CERN-TH.5674/90
IEM-FT-17/90

Extended Inflation in Scalar-Tensor Theories of Gravity

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ABSTRACT

Extended inflation based on a pure Brans-Dicke theory of gravity with constant parameter ω ($\omega < 30$ in order to avoid unacceptable distortions of the microwave background radiation) is ruled out by light-deflection and time-delay experiments ($\omega > 500$). We propose a dynamical mechanism to solve this problem, based on a general scalar-tensor theory of gravity with a function $\omega(\phi)$ having a singularity at $\phi = \phi_c$. We analyze a particular class of models, with a Brans-Dicke-like behaviour during inflation, which converge continuously to General Relativity during the radiation and matter-dominated eras. Solving numerically their equations of motion we find a subclass of models for which $\omega < 30$ at the end of inflation and $\omega > 500$ today, thus in agreement with all post-Newtonian experiments.

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CERN-TH.5674/90
IEM-FT-17/90
March 1990

The only known solution to the horizon, flatness and monopole problems of the Standard Cosmological Model is a period of expansion in a de Sitter phase, where the cosmic scale factor $a(t)$ increases by a factor 10^{28} , at least. The first scenario, called "old" inflation [1], was based on a supercooled first-order phase transition where $a(t)$ grew exponentially with time. The "old" inflationary model failed [2] since the false vacuum expanded too rapidly for the true vacuum to percolate. The solution to this problem, known as the "graceful exit" problem, gave rise to the so-called "new" inflationary scenarios [3]. New inflation is based on a slow-rollover phase transition and requires extreme fine-tuning of the effective potential parameters in order to become extremely flat near the false vacuum. Similarly, the flatness of the potential in chaotic inflation [4] is achieved by means of extremely small values of the parameters. This leads to values of Ω exponentially close to unity today while direct measurements suggest $\Omega \lesssim 0.5$, thus requiring an understanding of the "missing" matter.

"Extended" inflation [5,6] was recently proposed to overcome the fine-tuning problem of new inflation. It recovered the idea of old inflation under a Brans-Dicke theory of gravity [7] instead of General Relativity. The Brans-Dicke action is given by ¹:

$$S = \int d^4x \sqrt{-g} \left(-\phi R - \frac{\omega}{\phi} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + 16\pi \mathcal{L}_m \right) \quad (1)$$

where ϕ is the Brans-Dicke scalar field, ω a constant parameter and $\mathcal{L}_m$ contains a Higgs sector which undergoes a first-order phase transition with a finite energy density at the false vacuum $\rho_f \equiv M_f^4$. During inflation $a(t)$ does not grow exponentially but rather with a power-law, allowing for percolation [9,10] of the true vacuum phase after an expansion big enough to solve the horizon problem but not necessarily leading to $\Omega = 1$ today ².

On the other hand, the Brans-Dicke theory gives Einstein's theory in the limit $\omega \rightarrow \infty$ and so the value of ω is constrained by the classical tests of General Relativity. In particular, light-deflection and time-delay experiments require $\omega > 500$ [12].

The original idea of extended inflation has its own problems. Bubble thermalization could affect the predictions of the Standard Cosmological Model at a given time if too many bubbles of true phase larger than the horizon are produced. In particular, the bounds on the anisotropy of the microwave background radiation translate into an upper bound on ω ($\omega \lesssim 30$) which cannot be accommodated with light-deflection and time-delay experiments [12]. We will call it the " ω -problem" of extended inflation. Therefore, extended inflation under a pure Brans-Dicke gravity is ruled out.

In order to save extended inflation, some models have been suggested [10,13,15] having a potential ³ $V(\phi) \ll \rho_f$ with a minimum at $\phi = M_{Pl}^2$. The simplest possibility [13] is to assume the potential to be essentially flat up to some value $\phi_c < M_{Pl}^2$, such that the phase transition is completed when $\phi \leq \phi_c$. If the theory of gravity is a fundamental theory,

¹We will use the notation and conventions of Ref.[8].

²Though $\Omega < 1$ in extended inflation requires a rather narrow range of parameters [11].

