, 0)$ under $SU(3) \times SU(2) \times U(1)$. The problems of neutrino masses and proton stability are reconsidered and a possible simultaneous solution involving a discrete gauge symmetry is suggested.

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1. Introduction

It is well known that gauge symmetries in closed string theories come from Kac-Moody (K-M) algebras on the string world-sheet [1]. Representations of the K-M algebra are labelled by a free parameter, the level k . For ten-dimensional models, the $E_8 \times E_8$ and $SO(32)$ strings are known to be realized at level one, however the tachyonic E_8 model is realized at level two [2]. For the phenomenologically interesting 4-dimensional strings, it would be convenient to have them classified according to the level of the different gauge groups because several physical quantities depend on this value. In particular the gauge coupling constants are level-dependent and also the allowed representations of the groups in the spectrum of the models are a function of the level; for instance, for $SU(3)$ and $SU(2)$ the only allowed representations are the fundamentals if the level of the corresponding K-M's is one. Therefore knowing the level of the different gauge groups gives us substantial physical information and it is necessary in order to obtain the correct coupling constants at the Planck mass and compute their evolution under the renormalization group equations. Up until recently [3] this question was not addressed since most of the models constructed so far have level one. If the models are of highest rank (16 or 22), it is clear that the level is one, since for simply-laced groups their contribution to the Virasoro central charge equals the rank of the group at level one and increases with the level. However for models of lower rank it is not clear to which levels they might correspond.

In the present paper we present an explicit and systematic way to find 4- d strings of arbitrary level, starting from a level-one model and using the orbifold formulation of 4- d strings. We present constructions for which the higher level models are obtained from exactly marginal deformations of the original level-one model [4] and will provide a simple field theory interpretation of them in terms of flat directions for some matter fields in the theory. We emphasize that this can be done in any of the current approaches to constructing 4- d models, including symmetric and asymmetric compactifications.

Once we can parametrize the models from the different levels, it is natural to ask what kind of constraints can be imposed on these originally free parameters, either from string consistency conditions or from phenomenological requirements. In particular, for a Standard Model (SM)-like string with symmetries $SU(3) \times SU(2) \times U(1)$, there are three parameters k_3, k_2, k_1 which we would like to see if they are constrained or are completely arbitrary. A consistency requirement is unitarity, also conformal invariance. As phenomenological ones

we have to impose that the usual SM particles should be in the massless spectrum and that the gauge coupling constants when evolved by a renormalization group (RG) flow, give the correct values for $\sin^2\theta_W$ and M_{GUT} . We have performed a systematic analysis of these constraints and found the possible values that the levels can take.

Finally, the existence of higher level KM models open new possibilities for phenomenology of strings. The most direct is that Grand Unified Theories (GUT's) [5] can be obtained from string theory since scalars in the adjoint of the gauge group are only allowed for level ≥ 2 . But also higher order representations have been considered in the GUT model building in the past in order to solve different problems. For instance the "see-saw" mechanism [6] to give small neutrino masses and the "missing partner" mechanism [7] to split triplets and doublets in GUT's, both require higher dimensional representations. We study here under which conditions these mechanisms can be implemented in string theory. In particular we conclude that it will be impossible to have the standard Gell-Mann-Ramond-Slansky-Yanagida "see-saw" mechanism in E_8 GUT's since that would be in conflict with conformal invariance.

We also consider the realization of discrete local symmetries à la Krauss and Wilczek [8] in order to protect proton stability and at the same time provide Majorana neutrino masses to solve the neutrino mass problem [9]. This requires the introduction of a field with a charge being a multiple of the fundamental one with respect to a $U(1)$ group (like $B-L$) which, after having a non-vanishing vev breaks $U(1)$ to a discrete group. In order to have this field in the spectrum, a higher level Kac-Moody model is in general required.

The organization of the present paper is as follows. In the next chapter we provide some basic formulae and conventions for Kac-Moody algebras that are needed in the subsequent chapters. In chapter 3, we describe how higher level models can be obtained from the available formalisms on symmetric orbifold models with lower rank. We in fact present two alternative ways of obtaining these models. One is by reducing the rank in a discrete way and the other by doing it in terms of a free continuous parameter following reference [4]. A simple field theory interpretation of this latter approach is also provided where the effects of increasing the level on the allowed representations and the gauge coupling constants can be understood by field theoretical arguments.

Chapter 4 is devoted to describing the general constraints that can be imposed on the levels of the 4- d gauge group in order to satisfy general requirements like unitarity and

conformal invariance, as well as general constraints imposed by a renormalization group analysis of $SU(3) \times SU(2) \times U(1)$ with arbitrary levels. We show how the Weinberg angle and unification mass depend on the different levels and how their values can restrict the allowed values of the levels and representations. A similar analysis is presented for the existence of an intermediate scale in the case of a two-stage symmetry breaking. We also discuss general constraints that can be found by imposing integer electric charges for colour singlet fields. A recent analysis by Schellekens [10] is reviewed and generalized for different possible gauge groups where the higher-level models are required in order to have a reasonable Weinberg angle at the Planck scale.

Finally, in chapter 5 we make a general analysis of the possible relevance of higher-level models in phenomenology. Here we discuss the possibility of realizing usual model building mechanisms which require higher dimensional representations, like the "see-saw" mechanism for neutrino masses and the "missing partner" one, which for $SU(5)$ provided an explanation of the triplet-doublet mass splitting. We discuss how the inclusion of higher-dimensional representations can be used for phenomenological questions, in particular the existence of strings theory GUT's, and show that quite generically that implies the survival at lower energies of $SU(3)$ octets as well as $SU(2)$ triplets. We discuss at the end of this chapter the realization of the discrete local symmetries mentioned above in relation with the proton stability and neutrino mass problems. We present in chapter 6 our conclusions and general outlook.

2. Some basic notation

In this chapter we collect our conventions for KM algebras and review the relevant relations needed for the subsequent chapters [1].

We are interested in the sector of a $4-d$ heterotic string that provides the gauge symmetries in space-time. This is given by KM currents which satisfy the operator product expansion (OPE)

$$J^a(z)J^b(w) \sim \frac{k\delta^{ab}}{(z-w)^2} + \frac{if^{abc}J^c(w)}{(z-w)} \quad (2.1)$$

The level k of the algebra is defined as $k \equiv 2\tilde{k}/\psi^2$, where ψ is the highest root. The coefficients f^{abc} are the structure constants of the corresponding gauge group G . We will

only deal with simply-laced groups and will use the normalization $\psi^2 = 1$ unless otherwise explicitly stated. The numeric coefficient of the double pole in (2.1) then becomes $k/2$. For non-Abelian algebras, k must be a positive integer in order to have unitary highest weight representations. For $U(1)$ factors we define the "level" k_1 and fix the normalization by choosing the double pole coefficient to be $k_1/2$. For example, for the $U(1)_Y$ inside $SU(5)$ with the standard hypercharge assignments we find $k_1 = 5/3$. k_1 is only restricted to be positive.

The quantity $T(R)$ is as usual defined by

$$Tr T_a^{(R)} T_b^{(R)} = T(R)\delta_{ab} \quad (2.2)$$

With the choice $\psi^2 = 1$ we find the standard results $T(R) = 1/2$ and $T(R) = N$ for the fundamental and adjoint representations of $SU(N)$.

From the Sugawara construction, equations (2.1) induce a representation of the Virasoro algebra with energy-momentum tensor given by

$$T(z) = \frac{1}{k + C(A)} \sum_a : J^a(z) J^a(z) : \quad (2.3)$$

where $C(A)$ represents the Casimir invariant in the adjoint representation. The corresponding central charge is,

$$c = \frac{k \dim G}{k + C(A)} \quad (2.4)$$

The conformal weight of a primary field (highest weight representation) ϕ can be obtained from:

$$T(z)\phi(w) = \frac{h}{(z-w)^2} \phi(w) + \frac{\partial \phi(w)}{z-w}, \quad (2.5)$$

and from (2.3) it can be found to be

$$h = \frac{C(R)}{k + C(A)}, \quad (2.6)$$

where $C(R)$ is the Casimir of the representation R related to $T(R)$ by $C(R)\dim(R) = T(R)\dim G$.

The previous equations applied to non-Abelian groups. For the case of $U(1)$ factors of the gauge group, we can represent their generators as in the Frenkel-Kac construction by $J(z) = i\eta \cdot \partial P$, where the P 's are free boson fields in 2-D. From the OPE of two free bosons it is easy to see that $k_1 = 2\eta^2$, and the contribution to the conformal weight from the $U(1)$ factor is then

$$h = \frac{Q^2}{k_1} \quad (2.7)$$

where Q is the $U(1)$ charge of the corresponding state.

The double pole term in (2.1) fixes the coupling of two gauge bosons to one graviton. In ref. [11] it is shown that the gauge coupling g is related to the gravitational coupling κ by

$$\frac{\kappa^2}{g^2} = \frac{k}{4} \quad (2.8)$$

where string units $\alpha' = 1/2$ are used. Eq. (2.8) implies that the string and Planck scales are related by

$$\frac{M_S^2}{M_P^2} = \alpha k \quad (2.9)$$

where $\alpha = g^2/4\pi$.

Equation (2.8) is valid separately for each simple and $U(1)$ factor in the gauge group. It holds only at the string scale related to M_P by (2.9). At scales below M_S , renormalization group equations for the coupling constants must be used.

3. Higher-level four-dimensional strings

The first known example of a heterotic string in which the observable gauge sector arises from a higher level Kac-Moody algebra was the 10-d non-supersymmetric E_8 string. This model was originally found in ref. [12] by orbifoldizing the $E_8 \times E_8$ string by the combined action of a 2π spatial rotation and a Z_2 permutation of the two E_8 's. It was shown later [2] that the spectrum of the rank-reduced theory corresponded to that of an E_8 KM algebra at level $k=2$ tensored with a ($c=1/2$) Ising model. The derivation of the spectrum using orbifold techniques was spelled out in ref. [13]. In section 3.1 we review this construction as we will follow a similar approach in our subsequent analysis.

The E_8 string construction suggests that it should be possible to obtain 4-d string models with higher-level KM algebras by appropriately orbifoldizing 4-d heterotic strings. In section 3.2 we show how this idea works. We exemplify this by considering a particular 4-d model with gauge group $SU(3)^3 \times U(1)^2 \times SO(1,1)' \times U(1)'$ where all factors are realized as level 1 KM algebras. This model happens to have a Z_3 internal symmetry that can be combined with a Z_3 permutation of the three $SU(3)$'s to construct a rank-reduced model with gauge group $SU(3) \times U(1)^2 \times SO(1,1)' \times U(1)'$ where now the $SU(3)$ is realized as a level 3 KM algebra. Of course a similar approach can also be used to obtain higher level models in other types of constructions. All presently known four-dimensional string constructions, including both symmetric and asymmetric orbifolds as well as models obtained by tensoring $N=2$ superconformal models, are usually invariant under a number of Z_k symmetries. In all these constructions one can embed these symmetries into the gauge degrees of freedom by permuting cyclically identical gauge factors. One loop modular invariance with respect to the T transformation is verified by checking level matching, whereas S-invariance is insured by including the twisted sectors. The final outcome will be four-dimensional strings with some of the gauge symmetries realized at a higher level.

In section 3.3 we reproduce our rank-reduced model by directly orbifoldizing the $E_8 \times E_8$ string by the action of a Z_3 space group embedded in a non-Abelian way in the gauge group. As shown in ref. [4] this type of embedding does lower the rank in general. Thus some of the higher level models obtained twisting by permutations an original orbifold model, can be alternatively constructed in this way. Finally, we also prove that in the rank-enhanced model there are supersymmetric flat directions along which $SU(3)^3$ is broken to the diagonal $SU(3)$. This shows a third, perhaps simpler, mechanism to obtain higher-level string models.

3.1 THE E_8 STRING. A SHORT REVIEW

The E_8 string is obtained by quotienting the $E_8 \times E_8$ theory by a Z_2 symmetry of the form $\mathcal{R} \otimes \mathcal{P}_2$ where $\mathcal{R}$ is a 2π spatial rotation and $\mathcal{P}_2$ is a permutation of the two E_8 's. To describe the action of $\mathcal{P}_2$ we use the bosonic formulation, i.e. the gauge degrees of freedom are represented by 16 free bosons living on the $E_8 \times E_8$ torus. These bosons are denoted by $F^I, F'^I, I = 1, \dots, 8$.

The Hilbert space of the quotient (orbifold) theory is divided into untwisted and twisted sectors. In the untwisted sector all states not invariant under the joint action of $\mathcal{R}$ and $\mathcal{P}_2$

must be projected out of the spectrum. Thus, space-time bosons can only combine with left-moving states symmetric under $\mathcal{P}_2$. Similarly, space-time fermions can only combine with left-moving states anti-symmetric under $\mathcal{P}_2$. The charged massless states of the untwisted sector are then the 2×8 gauge bosons of E_8 plus fermions transforming in the adjoint of E_8 . The gravity multiplet— $(\phi, B_{\mu\nu}, g_{\mu\nu})$ also appears in this sector.

