CERN-TH-5682/90

STRINGS IN ULTRARELATIVISTIC COLLISIONS

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Lectures presented at the Summer School on
Phase Structure of Strongly Interacting Matter,
Cape Town, South Africa, January 1990

CERN-TH-5682/90
March 1990

§ On leave of absence from Physics Department, Brookhaven National Laboratory

CONTENTS

CONTENTS

PREFACE

CHAPTER 1 INTRODUCTION

- 1.1 Natural Units
- 1.2 Relativistic Mechanics

CHAPTER 2 EVOLUTION AND FRAGMENTATION OF STRINGS

- 2.1 A gauge invariant string action
- 2.2 Equations of motion, conservation laws
- 2.3 Solutions
- 2.4 The yo-yo string
- 2.5 String breaking
- 2.6 A string fragmentation model (AMOR)
- 2.7 Other fragmentation models
- 2.8 Comparison with data
- 2.9 References

CHAPTER 3 STRING FRAGMENTATION IN A MEDIUM

- 3.1 Model for string fragmentation in a medium
- 3.2 Parton model of neutrino-nucleus scattering
- 3.3 Comparison with data
- 3.4 References

CHAPTER 4 SOFT HADRON-HADRON INTERACTIONS

- 4.1 String formation procedures
- 4.2 The string model VENUS
- 4.3 KNO scaling
- 4.4 References

CHAPTER 5 NUCLEUS-NUCLEUS SCATTERING

- 5.1 VENUS model for nucleus-nucleus scattering
- 5.2 Cascading in nuclear collisions
- 5.3 References

PREFACE

Classical relativistic strings provide a very successful concept, used in models for lepton-induced reactions (e^+e^- annihilation, deep inelastic lepton-nucleon and lepton-nucleus scattering) as well as for hadron or ion induced collisions (hadron-hadron, hadron-nucleus and nucleus-nucleus scattering). The applicability is restricted to ultrarelativistic energies, i.e. the string energies should be in general considerably larger than the mass of the hadrons with the same flavour content as the strings (so for example a uu - d string, having the flavour content of a proton, should have a mass considerably larger than the proton mass).

The diversity of applications for string models is very fortunate: one may build up a hierarchy of models, starting with the easiest case of e^+e^- annihilation, towards the very complex models for nucleus-nucleus scattering. In e^+e^- there is in the simplest case just one string produced, most likely a quark-antiquark ($q\bar{q}$) string, so the fragmentation of such strings may be studied. In lepton-nucleon scattering in addition to fragmentation we get information about the parton structure of the nucleon. Also we get a new kind of string: diquark-quark (qq - q) strings and even more complicated fellows like quadriquark-antiquark ($qqqq$ - $\bar{q}$) strings. Proceeding towards lepton-nucleus scattering we have to deal with two complications: (a) the parton structure of nucleons inside a nucleus may be altered by the nuclear medium and (b) the string fragmentation occurs inside a nucleus, so string fragments may interact with (so far) spectator nucleons. Despite the increasing complexity, for all these lepton-induced reactions at least the basic interaction vertex (involving an exchange-boson and an (anti)quark) can be calculated in perturbation theory. This is not possible in hadron-hadron interactions; so in this case we need a model for string formation before we can use the string fragmentation procedures tested in the models for lepton-induced scattering. The next step to proceed to hadron-nucleus or nucleus-nucleus models is an (almost) straightforward extrapolation of hadron-hadron models; essentially one has to consider in addition just the well known nuclear geometry of nuclei. These simple extrapolations ignore completely string-string interactions or interactions of string fragments with spectator nucleons. Only recently these processes have been included in string models. This very last step is very important, because it provides the link towards collective behaviour (like a Quark Gluon Plasma, an equilibrated system of deconfined quarks and gluons). So string models are nowadays not any more just "background" models, concerning interesting and new physics, they are rather important tools to study the evolution of a nucleus-nucleus collision towards a plasma state.

It is the purpose of this lectures to introduce the above mentioned hierarchy of string models for ultrarelativistic collisions of increasing complexity. We are interested in lepton-induced reactions mainly in order to learn about aspects needed later for the more complicated hadron-induced reactions. Therefore this article should not be considered a complete overview over string fragmentation models for e^+e^- , we are for example not interested in parton showers and multi-jet events, since presently the energies for nucleus-nucleus collisions are not high enough for such details to be relevant.