³For a different solution based on chaotic inflation see Ref.[14].

having such a potential is a technical, though unnatural, solution since it requires additional fine-tunings. If the theory of gravity is an effective theory of a more fundamental theory valid at scales beyond M_{Pl} , the presence of $V(\phi)$ may not even be a technical solution. In particular, supersymmetric strings [16] predict $V(\phi) \equiv 0$, while the scale of supersymmetry breaking, in non-supersymmetric strings, is associated with the radius of compactification [17] and is therefore expected to be of order M_{Pl} . Any scale much smaller than M_{Pl} would be unnatural and so is the condition $V(\phi) \ll \rho_f$.

In this letter we propose a dynamical solution to the ω -problem of extended inflation based on more general scalar-tensor theories of gravity. Brans-Dicke theory [7] is just the simplest of these. A more general version, with $\omega = \omega(\phi)$ an arbitrary function, was proposed by Bergmann [18], Wagoner [19] and Nordtvedt [20], particular cases being the variable mass [21] and constant- G [22] theories. It is possible that in many of those theories extended inflation could hold, with $\omega(\phi) < 30$ during inflation, and $\omega(\phi) > 500$ after inflation, during matter- and even radiation-dominated eras.

We will work out a particular class of models based on the function $\omega(\phi)$ defined by ⁴

$$2\omega(\phi) + 3 = \frac{2\beta}{(1 - \frac{\phi}{\phi_c})^\alpha} \quad (2)$$

where $\alpha, \beta > 0$ are constant parameters, and $\phi_c \equiv M_c^2$ with M_c a mass scale close to the Planck mass today, M_{Pl} .

The gravitational field equations corresponding to (1) are given by:

$$R_{\mu\nu} + \frac{\omega}{\phi^2} \partial_\mu \phi \partial_\nu \phi + \frac{1}{\phi} \left(D_\mu D_\nu \phi + \frac{1}{2} g_{\mu\nu} D_\lambda D^\lambda \phi \right) = 8\pi \frac{1}{\phi} \left(\frac{1}{2} g_{\mu\nu} T^\lambda{}_\lambda - T_{\mu\nu} \right) \quad (3)$$

$$(2\omega + 3) D_\mu D^\mu \phi + \omega' \partial_\mu \phi \partial^\mu \phi = 8\pi T^\lambda{}_\lambda \quad (4)$$

where $\omega' \equiv \frac{d\omega}{d\phi}$. In the presence of a homogeneous and isotropic Robertson-Walker metric, Eqs. (3) and (4) are equivalent to:

$$\left(\frac{\dot{a}}{a} \right)^2 + \frac{k}{a^2} = \frac{8\pi \rho}{3\phi} + \frac{\omega}{6} \left(\frac{\dot{\phi}}{\phi} \right)^2 - \frac{\ddot{a}}{a\phi} \quad (5)$$

$$\ddot{\phi} + 3 \frac{\dot{a}}{a} \dot{\phi} + \frac{\omega'}{2\omega + 3} \dot{\phi}^2 = \frac{8\pi}{2\omega + 3} (\rho - 3p) \quad (6)$$

with the energy conservation equation

$$\dot{\rho} = -3 \frac{\dot{a}}{a} (\rho + p) \quad (7)$$

We will try to solve Eqs. (5) and (6) as the universe supercools in the false vacuum and $\rho = \rho_f$ acts as an effective cosmological constant. During inflation, $p = -\rho$ from Eq. (7), while Eqs. (5) and (6) can be approximated by

⁴Barker's theory [22] coincides with (2) for $\beta = -1/2$, $\alpha = 1$ while pure Brans-Dicke theory is recovered for $\alpha = 0$.

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} \frac{\rho_I}{\phi} \quad (8)$$

$$3 \frac{\dot{a}}{a} \dot{\phi} = \frac{32\pi}{2\omega + 3} \rho_I \quad (9)$$

where we have taken $k = 0$. During inflation $a(t)$ is going to increase very fast and the solution for $k \neq 0$ will converge very rapidly towards the $k = 0$ solution. We are assuming in (8) and (9) that the terms $\frac{\omega}{6} \left(\frac{\dot{\phi}}{\phi}\right) - \frac{\dot{a}}{a} \dot{\phi}$ and $\frac{\omega'}{2\omega + 3} \dot{\phi}^2$ in (5) and (6), respectively, can be neglected during the inflationary era. This assumption should be verified *a posteriori*.