In the twisted sector the boundary conditions on the $E_8 \times E_8$ left-handed bosons take the form

$$\begin{aligned} F((\sigma + \pi) - t) &= F'(\sigma - t) + \pi W \\ F'((\sigma + \pi) - t) &= F(\sigma - t) + \pi W' \end{aligned} \quad (3.1)$$

where W and W' are vectors in the E_8 lattice Λ . The mode expansions for F and F' can be written as

$$\begin{aligned} F(\sigma - t) &= F_0 + M(\sigma - t) + \frac{i}{2} \sum_r \frac{f_r}{r} e^{-2ir(\sigma - t)} \\ F'(\sigma - t) &= F'_0 + M'(\sigma - t) + \frac{i}{2} \sum_r \frac{f'_r}{r} e^{-2ir(\sigma - t)} \end{aligned} \quad (3.2)$$

Imposing the boundary conditions (3.1) gives

$$\begin{aligned} M &= M' = \frac{W + W'}{2} \equiv P \\ F'_0 &= F_0 + \pi P \pmod{\pi\Lambda} \\ f_r &= e^{2i\pi P} f'_r = e^{4i\pi P} f_r \end{aligned} \quad (3.3)$$

The last equation in (3.3) implies that r can only take integer or half-integer values. Thus, f_r splits into two types of oscillators denoted by α_n and $\beta_{n+\frac{1}{2}}$, $n \in \mathbb{Z}$. f'_r splits according to eq. (3.3).

The mass formula for the left-movers is then given by

$$\frac{m_L^2}{8} = N + P^2 - \frac{1}{2} \quad (3.4)$$

The term P^2 arises from $(M^2 + M'^2)/2$. N is an oscillator number that can take integer and half-integer values. The normal ordering constant includes the contributions of the α and β oscillators plus the oscillators associated to the space-time left coordinates. There are 8 + 240 massless states corresponding to $(N = 1/2, P = 0)$ plus $(N = 0, P^2 = 1/2)$. There is also a state $(N = 0, P = 0)$ with $m_L^2/8 = -1/2$.

For the right-movers the Ramond and Neveu-Schwarz sectors are treated separately. In the former $m_R^2/8 = \bar{N}_R$ and $\bar{N}_R \in \mathbb{Z}$. In the latter $m_R^2/8 = \bar{N}_{NS} - 1/2$ and $\bar{N}_{NS} \in \mathbb{Z}$ because in the twisted sector the GSO projection becomes $\{1 + (-1)^F\}$ requiring F even. Combining the above results and imposing left-right level matching we find massless fermions transforming in the adjoint of E_8 plus a tachyon.

Before leaving this section we would like to prove that the E_8 KM algebra of the orbifold theory is realized at level $k = 2$. For the two original E_8 's at level $k = 1$ as usual we can take the Cartan currents as $H_1^I = \frac{i}{\sqrt{2}} \partial F^I$ and $H_2^I = \frac{i}{\sqrt{2}} \partial F'^I$. Since F and F' are free bosons the currents H_1^I satisfy the OPE

$$H_1^I(z) H_1^J(w) \sim \frac{\frac{1}{2} \delta_{IJ} \delta^{IJ}}{(z - w)^2} \quad (3.5)$$

Now, in the orbifold theory the gauge group appears in the untwisted sector and the left currents must be symmetric under the exchange of the two E_8 's. Hence, for the E_8 theory we can take the Cartan currents to be $H^I = H_1^I + H_2^I$. We can check that the H^I satisfy the correct OPE with the step operators $E^\beta = E_1^\beta + E_2^\beta$. Finally, using (3.5) we can compute the $H^I(z) H^J(w)$ OPE and from it read off the level $k = 2$. The E_8 KM algebra at $k = 2$ leads to a Virasoro algebra with $c = 31/2$. The missing $c = 1/2$ is provided by the coset theory $E_8^1 \times E_8^1 / E_8^3$, where superscripts indicate levels. This coset theory is equivalent to an Ising model [2].

3.2 A LEVEL $k = 3$ EXAMPLE

As shown in ref. [14] many 4- d string models can be constructed by means of orbifold compactifications with Wilson line backgrounds. This formalism is particularly suitable for relating higher Kac-Moody level strings to more familiar notions like gauge symmetry breaking through flat directions, as we will see later on. However the results discussed in this section really apply to all analogous four-dimensional string constructions. Here we would like to consider a particular example based on a Z_3 orbifold for which the lattice defining the internal torus is the product of three $SU(3)$ root lattices. We believe this example shows the most important features of the modelling by permutations method and thus we discuss it in some detail. The reader can easily construct other models using the same approach.

In this example the embedding shift in the $E_8 \times E_8$ lattice is taken to be

$$\nu = \frac{1}{3}(2, 1, 1, 1, 0, 0, 0) \times \frac{1}{3}(2, 0, \dots, 0) \equiv \nu' \times \nu' \quad (3.6)$$

The resulting model has gauge group $SU(9) \times SO(14)' \times U(1)'$. Further gauge breaking is achieved by turning on the Wilson line

$$A_1 = \frac{1}{3}(0, 1, 0, 0, -1, 2, 0, 0) \times (0, 0, \dots, 0) \equiv A_1 \times A_1' \quad (3.7)$$

The gauge group then becomes $SU(3)^3 \times U(1)^2 \times SO(14)' \times U(1)'$.

The untwisted charged matter content consists of 3 copies of

$$(3, 3, 3) + 3(1, 1, 1) + 14' + 64' \quad (3.8)$$

The twisted sector splits into three groups with shifts $\nu + n_1 A_1$, $n_1 = 0, \pm 1$. Each of these groups has multiplicity nine and in this particular example the matter content of each is only different in the $U(1)$ charges. Altogether the twisted matter content consists of 27 copies of

$$(\bar{3}, 1, 1) + (1, \bar{3}, 1) + (1, 1, \bar{3}) \quad (3.9)$$

It can be checked that all non-Abelian factors are realized at level $k=1$. This simply follows because the gauge currents of the unbroken group are just a subset of the original $E_8 \times E_8$ currents.

For our subsequent analysis it will be useful to write vectors in the E_8 lattice in terms of the roots and weights of the maximal $SU(3)^4$ subgroup of E_8 . The 240 roots of E_8 are given by the 6×24 roots of $SU(3)^4$:

$$(\rho, 0, 0, 0) + \text{permutations} \quad (3.10)$$

and the 8×27 weights:

$$\pm(d, d, d, 0) ; \pm(-d, d, 0, -d) ; \pm(d, 0, -d, -d) ; \pm(0, -d, d, -d) \quad (3.11)$$

where ρ stands for any of the $SU(3)$ roots and d for any of the weights $d_1 = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{6}})$, $d_2 = (-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{6}})$ and $d_3 = (0, -\frac{2}{\sqrt{6}})$. Also $\rho_1 = (\sqrt{2}, 0)$, $\rho_2 = (-\frac{1}{\sqrt{2}}, \frac{2}{\sqrt{6}})$ and $\rho_3 = \rho_1 + \rho_2$.

This change of basis amounts to switching from E_8 coordinates F^I , $I = 1, \dots, 8$ to $SU(3)^4$ coordinates denoted by Z^i , $i = 1, \dots, 4$, $a = 1, 2$. Using the above notation in our model, we can take the shift and Wilson line of the first E_8 to be

$$\begin{aligned} V &= (0, 0, -d_2, \frac{\rho_1}{3}) \\ A_1 &= (0, 0, 0, d_3) \end{aligned} \quad (3.12)$$

For each state in the spectrum we can construct its vertex operator. For our purposes we only need to focus on the piece of the vertex that comes from the first E_8 degrees of freedom. In general this piece is the product of an exponential $e^{iF \cdot Z}$ times a string of derivatives $\partial^\mu Z$. Here $P = W + V + n_1 A_1$ and W is a vector in the E_8 lattice Λ . The derivatives indicate whether the states involve Z oscillators acting on the vacuum. For example, the untwisted states transforming as $(3, 3, 3)$ have $P = (d, d, d, 0)$. The twisted states in (3.9) (with $n_1 = 0$) have $P = (-d, 0, 0, d_2 + \frac{\rho_1}{3})$, $P = (0, -d, 0, -d_1 + \frac{\rho_1}{3})$ and $P = (0, 0, -d, \frac{\rho_1}{3})$ respectively. Similar vectors appear when $n_1 = \pm 1$.

At this point it is worth reviewing some features of the plain Z_3 orbifold defined on the $SU(3)^3$ root lattice. Each state in the twisted sector is labelled by (n_1, n_2, n_3) where $n_i = 0, \pm 1$ refers to the fixed point at which the state sits in the i^{th} sub-lattice. Correlations involving twisted and/or untwisted states vanish unless each n_i is conserved mod 3 (untwisted states have $n_i \equiv 0$). This just means that there are $Z_3^{(j)}$ symmetries under which states are multiplied by a phase $\exp(i2\pi n_j/3)$. These symmetries can be used to orbifoldize once again. For instance, to $Z_3^{(1)}$ we can assign an embedding $\mathcal{E}$ into $E_8 \times E_8$ and mod by the combined action of $Z_3^{(1)} \times \mathcal{E}$. Clearly, the embedding must have order 3. In particular, $\mathcal{E}$ can be realized by a shift $n_1 A_1$ with $3A_1$ belonging to the $E_8 \times E_8$ lattice.

Turning on the Wilson line A_1 in (3.7) can thus be understood in the manner explained above. The structure of the Hilbert space of the resulting model is worked out as follows. In the untwisted sector there is no action on the gauge degrees of freedom and we just demand that the gauge piece of a given state be invariant under the embedding action. To each untwisted state there corresponds a $E_8 \times E_8$ weight $\mathcal{W}$ determined according to the mass formula

$$\frac{m_L^2}{8} = N_\star + \frac{1}{2} \mathcal{W}^2 - 1 \quad (3.13)$$

$N_\star$ is an oscillator number that takes integer values. The normal ordering constant includes the contributions of the left-moving space-time and internal coordinates plus the $E_8 \times E_8$

coordinates F, F' . Requiring invariance under the embedding realized by a shift $n_1 A_1$ implies $\mathcal{W} \cdot A_1 = \text{integer}$ on top of the already imposed conditions $\mathcal{W} \cdot \mathcal{V} = \text{integer}$ for gauge particles and neutral matter or $\mathcal{W} \cdot \mathcal{V} = \frac{1}{3} \text{ mod integer}$ for charged matter.

The twisted sector is divided in three groups, each with multiplicity 9, according to n_1 . In each group the net shift is $\mathcal{V} + n_1 A_1$. The mass formula is given by

$$\frac{m_L^2}{8} = N_L + \frac{1}{2} (\mathcal{W} + \mathcal{V} + n_1 A_1)^2 - \frac{2}{3} \quad (3.14)$$

Here N_L can take integer and third-integer values because the twisted boundary conditions for the internal coordinates allow third-integer modded oscillators. The change in the normal ordering constant is precisely due to these oscillators. States in the twisted sector must survive a generalized GSO projection that for non-oscillator states in the Z_3 orbifold takes the form

$$(\mathcal{W} + \mathcal{V} + n_1 A_1) \cdot (\mathcal{V} + n_1 A_1) - \frac{1}{2} (\mathcal{V} + n_1 A_1)^2 = -\frac{1}{3} \text{ mod integer} \quad (3.15)$$

There is a similar projection for oscillator states. It can be proved that in the Z_3 orbifold all twisted massless states automatically verify the generalized GSO projection.

After turning on A_1 the resulting model still has $Z_3^{(2)}$ and $Z_3^{(3)}$ symmetries left to which we can assign appropriate embeddings. For example, in our model containing $SU(3)^3$ in the gauge group to $Z_3^{(2)}$ we can assign an embedding realized as a cyclic permutation P_3 of the three $SU(3)$'s. We now wish to compute the spectrum resulting after orbifolding by $Z_3^{(2)} \times P_3$.

As in the case of embedding by a shift, in the untwisted sector we must demand that the gauge piece of a given state be invariant under P_3 . In particular, invariant combinations of the states corresponding to the $SU(3)^3$ gauge bosons are just the gauge bosons of the diagonal $SU(3)$. Likewise, from the untwisted matter fields transforming as $(3, 3, 3)$ we must construct combinations symmetric under the exchange of the $SU(3)$'s. There are 11 such combinations and it is straightforward to check that they transform as $10 + 1$ under the diagonal $SU(3)$. In the untwisted sector there are also 3 $SU(3)_{diag}$ singlets plus the matter in (3.8) transforming under the "hidden" $SO(1)' \times U(1)'$. All these matter fields appear in three copies.

The twisted states in sectors $\mathcal{V} + n_1 A_1$ now split into three groups. Each group has multiplicity 3 due to the freedom in n_3 . Since the embedding is now being realized by a permutation its effect is not as simple as adding a shift $n_2 A_2$. Rather, we must consider boundary conditions that close up to permutations of the three $SU(3)$'s, as is done in the E_9 model discussed in the previous section. To write out the "permuted" boundary conditions it is convenient to work in the $SU(3)^4$ basis mentioned before. P_3 only affects Z_1, Z_2, Z_3 as these are the ones giving rise to the gauge group $SU(3)^3$. The boundary conditions are given by

$$\begin{aligned} Z_1((\sigma + \pi) - t) &= Z_2(\sigma - t) + \pi W_1 + \pi B_1 \\ Z_2((\sigma + \pi) - t) &= Z_3(\sigma - t) + \pi W_2 + \pi B_2 \end{aligned} \quad (3.16)$$

$$Z_3((\sigma + \pi) - t) = Z_1(\sigma - t) + \pi W_3 + \pi B_3$$

whereas for the unaffected coordinates we have

$$Z_4((\sigma + \pi) - t) = Z_4(\sigma - t) + \pi W_4 + \pi B_4 \quad (3.17)$$

In the above B is the net shift $B = V + n_1 A_1$ where V and A_1 are given in (3.12) and B_i denotes the components in the $SU(3)^4$ basis. Recall that $3B \in \Lambda$. Likewise, the W_i are the $SU(3)^4$ components of a vector $W \in \Lambda$.