CHAPTER 1 INTRODUCTION

For all theoretical considerations we use so-called "natural units" for physical quantities like time, length, energy, momentum etc. Such units have the advantage of making formulas simpler: factors of c and $\hbar$ disappear. In chapter 1 we introduce these natural units and discuss how they are related to "conventional" ones. In chapter 2 we give a very brief introduction to relativistic mechanics of point particles (zero-dimensional objects). Relativistic string dynamics, to be discussed later, is a generalization which deals with the space-time evolution of one-dimensional objects (=strings). In both cases our objects are elements of the four-dimensional Minkowski space; in the case of point particles we consider trajectories, in the case of strings we consider surfaces in Minkowski space.

1.1 Natural Units

"Conventional" units for length l and energy E are:

$$[l] = \text{fm} \equiv 10^{-10} \text{ m} \quad (1.1)$$

and

$$[E] = \text{GeV} = 1.610 \cdot 10^9 \text{ Joule.} \quad (1.2)$$

In order not to introduce new units we multiply time t and momentum p with the velocity of light c , since ct and cp have the dimensions of length and energy, and correspondingly we multiply masses by c^2 . So we use ct , cp and c^2m with dimensions

$$\begin{aligned} [ct] &= \text{fm} \\ [cp] &= \text{GeV} \\ [c^2m] &= \text{GeV}. \end{aligned} \quad (1.3)$$

Formulas become considerably simpler by using "natural units", i.e., considering length l and time ct in units of $\hbar c$

$$\begin{aligned} \tilde{l} &\equiv \frac{l}{\hbar c} \\ \tilde{t} &\equiv \frac{ct}{\hbar c} \end{aligned} \quad (1.4)$$

with Planck's constant being

$$\hbar c = 0.197 \text{ GeV fm.} \quad (1.5)$$

So we find the dimensions

$$\begin{aligned} [\tilde{l}] &= \left[\frac{l}{\hbar c} \right] = \text{GeV}^{-1} \\ [\tilde{t}] &= \left[\frac{ct}{\hbar c} \right] = \text{GeV}^{-1}. \end{aligned} \quad (1.6)$$

Hence by using $\frac{l}{\hbar c}$, $\frac{ct}{\hbar c}$, cp , E and c^2m the only dimension we have to deal with is GeV . In natural units c and $\hbar c$ are obviously unity, i.e.

$$\begin{aligned} c &= 1 \\ \hbar &= 1, \end{aligned} \quad (1.7)$$

which means that these constants do not appear in equations any more. Consider for example the Schrödinger equation

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2}. \quad (1.8)$$

By using natural units, i.e., defining $\tilde{t} = \frac{ct}{\hbar c}$, $\tilde{x} = \frac{x}{\hbar c}$, and $\tilde{m} = mc^2$ we get

$$i \frac{\partial \psi}{\partial \tilde{t}} = -\frac{1}{2\tilde{m}} \frac{\partial^2 \psi}{\partial \tilde{x}^2} \quad (1.9)$$

where Planck's constant $\hbar$ has disappeared. In the following, unless otherwise noted, we are going to use natural units.

1.2 Relativistic Mechanics

The main postulate of Special Relativity is that for a given space-time point $(t, \vec{x}) = (x_0, \vec{x})$ the quantity

$$s^2 = (x_0)^2 - (\vec{x})^2 \equiv x_0 x_0 - x_i x_i \quad (1.10)$$

is invariant in all inertial frames. By introducing the metric g as the 4×4 diagonal matrix

$$\{g^{\mu\nu}\} \equiv \{g_{\mu\nu}\} \equiv \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} \quad (1.11)$$

we can write eq. (1.10) as

$$s^2 = x_\mu g^{\mu\nu} x_\nu \quad (1.12)$$

or by using the property of g

$$x^\nu = x_\mu g^{\mu\nu} \quad (1.13)$$

we write

$$s^2 = x^\nu x_\nu. \quad (1.14)$$

So the invariance condition is

$$x'^{\mu} x'^{\mu} = x^{\mu} x_{\mu} \quad (1.15)$$

or

$$x'^{\mu} g_{\mu\nu} x'^{\nu} = x^{\mu} g_{\mu\nu} x^{\nu} \quad (1.16)$$

The linear ansatz

$$x'^{\mu} = \Lambda^{\mu}_{\nu} x^{\nu} \quad (1.17)$$

leads to

$$\Lambda^{\mu}_{\rho} x^{\rho} g_{\mu\nu} \Lambda^{\nu}_{\sigma} x^{\sigma} = x^{\rho} g_{\rho\sigma} x^{\sigma} \quad (1.18)$$

so the Λ has to fulfil

$$\Lambda^{\mu}_{\rho} g_{\mu\nu} \Lambda^{\nu}_{\sigma} = g_{\rho\sigma} \quad (1.19)$$

which reads in matrix notation

$$\Lambda^T g \Lambda = g. \quad (1.20)$$

There are four types of transformations (fulfilling eq. (1.20)):