For illustrative purposes we will analyze Eqs. (8) and (9) with $\omega(\phi)$ in (2) for $\alpha = 1$, where solutions can be found analytically. They are given by:

$$\phi(t) = \phi_c \tanh^2 F(t) \quad (10)$$

$$a(t) = \left[\frac{M_c}{m_{Pl}} \sinh F(t) \right]^\beta \quad (11)$$

$$F(t) = \frac{m_{Pl}}{M_c} \left(1 + \frac{\chi^t}{\beta} \right) \quad (12)$$

where $\chi^2 = \frac{8\pi}{3m_{Pl}^2} \rho_I$ and $m_{Pl} < M_{Pl}$ is an arbitrary integration constant corresponding to the effective Planck mass at the beginning of inflation, $t = 0$. Using Eqs. (10)-(12) we can now evaluate the terms we have neglected in (5) and (6):

$$\left| \frac{\omega}{6} \left(\frac{\dot{\phi}}{\phi} \right) - \frac{\dot{a}}{a} \dot{\phi} \right| < \epsilon_1(\beta) \frac{8\pi}{3} \frac{\rho_I}{\phi} \quad (13)$$

$$\left| \dot{\phi} + \frac{\omega'}{2\omega + 3} \dot{\phi}^2 \right| < \epsilon_2(\beta) 3 \frac{\dot{a}}{a} \dot{\phi} \quad (14)$$

where

$$\epsilon_1(\beta) = \frac{4}{3\beta} + \frac{1}{\beta^2} \quad (15)$$

$$\epsilon_2(\beta) = \frac{1}{3\beta} \quad (16)$$

We then see that for $\beta \gg 1$, Eqs. (8) and (9) are a good approximation to (5) and (6) during inflation. For instance, if $\beta = 25$, $\epsilon_1 = 5.5 \times 10^{-2}$ and $\epsilon_2 = 1.3 \times 10^{-2}$ so the error is bounded to less than 6%.

In the regime where $F(t) < 1$ the solution (10)-(12) behaves like in Brans-Dicke inflation with parameter $\omega = \beta$, that is:

$$\phi(t) = m_{Pl}^2 \left(1 + \frac{\chi^t}{\beta} \right)^2 \quad (17)$$

$$a(t) = \left(1 + \frac{\chi^t}{\beta} \right)^\beta \quad (18)$$

In the regime where $F(t) > 1$ the solution (10)-(12) approaches very quickly Einstein inflation with exponential growth of the scale factor, that is:

$$\phi(t) = M_c^2 \quad (19)$$

$$\frac{a(t_1)}{a(t_2)} = \exp \left[\frac{m_{Pl}}{M_c} \chi(t_1 - t_2) \right] \quad (20)$$

A quick glance at Eqs. (8) and (9) shows that functions (17)-(20) are generic solutions for arbitrary values of α in (2). In the range $\phi \ll \phi_c$, ω behaves like a constant β as in Brans-Dicke theory and the solution to (8) and (9) is easily found to be given by (17)-(18). On the other hand, in the limit $\phi \rightarrow \phi_c$ (i.e. $\omega \rightarrow \infty$) Eq. (9) is solved by $\dot{\phi} = 0$ while Eq. (8) provides the exponential behaviour of (20). In between the two regimes, the scale factor evolves continuously from the power-law (18) to the exponential behaviour (20).

If the phase transition happens during the regime where the solution can be approximated by (18), percolation of the true vacuum will occur. We will now analyze the conditions under which this will happen, and will find that they impose only an extra mild constraint on m_{Pl} with respect to the Brans-Dicke case [10].