The mode expansion for the Z_j is given by

$$Z_j(\sigma - t) = Z_{j0} + M_j(\sigma - t) + \frac{i}{2} \sum_r \frac{Z_j r}{r} e^{-2ir(\sigma - t)} \quad (3.18)$$

Imposing the boundary conditions (3.16) leads to

$$\begin{aligned} M_1 &= M_2 = M_3 = M \\ 3M &= (W_1 + W_2 + W_3) + (B_1 + B_2 + B_3) \end{aligned} \quad (3.19)$$

The above means that $3M$ is to be understood as a vector in the shifted $SU(3)$ weight lattice denoted Ω . Conditions (3.16) together with (3.19) also imply

$$\begin{aligned} Z_{20} &= Z_{10} + \pi M \pmod{\pi\Omega} \\ Z_{30} &= Z_{10} + \pi M \pmod{\pi\Omega} \end{aligned} \quad (3.20)$$

Finally, the oscillators are forced to satisfy

$$z_{1r} = e^{2\pi i r} z_{2r} = e^{4\pi i r} z_{3r} = e^{6\pi i r} z_{1r} \quad (3.21)$$

This last condition implies $6\pi r = \text{integer}$. Thus, there are three inequivalent possibilities

for τ , namely $\tau = m$, $\tau = m + \frac{1}{3}$ and $\tau = m + \frac{2}{3}$ with $r \in Z$. Therefore, there are three types of oscillators z_{1r} denoted by α_m , $\beta_{m+\frac{1}{3}}$ and $\gamma_{m+\frac{2}{3}}$. The z_{2r} and z_{3r} oscillators are related to these according to eq. (3.21). Finally, for Z_4 eq. (3.17) just implies integer-modded oscillators and $M_4 = W_4 + B_4$. Recall that the W_i are constrained by $W \in \Lambda$.

The left mass formula in the “permuted” sectors is given by

$$\frac{m_L^2}{8} = N + \frac{3}{2}M^2 + \frac{1}{2}(W_4 + H_4)^2 + \frac{1}{2}P^2 - \frac{4}{9} \quad (3.22)$$

N can take integer and third-integer values because of the twisted internal coordinates and the permuted E_8 coordinates. The change in the normal ordering constant arises from the β and γ oscillators. P' takes care of the second E_8 degrees of freedom and it is given by $P' = W' + B'$ where $W' \in \Lambda$ and $H' = V' + n_1 A'_1$. V' and A'_1 are given in (3.6) and (3.7).

Using the explicit values of B and B' is easy to check that the massless solutions of (3.22) all have $3M = -d$ where d is any of the weights d_1, d_2, d_3 , whereas for $M_4 = W_4 + \frac{\theta_1}{3} + n_1 d_3$ we find the possibilities given by

$$\begin{aligned} W_4 &= 0, -d_1, d_2 & ; & \quad n_1 = 0 \\ W_4 &= -d_3, d_2, \rho_2 & ; & \quad n_1 = 1 \\ W_4 &= d_3, -d_1, \rho_3 & ; & \quad n_1 = -1 \end{aligned} \quad (3.23)$$

The corresponding states clearly transform as $\bar{\mathbf{3}}$ of $SU(3)_{diag}$. Their observable $U(1)$ charges depend on M_4 . In fact, these charges are just the components of M_4 .

Equation (3.22) is really valid in the case $n_2 = \pm 1$. When $n_2 = 0$ the boundary conditions for all Z_i are of type (3.17), i.e. without permutations. The left mass formula is thus of the form (3.14), that is

$$\frac{m_L^2}{8} = N_t + \frac{1}{2}P^2 + \frac{1}{2}P'^2 - \frac{2}{3} \quad (3.24)$$

where P has $SU(3)^4$ components $P_i = W_i + B_i$ and $P' = W' + B'$. The $SU(3)_{diag}$ charges of these states are given by the vector $\vec{M} = P_1 + P_2 + P_3$ that belongs to the shifted $SU(3)$ lattice. It can be checked that the massless solutions of (3.24) all have $\vec{M} = -d$, whereas for $M_4 = W_4 + \frac{\theta_1}{3} + n_1 d_3$ we again find the possibilities shown in (3.23). The corresponding states transform as $\bar{\mathbf{3}}$ of $SU(3)_{diag}$.

Taking into account the multiplicity due to the freedom in n_3 we see that the massless content of the twisted sector altogether consists of 81 states transforming as $\bar{\mathbf{3}}$ of $SU(3)_{diag}$. As we already explained, in the untwisted sector the massless matter includes 3 copies of a $\mathbf{10}$ of $SU(3)_{diag}$. The total $SU(3)_{diag}$ anomaly cancels since a $\bar{\mathbf{3}}$ and a $\mathbf{10}$ contribute -1 and 27 respectively. We have also checked that the observable $U(1)$ anomalies cancel.

Finally, we notice that $SU(3)_{diag}$ is realized at level $k = 3$. This follows, for instance, from the fact that the $SU(3)_{diag}$ Cartan currents H^a are symmetric combinations of the Cartan currents of the three original $SU(3)$'s. The $H^a H^b$ OPE readily shows that $k = 3$. That $k = 3$ can also be inferred from the observation that for this level a $\mathbf{10}$ of $SU(3)$ has conformal dimension $h = 1$ which, given that it is neutral under the remaining gauge symmetries, is precisely the value required for it to be massless in the untwisted sector. We also notice that the $SU(3)$ KM algebra at $k = 3$ gives a conformal algebra with $c = 4$. The missing $c = 2$ is provided by the coset $SU(3)_1 \times SU(3)_1 / SU(3)_3$, where subscripts indicate levels.

3.3 MODDING BY A NON-ABELIAN EMBEDDING OF THE ORBIFOLD SPACE GROUP

In the previous section we have shown how one can obtain a higher level Kac-Moody four-dimensional string by modding by a permutation symmetry. This was to some extent a generalization of the analogous construction for the ten-dimensional E_8 string. We would like to discuss now a less familiar method to construct such a kind of higher level theories. Unlike the method described in the previous section which can be used in any four-dimensional string construction, this second method specifically makes use of the toroidal geometrical nature of orbifolds. As remarked in ref. [4] it is possible to lower the rank of the gauge group in orbifold compactifications by implementing non-Abelian embeddings of the space group. A good example may be obtained by embedding the Z_3 twist of the Z_3 orbifold into the gauge degrees of freedom by a Weyl automorphism Θ of the $E_8 \times E_8$ lattice instead of by a shift $\mathcal{V}$. Next, Wilson lines $\mathcal{A}_i$ are turned on. The Θ and $\mathcal{A}_i$ actions do not commute, in particular $\Theta \mathcal{A}_i = \tilde{\mathcal{A}}_i \neq \mathcal{A}_i$. This fact enables a lowering of the rank, because would-be Cartan currents are projected out by the condition $\mathcal{W} \cdot \tilde{\mathcal{A}}_i = \text{integer}$ on top of $\mathcal{W} \cdot \mathcal{A}_i = \text{integer}$. This lowering of the rank is in fact signalling in some cases that the unbroken gauge symmetry is realized at a higher level.

The automorphism Θ is best described in the $SU(3)^4$ basis introduced in the previous

section. In the example we want to consider Θ is given by Z_3 Coxeter rotations of the 4 $SU(3)$'s inside the first E_6 and 2 of the $SU(3)$'s in the second E_6 . This embedding leads to gauge group $SU(9) \times SU(1)^4 \times U(1)^4$ as explained in [1]. This is just, up to this point, an equivalent way of constructing the same model we started with in the previous section. In the following we only deal with the "observable" $SU(9)$.

The $SU(9)$ Cartan currents are no longer given by combinations of derivatives ∂Z because the Z 's are rotated by Θ and the gauge currents of the unbroken gauge group must be invariant under Θ . The Cartan sub-algebra now arises from combinations of $SU(3)^4$ root states left invariant by Θ . For example, one such combination is given by

$$e^{i(p_1,0,0,0)Z} + e^{i(p_2,0,0,0)Z} + e^{i(-p_1,0,0,0)Z} \quad (3.25)$$

since under a Z_3 rotation $p_1 \rightarrow p_2 \rightarrow -p_3$. There are two states of type (3.25) for each $SU(3)$. Hermitian combinations of them give the 8 Cartan generators of $SU(9)$. The remaining 72 generators come from states of the form

$$e^{i(d_1,d_1,d_1,0)Z} + e^{i(d_2,d_2,d_2,0)Z} + e^{i(d_3,d_3,d_3,0)Z} \quad (3.26)$$

etc. The untwisted charged massless states are of the form

$$e^{iWZ} + \delta^2 e^{i(\Theta W)Z} + \delta e^{i(\Theta^2 W)Z} \quad (3.27)$$

where $\delta = e^{2\pi i/3}$ and $W \in \Lambda$ is of type (3.10) or (3.11). If we denote the above combination by $|S_L\rangle$ then $\Theta|S_L\rangle = \delta|S_L\rangle$ which is the required transformation so that the full state $|S_R\rangle \times |S_L\rangle$ is invariant under the full orbifold action. The four oscillator states

$$V_j = \frac{\partial Z_j^4 - i\partial Z_j^2}{\sqrt{2}} \quad (3.28)$$

are also massless and transform appropriately under Θ . Altogether one has $8+72+4=84$ states corresponding to a 84 dimensional rep. under $SU(9)$. These states come in three copies due to the three right moving indices.

In the twisted sector it is necessary to consider boundary conditions for the Z_j that close up to the rotations in Θ . This leads to third-integer modded oscillators and to a mass formula in which the zero point energy vanishes altogether. There is also a fixed point degeneracy that in this case is 9. In fact, the vacuum is massless and it corresponds to a $\bar{9}$ of $SU(9)$. Since in the Z_3 orbifold there are 27 fixed points we have altogether 27 $\bar{9}$'s in the twisted sector.

We next turn on the Wilson line

$$\begin{aligned} A_1 &= (d_1, 0, 0, 0) \\ \Theta A_1 &= \tilde{A}_1 = (d_1, 0, 0, 0) \end{aligned} \quad (3.29)$$

In the untwisted sector we must project on states satisfying $W \cdot A_1 = \text{integer}$ and $W \cdot \tilde{A}_1 = \text{integer}$. This projection kills most of the generators of type (3.26) but none of type (3.25). $SU(9)$ is thus broken down to $SU(3)^3 \times U(1)^2$. Likewise, most states of type (3.27) disappear and the untwisted matter reduces to that in (3.8).

Concerning the twisted sector, the massless matter is again found to be that in (3.9) with multiplicity 27. In fact, when the Wilson line is completely rotated by Θ the number of twisted massless states does not change because the mass formula is not modified, since the boundary conditions forbid winding states and a shift does not affect the zero-point energy. All that changes is the way these states fall into representations of the unbroken gauge group.

We now turn on a second Wilson line given by

$$\begin{aligned} A_3 &= \lambda(0, d_3, d_3, 0) \\ \Theta A_3 &= \tilde{A}_3 = \lambda(0, d_3, d_3, 0) \end{aligned} \quad (3.30)$$

As shown in [4] λ can be an arbitrary parameter. If $\lambda \neq 1$, A_3 and $\tilde{A}_3$ are not related by a lattice vector. Thus, imposing

$$\begin{aligned} W \cdot A_3 &= \text{integer} \\ W \cdot \tilde{A}_3 &= \text{integer} \end{aligned} \quad (3.31)$$

leads to rank reduction. In fact, now only 4 states of type (3.25) and 6 of type (3.26) survive.

These are shown below

$$\begin{aligned}
 |U_1\rangle &= e^{i(0,-d_1,d_1,-d_1)Z} + e^{i(0,-d_2,d_2,-d_2)Z} + e^{i(0,-d_3,d_3,-d_3)Z} \\
 |U_3\rangle &= e^{i(0,-d_1,d_1,-d_2)Z} + e^{i(0,-d_2,d_2,-d_3)Z} + e^{i(0,-d_3,d_3,-d_1)Z} \\
 |U_5\rangle &= e^{i(0,-d_1,d_1,-d_3)Z} + e^{i(0,-d_2,d_2,-d_1)Z} + e^{i(0,-d_3,d_3,-d_2)Z} \\
 |U_7\rangle &= e^{i(0,0,0,\rho_1)Z} + e^{i(0,0,0,\rho_2)Z} + e^{i(0,0,0,-\rho_3)Z} \\
 |U_{10}\rangle &= e^{i(\rho_1,0,0,0)Z} + e^{i(\rho_2,0,0,0)Z} + e^{i(-\rho_3,0,0,0)Z}
 \end{aligned} \tag{3.32}$$

and $U_{i+1} = U_i^\dagger$ for the rest. Clearly, U_9 and U_{10} are orthogonal to the other U_i states. The Hermitian combinations $(U_9 + U_{10})$ and $i(U_9 - U_{10})$ are the generators of the two observable $U(1)$'s.

We can also prove that $U_1, \dots, U_8$ provide a realization of the $SU(3)$ KM algebra at level $k=3$. The Cartan currents turn out to be

$$\begin{aligned}
 H_1 &= \frac{U_7 + U_8}{\sqrt{2}} \\
 H_2 &= \frac{i(U_7 - U_8)}{\sqrt{2}}
 \end{aligned} \tag{3.33}$$

The step operators are given by

$$\begin{aligned}
 E_{\rho_1} &= U_1 + U_3 + U_5 \\
 E_{\rho_2} &= U_1 + \delta^2 U_3 + \delta U_5 \\
 E_{-\rho_3} &= U_1 + \delta U_3 + \delta^2 U_5
 \end{aligned} \tag{3.34}$$

and $E_{-\alpha} = E_{\alpha}^\dagger$ for the rest. In proving that the above satisfy the $SU(3)$ current algebra at $k=3$ we have used the normalization $\psi^2 = 2$ for simplicity. We thus conclude that $SU(3)^3 \times U(1)^2$ breaks down to $SU(3) \times U(1)^2$.