(1) Rotations

$$\Lambda = \begin{pmatrix} 1 & \\ & R \end{pmatrix} \quad (1.21)$$

with an orthonormal ($R^T R = 1$) 3×3 matrix R .

(2) Boosts:

$$\Lambda = \begin{pmatrix} \cosh y & & \sinh y \\ & 1 & \\ \sinh y & & \cosh y \end{pmatrix} \quad (1.22)$$

(3) Time inversion

$$\Lambda = \begin{pmatrix} -1 & & \\ & 1 & \\ & & 1 \end{pmatrix} \quad (1.23)$$

(4) Full inversion

$$\Lambda = \begin{pmatrix} -1 & & \\ & -1 & \\ & & -1 \end{pmatrix} \quad (1.24)$$

We can easily understand the meaning of the "angle" y characterizing Lorentz Boosts. Consider the transformation from a comoving system $(t, x, y, z) = (d\tau, 0, 0, 0)$ to some reference frame with $(t', x', y', z') = (dt, dx, dy, dz)$, the differentials indicating that we

are in the vicinity of $(0, 0, 0, 0)$, where the two systems coincide. We perform a boost, eq. (1.22), to obtain

$$dt = \cosh y \, d\tau \quad (1.25)$$

$$dz = \sinh y \, d\tau$$

from which we obtain a relation between y and the velocity $v = dz/dt$:

$$v = \tanh y \quad (1.26)$$

which can be inverted as

$$y = \frac{1}{2} \ln \frac{1+v}{1-v} \quad (1.27)$$

which is just the definition of the rapidity y .

In order to express energy and momentum in terms of rapidity y we assume that the energy, being the zero component of an energy momentum four-vector $p = (E, \vec{p})$ is equal to the mass of a system in the comoving frame. So by transforming $(m, 0, 0, 0)$ we obtain the general expression for p . To be a little bit more general, we are not considering the rest frame of a system (particle), but a frame where the longitudinal component p_z vanishes. So we have $p = (m_t, \vec{p}_t, 0)$ with the transverse mass defined as

$$m_t = \sqrt{m^2 + p_t^2}. \quad (1.28)$$

We transform p , using eq. (1.22), to obtain

$$p = (E, p_t, p_z) \quad (1.29)$$

with

$$E = m_t \cosh y \quad (1.30)$$

$$p_z = m_t \sinh y$$

which, of course, fulfils the mass-shell condition

$$E^2 - p_t^2 - p_z^2 = m^2. \quad (1.31)$$

Using eqs. (1.26, 27, 30) we can write the rapidity as

$$y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z}. \quad (1.32)$$

One of the most important properties of the rapidity variable is its behaviour under Lorentz boosts. Let y and y' be the rapidities of some particle in the frames F and F' , where F moves with the rapidity $\tilde{y}$ relative to F' . We find:

$$\begin{aligned} y' &= \frac{1}{2} \ln \frac{E' + p'_z}{E' - p'_z} \\ &= \frac{1}{2} \ln \frac{E \cosh \tilde{y} + p_z \sinh \tilde{y} + E \sinh \tilde{y} + p_z \cosh \tilde{y}}{E \cosh \tilde{y} + p_z \sinh \tilde{y} - E \sinh \tilde{y} - p_z \cosh \tilde{y}} \\ &= \frac{1}{2} \ln \frac{(E + p_z)(\cosh \tilde{y} + \sinh \tilde{y})}{(E - p_z)(\cosh \tilde{y} - \sinh \tilde{y})} \\ &= y + \tilde{y} \end{aligned} \quad (1.33)$$

which means that rapidity (not velocity!) is additional under Lorentz boosts. This makes the rapidity extremely useful for applications.