The probability of a point remaining in the false vacuum is $P(t) = e^{-\epsilon(t)}$ where

$$\epsilon(t) = \frac{4\pi}{3} \int_0^t dt' \lambda(t') a(t')^3 \left[\int_{t'}^t \frac{d\tau}{a(\tau)} \right]^3 \quad (21)$$

and $\lambda(t)$ is the nucleation rate per unit time per unit volume, which can be taken to be constant during the inflationary phase. In the power-law regime (18) the Hubble function during inflation is $H(t) = \chi \left(1 + \frac{\chi^t}{\beta} \right)^{-1}$ and the exponent (21) can be approximated, for $\frac{\chi^t}{\beta} > 1$, by

$$\epsilon(t) = \frac{\pi}{3} \epsilon \beta \left(\frac{H(0)}{H(t)} \right)^4 \quad (22)$$

where $\epsilon \equiv \frac{\lambda}{\chi^4} \ll 1$ is a parameter calculable from the effective potential. Following Ref.[10]

we will fix the end of inflation at a time t_f such that $\epsilon \left(\frac{H(0)}{H(t_f)} \right)^4 > \epsilon_\sigma = \frac{3}{4\pi}$. At that

time the nucleation rate begins to outrun the expansion rate and percolation of the true vacuum takes place.

The condition satisfied by t_f

$$1 + \frac{\chi t_f}{\beta} < \left(\frac{3}{4\pi\epsilon} \right)^{1/4} \quad (23)$$

along with the condition $F(t_f) < 1$ to be in the Brans-Dicke regime before percolation, translates into a condition for m_{PI}

$$m_{PI} < \left(\frac{4\pi\epsilon}{3} \right)^{1/4} M_c \quad (24)$$

Notice that in the exponential regime (20) the probability for a point to remain in the false vacuum is given by $\epsilon(t) = \frac{4\pi}{3}\epsilon \left(\frac{M_c}{m_{PI}} \right)^4 \left(\frac{m_{PI}}{M_c} \chi t \right)$. Therefore, condition (24) guarantees that the rate of bubble formation is faster than the rate of expansion of the universe even during the exponential regime (20). Without more detailed calculations, this fact gave us confidence in assuming that percolation could take place even in the intermediate region between the power-law and the exponential regimes.

Using the condition for the beginning of inflation [10] as $H^4(0) < \rho_f \equiv M_f^4$, at $t = 0$, i.e.

$$M_f < \left(\frac{3}{8\pi} \right)^{1/2} m_{PI} \quad (25)$$

along with (24), one obtains

$$M_f < \left(\frac{3\epsilon}{16\pi} \right)^{1/4} M_c \quad (26)$$

We observe that conditions (25) and (26) were already required in Brans-Dicke inflation [10] (with $M_c \approx M_{PI}$). In particular, condition (26) along with the range $10^2 \text{ GeV} < M_f < 10^{16} \text{ GeV}$, coming from baryogenesis and monopole dilution after inflation [10], does represent a non-trivial constraint on ϵ : acceptable values $10^{-64} < \epsilon < 10^{-4}$ can be obtained in models with moderate couplings. Given the acceptable ϵ -range, the extra condition (24) does not imply any extra tuning of the parameters. The main difference is that in pure Brans-Dicke inflation m_{PI} can be close to M_c , while in our model Eq. (24) imposes the condition $m_{PI} \ll M_c$.

From (23), the total number of e -folds of inflation during the Brans-Dicke regime before the phase transition is

$$N_e > \left(\frac{3}{4\pi\epsilon} \right)^{\beta/4} \quad (27)$$

which can provide the 28 necessary orders of magnitude to solve the horizon and flatness problems even for moderate values of β , depending on the particular value of ϵ .