The constraints in (3.31) also project out many untwisted charged states. There survive 10 states of type (3.27) that are just like the U_i in (3.32) with appropriate phases δ^2 and δ inserted. We denote these by $C_1, \dots, C_{10}$. The states V_j in (3.28) also survive. Given the gauge currents in (3.33) and (3.34) we can prove that $C_1, \dots, C_8$ together with V_1 and $(V_2 - V_3)$ furnish a representation with highest weight $3d_1$ of the $SU(3)$ algebra, i.e. these states transform as a 10 of $SU(3)$. The remaining states are singlets. All these appear in three copies as usual in the Z_3 orbifold.

Concerning the twisted sector, the number of states does not change since the Wilson line A_3 is completely rotated by Θ . Taking into account the 27 fixed points we find 81 states transforming as $\bar{3}$ of $SU(3)$. Hence, we conclude that the model obtained after turning on A_3 coincides with the one resulting after turning on the permutation $\mathcal{P}_3$ in the example presented in the previous section. Thus this method provide us with an alternative way of constructing higher Kac-Moody orbifold models.

There is an intuitive way to understand how these two constructions are equivalent. The permutation of the three $SU(3)$ factors in the first construction is, of course, an outer automorphism of $SU(3)^3$. However, from the E_8 point of view, that automorphism is inner (E_8 has no outer automorphism) and hence one should be able to construct the permutation in terms of Weyl rotations. This is precisely what this second construction does.

3.4 FLAT DIRECTIONS

In ref. [15] it was pointed out that reducing the rank via non-Abelian embeddings as in the previous section can be understood in terms of the effective 4- d effective field theory. The key observation is that Wilson lines correspond to "gauge fields in the compactified dimensions" and hence are related to untwisted matter fields in the 4- d theory. Turning on continuous Wilson lines such as (3.30) should be equivalent to turning on non-zero vevs along flat directions in the potential of untwisted massless charged fields. In this way both the permutation modding of section 3.2 and the "non-Abelian embedding" method of section 3.3 turn out to be related to something much more intuitive: the breaking of gauge symmetries of the form $G \times G \times \dots \times G$ to the diagonal subgroup $G^{diagonal}$ by taking symmetry breaking flat directions in the effective massless scalar potential. We will now discuss this relationship.

Restricting to vacua with unbroken supersymmetry and vanishing cosmological constant the flatness conditions are given by^{*}

$$\begin{aligned}
 \langle D_\alpha \rangle &= \langle g_\alpha \phi_i^\dagger (T_\alpha)_j^i \phi^j \rangle = 0 \\
 \langle W \rangle &= \langle \frac{\partial W}{\partial \phi_i} \rangle = 0
 \end{aligned} \tag{3.35}$$

where ϕ_i are the scalar fields, W the superpotential, and g_α and T_α the couplings and

^{*} In the presence of an anomalous $U(1)$, the D-term has an additional dilaton-dependent Fayet-Iliopoulos term.

generators of the gauge group.

In the example, discussed in section 3.2, with observable gauge group $SU(3)^3 \times U(1)^2$ the untwisted matter includes 3 copies of a field transforming as $(3, 3, 3)$. We denote these by ϕ_{ijk}^a , where a is a right-moving index and i, j, k are $SU(3)$ indices. It is easy to check that the condition $\langle D_a \rangle = 0$ is compatible with vevs

$$\begin{aligned} \langle \phi_{123}^1 \rangle &= \langle \phi_{312}^1 \rangle = \langle \phi_{231}^1 \rangle = v \\ \langle \phi_{132}^1 \rangle &= \langle \phi_{313}^1 \rangle = \langle \phi_{321}^1 \rangle = -v \end{aligned} \quad (3.36)$$

These vevs break $SU(3)^3$ to the diagonal $SU(3)$. The D -terms lead to a mass matrix for the ϕ_{ijk}^1 with zero eigenvalue for the 14 symmetric combinations transforming as $10 + 1$ under $SU(3)_{diag}$. The remaining 16 combinations disappear through the Higgs mechanism.

Concerning the F -terms, they are protected by orbifold selection rules on couplings. Potentially dangerous cubic terms in the superpotential are of type $\phi^1 \phi^1 U^a$ or $\phi^1 \phi^1 T$ where U (T) denotes untwisted (twisted) fields. The former are forbidden by the rule $U^a U^b U^c \sim e^{i\theta^a}$, in particular, $\phi^1 \phi^1 \phi^1$ is not allowed. The latter are forbidden by the space group selection rule since ϕ^1 is in the untwisted sector (they are also forbidden by gauge invariance). Similarly, higher-order dangerous terms are not allowed.

Allowed couplings of the type

$$e^{i(p-q)r} \phi_{ijk}^{1m} \phi_{lmn}^2 \phi_{pqr}^3 \quad (3.37)$$

give masses to non symmetric combinations of ϕ_{lmn}^2 and ϕ_{pqr}^3 . The twisted fields remain massless since couplings of type $\phi^1 T$ are forbidden by the space group selection rule. Altogether we again find a charged $SU(3)_{diag}$ massless content consisting of 3 fields transforming as 10 plus 81 fields transforming as $\bar{3}$.

That $SU(3)_{diag}$ has $k = 3$ can also be understood in field theory language. We know that when a group G^N breaks to G_{diag} the coupling constant must be rescaled as $g \rightarrow g/\sqrt{N}$. Now, eq. (2.8) implies that the level must be rescaled as $k \rightarrow kN$. Finally, we conclude that the model obtained through flat directions coincides with the models constructed in the previous sections through modding by permutations and modding by non-Abelian embeddings. Thus, taking flat directions breaking $G^N \rightarrow G_{diag}$, provides a simple way to generate higher-level theories in a way which is equivalent (at least perturbatively) in the

vevs of the matter fields) to the permutation and the "non-Abelian embedding" method. There is nothing explicitly dependent on the fact that we used four-dimensional orbifold models to exemplify this fact. Thus we believe that this conclusion is general and applies to all four-dimensional string constructions. The idea is to start with level-one models in which a certain group appears replicated N times. If one finds a scalar field flat direction which breaks the symmetry down to G_{diag} , we know that the resulting Kac-Moody symmetry will be realized at level N .

4. General constraints on the Kac-Moody levels

We have discussed in the previous chapter how one can explicitly obtain four-dimensional strings with gauge symmetries realized at higher Kac-Moody levels. We now address in this chapter the question of the constraints on the Kac-Moody levels coming from both theoretical and phenomenological requirements.

4.1 CONSISTENCY CONDITIONS

We are assuming a four-dimensional string with a gauge group G originated from a Kac-Moody algebra with level k . G in general can be the product of several semisimple factors and each of them will be parametrized by its own level. Therefore we have a set of parameters k_i which label the model. In order to have a better control of the possibilities of these string models, we are interested in knowing what are the possible values that these parameters can have. We can list as consistency constraints the following.

(i) Unitarity. At level k unitary highest weight representations allowed to appear must satisfy the condition

$$\sum_{j=1}^{\text{rank } G} n_j m_j \leq k \quad (4.1)$$

where n_j are the Dynkin labels of the highest weight of the representation of G , and m_j are positive integers (≤ 6) known for every simple Lie group [16]. In particular, for $SU(N)$ groups for which $m_j = 1$, the only allowed representations for level one are those with Dynkin labels $(1, 0, 0, \dots)$ and permutations, corresponding to the fundamental and the completely antisymmetric representations only. Therefore the adjoint representation, for instance, cannot appear, and that was the main reason why it was previously believed that GUT's were

not realizable in string theory. Nevertheless as we increase the level more representations are allowed and there is no reason to prohibit GUT's with massless adjoint Higgses in four-dimensional strings.

(ii) Conformal invariance. Since the gauge groups come only from the left-moving sector of the heterotic string, they have to contribute to the corresponding Virasoro central charge, which from conformal invariance has to be equal to 22. Therefore the contribution from the gauge group(s) cannot exceed that value,

$$c = \sum_i \frac{k_i(\dim G_i)}{k_i + C_i(A)} \leq 22 \quad (4.2)$$

where the sum is over the different group factors and every $U(1)$ factor contributes 1 to the sum. This is the same condition that restricts the rank of the gauge group to be no greater than 22, since for simply-laced Lie groups at level one $c = \text{rank } G$, and c increases with the level. We can also conclude from this that higher level models necessarily have to have lower rank [3].

Notice that condition (4.2) gives, for sufficiently large gauge groups (any GUT), an upper bound on the levels. For instance if the whole gauge group is $SO(10)$, the maximum value that k could take is 7. For smaller gauge groups the bound is less restrictive and for the Standard Model case, $SU(3) \times SU(2) \times U(1)$, this constraint is practically useless since the maximum that this group can contribute for the central charge is $12 = 8 + 3 + 1$. Then, this gives no upper bounds to the values of the different levels k_3, k_2, k_1 , so we will have to look for other constraints to impose in order to restrict the values of these parameters. We have not considered more consistency conditions. An important one is of course modular invariance, but it is not known how this requirement can constrain the values of the levels, except for the fact that it seems more difficult to construct explicit modular invariants for levels higher than one. Notice that the only consistent models in ten dimensions have level one, but as discussed in the previous chapter, in four dimensions there is more freedom and there are consistent models with higher levels.

4.2 PHENOMENOLOGICAL CONSTRAINTS

We will consider now the case of a four-dimensional string with gauge group $SU(3) \times SU(2) \times U(1)$ with arbitrary levels and try to restrict the value of the levels by imposing known low-energy relations. The first condition we have to impose is that the Standard Model spectrum is part of the massless spectrum of the theory. We know that a massless state has to have conformal weight $h = 1$ in the left moving sector, so the contribution to h from the gauge group cannot exceed that value. Therefore any massless state belonging to a representation (R_3, R_2, Y) of the SM group, has to satisfy the constraint,

$$\frac{C(R_3)}{k_3 + 3} + \frac{C(R_2)}{k_2 + 2} + \frac{Y^2}{k_1} \leq 1 \quad (4.3)$$

where we have used eqs. (2.6) and (2.7). In particular, the existence of the right-handed electron, that transforms as $(1, 1, 1)$, implies that $k_1 \geq 1$.

The levels of the groups play an important role in determining the gauge coupling constants at the Planck mass as discussed above. Therefore they will also affect the low-energy couplings after the renormalization group equations are considered [17]. We will then proceed to express the RG equations in terms of the levels and then, using the experimental values of the known parameters, like $\sin^2 \theta_W$ and the electromagnetic coupling constant at the Weinberg-Salam scale, we can find constraints on the levels. Also we have to impose that the couplings are not too big at the unification scale in order that the perturbative approach makes sense.

From equation (2.8) we can see that at the scale of unification,

$$\sin^2 \theta_W = \frac{k_2}{k_1 + k_2} \quad (4.4)$$

The gauge couplings will run according to the RG equations:

$$\frac{1}{\alpha_s(M_W)} = \frac{k_i}{4\alpha_X} + \frac{b_i}{2\pi} \log \frac{M_X}{M_W} \quad (4.5)$$

where $\alpha_i = g_i^2/4\pi$, $\alpha_X = \kappa^2/4\pi$, M_X is the scale ($\sim M_P$) for which $\alpha_i k_i = \alpha_X k_X$. M_W represents the Weinberg-Salam scale and the b_i are the coefficients of the one-loop β functions.

For an $SU(N)$ group they are given by

$$b_N = -3N + n_H T(R) \quad (4.6)$$

where n_H is the number of chiral fields transforming in the representation R and $T(R)$ is defined in (2.2). The electromagnetic coupling constant is as usual given by

$$\alpha_{em} \equiv \alpha = \frac{\alpha_1 \alpha_2}{\alpha_1 + \alpha_2} \quad (4.7)$$

From equations (4.5) and (4.7) we can derive two expressions for $\sin^2 \theta_W$ (using $\frac{1}{\alpha_2} + \frac{1}{\alpha_1}$ or $\frac{\alpha_2}{\alpha_1}$), which can be written as

$$\sin^2 \theta_W = \frac{1}{1+x} \left\{ 1 + \frac{\alpha_2}{2\pi} \left(b_2 - \frac{b_1}{x} \right) \log \frac{M_X}{M_W} \right\} \quad (4.8)$$

and

$$\sin^2 \theta_W = \alpha \left\{ \frac{z}{\alpha_3} + \frac{1}{2\pi} (b_2 - z b_3) \log \frac{M_X}{M_W} \right\} \quad (4.9)$$

where $x \equiv \frac{k_1}{k_2}$ and $z \equiv \frac{k_1}{k_3}$. From these two equations we can find:

$$\log \frac{M_X}{M_W} = \frac{2\pi \left\{ \frac{1}{\alpha} - \frac{z(x+1)}{\alpha_3} \right\}}{b_1 + b_2 - z(x+1)b_3} \quad (4.10)$$

Note that all the equations depend only on the two parameters x and z instead of the three levels. This will imply that we cannot find absolute constraints on the values of the levels but only on the relative values given by x and z . We will try to restrict these parameters using the following conditions.