CHAPTER 2

EVOLUTION AND FRAGMENTATION OF STRINGS

In order to keep the article self-contained we review classical string theory in general, i.e. we discuss how to obtain a string action (2.1), we derive equations of motion for the space-time evolution of strings (2.2), and we discuss the general solution as well as conservation laws (2.3). We then treat the simplest possible string solution, the so called "yo-yo" string (2.4). In section 2.5 we discuss the rules for string breaking, in the general case and in particular for yo-yo strings. Although in classical string theory the time evolution is fixed once a string breakpoint is known, the determination of locations of breakpoints requires further input. For this purpose we introduce in 2.6 a model, where the same symmetry arguments are used which led earlier to the string action (leading to an "area law" for string breaking). We discuss relations of this fragmentation model to others in 2.7. We finally apply our model in 2.8 to compare with e^+e^- , $\bar{\nu}p$, and μp data. e^+e^- annihilation produces essentially quark-antiquark strings, whereas $\bar{\nu}p$ and μp provides a test for diquark-quark fragmentation. The actual weights for the string superpositions are given by the parton-model, a description of which is postponed to chapter 3.

2.1 A gauge invariant string action

In the following chapters we discuss classical string theory (see ref. [2.1]), as the basic tool for later applications. A classical string is a two-dimensional surface in the four-dimensional Minkowski space

$$x = x(\tau, \sigma) \quad (2.1)$$

with a spacelike parameter σ and a timelike one τ . Of course this is only one of infinitely many parametrizations of this surface. A transformation from one parameter space to another

$$\begin{pmatrix} \tau \\ \sigma \end{pmatrix} \longrightarrow \begin{pmatrix} \tilde{\tau}(\tau, \sigma) \\ \tilde{\sigma}(\tau, \sigma) \end{pmatrix} \quad (2.2)$$

is called a gauge transformation, and the group of such transformations is called a gauge group. One assumes that the string action should not depend on the parametrization, so gauge invariance is a necessary requirement. Further restrictions should be locality and covariance. Concerning the question of gauge invariance, it is useful to relate a metric g to a certain string parametrization via (using $\partial_1 \equiv \frac{\partial}{\partial \tau}$ and $\partial_2 \equiv \frac{\partial}{\partial \sigma}$):

$$g_{\alpha\beta} = \partial_\alpha x^\mu \partial_\beta x_\mu \quad (2.3)$$

where α and β assume the values 1 and 2. By using "dot" and "prime" as abbreviations for $\frac{\partial}{\partial \tau}$ and $\frac{\partial}{\partial \sigma}$, the metric can be written as

$$g = \begin{pmatrix} \dot{x} \cdot \dot{x} & \dot{x} \cdot x' \\ x' \cdot \dot{x} & x' \cdot x' \end{pmatrix} \quad (2.4)$$

How does the metric g transform under gauge transformations eq. (2.2)? Defining the two component variable ξ via

$$\xi_1 \equiv \tau; \quad \xi_2 \equiv \sigma \quad (2.5)$$

and using

$$\partial_\alpha \equiv \frac{\partial}{\partial \xi_\alpha} \quad (2.6)$$

we get

$$\begin{aligned} g_{\alpha\beta} &= \partial_\alpha x^\mu \partial_\beta x_\mu \\ &= \tilde{\partial}_i x^\mu \partial_\alpha \tilde{\xi}_i \tilde{\partial}_j x_\mu \partial_\beta \tilde{\xi}_j \\ &= \partial_\alpha \tilde{\xi}_i \tilde{g}_{ij} \partial_\beta \tilde{\xi}_j. \end{aligned} \quad (2.7)$$

Since the components of the Jacobi matrix M of the gauge transformation eq. (2.2) are given as

$$M_{ab} = \partial_b \tilde{\xi}_a \quad (2.8)$$

we can write eq. (2.7) in matrix notation as

$$g = M^T \tilde{g} M \quad (2.9)$$

This leads to the identity

$$\sqrt{|\det g|} = \sqrt{|\det \tilde{g}|} |\det M|. \quad (2.10)$$

On the other hand we have

$$d^2 \tilde{\xi} = |\det M| d^2 \xi, \quad (2.11)$$

which together with eq. (2.10) immediately suggests that a ξ integration over $\sqrt{|\det g|}$ is invariant under gauge transformations:

$$\tilde{I} \equiv \int \sqrt{|\det \tilde{g}|} d^2 \tilde{\xi} = \int \sqrt{|\det g|} d^2 \xi \equiv I. \quad (2.12)$$