We will conclude this paper with a numerical analysis of the theory defined by (1) and (2) in the inflationary and radiation-dominated eras. However, before integrating

numerically the equations of motion (5)-(7), we will examine analytically the convergence of our theory to General Relativity. This convergence is considered 'good' if $\phi = \phi_c$ is an attractor of the equations of motion. In this case, no matter what the initial conditions are, ϕ will evolve continuously towards ϕ_c (and so $\omega(\phi) \rightarrow \infty$) recovering General Relativity in the limit and making this theory indistinguishable from Einstein's theory at present times. To prove analytically that $\phi = \phi_c$ is an attractor, we will write Eq. (6) in integrated form for $\omega(\phi)$ in Eq. (2) as

$$\dot{\phi}(t) = \frac{4\pi}{\beta} a^{-3}(t) \left[1 - \frac{\phi(t)}{\phi_c} \right]^{\alpha/2} \left(\int_{t_0}^t dt' [\rho(t') - 3p(t')] a^3(t') \left[1 - \frac{\phi(t')}{\phi_c} \right] + C \right) \quad (28)$$

During inflation $\dot{\phi}(t) > 0$ (see Eq. (9)) and therefore $\phi(t_f) > 0$. This gives $C > 0$ during the radiation-dominated era ($T_\lambda(\text{rad}) = \rho - 3p = 0$) from which we deduce also $\dot{\phi}(t) > 0$ and therefore $C > 0$ during the matter-dominated era. However $T_\lambda(\text{matter}) = \rho(t) > 0$ and again $\dot{\phi}(t) > 0$ at the matter-dominated era. We conclude that $\phi(t) > 0$ at any time and therefore $\phi = \phi_c$ is an attractor of the equations of motion.

The second test for convergence to General Relativity concerns the post-Newtonian experiments. There are three quantities with a direct physical meaning. The most important one is $\omega(\phi)$ itself. It should be large enough today [12] ($\omega > 500$) for the theory to agree with post-Newtonian experiments. The second physical quantity is the present rate of change of Newton's gravitational constant ⁵

$$t_G = -\frac{\dot{G}}{G} \quad (29)$$

as compared to the Hubble time $t_H \equiv \frac{a}{\dot{a}}$. From the standard post-Newtonian analysis [12]:

$$\dot{G} = \frac{1}{\phi} \left(1 + \frac{1}{2\omega + 3} \right) \quad (30)$$

and so

$$t_G = \left(1 + \frac{1}{2\beta} \frac{\phi}{\phi_c} \frac{2\omega + 3}{2\omega + 4} \right) \frac{\phi}{\phi_c} \quad (31)$$

The experimental constraint is that $t_G/t_H \gg 1$ at present times [12,23]. The third quantity of interest is the ratio between the Hubble and cosmic times. It is known to be of order one today [12,24]. The 'good' convergence of our theory ensures that if the experimental constraints ($\omega > 500$, $t_G/t_H \gg 1$, $t_H/t \simeq 1$) are satisfied at early times (during the radiation-dominated era, as we actually found) they will be satisfied at present times.

⁵The choice of the minus sign in (29) comes from the fact that $\dot{G} < 0$ in our theory.

still be compatible with $\omega(\text{today}) > 500$. This means that the end of inflation occurs in an intermediate region much below the exponential regime but past the pure Brans-Dicke behaviour. We have given arguments in favour of percolation in this region. A detailed calculation of the conditions for percolation and homogeneity and isotropy after thermalization is under work. On the other hand, it is clear that the general mechanism proposed in this paper could work more efficiently for a different class of models with a stronger singularity at the attractor and a faster growth of $\omega(\phi)$ during the radiation-dominated era. Investigating a class of models leading to smaller values of ϕ_f is an open possibility that will be explored in the future. Being optimistic one would conclude that gravity is better described by one of those theories, allowing inflation at early times and evolving gradually to General Relativity.

Acknowledgements

We would like to thank B. Lynn for supplying us with the integration program COLSYS and M. Traseira for helping us with the computer. We also thank A. Linde for a careful reading of the manuscript and helpful discussions.

We have solved numerically Eqs. (5) and (6) for $k = 0^6$ using the integration program COLSYS [25] (with relative error chosen to be $\epsilon = 10^{-6}$) which was run on the IBM 3090 at CERN.