(i) Perturbative unification. In order that this one-loop analysis can make sense, the coupling constants of the gauge groups have to be small; a conservative condition is then :

$$\alpha_i = \frac{4\pi x}{k_i} \leq \frac{2}{5} \Rightarrow \alpha_X \leq \frac{k_1}{10} \quad (4.11)$$

(ii) Experimental constraints:

$$10^{13} \text{ GeV} \leq M_X \leq 10^{19} \text{ GeV}, \quad M_W = 80 \text{ GeV}$$

$$0.22 \leq \sin^2 \theta_W \leq 0.24$$

and $\alpha = \frac{1}{128}$, $\alpha_X = \frac{1}{9}$ at the scale M_W .

(iii) We will also use

$$b_3 = -3 + b'_3 \quad (4.12)$$

$$b_2 = 1 + b'_2 \quad (4.13)$$

$$b_1 = 11 + b'_1 \quad (4.14)$$

with $b'_i \geq 0$, where we have already used the contributions to the β functions from the three families of the supersymmetric Standard Model plus the minimal two Higgses [18].

Using these conditions and equations (4.8) and (4.9) we can easily find bounds on x and z to be

$$0.2 \leq k_1/k_2 \leq 9.5$$

$$0.2 \leq k_2/k_3 \leq 5.2$$

which, as we mentioned before, will restrict the relative values of the three levels, that is given the value of one of the levels, the other two cannot differ that much from the first one, so we do expect the levels to have similar values. Also, from equations (4.5) and (4.11) we can find limits on the values of the coefficients b_i . So for instance we find that $b'_3 \leq 3.8$. From equation (4.6) we know that this will also restrict the value of $T(R)$, but these values are known for the groups we are considering ($T = \frac{1}{2}, 3, \frac{5}{2}$ for the triplet, octet and sextet of $SU(3)$ respectively, and essentially increases with the dimension of the representation). Therefore in order that the conditions above are satisfied we can have in the massless spectrum, beyond the standard model states, less than 8 triplets, one or zero octets and sextets and nothing more. Notice that even though arbitrarily high levels are allowed, which in principle would allow very large representations, the extra matter that we can have is highly restricted if we insist on having perturbative unification. A similar result is found for the $SU(2)$ representations, for which no more than 8 doublets or 2 triplets are allowed but nothing more. For the hypercharges they have to be less than 3.

Notice that the constraints found on b'_i were independent of the levels k_i , we have not used all the possible constraints yet. For instance, from equation (4.9) we can see that given a value of z , there is a constraint on the relative values of b'_2 and b'_3 . For $z = 1$ we find $b'_2 - b'_3 \leq 0.52$, which will indicate that the number of doublets of $SU(2)$ and triplets of

$SU(3)$ can differ at most by one which is much more restrictive than what we had before. This coincides with the analysis of reference [19] in the case $k_3 = k_2 = 1$. For general values of z and x , we can only say that given those values, equations (4.9) and (4.10) will constrain the relations on the values of b'_1, b'_2 and b'_3 .

4.3 INTERMEDIATE SCALE ANALYSIS

The existence of an intermediate scale will modify the previous analysis. We will briefly discuss what kind of changes can be expected in that case. We assume that at the Planckian scale (M_X) there is a symmetry group G which breaks to the Standard Model one at an scale M_I intermediate between M_X and the Weinberg-Salam scale (M_W). G is taken to be a product of simple factors and $U(1)$'s. The generators of $SU(3) \times SU(2) \times U(1)$ can be written as linear combinations of the ones in G as:

$$T_i = \sum_{\mu} a_{i\mu} T_{\mu} \quad (4.15)$$

where T_i ($i = 1, 2, 3$) are the $SU(3) \times SU(2) \times U(1)$ generators and T_{μ} are the G generators (μ runs over the number of factors in G).

We will first consider the running of the coupling constants in the range M_X to M_I . In this region the gauge group is G and the Standard Model parameters can be written in terms of those of G , using (4.15).

$$\frac{1}{\alpha_i} = \sum_{\mu} a_{i\mu}^2 \left(\frac{1}{\alpha_{\mu}} \right), \quad k_i = \sum_{\mu} a_{i\mu}^2 k_{\mu} \quad (4.16)$$

The running of the coupling constants in the region M_I to M_W is similar to equation (4.5). Therefore the RGE's for the total range M_X to M_W are,

$$\frac{1}{\alpha_i(M_W)} = \frac{k_i}{4\alpha_X} + \frac{b'_i}{2\pi} \log \frac{M_X}{M_I} + \frac{b_i}{2\pi} \log \frac{M_I}{M_W} \quad (4.17)$$

where $b'_i \equiv \sum_{\mu} a_{i\mu}^2 b_{\mu}$. Using a similar analysis as for the case without intermediate mass

scale we can find expressions for the Weinberg angle and the unification mass as,

$$\sin^2 \theta_W = \frac{1}{1+x} \left\{ 1 + \frac{\alpha}{2\pi} \left[(b_2 x - b_1) \log \frac{M_X}{M_I} + (b_2 x - b_1) \log \frac{M_I}{M_W} \right] \right\} \quad (4.18)$$

$$\sin^2 \theta_W = \alpha \left[\frac{z}{\alpha_3} + \frac{1}{2\pi} (b_2 - z b_3) \log \frac{M_X}{M_I} + \frac{1}{2\pi} (b_2 - z b_3) \log \frac{M_I}{M_W} \right] \quad (4.19)$$

which imply

$$\log \frac{M_X}{M_I} = \frac{2\pi \left(\frac{1}{\alpha} - \frac{z(1+x)}{\alpha_3} \right) - (b_1 + b_2 - z(1+x)b_3) \log \frac{M_L}{M_W}}{b_1 + b_2 - z(1+x)b_3} \quad (4.20)$$

Given a group G , we can find the coefficients $a_{i\mu}$ and so we can use equations (4.18)-(4.20) to find the new constraints on the levels and spectrum of the low-energy theory. For instance, if we take G to be any of the subgroups of $SO(10)$ and E_6 containing the Standard Model group, it is easy to see that the constraints on x and z do not change substantially compared with the results of the previous sections, using standard values for the intermediate scale ($M_I = \sqrt{M_X M_W}$ up to two orders of magnitude). Similarly, the limits on b'_i remain about the same. However the relative values of the coefficients b_i can change radically depending on the part of the spectrum that acquires a mass at M_I , as can be seen from equation (4.19). For instance the constraint $b'_2 - b'_3 \simeq 0.5$ for $z = 1$ found above can be substantially relaxed depending on the values of b_2, b_3 and M_I . We conclude then that the constraints in the case of an intermediate scale are less restrictive but the parameters are not completely arbitrary and model dependent restrictions can be obtained using the general equations (4.18)-(4.20).

4.4 CONSTRAINTS ON LEVELS FROM INTEGRALITY OF GAUGE QUANTUM NUMBERS

One could also be interested in imposing some restriction on the possible gauge quantum numbers of the string states. A typical example is charge. One might want to request that all (colour singlet) string states should have integer charge since, after all, no violation of charge quantization has been experimentally observed. Schellekens [10] has recently shown how imposing that condition leads to constraints on the standard model levels. One can apply similar arguments to other gauge quantum numbers existing in extensions of the standard model like $B-L$ and others. We consider those cases below.

Let us briefly review the results of ref. [10] concerning charge quantization of the standard model. Given an e.g. $SU(N)$ Kac-Moody algebra at level k one can define "simple currents" [20] $J_N^{(i)}$ with conformal dimension $h = k(N-i)/2N$. The monodromy of $J_N^{(i)}$ with some field Φ is given by $\alpha = i/N$ where i is the N^{th} -ality of Φ . One can see that if a simple current J has integer monodromy with respect to all fields in the spectrum then J itself must be in the spectrum. In this case J must also have integral conformal spin so that its presence does not violate the T modular transformation.

In the case of the standard model, consider the simple current $J = (J_3^{(1)}, J_2^{(1)}, J_1)$ has hypercharge $Y = k_1/2$. The monodromy α_J of J with any field is given by

$$\alpha_J = \frac{k_3}{3} + \frac{k_2}{2} + Y \quad (4.21)$$

Since $\alpha_J = \text{integer}$ happens to be the condition for charge integrality, if we impose such a condition then according to the statements in the previous paragraph the current J itself must be in the spectrum and $h_J = \text{integer}$. For the latter one gets

$$h_J = \frac{k_3}{3} + \frac{k_2}{4} + \frac{k_1}{4} \quad (4.22)$$

thus one gets the condition

$$4k_3 + 3k_2 + 3k_1 = 0 \pmod{12} \quad (4.23)$$

In particular, if $k_3 = k_2 = 1$, $k_1 = 5/3$, the conformal dimension $h_J = 1$ and J can give rise to a massless gauge particle with quantum numbers $(3, 2, 5/6)$. But these gauge particles, together with their conjugates, enhance the gauge symmetry to $SU(5)$. Thus, as remarked in ref. [10], either one gives up charge quantization and/or the canonical hypercharge normalization $k_1 = 5/3$ and/or one considers higher level $SU(3) \times SU(2) \times KM$ algebras. We will comment on the different advantages and shortcomings of each of these possibilities later on.

We should perhaps remark that requiring that all the spectrum of a string (massive as well as massless states) obeys a charge quantization rule is probably too strong a constraint. If one finds possible arguments against having relatively light fractionally charged colour singlet particles ($F\bar{C}P$), it is enough to impose that all such $F\bar{C}P$'s are superheavy. This

may come about in a string theory in several possible different ways. To start with, it may happen that the internal contribution to the conformal dimension of the given $F\bar{C}P$ could be big so that, even though the Kac-Moody contribution to it was small, the total $h > 1$. This is precisely what happens in the $(2,2)$ E_8 compactifications in which the gauge symmetry is broken through the "Hosotani-Witten" mechanism [21]. In these models all the gauge symmetries are realized at level=1 and when broken to the standard model one has the standard normalization $k_1 = 5/3$. There is no contradiction with the arguments in ref. [10] because there are in general $F\bar{C}P$'s in the string spectrum but they are massive. This is true if the manifold discrete isometry associated to the gauge breaking ("Wilson line") acts freely, i.e. there are no fixed points. In this case the massless spectrum is just a projection of the original E_8 spectrum, which obeyed the standard charge quantization rule. There are $F\bar{C}P$'s in the spectrum but they are massive since they involve some winding in the internal manifold and hence the associated conformal dimension is too high.

There could also exist dynamical reasons which force the $F\bar{C}P$'s to be superheavy. An example of this is the three-generation $SU(3) \times SU(2)_L \times U(1)$ orbifold model of refs. [22]. In that model $k_1 = 11/3$ and there are massless $F\bar{C}P$'s in the spectrum. However in this model the classical vacuum is unstable due to the existence of an anomalous $U(1)$ which generates a one-loop D -term in the scalar potential [23]. This term necessarily forces some massless singlets to get a vev. Those vev's in turn inescapably give mass to all the massless $F\bar{C}P$'s in the model through Yukawa couplings [24]. There are possibly many other dynamical mechanisms which could give masses to otherwise massless $F\bar{C}P$'s. The above discussion just illustrates how level 1 models with standard $k_1 = 5/3$ normalization do not necessarily imply the existence of massless $F\bar{C}P$'s.

It is anyway interesting to see how the arguments above can also be used to restrict the level of the gauge factors in extended gauge models. This is interesting since in the specific realistic 4-d strings built up to now extra gauge factors do generically appear.

i) $SU(3) \times SU(2) \times U(1)_R \times U(1)_{B-L}$. This is the simplest possible gauge extension of the standard model since it only involves one extra gauge boson. Three right-handed neutrinos are the minimal extra matter content needed to cancel $U(1)$ anomalies. This model has certain phenomenological virtues which we discuss below [9]. As could be expected, the conditions for charge integrality are essentially the same since the standard model is just a subgroup of this one. Since the standard hypercharge is obtained from the linear

combination $Y = Q_R + (B - L)/2$ the charge quantization condition is now

$$\alpha_Q = \frac{l_3}{3} + \frac{l_2}{2} + Q_R + \frac{(B-L)}{2} = \text{integer} \quad (4.24)$$

The associated simple current is $J = (J_3^{(1)}, J_2^{(1)}, J_R, J_{B-L})$ where the current J_R has charge $Q_R = k_R/2$ and the current J_{B-L} has charge $(B-L) = k_{B-L}/4$. The conformal dimension of this current is

$$h_J = \frac{k_3}{3} + \frac{k_{2_L}}{4} + \frac{k_R}{4} + \frac{k_{B-L}}{16} \quad (4.25)$$

and requiring integrality of charge would imply

$$4k_3 + 3k_{2_L} + 3k_R + \frac{3}{4}k_{B-L} = 0 \text{ mod } 12 \quad (4.26)$$

Since the relation between Y and $(B-L)$, Q_R implies that

$$k_1 = \frac{k_{B-L}}{4} + k_R \quad (4.27)$$

one of course recovers the condition in eq. (4.23). For $k_{B-L} = 8/3$ and $k_R = 1$ one gets $k_1 = 5/3$ and if furthermore $k_3 = k_2 = 1$ the gauge symmetry is enhanced to $SU(5) \times U(1)_5$ where

$$Q_5 = (B-L) - \frac{4Y}{5}. \quad (4.28)$$

However, in this case this is not the only enhancing possibility. Instead of charge quantization one might be interested in a somewhat weaker condition, $B-L$ quantization, i.e. $B-L$ integrality of colour singlets. This condition is obtained if

$$\alpha_J = \frac{l_3}{3} - (B-L) = \text{integer}.$$

An associated simple current $J = (J_3^{(1)}, 1, 1, J_{B-L}')$ where J_{B-L}' has $(B-L) = -k_{B-L}/2$ exists. If we insist on having integral conformal spin one gets the constraint

$$4k_3 + 3k_{B-L} = 0 \text{ mod } 12$$

For $k_{B-L} = 8/3$ and $k_3 = 1$ the symmetry is then enhanced to $SU(4) \times SU(2)_L \times U(1)_R$. In this case the charge is not in general quantized. If we further had $k_R = 1$ the symmetry

would be enhanced to $SO(10)$. In the same way, when the non-Abelian $SU(3)$ and $SU(2)$ factors have $k_3 = k_2 = 1$ different rational values of k_R and k_{B-L} lead to different enhanced rank 5 gauge groups.

