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THE SPIN DENSITY FUNCTIONS OF THE NUCLEON,
FLAVOUR SYMMETRY BREAKING, AND CHIRAL DYNAMICS^{*)}

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ABSTRACT

It is shown that the flavour dependence of the matrix elements of the anomalous divergence and the flavour dependence of the matrix elements of the pseudoscalar quark bilinears correspond to each other. This can be used to define flavour-independent matrix elements of the anomalous divergence, which are related to the axial $U(1)$ charge and to the spin densities inside the nucleon.

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The results of the experiments on the deep inelastic scattering of polarized leptons off polarized nucleons¹⁾ have led a number of authors to investigate the connection between the observed smallness of the SU(3) singlet axial charge (in a naive sense the portion of the nucleon spin due to quarks) and the so-called U(1) problem of QCD^{2...6)}. In ref. (3) the author has given arguments why in QCD the SU(3) singlet axial charge of the nucleon is expected to depart substantially from "naive constituent quark model" estimates, unlike the SU(3) octet charges, as a consequence of the U(1) anomaly and the spontaneous breaking of the chiral symmetry in QCD.

In these considerations the nucleon matrix element of the anomalous divergence of the singlet axial current, proportional to $\alpha_s \cdot \tilde{G}G$ (G: gluonic field strength), plays an important rôle. Once the effects of chiral symmetry breaking are taken into account, this matrix element is highly dependent on the quark mass ratios, in particular also very sensitive to the effects of isospin breaking. Hence one must doubt its relevance for the nucleon spin as a well-defined and flavor symmetric quantity, and a confusion has arisen about the interpretation of these effects. In this note we shall clarify the issue.

For the sake of clarity we discuss first the simplified case of only two quark flavors, i.e. the s-quarks and moreover other heavy quarks are neglected.

The first moments of the quark spin density functions Δq ($q = u, d$) of the nucleon are defined by:

$$\Delta q \cdot s_\mu = \langle p, s | \bar{q} \gamma_\mu \gamma_5 q | p, s \rangle \quad (1)$$

(s_μ : spin vector).

In the case of two flavors the sum $\Delta\Sigma = \Delta u + \Delta d$ is given by the matrix element of the axial baryon current j_μ^{05} :

$$\begin{aligned} \Delta\Sigma_s &= \langle p, s | \bar{u} \gamma_\mu \gamma_5 u + \bar{d} \gamma_\mu \gamma_5 d | p, s \rangle \\ &= \langle p, s | j_\mu^{05} | p, s \rangle \end{aligned} \quad (2)$$

This quantity defines the portion of the nucleon spin carried by the quark spins. In a naive SU(6) type wave function model of the nucleon $\Delta\Sigma$ is equal to one. In reality it may depart significantly from one due to the contributions of orbital angular momenta and of polarized gluons to the spin.

In the chiral limit the singlet axial current is not conserved, due to the QCD anomaly:

$$\partial_\mu j_\mu^{05} = 2\alpha_s / 2\pi \text{tr } \tilde{G}G = a. \quad (3)$$

However the three isospin triplet currents are conserved, and the chiral symmetry is realized through the existence of the three massless Goldstone bosons π^+, π^-, π^0 . The fourth meson $\eta = (\bar{u}u + \bar{d}d)/\sqrt{2}$ acquires a large mass (of the order of the nucleon mass).

The matrix elements of the isovector axial currents are described by two form factors G_1 and G_2 (we do not denote explicitly the isospin index, $q = p-p'$):

$$\bar{v}(p) (G_1(q^2) \gamma_\mu \gamma_5 - G_2(q^2) q_\mu \gamma_5) v(p') \quad (4)$$

The induced form factor G_2 acquires a pole at $q^2 = 0$, the pion pole.

Due to the current conservation one finds the Goldberger-Treiman relation⁷⁾: $2M G_1(0) = 2M g_A = 2F_\pi g_\pi N N$.