During inflation we used as initial conditions the values $\phi(0)$, $\dot{\phi}(0)$ and $\alpha(0)$ obtained from Eqs. (17)-(18) at $t = 0$. Solving the coupled differential equations we obtained a characteristic feature for different α 's. For $\alpha \neq 0$ the ϕ field converged asymptotically to ϕ_∞ while for Brans-Dicke it increased forever. A plot of ϕ/ϕ_f for different values of α is shown in Fig. 1. The end of inflation at t_f was determined by ϵ through Eq. (23). The values of $\phi_f \equiv \phi(t_f)$ and $\dot{\phi}(t_f)$ at the end of inflation were taken as initial conditions for the radiation-dominated era. Assuming that all the energy ρ_f of the false vacuum is converted into thermal radiation ('good reheating' condition) by bubble collisions, then $\frac{\rho_f}{\rho_f} \equiv \left(\frac{a(t_f)}{a(t)}\right)^4$ and we can rescale the cosmic scale $a(t)$ during the radiation-dominated era with the initial condition $a(t_f) = 1$.

We want to find out numerically the range of models of kind (2) that admit $\omega_f \equiv \omega(t_f) < 30$ and $\omega(\text{today}) > 500$ for a given value of ϕ_f and α , by fixing $\beta = \beta(\phi_f, \alpha)$ through Eq. (2) such that $\omega(\phi_f) = 25$. We ran the program for different values of ϕ_f and α until $\phi(t)$, and $\omega(t)$, stabilized to a particular value. This would happen just within a few orders of magnitude of the dimensionless variable χt (typically $\sim 10^7$). This way we could select the cases for which ω stabilized above 500. This gave the region above the line $\phi_f(\alpha)$ shown in Fig. 2.

To illustrate the evolution of the different physical quantities during the radiation-dominated era, we analyzed a representative case corresponding to $\frac{m_{\text{Pl}}}{M_*} = \frac{1}{30}$, $\phi_f = 0.2M_*^2$ and $\alpha = 16$. In particular, the evolution of $\phi(t)$, $\omega(t)$ and t_G/t_H , t_H/t is shown in Figs. 3, 4 and 5, respectively. We can see from all of them that their values are in agreement with post-Newtonian experiments already at very early times. In particular, at the time of nucleosynthesis, the Newtonian gravitational constant (30) will be so close to its present value that the calculation of primordial elements abundance would yield the same results as in General Relativity.

In conclusion, we have proposed a dynamical mechanism to solve the ω -problem of extended inflation, based on general scalar-tensor theories of gravity. During inflation, the theory should have a Brans-Dicke-like behaviour with parameter $\omega < 30$ (not to disturb the isotropy of the cosmic microwave radiation) and a power-law expansion of the scale factor in order to allow percolation of the true phase after the phase transition. After inflation, the scalar field should evolve continuously towards an attractor of the equations of motion, giving $\omega > 500$ today, in order to agree with post-Newtonian experiments. We have examined a general class of models defined by a function $\omega(\phi)$, (2). Numerical analysis gives the subclass of models, shown in Fig. 2, satisfying the above constraints. This analysis shows that it is hard to get values of ϕ_f much smaller than $0.1M_*^2$ and

⁶ The huge increase of $a(t)$ will make the k/a^2 -term negligible during inflation. In the post-inflationary era, numerical results for $k = \pm 1$ are very close to those for $k = 0$, due to the expansion of the universe, as numerical calculations [21] have proved.

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Figure Captions

- Fig. 1: Evolution of $\phi(t)$ during inflation for $m_{Pl}/M_c = 1/30, \beta = 25$, different values of α and initial conditions fixed from (10)-(12) in the text.
- Fig. 2: Region in the (α, ϕ_f) -plane, where ϕ_f is the value of ϕ at the beginning of the radiation-dominated era, such that $\omega(t_{today}) > 500$. The parameter β is fixed to $\beta = 26.5(1 - \phi_f/M_c^2)^\alpha$, giving $\omega(\phi_f) = 25$.
- Fig. 3: Evolution of $\phi(t)$ during the radiation-dominated era corresponding to the initial condition $\phi_f = 0.2M_c^2$, $m_{Pl}/M_c = 1/30$, $\alpha = 16$ and β fixed by $\omega(\phi_f) = 25$.
- Fig. 4: Evolution of $\omega(t)$ with the same parameters as in Fig. 3.
- Fig. 5: Evolution of t_C/t_H and t_H/t with the same parameters as in Fig. 3.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5