ii) $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ This is a straightforward non-Abelian generalization of the previous example. Essentially all the arguments and formulae go through. One can define a simple current $J = (J_3^{(1)}, J_{2_L}^{(1)}, J_{2_R}^{(1)}, J_{B-L})$ with conformal dimension as in eq. (4.23) and another one $J = (J_3^{(1)}, 1, 1, J_{B-L}')$ as above yielding similar results. In the present case, for $k_{B-L} = 8/3$ and $k_3 = 1$ the symmetry is enhanced to the semisimple group $SU(4) \times SU(2)_L \times SU(2)_R$. If furthermore $k_{2_R} = 1$ the symmetry is again enhanced to $SO(10)$.

iii) $SU(4) \times SU(2)_L \times SU(2)_R$ This is the simplest semisimple extension of the standard model. A standard quark-lepton generation has quantum numbers $(4, 2, 1) + (\bar{4}, 1, 2)$. However, a 4-d string model with the above group will possibly have other massless chiral fields transforming in a different way, like, e.g., exotic quarks and leptons with quantum numbers $(4, 1, 1) + (\bar{4}, 1, 1)$. These fields do not obey the charge quantization rule (although they do obey a $(B-L)$ quantization rule). One way to get rid (if we wanted to) of those exotic states is to project the string spectrum on states possessing integral monodromy with respect to the simple current $J = (J_4^{(2)}, J_{2_L}^{(1)}, J_{2_R}^{(1)})$, that is

$$\alpha_J = \frac{2l_4}{4} + \frac{l_{2_L}}{2} + \frac{l_{2_R}}{2} = \text{integer}. \quad (4.29)$$

One can check that the ordinary quark-lepton generations survive this projection but the exotics above do not. The condition on the levels to guarantee absence of FCP 's in the whole string spectrum is

$$2k_4 + k_{2_L} + k_{2_R} = 0 \text{ mod } 4. \quad (4.30)$$

In particular, when $k_4 = k_{2_L} = k_{2_R} = 1$ one has $h_J = 1$ and J enhances the group to $SO(10)$ by providing gauge bosons transforming like $(6, 2, 2)$. There are however choices like e.g. $k_4 = 3, k_{2_L} = k_{2_R} = 1$ which do not lead to any enhancement and still guarantee charge quantization.

iv) $SU(3)_C \times SU(3)_L \times SU(3)_R$ This is the simplest rank-6 semisimple extension of the standard model. This gauge group often appears in string models since it is a maximal subgroup of E_6 . A standard quark-lepton generation has quantum numbers $(\bar{3}, \bar{3}, 1) +$

$(1, 3, 3) + (3, 1, 3)$. However a string model with the above gauge group will have in general massless states with quantum numbers like e.g. $(3, 1, 1)$ and permutations. These states contain generically FCP 's. One can construct a simple current $J = (J_{3_C}^{(0)}, J_{3_L}^{(0)}, J_{3_R}^{(0)})$ whose monodromy with any string state is given by

$$\alpha_J = \frac{I_{3_C}}{3} + \frac{I_{3_L}}{3} + \frac{I_{3_R}}{3} \quad (4.31)$$

Imposing α_J to be integer, all FCP 's in the string spectrum are eliminated. However if $k_{3_C} = k_{3_L} = k_{3_R} = 1$ the current J which has quantum numbers $(3, 3, 3)$ has $h = 1$ and the symmetry is enhanced to E_6 since the projection on integer α_J in eq. (4.31) kills all states not having definite E_6 -triality. The condition on the levels to guarantee the absence of FCP 's in the string spectrum is now

$$k_{3_C} + k_{3_L} + k_{3_R} = 0 \pmod{3}. \quad (4.32)$$

This ends up our investigation of the constraints on Kac-Moody levels of extended gauge models coming from requesting integrality of some gauge quantum numbers. This analysis allows us to have some constraints on string model building using the simplest extensions of the standard model. Let us however remark again that if due to some argument one wants to get rid of possible FCP 's or other exotic states it is probably too strong a constraint to project them from all the string spectrum and not just from the massless one.

5. Higher level Kac-Moody algebras and phenomenology

We have shown in the previous chapter how the ratios of standard model gauge levels k_3, k_2, k_1 are constrained by the renormalization group evolution of coupling constants. We also saw how the fact that the conformal dimension of string states depends on the quadratic Casimir of the given representation makes that, at a given level k , only a few of those have any chances of being light. This constraint is in fact stronger than the one coming from unitarity. We will see in the present chapter how the above arguments also restrict the simplest extensions of the standard model (including GUT's). The possibilities of encountering different mechanisms suggested in GUT theories in order to avoid a number of phenomenological problems, like proton instability or neutrino masses, are also severely limited.

5.1 SOME PHENOMENOLOGICAL ASPECTS OF NON-SEMISIMPLE MODELS

By these we mean the standard model or extensions involving $U(1)$ factors like models i) and ii) in previous section. Models of these type have in principle arbitrary integer levels for the non-Abelian factors k_3, k_2 etc. and arbitrary real levels $k_1, k_{n-L}, k_{n-...}$ etc. for the $U(1)$ factors. We learnt in the previous chapter that for the standard model (and for model i) the in previous section) one has $1/5 \leq k_2/k_3 \leq 5$ and $1/5 \leq k_1/k_2 \leq 10$ approximately. The first of these constraints is still valid for model ii) in the previous section and similar limits can be derived for k_R and k_{n-L} . All the orbifold models constructed in refs. [22] [24] are of this non-semisimple type. The flipped- $SU(5)$ [25] type of models [26] are also non-semisimple since the "visible" gauge symmetry is really $SU(5) \times U(1)_S$ and the hypercharge lives both in $SU(5)$ and $U(1)_S$. As in essentially all realistic models constructed to date, the non-Abelian symmetries in both the orbifold and the latter models are realized at level 1. Thus in order to avoid a possible enhancement to a GUT (which would be a disaster, since at level 1 there are no massless adjoints to break the symmetry) one has to choose either $k_1 \neq 5/3$ or else admit the presence of FCP 's or both (which is the case of the orbifold models). The FCP 's need not be massless though, as we discussed above.

The problem with the models having $k_3 = k_2 = 1$ and $k_1 > 5/3$ is usually the renormalization group equations. Notice that since $\sin^2 \theta_W = k_2/(k_1 + k_2)$ at the unification (or Planck) scale, as k_1 grows that value decreases sharply. What happens is that g_1 at M_X is too small. This may be compensated if g_2 and g_3 receive big renormalization contributions when running them to low energies. This is the case if (apart from the standard families) there are some massless chiral fields with $SU(3)$ and $SU(2)$ charges but no (or very small) hypercharge contributing through loops. But no particles obeying the charge quantization rule have this property. For example, colour triplets have hypercharge $2/3$ or $1/3$ and $SU(2)$ doublets have $Y = 1/2$ but nothing smaller. However, if $k_1 \neq 5/3 \pmod{4}$, massless particles not obeying the charge quantization rule (FCP 's) may exist and solve the problem. For example, one can have a model with e.g. $k_1 = 10/3$ if, apart from the standard three generations, one has the following additional massless chiral content: four multiplets transforming as $(1, 2, \pm 1/2)$, one like $(3, 1, -1/3) + (\bar{3}, 1, 1/3)$ (these fields obey the charge quantization rule), and two like $(3, 1, 0) + (\bar{3}, 1, 0) + (1, 2, 0) + (1, 2, 0)$ (these ones do not). With this particle content one can compute that at the weak scale $\sin^2 \theta_W = 0.23$ and $M_X = 0.3 \times 10^{17} \text{ GeV}$ which are reasonable values. Thus in a model with $k_3 = k_2 = 1$

and $k_1 > 5/3$ one can only get reasonable values for $\sin^2\theta_W$ and M_X if there are particles with non-standard charge quantization relatively close to the weak scale. Notice that this is a strong prediction applicable to a large class of models. It would be interesting to do a systematic analysis of cosmological constraints on such states since at least the lightest of them should be stable.

The strings with gauge group just $SU(3) \times SU(2) \times U(1)$ (plus may be some “hidden” group) have in our opinion a difficult naturalness problem, as we discussed elsewhere [9]. In the same way that the charge does not obey the standard quantization unless certain conditions are met, the proton is generically unstable in such types of strings unless some B or L or $B-L$ symmetry is implemented. The reason is that the $SU(3) \times SU(2) \times U(1)$ symmetry allows for dimension-four couplings of the form

$$u_L^c d_L^c d_L^c; Q d_L^c L; L L L^c L, \quad (5.1)$$

which violate B or L symmetries. We have used here standard notation for quark and lepton superfields. In particular, if the first two couplings are simultaneously present the proton would decay in a few minutes. One could try to implement B or L or some combination as global continuous symmetries in the string spectrum. However, as shown in ref. [27], if we want these global symmetries to be respected by all correlation functions they should have associated world-sheet currents, which in turn can be used to construct vertex operators corresponding to gauged symmetries. Thus global continuous symmetries acting on the whole string are necessarily promoted to gauge symmetries. It seems then that the simplest way to achieve proton stability in string models is to gauge B or L or some combination of both. With the content of the standard model those two symmetries are anomalous and the minimal anomaly free model has gauged $B-L$ with the simple addition of the right-handed neutrinos.

Of course one can fiddle around by choosing some “flat direction” in the scalar potential in which some effective discrete symmetry is generated which guarantees sufficient proton stability. This is in fact the approach followed in the “realistic” models built up to now [22], [28], [26]. We believe however that that approach, although interesting in some aspects, is rather artificial and equivalent to some sort of fine-tuning. This line of argumentation suggests that models incorporating a gauged $B-L$ are perhaps the most elegant solution to the proton stability problem in string models.

If one considers the non-semisimple models and compares them with good old grand unified theories one feels a bit disappointed about the predictions for the Weinberg angle. In the string models one has at the unification scale an essentially arbitrary value $\sin^2\theta_W = k_2/(k_1 + k_2)$, with $k_2 = \text{integer}$ and k_1 an arbitrary real number. In an $SU(5)$ model one has the definite prediction $\sin^2\theta_W = 3/8$ which, after renormalization, agrees remarkably well (in the supersymmetric case) with the experimental measurements. This would suggest to look for strings models in which $k_1/k_2 = 5/3$ is automatically guaranteed. This leads us to models with semisimple gauge symmetries or models incorporating some grand unification gauge group.

5.2 SEMISIMPLE AND GRAND-UNIFICATION GROUPS IN STRING MODELS

The minimal way of obtaining $k_1/k_2 = 5/3$ in a natural way is to have a string model in which the hypercharge is inside some simple groups with the same Kac-Moody levels. The simplest example is the gauge group $SU(4) \times SU(2)_L \times SU(2)_R$. In this case the hypercharge is embedded as

$$Y = \sqrt{2/3}T_C - T_R$$

where T_C is the $SU(4)$ Cartan generator which commutes with the $SU(3)$ subgroup. It is related to the $B-L$ generator through $T_C = \sqrt{3/8}(B-L)$. If we have $k_4 = k_{2_L} = k_{2_R}$ then automatically $k_1/k_2 = 5/3$. This is still, in some sense, a fine-tuning but a more reasonable one since there are only a discrete number of possibilities, whereas in the case of non-semisimple models k_1 can in principle be any positive real number. Furthermore, there are simple situations like the standard orbifold or other type of simple free fermionic or bosonic constructions in which the possibility $k_4 = k_{2_L} = k_{2_R} = 1$ is easily realized. In a model like this there will be particles with non-standard fractional charges (FCP s) in the string spectrum since, as we explained in the previous chapter, the gauge symmetry would otherwise be enhanced to $SO(10)$. For example, the fields with quantum numbers $(4, 1, 1)$, $(1, 2, 1)$ and $(1, 1, 2)$ have Kac-Moody conformal dimensions $h = 3/8, 1/4$ and $1/4$ respectively, certainly smaller than those of standard fields like $(4, 2, 1)$ (with $h = 5/8$) and hence, in the absence of some peculiar dynamical effect, there is in general no reason to prevent these particles from being massless. In fact, using the result that the contribution

to the conformal dimension of the fields coming from the Kac-Moody sector is given by

$$h = \frac{C_4}{(k_4 + 1)} + \frac{C_L}{(k_L + 2)} + \frac{C_R}{(k_R + 2)} \quad (5.2)$$

one can easily see that the condition $h \leq 1$ does in principle allow all representations compatible with unitarity to be light. Those are the following

$$(1, 1, 1); (1, 2, 1); (1, 1, 2); (4, 2, 2); (6, 2, 1); (6, 1, 2). \quad (5.3)$$

One can easily see that the colour singlet states (including bound states) which one can form with the particles in (5.3) have charge = (integer)/2. This is in fact general and will apply to all the string spectrum independently of the Kac-Moody levels, because the phase in eq. (4.29) is always a half integer. The particles in eq. (5.3) as well as other examples that we discuss below, provide us with prototype examples of FCP's which could be experimentally searched or constrained through cosmological arguments.