Let us consider the corresponding matrix elements of the singlet current. Here the induced form factor G_2^0 does not exhibit a Goldstone pole. Instead one finds^{3,4)}:

$$G_1^0(0) = \Delta\Sigma = A(0), \quad (5)$$

where A is the form factor of the anomalous divergence:

$$\langle p | a | p' \rangle = 2M A(q^2) \bar{v}(p) i \gamma_5 v(p'). \quad (6)$$

As already emphasized in ref. (3), the axial baryon charge, i.e. the quark part of the nucleon spin, is nothing but the nucleon matrix element of the anomalous divergence, i.e. a gluonic quantity.

Thus this quantity cannot be related to the octet axial charges by arguments based on naive quark model considerations (i.e. extensions of SU(3) to U(3) using the "Zweig" rule). Experimentally it seems that the quantity $A(0)$ is rather small, perhaps even zero. We see no compelling reason why $A(0)$ should be exactly zero, in which case one might consider an extended symmetry⁸⁾.

In a "naive" parton approach this quantity can be related to the first moment of the gluonic helicity distribution in a nucleon²⁾:

$$\Delta\Sigma = A(0) = -2\alpha_s/2\pi \cdot \Delta g. \quad (7)$$

However this identification is doubtful, due to the subtle effects of the chiral symmetry breaking in the singlet channel^{4,5)}. Taking (7) for granted, one would conclude: If in a polarized nucleon the gluonic sea were unpolarized, one would have $\Delta\Sigma = \text{const.}$ $\alpha_s \cdot \Delta g = 0$, a situation which seems not far from the experimental results³⁾.

We note that the gluonic quantity A is well defined in the chiral limit. In a next step we introduce the violation of the chiral symmetry by setting $m = m_u = m_d = 0$. In this case isospin is still a good symmetry. The pions acquire a mass: $M_\pi^2 = \text{const.} \cdot A \cdot m$ (A : QCD scale parameter).

The isotriplet axial vector current is no longer conserved:

$$\partial^\mu (\bar{u} \gamma_\mu \gamma_5 u - \bar{d} \gamma_\mu \gamma_5 d) = 2 \text{im}(\bar{u} \gamma_5 u - \bar{d} \gamma_5 d) \quad (8)$$

The r.h.s. of this relation is proportional to the pion field. The induced form factor G_2 of the axial vector current exhibits a pole at M_π^2 but this pole does not contribute to the divergence in the forward direction ($q^2 \rightarrow 0$). Instead the Goldberger-Treiman relation follows since the m.e. of the divergence is given by

$2M_\pi^2 F_\pi g_{\pi N}/(-q^2 + M_\pi^2) + \text{corrections}$. The corrections are due to the violation of the chiral symmetry.

Let us parametrize the violation of the Goldberger-Treiman relation by

$$\Delta = 1 - \frac{M \cdot g_A}{F_\pi \cdot g_{\pi NN}} \quad (9)$$

Using the measured values of g_A , F_π and $g_{\pi NN}$, one finds $\Delta = +5.5\%$. The order of magnitude of this deviation is slightly larger than expected from specific studies of the effects of the chiral symmetry breaking. However it must be an effect of the violation of the chiral SU(2) symmetry, i.e. of order m/Λ , and not, as discussed in ref. (5) due to a violation of the isospin symmetry. As emphasized in ref. (9), a large part of the deviation can be understood as an effect due to chiral loops. Including these effects, the Goldberger-Treiman relation gives $g_A = 1.28 \dots 1.29$, thus deviating from the observed value by only 2%, i.e. it is consistent with the typical order of magnitude for an isospin breaking effect.

We proceed to consider the analogous situation for the singlet case. Here we find:

$$\partial^\mu (\bar{u} \gamma_\mu \gamma_5 u + \bar{d} \gamma_\mu \gamma_5 d) = 2\text{im}(\bar{u} \gamma_5 u + \bar{d} \gamma_5 d) + a.$$

Assuming an unsubtracted dispersion relation for the divergence formfactor $D(q^2)$, one finds:

$$D(q^2) = \frac{2M_\pi^2 \cdot F_\pi \cdot g_{\eta N}}{-q^2 + M_\pi^2} + \frac{2}{\pi} \int \frac{dq'^2}{q'^2 - q^2} \text{Im } D(q'^2) \quad (10)$$

Note that we have $D(0) = 2M\Delta\Sigma = 2M g_A^0$.