Writing the integral I explicitly as

$$I = \int \sqrt{(\dot{x} \cdot \dot{x})^2 - x'^2 \dot{x}^2} d\tau d\sigma \quad (2.13)$$

shows that I is also local and covariant. In fact I is the simplest local, covariant and gauge invariant expression, so a very attractive candidate for a string action. Therefore we define the action of a relativistic string as [2.2]

$$S = \int L d\tau d\sigma \quad (2.14)$$

with

$$L = -\kappa \sqrt{-\det g} = -\kappa \sqrt{(x' \dot{x})^2 - x'^2 \dot{x}^2} \quad (2.15)$$

(we used $|\det g| = -\det g$). We will see later that the proportionality constant κ can be identified with the “string tension”, the energy per unit length of the string. This action in fact measures the area of the string surface:

$$S = -\kappa \int d^2A \quad (2.16)$$

with d^2A being a string surface element. This becomes obvious when we choose τ to be the time t and σ to be the length of the string: by defining

$$\gamma = (1 - v_\perp^2)^{-1/2}, \quad \vec{v}_\perp \equiv \frac{\partial \vec{x}}{\partial t} - \frac{\partial \vec{x}}{\partial l} \left(\frac{\partial \vec{x}}{\partial t} \frac{\partial \vec{x}}{\partial l} \right) \quad (2.17)$$

and using $|\frac{\partial \vec{x}}{\partial l}| = 1$ we find

$$S = -\kappa \int_\gamma \frac{1}{\gamma} dl dt$$

which is a surface integral as stated in eq. (2.16).

2.2 Equations of motion, conservation laws

We rewrite the action defined in the last chapter more explicitly as

$$S = - \int_{\tau_1}^{\tau_2} d\tau \int_0^\pi d\sigma L \quad (2.18)$$

with

$$L = -\kappa \sqrt{(x' \dot{x})^2 - x'^2 \dot{x}^2} \quad (2.19)$$

We use the convention $\sigma_{\min} = 0$ and $\sigma_{\max} = \pi$, τ_1 and τ_2 are initial and final time. To obtain the equations of motion we require

$$\delta S = 0 \quad (2.20)$$

under infinitesimal variations $\delta x(\sigma, \tau)$ of the string surface. We obtain

$$\delta S = \int_{\tau_1}^{\tau_2} dt \int_0^\pi d\sigma \left(\frac{\partial L}{\partial \dot{x}_\mu} \frac{\partial}{\partial \tau} \delta x_\mu + \frac{\partial L}{\partial x'_\mu} \frac{\partial}{\partial \sigma} \delta x_\mu \right). \quad (2.21)$$

Integration by parts yields

$$\delta S = \int_0^\pi d\sigma \left[\frac{\partial L}{\partial \dot{x}_\mu} \delta x_\mu \right]_{\tau_1}^{\tau_2} + \int_{\tau_1}^{\tau_2} d\tau \left[\frac{\partial L}{\partial x'_\mu} \delta x_\mu \right]_0^\pi - \int_{\tau_1}^{\tau_2} d\tau \int_0^\pi d\sigma \left(\frac{\partial}{\partial \tau} \frac{\partial L}{\partial \dot{x}_\mu} + \frac{\partial}{\partial \sigma} \frac{\partial L}{\partial x'_\mu} \right) \delta x_\mu. \quad (2.22)$$

Keeping the initial and final position of the string fixed

$$\delta x_\mu(\tau_1, \sigma) = \delta x_\mu(\tau_2, \sigma) = 0 \quad (2.23)$$

we find the equations of motion

$$\frac{\partial}{\partial \tau} \frac{\partial L}{\partial \dot{x}_\mu} + \frac{\partial}{\partial \sigma} \frac{\partial L}{\partial x'_\mu} = 0 \quad (2.24)$$

and the boundary conditions

$$\frac{\partial L}{\partial x'_\mu} = 0 \quad \text{at } \sigma = 0, \pi. \quad (2.25)$$