Eq. (5.2) gives further constraints on possible massless fields when there are higher levels. For example, one may try to realize the Gell-mann - Ramond-Slansky-Yanagida (GRSY) [6] mechanism to give a large Majorana mass to the right-handed neutrinos. In this model this requires the existence of chiral multiplets transforming like $(10, 1, 3)$ under the gauge group and coupling in a symmetric way to the $(\bar{4}, 1, 2)$ right-handed quarks and leptons. Unitarity already tells us that we need $k_4 \geq 2$ and $k_{2R} \geq 2$. However, the condition $h \leq 1$ gives us, using eq. (5.2), the constraint

$$k_{2R}(2k_1 - 1) \geq 18$$

which e.g., for equal levels of the different factors will imply $k \geq 4$. Thus one would need to go to rather high levels in order to realize this type of mechanism in this model. The same happens with possible chiral fields $(10, 3, 1)$ which have the same conformal dimension and have "Majoron" like quantum numbers.

If one wants to use some sort of "see saw" mechanism to get rid of the right-handed neutrinos in models with $k < 4$ one will have to use (if available) Higgs multiplet representations transforming like $(1, 1, 2)_H + (\bar{4}, 1, 2)_H$ to break the gauge symmetry. If there are Yukawa couplings of the form $(1, 1, 2)^i \times (4, 1, 2)_H \times (1, 1, 1)^j$ where i is a family index,

once the Higgs gets a vev the right-handed neutrinos will combine with the singlets and will decouple from the low energy spectrum. Of course, this requires the string model to have in its massless spectrum three singlets with the appropriate Yukawa couplings. That might be difficult to obtain in specific models, particularly for the three neutrinos at the same time.

Although the $B - L$ symmetry contained in $SU(1)$ forbids dangerous couplings like those in eq. (5.1), there could be massless chiral multiplets transforming like $(6, 1, 1)$ and coupling to quarks and leptons. These include lepto-quark and diquark types of fields which will mediate fast proton decay through dimension-four and -five operators unless they are superheavy. At levels $k_4 = k_{2L} = k_{2R} = 1$ the conformal dimension of these fields and the Weinberg-Salam Higgses $(1, 2, 2)$ is the same $h = 1/2$, and there is a priori no reason why the latter should be light but not the former. In fact this equality of conformal dimension reflects the possibility of enhancing the symmetry to $SO(10)$ so that $(6, 1, 1)$ and $(1, 2, 2)$ fit inside a 10-dimensional representation. If, on the other hand, one considers higher and higher levels (with $k_4 = k_{2L} = k_{2R}$) the conformal dimension of the $(6, 1, 1)$ increases compared to the one of $(1, 2, 2)$. This would suggest that the possibility of having light Weinberg-Salam doublets but superheavy sextets becomes more and more likely, although this should not be taken very seriously since that depends on the contribution of the other gauge and internal degrees of freedom to the total conformal dimension.

Something very similar happens with the simplest rank-6 semisimple model, $SU(3)_C \times SU(3)_L \times SU(3)_R$. Here the standard embedding of the hypercharge is

$$Y = -T_R - \frac{1}{6}(Y_L + Y_R) \quad (5.4)$$

where $Y_{L,R}$ denote the "hypercharge" Cartan generators of the $SU(3)_{L,R}$ factors. This type of model is easily realized by adding some order 3 "Wilson line" in $(2, 2)$ string compactifications like the ones based on Yau's three-generation manifold [28]. If the isometry associated to the Wilson line is freely acting the massless spectrum does not contain FCP's whereas if that is not the case massless particles with quantum numbers $(3, 1, 1)$ or $(\bar{3}, 3, 1)$ and permutations will be generically present in the massless spectrum. Models of this type can also be easily built using orbifold models with underlying Wilson line backgrounds [14]. Notice that the FCP's in this case will obey a charge quantization rule such that all colour singlets will have charge=integer/3. Again this is a consequence of eq.(4.31) which tells us that the monodromy of the fields will always be of order three. These FCP's together with

those in eq. (5.3) provide us, as we mentioned above, with prototypes of particles with unconventional charge quantization. In fact, in the orbifold models of ref. [22] the charges of the FCP's appearing in the spectrum belong also to these classes.

The implementation of the GRSY mechanism would require in this case the presence of massless $(1, \bar{6})$ representations coupling to the leptons $(1, 3, \bar{3})$. These fields can only be present in the massless spectrum for $k \geq 4$ (in order to have $h \leq 1$) so again will not be available unless we go to quite complicated theories. The alternative, already available at level one, is again the existence of massless $(1, 3, \bar{3})_H + (1, \bar{3}, 3)_H$ fields and extra singlets coupling like $(1, 3, \bar{3})^j \times (1, \bar{3}, 3)_H \times (1, 1, 1)^j$.

Proton stability, as is well known from the study of "superstring-inspired" [29] models, is more problematic in these models than in those one might build with the $SU(4) \times SU(2)_L \times SU(2)_R$ symmetry, since the massless states necessarily involve massless lepto-quark and diquark fields. Getting rid of them usually requires a clever choice of gauge symmetry breaking flat directions which does not seem very natural. From this point of view possible models based on $SU(1) \times SU(2)_L \times SU(2)_R$ seem more promising.

If one wants to have automatically $k_1/k_2 = 5/3$ without involving any discrete choice of levels one has to construct string theories with a grand unification group. This includes the $(2, 2)$ compactifications which have (in a sense) E_6 as a grand unification group. In these models the symmetry is broken down to some subgroup like $SU(3)^3$ or $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ etc. by Wilson line breaking [21] associated to some discrete isometry of the compactifying variety. As we mentioned above, in these models there will necessarily be in the massive spectrum particles with non-standard charge quantization [30]. If the isometry associated to the Wilson line is not freely acting, those particles may be present in the massless spectrum. In principle $(0, 2)$ models with other GUT symmetries like $SO(10)$ or $SU(5)$ and further gauge symmetry breaking through Wilson lines may also be built although very few examples of this type are available in the literature [31]. E_6 models may have the potential problem of the extra colour triplets present in their massless spectrum giving rise to fast proton decay. In the $SU(5)$ case the absence of operators like those in eq.(5.1) is not guaranteed. $SO(10)$ models would in this respect be more interesting. However, in all these models the gauge symmetry is realized at level one whereas we are interested here in a what novel features are available when the gauge symmetries are realized at higher levels.

When the level $k \geq 2$ one can construct real Grand Unification String Models (GUSM's), i.e. models in which the GUT symmetry is broken through the standard Higgs mechanism involving massless chiral superfields. The simplest possibility is having an $SU(5)$ gauge GUT [32] whose symmetry is broken through adjoint Higgses. In this case $k_1/k_2 = 5/3$ is automatically guaranteed and with $k_5 \geq 2$ one can have massless adjoint fields to break the symmetry down to the standard model. One of the potential problems of GUSM's is the question, well known since the Eighties, of the mass-splitting of the Weinberg-Salam doublets (which we want to remain light) and the accompanying colour triplets (which we want to maintain heavy in order to avoid their mediating fast proton decay) they come along with in representations of the GUT symmetry (e.g. 5-plets in $SU(5)$, 10-plets in $SO(10)$). In $SU(5)$ only three solutions were known. First is the obvious possibility of fine-tuning a mass term for the 5-plets $\bar{H}, \bar{H}$ against a coupling to the adjoint Higgses ϕ in the superpotential: $W = m\bar{H}\bar{H} + \lambda\bar{H}\phi\bar{H}$. Obviously choosing appropriately m and λ one can make the doublets light and the colour triplets heavy. This solution is certainly quite artificial. A more natural possibility would be to have a massless singlet S coupling with a term $S\bar{H}\bar{H}$ instead of the mass term. Then, once $SU(2) \times U(1)$ is broken, one can see that the minimization condition $\partial W/\partial H = 0$ forces the $SU(2)$ doublets to be massless ("sliding singlet mechanism" [33]). Unfortunately it was soon realized that once supersymmetry is broken the massless singlet necessarily induces large unacceptable one-loop masses ($\sim \sqrt{M_W M_X}$) to the Higgs doublets [34]. The other known mechanism for splitting doublets from colour triplets is the "missing partner mechanism" [7]. This requires the existence of massless chiral superfields with dimension 50. The conformal dimension of these fields is $h = 42/(5k + 25)$ and hence only for $k \geq 4$ can this type of multiplet be light. But with such high level it would be difficult to avoid the presence of many other massless fields which have smaller conformal dimension like adjoints, 40's, 15's etc. which may be massless already at level two. Furthermore, again terms like those in eq.(5.1) are in principle allowed by gauge invariance and may give additional problems to proton stability. Thus we see that using higher level Kac-Moody string models to build a standard $SU(5)$ GUT does not seem very promising.

In the case of $SO(10)$ [35] two different sets of Higgses are required to do the breaking down to the standard model. One needs Higgs chiral multiplets with dimension 45 or 54 to give masses to the diquark and/or lepto-quark gauge bosons and multiplets with dimensions 16 or 126 essentially to lower the rank. The first two may be massless in a level $k \geq 2$ $SO(10)$ model, and of course, 16-plets are already available at level one. The Kac-Moody

conformal dimension of the 126-plets is $h = 25/(2k + 16)$ and hence only for $k \geq 5$ may they be massless. Notice that these are also the chiral multiplets which are required to implement the GRSY mechanism and give a large mass to the right-handed neutrinos in the context of $SO(10)$. If we recall that, as we mentioned in chapter 3, the level in an $SO(10)$ string is necessarily $k \leq 7$ in order not to get a total central charge bigger than 22, one is not left with much space. Furthermore, it would again be difficult to avoid the presence of other massless multiplets e.g. with dimensions 45, 54 (available at level 2) or 120 (which can be massless at level 3). The simplest solution to the neutrino mass problem would involve three extra singlets and one $16, \overline{16}$ multiplet in an obvious extension of what we discussed for the $SP(4) \times SU(2)_L \times SU(2)_R$ case.

Concerning the doublet-triplet splitting problem, there is no "missing partner" mechanism for $SO(10)$ and one can construct a "sliding single-1" mechanism with the same problem as $SU(5)$ had. There is however an interesting possibility [36] in $SO(10)$, which in some measure or less disguised form is at the basis of the doublet-triplet splitting obtained through Wilson lines in the "Hassani-Witten" mechanism [21]. If $SO(10)$ is spontaneously broken through a vev of an adjoint Higgs ϕ along the $B - L$ direction, the symmetry is broken down to $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. Then, if there is a coupling of the Higgs 10-plets Π (which contain the Weinberg-Salam doublets and their colour triplet partners) with the adjoint ϕ of the form $\Pi\phi\Pi$, the colour triplets will become massive, whereas the $SU(2)$ doublets will remain massless since the coupling will be proportional to $B - L$ and the doublets have $B - L = 0$. This is a quite elegant way of achieving the splitting since it results only on the gauge quantum numbers involved. In fact this is what is underlying the Wilson line doublet-triplet splitting mechanism discussed in ref. [21].

The constraints on the particles which can be massless from imposing $h \leq 1$ are rather severe in the E_6 GUT case. In order to have $c \leq 22$ one has $78k/(k + 12) \leq 22$, implying $k \leq 1$. On the other hand, with that constraint only the fundamental 27-plet and the adjoint verify $h \leq 1$, the two representations with dimension 351 can never be massless. Thus one cannot make the trivial but considerably strong statement that one cannot construct a string E_6 GUT with representations other than the fundamental or the adjoint.

There is one feature which may be quite common in GUSM's of the type discussed above. In those models it is assumed that adjoint Higgses ϕ will acquire a vev and break the GUT symmetry close to the Planck mass. Thus there should be flat directions in the ϕ field

scalar potential in which the latter vanishes (or is very much suppressed). If this is the case and ϕ has vanishing superpotential, once the GUT symmetry is broken, the non-Goldstone bosons in ϕ will typically remain massless. For example, in $SU(5)$ if the adjoint Higgs ϕ gets a vev, chiral superfields transforming like

$$(8, 1, 0) + (1, 3, 0) + (1, 1, 0) \quad (5.5)$$

under $SU(3) \times SU(2) \times U(1)$ will remain massless (although, of course, they may get masses close to the weak scale once supersymmetry is broken). The existence of these relatively light superfields is not inconsistent with the renormalization group constraints. For example, if the states in (5.5) as well as the standard two Weinberg-Salam doublets plus a couple of singlets with quantum numbers of right-handed electrons remain light, one gets $\sin^2\theta_W = 0.22$ and $M_X = 4 \times 10^{19}$ which are not unreasonable. Many other possibilities yielding equally admissible values exist. Similar conclusions apply to $SO(10)$. Thus, e.g. if $SO(10)$ was broken through an adjoint 45-plet of Higgses with no superpotential down to $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, matter transforming like $(8, 1, 1, 0) + (1, 3, 1, 0) + (1, 1, 3, 0) + (1, 1, 1, 0)$ would remain light. It is amusing that $\sim \text{TeV}$ multiplets with $(1, 3, 0)$ quantum numbers (in particular its neutral component) have been recently proposed as dark matter candidates [37].