A similar dispersion relation holds for the isotriplet case, where the η must be replaced by the π meson. There one can show that as a result of the chiral

symmetry the continuum integral must vanish separately as $m \rightarrow 0^{(10)}$. In reality it is very small. Thus the pion pole dominance is assured.

This is not true in the singlet case. If it were justified to use the η -pole dominance in eq. (10), we would find a relation analogous to the Goldberger-Treiman relation:

$$2 M \Delta \Sigma = 2 M g_A^{\eta} = 2 F_{\eta} g_{\eta N}. \quad (11)$$

The fact that the singlet meson is not a Goldstone boson in the limit $m \rightarrow 0$ would not matter—provided the continuum contribution is disregarded.

Due to the presence of the anomaly it follows:

- a) The singlet meson η does not become massless in the limit $m \rightarrow 0$.
- b) The continuum cannot be shown to vanish in the limit $m \rightarrow 0$.

Thus the relation (11) must be replaced by

$$2 M \Delta \Sigma = 2 F_{\eta} g_{\eta N} + C, \quad (12)$$

where C is the contribution due to the continuum. Unlike in the triplet case this contribution can be arbitrarily large, and its sign is unknown. It originates from the presence of the continuum of gluonic states in the pseudoscalar channel above the η -meson.

We conclude that it is not possible, without further assumptions, to relate the axial singlet charge $g_A^{\eta} = \Delta \Sigma$ to the η -nucleon coupling. In fact, we find it quite likely that the continuum contribution C is large in the singlet channel, which would signal the violation of the "Zweig" rule in this channel.

Subsequently we study what happens if the chiral symmetry and the isospin symmetry are broken at the same time, i.e. the case $m_u \neq m_d$. For the divergence of the isovector and isoscalar currents one finds:

$$\partial^{\mu}(\bar{u}\gamma_{\mu}\gamma_5 u - \bar{d}\gamma_{\mu}\gamma_5 d) = 2m_u \bar{u}\gamma_5 d - 2m_d \bar{d}\gamma_5 u \quad (13)$$

$$= (m_u + m_d)(\bar{u}\gamma_5 u - \bar{d}\gamma_5 d) + (m_u - m_d)(\bar{u}\gamma_5 u + \bar{d}\gamma_5 d)$$

$$\partial^{\mu}(\bar{u}\gamma_{\mu}\gamma_5 u + \bar{d}\gamma_{\mu}\gamma_5 d) = a + (m_u + m_d)(\bar{u}\gamma_5 u + \bar{d}\gamma_5 d)$$

$$+ (m_u - m_d)(\bar{u}\gamma_5 u - \bar{d}\gamma_5 d)$$

Note that the field $(\bar{u}\gamma_5 u - \bar{d}\gamma_5 d)$ is proportional to the π^0 -field.

We find by considering the matrix element of the singlet current between the π -meson and the vacuum which must vanish, apart from small corrections due to the violation of the isospin symmetry:

$$\langle 0 | (m_u - m_d)(\bar{u}\gamma_5 u - \bar{d}\gamma_5 d) | \pi^0 \rangle \quad (14)$$

$$+ \langle 0 | (m_u + m_d)(\bar{u}\gamma_5 u + \bar{d}\gamma_5 d) + a | \pi^0 \rangle = 0.$$

From the isovector divergence we have:

$$F_{\pi} M_{\pi}^2 = \langle 0 | (m_u + m_d)(\bar{u}\gamma_5 u - \bar{d}\gamma_5 d) | \pi^0 \rangle.$$

Combining both relations, and using isospin implying $\langle 0 | \bar{u}\gamma_5 u + \bar{d}\gamma_5 d | \pi^0 \rangle = 0$ we obtain:

$$\langle 0 | a | \pi^0 \rangle = \frac{m_d - m_u}{m_d + m_u} F_{\pi} M_{\pi}^2. \quad (15)$$

This relation, derived previously in ref. (11), implies that the gluonic anomaly has a large m.e. between the vacuum and the π^0 -meson, since the ratio $(m_d - m_u) / (m_d + m_u)$ is expected to be of order one (in reality it seems about 0.3).