Let us finally mention a potential problem which is likely to appear in this type of GUSM's. They are supposed to have an adjoint Higgs in the massless spectrum. However there is no reason not to have more than one. This would be catastrophic for specific models since, unless the extra adjoints somehow get a mass, their presence would ruin the renormalization group predictions and would necessarily imply that the gauge coupling constants would become very large much below the Planck mass. In fact this is what happens in the $SO(10)$ models discussed in [3].

5.3 A COMMENT ON PROTON STABILITY AND THE NEUTRINO MASS PROBLEM

We have already mentioned the generic difficulties that the string models with gauge group just $SU(3) \times SU(2) \times U(1)$ (or $SU(5)$ for that matter) have with proton stability. Another generic problem essentially for all models is that of neutrino masses. Generically in four-dimensional strings there are singlets S (usually denoted ν_i^c) which couple to the lepton L and Higgs doublets $\overline{H}$ through terms of the form $S L \overline{H}$. These generate unwanted

$\sim M_W$ Dirac neutrino masses once $\overline{H}$ gets a vev. It is important to realize that this not only happens in left-right symmetric models like $SO(10)$ but essentially in all models, just because such couplings are allowed by the gauge symmetries of the standard model. Of course, for some peculiar vacuum solution (flat direction) they may be absent. Still, this seems to be a generic feature of most realistic models built up to now.

We already discussed in the previous chapter (see also ref [9]) how the simplest solution to the proton stability (against dimension-four operators) problem is the existence of an additional gauge boson at low energies forbidding this B -violating process. The simplest possibility is having a gauged $B - L$ symmetry on top of the standard model gauge group. This is an interesting possibility which could have measurable experimental consequences at future accelerators. However one might dislike this possibility on the grounds that already present experimental data require [38] a gauge boson mass for this extra Z^0 $M_{Z'} \geq 500$ GeV, and improvements on this limit are soon to be expected. Although a ~ 1 TeV $B - L$ gauge boson is perfectly sufficient to guarantee proton stability, one would have to explain why it is ten times (or more) heavier than the observed standard Z_0 . The question would then be whether there is a way by which one can make the $B - L$ gauge boson arbitrarily heavy (even $\sim M_{Planck}$) and still obtain a natural absence of B -violating dimension-four couplings. In this way essentially just the standard gauge group would be observable at low energies.

Krauss and Wilczek [8] have recently suggested that a *discrete* gauge symmetry remnant of a gauged $U(1)$ like the one of eq. (4.28) could be sufficient to guarantee proton stability. In order for their mechanism to work the Q_5 symmetry has to be broken by a singlet field σ with $q_5 = 4/5$ in such a way that once σ gets a vev a Z_4 subgroup remains unbroken. All quark-lepton fields transform with a phase $\exp(2i\pi/4)$ and hence the dangerous couplings in eq. (5.1) are all forbidden. In ref.[9] we briefly suggested as a possibility a slight modification which seems more interesting. One could break the Q_5 symmetry by a field σ with quantum numbers $L = \pm 2, B = Y = 0$. It is then clearer to consider the $U(1)$ linear combination $Q_R = Y - (B - L)/2$. Once σ gets a vev, an unbroken Z_2 symmetry remains under which all right-handed fields and Higgses are odd and left-handed fields are even. This is again sufficient to forbid all dangerous couplings in eq. (5.1). The interesting point here is that in this case σ has the appropriate quantum numbers to couple to the right handed neutrinos through a term $\sigma \nu_L^c \nu_L^c$, giving rise to the standard GRSY solution to the neutrino mass problem. Thus one would in principle solve both the proton stability and neutrino mass problems.

It is important to remark here that a σ field with the quantum numbers of a right-handed neutrino ($q_R = \pm 1/2$), as in the alternative solution to the neutrino mass problem discussed in the previous section, does not work here. In this case there is no unbroken discrete gauge symmetry left and there is no reason why the couplings in eq. (5.1) should be absent. For example, the non-renormalizable couplings

$$u_L^c d_L^c d_L^c \sigma + Q d_L^c L \sigma \quad (5.6)$$

are allowed by the gauge symmetries (including Q_R) and will generically be generated. If σ gets a superlarge vev so that we give a very large mass to the extra Z_0 , effective renormalizable couplings like the dangerous ones in eq. (5.1) are generated. This is however not the case if $q_R = \pm 1$ as in the alternative we suggest above.

In principle the scenario with the discrete Z_2 gauge symmetry can be realized in an $SU(3) \times SU(2)_L \times U(1)_L \times U(1)_{B-L}$ string model, although necessarily the level of the $U(1)$'s has to be higher than the one corresponding to the canonical normalization of hypercharge. This is easy to see. For an $SU(3) \times SU(2)_L$ -singlet to be light its conformal dimension must be

$$h = \frac{q_R^2}{k_R} + \frac{(B-L)^2}{k_{B-L}} \leq 1 \quad (5.7)$$

In our case $q_R = \pm 1$ and $B-L = \pm 2$ and hence one has the condition $1/k_R + 4/k_{B-L} \leq 1$. The standard normalization yielding $k_1 = 5/3$ is obtained for $k_{B-L} = 8/3$ and $k_R = 1$, which is incompatible with the constraint (recall $k_1 = k_{B-L}/4 + k_R$). The minimum value of k_1 compatible with eq. (5.7) is $k_1 = 4$ which is obtained for $k_{B-L} = 8, k_R = 2$. Then equation (2.6) tells us that in principle singlet fields with hypercharge = 2 (charge = 2) can also be light. Notice that one has for the Weinberg angle

$$\sin^2 \theta_W \leq \frac{k_2}{k_2 + 4} \quad (5.8)$$

and hence $\sin^2 \theta_W \leq 0.2$ for $k_2 = 1$ and the minimum k_1 value. The renormalization group corrections can only decrease this value, unless there are some light FCP's contributing in the loops. For e.g. $k_2 = 3, k_R = 3$ and $k_{B-L} = 8$ (implying $k_1 = 5$) one can get the standard $SU(5)$ value $\sin^2 \theta_W = 3/8$.

way, the important point is that one can in principle construct string models in which discrete Z_2 gauge symmetry is generated forbidding all proton decay operators of dimension 1 if the relevant $U(1)$'s have sufficiently high level. In this way the only observable violations of low energy would be those of the standard model and at the same time bounded neutralinos may get large masses through couplings with σ . The mechanism requires higher level non-Abelian factors, although in this case the Weinberg angle unification scale is small and one has to rely on renormalization group corrections as a result consistent with experiment. It may also work in models with bigger gauge groups like $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. In this case the σ field should have quantum numbers $(1, 1, 3, 2)$ and the corresponding conformal dimension is given by

$$h = \frac{2}{k_2 + 2} + \frac{4}{k_{n-1}}. \quad (5.9)$$

could have $h \leq 1$ for this multiplet to be light. Now, since the $SU(2)_R$ is broken by an adjoint, the centre Z_2 of the group remains unbroken and provides us with the discrete gauge symmetry.

6. Final comments and conclusions

have studied in this paper both the construction and phenomenological implications of dimensional strings including gauge interactions realized through higher-level Kac-Moody algebras.

have shown how one can construct level k theories by modding G^k factors by order automorphisms. We do this in the context of the orbifold formalism since in this way we can relate the obtained models with more familiar concepts like gauge symmetry through flat directions and Wilson lines. In particular, we show how one can construct the same higher level models by taking scalar flat directions in a level one G^k theory. We analyze in full detail an $SU(3)$ level three model which is modded by permutations a level one $SU(3)^3$ theory. The latter is a Z_3 orbifold realized Wilson line backgrounds. Interestingly enough one can obtain the same $k=3$ by performing a non-Abelian gauge embedding both of the twist and Wilson lines in

¹⁾ but this method can also be used not only for G^k but for products of several different simple groups or subgroup in common. In this case one twists by permutations of these common subgroups.

the original Z_3 model. This is in fact equivalent (at least perturbatively) to taking flat scalar field directions in the original $SU(3)^3$ model. Thus, there are marginal deformations in the theory which continuously connect the level-one and level-three theories. This is interesting since it shows an easy and systematic way to get higher level Kac-Moody models by just taking some specific flat directions in the moduli space of level-one theories, which are much easier to construct.

With the above techniques one could try to search for specific 4-d strings with some phenomenological promise. We have preferred instead to look first for general constraints on the higher level models and second, examine the possible phenomenological prospects higher level models can offer.

There are several sources of constraints on higher level Kac-Moody models. There is the obvious conformal invariance constraint $c \leq 22$ which already tells us interesting information like the fact that $SO(10)$ and E_6 can only be realized at levels ≤ 7 and ≤ 4 respectively. Another obvious constraint, the fact that the conformal dimension h of a given field has to be smaller or equal to one for a field to be massless, turns out to be quite strong, much stronger than the unitarity constraint. A third constraint on the levels comes from imposing that the renormalization group predictions for the Weinberg angle and the unification scale do not get spoiled. This gives constraints for the ratios of the levels of the standard model gauge factors, k_3/k_2 and k_2/k_1 . Assuming the "big desert" scenario, one gets the approximate inequalities $1/5 \leq k_3/k_2 \leq 5$ and $1/5 \leq k_1/k_2 \leq 10$. We also provide the relevant formulae for the case in which there is an intermediate scale of gauge symmetry, for which analogue constraints can also be derived. Finally, more constraints on the levels can be obtained if one imposes integrality of certain gauge quantum numbers. The prototype of that is requiring that all the states of the string obey the standard charge quantization condition of quarks and leptons [10]. Other such type of quantizations can be imposed in extensions of the standard model and we discuss those in some detail.

With the above four types of constraints plus certain other phenomenological inputs we analyze the new possibilities offered by models with higher level Kac-Moody algebras. The most obvious of those is the possibility of having adjoint matter fields as well as other higher-dimensional representations in the massless spectrum. In this way one can build Grand Unified String Models (GUSM's) in which the gauge symmetry is broken through adjoint Higgs fields in addition to some other scalar fields which are also generally required.

Models with gauge groups $SU(5)$, $SO(10)$ and E_6 could in principle be constructed but the constraint $h \leq 1$ on the conformal dimension of the fields severely limits the phenomenological possibilities. This is particularly the case in trying to solve well-known problems like the “doublet-triplet splitting”, neutrino masses and fast proton decay (through dimension-four operators) problems. Thus in $SU(5)$ the “missing partner” mechanism [7] can only be available if $k_3 \geq 4$, whereas other known mechanisms for splitting Weinberg-Salam doublets from colour triplets do not seem to work. In $SO(10)$ the standard Gell-Mann - Ramond-Slausk-Yanagida mechanism for neutrino masses (which requires massless 126 multiplets) is only possible for $k \geq 5$ and for $k \leq 2$ only 45, 54 and 10 -dimensional chiral multiplets can be light (apart from the spinorials). In $SO(10)$ an elegant doublet-triplet mechanism is possible if the adjoint Higgs breaks the symmetry down to $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. In the case of E_6 one can see that the constraints $c \leq 22$ and $h \leq 1$ imply that only fundamentals or adjoints can be massless for any Kac-Moody level. This applies to any heterotic 4-d string either space-time supersymmetric or not, and explicitly shows that, although when considering higher level Kac-Moody algebras plenty of new possibilities are available, there are still strong constraints for model building.

From the point of view of the existence of dangerous dimension four-operators mediating fast proton decay, $SO(10)$ models seem to be in better shape than $SU(5)$ or E_6 . In $SO(10)$ the symmetry $B-L$ which forbids proton decay is gauged (unlike in $SU(5)$) whereas in E_6 , although $B-L$ is gauged, new B -violating dimension four operators involving exotic new quarks appear. This suggests that models based on an $SO(10)$ symmetry are a good starting point to get string models with a sufficiently stable proton. We also argue that in this type of Grand Unified String Models the existence of massless chiral multiplets transforming like $SU(3)$ octets and $SU(2)$ triplets could be quite generic. Another important consequence of GUTSM is the standard $SU(5)$ prediction $\sin^2 \theta_W = 3/8$ at the unification scale. This is in good agreement with experiment after loop corrections are taken into account. However, one must recall that in level 1 compactifications (leading to E_6 in the (2,2) case or $SO(10)$ or even smaller groups for (0,2) compactifications) with Wilson line symmetry breaking the same prediction holds. This is not generically the case in non-simple or even non-semisimple gauge groups, as we discuss in the text. In semisimple models (like e.g. $SU(1) \times SU(2)_L \times SU(2)_R$) getting the “canonical” $\sin^2 \theta_W$ requires a “discrete fine-tuning” (e.g. having $k_1 = k_L = k_R$ in the above example). In non-semisimple gauge groups (like in the standard model and some extensions) the Weinberg angle is not really predicted by the form of the gauge group

since k_1 can be any positive real number. Of course, in specific models it can be computed and will be a fractional number in rational conformal field theories. However, the obtained value will rarely be the canonical $3/8$ result. This may or may not be considered to be a problem (particularly since renormalization group corrections can correct for a “wrong” initial value). In the first case this would suggest the construction of GUTSM’s or even much more simply, level-one models in which a GUT symmetry is broken through Wilson lines. This is simpler and avoids the problems that higher-level models involve not only from the formal but from the phenomenological point of view.

We finally discussed the interesting possibility of having a discrete gauge symmetry à la Krauss and Wilczek forbidding dimension-four operators mediating fast proton decay. This can be achieved in rank ≥ 5 extensions of the standard model including gauged $B-L$ and $U(1)_R$ symmetries. At the same time the right-handed neutrinos may get large masses. This scenario requires higher level Kac-Moody algebras or gauged $U(1)$ ’s with higher k ’s (i.e. normalizations bigger than the canonical ones). The latter possibility can in principle be obtained starting with purely level-one strings.

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