Analogously we consider the nucleon m.e. of the axial vector currents. The divergence form factor $D^{I=1}(q^2)$ is given by:

$$D^{I=1}(q^2) = -\frac{2M_\pi^2}{q^2 + M_\pi^2} \frac{F_\pi g_{\pi N}}{M_\pi^2} + \frac{2}{\pi} \int \frac{dq'^2}{q^2 - q'^2} \text{Im } D^{I=1}(q'^2) \quad (16)$$

The contribution of the continuum describes the violation of the Goldberger-Treiman relation, just as in the previous case $m_u = m_d$. Safely we can neglect this in the limit $m_u \rightarrow 0, m_d \rightarrow 0$: there can be no large corrections of order $(m_d - m_u / m_d + m_u)$, which would be finite in the chiral limit.

Thus PCAC can be used for the pseudoscalar isovector quark density, and we find:

$$\langle N | (m_u - m_d)(\bar{u}\gamma_5 u - \bar{d}\gamma_5 d) | N \rangle = \bar{v}\gamma_5 v \, 2M g_A \left[\frac{m_u - m_d}{m_u + m_d} \right] \quad (17)$$

Likewise we have:

$$\begin{aligned} \langle N | (m_u + m_d)(\bar{u}\gamma_5 u + \bar{d}\gamma_5 d) + a | N \rangle \\ = \bar{v}\gamma_5 v \, 2M \left[g_A^0 - g_A \cdot \frac{m_u - m_d}{m_u + m_d} \right] \end{aligned} \quad (18)$$

The sum of the two terms corresponds to the divergence of the isoscalar current. As expected, the isospin dependent terms cancel exactly (apart from tiny corrections of order α , not considered here).

We conclude that the isospin dependent terms in the m.e. of the anomalous divergence cancel exactly with the isospin dependent terms in the m.e. of the pseudoscalar quark densities. In retrospect this is easily understood, since these isospin dependent terms arise from the large coupling of the anomalous divergence and of the pseudoscalar quark density to the pion. Thus it is not

surprising why the m.e. $\langle N | a | N \rangle$ shows a large isospin dependence. It is caused by the coupling of the anomalous divergence to the pion, which couples with its normal strength, given by $g_{\pi N}$, to the nucleon. We conclude: The isospin dependent term does not give information about the gluon content and about the spin or the axial singlet charge of the nucleon, but merely reflect a property of the pion, namely its coupling to the anomalous divergence.

However, the isospin dependent term can be calculated in terms of the quark mass ratios and subsequently subtracted in such a way that only the flavor independent term remains. The latter is identical to the one obtained in the isospin limit $m_u = m_d$ and, apart from a small correction due to the violation of the chiral $SU(2) \times SU(2)$ symmetry, to the nucleon matrix element of the anomalous divergence in the chiral limit. This is the matrix element which is of special interest, i.e. it determines the singlet axial charge.

However, the question whether this flavor independent term can be related to the gluonic spin density moment Δg , i.e. to a perturbative quantity, remains open.

Finally we should like to mention that a generalization to the case of three flavors and of the breaking of chiral $SU(3) \times SU(3)$ is straightforward. Also here the flavor dependent terms of the matrix element of the anomalous divergence, which are due to the coupling of the matrix element of the anomalous divergence to the octet of pseudoscalar mesons, can be consistently subtracted. The result, a flavor independent quantity, is identical to the one obtained in the $SU(3)$ limit. Again only the latter is of interest for the discussion of the axial singlet charge and of the spin problem.

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