One may obtain conservation laws, being the consequence of the invariance of the Lagrangian density L under the Poincaré group (Lorentz transformations plus translations). In the following we study the consequence of translational invariance. We again consider variations δx , but take into account the equations of motion. We get (see eq. (2.22)):

$$\delta S = \int_{\sigma_1}^{\sigma_2} d\sigma \left[\frac{\partial L}{\partial \dot{x}_\mu} \delta x_\mu \right]_{\tau_1}^{\tau_2} + \int_{\tau_1}^{\tau_2} d\tau \left[\frac{\partial L}{\partial x'_\mu} \delta x_\mu \right]_0^\pi \quad (2.26)$$

where we now consider a small surface $\sigma_1 \leq \sigma \leq \sigma_2$, $\tau_1 \leq \tau \leq \tau_2$. For an infinitesimal translation

$$\delta x_\mu = \epsilon_\mu \quad (2.27)$$

we get

$$\delta S = \left[\int_{\sigma_1}^{\sigma_2} d\sigma \left[\frac{\partial L}{\partial x'_\mu} \right]_{\tau_1}^{\tau_2} + \int_{\tau_1}^{\tau_2} d\tau \left[\frac{\partial L}{\partial x'_\mu} \right]_{\sigma_1}^{\sigma_2} \right] \epsilon_\mu. \quad (2.28)$$

Because of the invariance of S under arbitrary translations ϵ_μ we find

$$\int_{\sigma_1}^{\sigma_2} d\sigma \left[\frac{\partial L}{\partial x'_\mu} \right]_{\tau_1}^{\tau_2} + \int_{\tau_1}^{\tau_2} d\tau \left[\frac{\partial L}{\partial x'_\mu} \right]_{\sigma_1}^{\sigma_2} = 0. \quad (2.29)$$

With C_0 being the curve surrounding the area $\sigma_1 \leq \sigma \leq \sigma_2$, $\tau_1 \leq \tau \leq \tau_2$, we get

$$\int_{C_0} d\sigma \frac{\partial L}{\partial \dot{x}_\mu} + d\tau \frac{\partial L}{\partial x'^\mu} = 0. \quad (2.30)$$

This is the strings version of conserved currents according to Noether's Theorem. We define the energy momentum currents as:

$$P_\tau^\mu = - \frac{\partial L}{\partial \dot{x}_\mu}, \quad P_\sigma^\mu = - \frac{\partial L}{\partial x'_\mu}. \quad (2.31)$$

The energy momentum associated with an arbitrary curve C is defined as

$$P^\mu \equiv P^\mu(C) = \int_C d\sigma P^\mu_\tau + d\tau P^\mu_\sigma. \quad (2.32)$$

From eq. (2.30) it follows that the momentum of a closed curve is zero

$$P^\mu(C_0) = 0, \quad C_0 \text{ closed.} \quad (2.33)$$

The momentum of the string is defined as

$$P^\mu(\text{string}) = P^\mu(C_{\text{str}}) \quad (2.34)$$

where C_{str} is any curve from one boundary to the other (not necessarily at constant time). Because of $P^\mu(\text{closed } C) = 0$ and eq. (2.25) $P^\mu_\sigma(\sigma = 0, \pi) = 0$ the definition eq. (2.34) makes sense. This definition in particular implies that the string momentum is time independent, since C_{str} can be chosen at fixed but arbitrary time τ ($C_{\text{str}} = C_\tau$):

$$P^\mu(\text{string}) = P^\mu(C_\tau) = \int_{C_\tau} d\sigma P^\mu_\tau. \quad (2.35)$$

Using the currents defined in eq. (2.31) we may rewrite the equations of motion eq. (2.24) as

$$\frac{\partial}{\partial \tau} P^\mu_\tau + \frac{\partial}{\partial \sigma} P^\mu_\sigma = 0, \quad (2.36)$$

and the boundary condition eq. (2.25) reads

$$P^\mu_\sigma = 0 \quad \text{at } \sigma = 0, \pi. \quad (2.37)$$

Our next aim will be to solve these equations of motion.

2.3 Solutions

To solve the equations of motion we choose a gauge which simplifies the equations of motion. The orthonormal gauge

$$x' \dot{x} = 0, \quad \dot{x}^2 + x'^2 = 0 \quad (2.38)$$

does so. If we completely fix the gauge by setting

$$x_0 = \tau = t, \quad (2.39)$$

eq. (2.38) reads:

$$\ddot{x} \dot{x} = 0, \quad (\ddot{x})^2 + (\dot{x}')^2 = 1. \quad (2.40)$$

The currents eq. (2.31) are now

$$P_\tau = \kappa \dot{x}, \quad P_\sigma = -\kappa x' \quad (2.41)$$

and the equations of motion eq. (2.36) are simply wave equations

$$\ddot{x}_\mu - x''_\mu = 0, \quad (2.42)$$

and the boundary conditions eq. (2.37) are

$$x'(t, 0) = x'(t, \pi) = 0. \quad (2.43)$$

The solution of eqs. (2.42,43) is

$$\vec{x}(t, \sigma) = \frac{1}{2} [\vec{y}(t + \sigma) + \vec{y}(t - \sigma)] \quad (2.44)$$

where $\vec{y}$ is obviously the trajectory of one endpoint $\vec{y}(t) = \vec{x}(t, 0)$, called the directrix. The directrix has to be periodic

$$\vec{y}(t + 2\pi) - \vec{y}(t) = \frac{2\vec{P}}{\kappa}. \quad (2.45)$$

Eq. (2.44) means: each point on the string may be obtained by a simple geometrical construction once the directrix $\vec{y}(t)$ is known. From eqs. (2.35,41,44) we also see that the momentum of a piece of string is generated by the momenta of the two corresponding directrix pieces:

$$dP(t, \sigma) = \frac{\kappa}{2} [\dot{y}(t + \sigma) d\sigma + \dot{y}(t - \sigma) d\sigma] \quad (2.46)$$

so the momentum for the string piece corresponding to $[0, \sigma]$ (from A to B in fig. 2.1) is

$$P[0, \sigma] = \int_0^\sigma dP = \frac{\kappa}{2} \int_{t-\sigma}^{t+\sigma} dt' \dot{y}(t') = \frac{\kappa}{2} [y(t + \sigma) - y(t - \sigma)] \quad (2.47)$$

being proportional to the distance vector between two directrix points (DE in fig. 2.1). All this shows that the directrix piece from $D = \vec{y}(t - \sigma)$ to $E = \vec{y}(t + \sigma)$ determines the string piece from $A = \vec{x}(t, 0)$ to $B = \vec{x}(t, \sigma)$. We also find a section of the directrix related to the other string part from $B = \vec{x}(t, \sigma)$ to $C = \vec{x}(t, \pi)$. However we can equally well relate to this string piece the "antidirectrix" (trajectory of the other end $\vec{x}(t, \pi)$) from F over G to C . It is obvious from fig. 2.1 that putting together the directrix piece D to E and the antidirectrix piece F to G , corresponding to the two string pieces, we recover the full directrix (after a shift of the antidirectrix by $\frac{1}{\kappa} \vec{P}$, which is the constant vector by which directrix and antidirectrix differ).

2.4 The yo-yo string

We are now going to discuss a simple but important example: a one-dimensional directrix, one period of which consists of two linear segments ("yo-yo string"). For a one-dimensional directrix, straight lines with a tilt of 45° against vertical (in space-time) are mandatory, since the string end (represented by $\vec{y}(t)$) moves with the velocity of light (because of eqs. (2.40,43)). From eq. (2.44) it is clear that the corresponding string is a simple straight line ($q\bar{q}$ in fig. 2.2) stretched between directrix and antirectrix.

It is very instructive to investigate energy and momentum distribution along a one-dimensional yo-yo string. For the following we use $E \equiv p_0$ and $P \equiv p_3$ for energy and longitudinal momentum (no transverse momentum), the space-time coordinates are t and $z \equiv x_3$. From eqs. (2.35, 2.41) we obtain for an arbitrary string element

$$dE = \kappa d\sigma; \quad dP = \kappa \dot{z} d\sigma. \quad (2.48)$$

The gauge fixing condition $z'\dot{z} = 0$ (from eq. (2.40)) requires either z' or $\dot{z}$ to be zero, an ordinary yo-yo having exactly two points with $z' = 0$: the two endpoints of the string (because of the boundary condition eq. (2.37) every string has to fulfil $\vec{x}' = 0$ at the endpoints). So we obtain from the gauge fixing conditions $z'\dot{z} = 0$ and $\dot{z}^2 + z'^2 = 1$, inside the string

$$\dot{z} \equiv \frac{\partial z}{\partial t} = 0; \quad |z'| \equiv \left| \frac{\partial z}{\partial \sigma} \right| = 1; \quad \text{for } \sigma \neq 0, \pi \quad (2.49)$$

and at the endpoints

$$z' \equiv \frac{\partial z}{\partial \sigma} = 0; \quad |\dot{z}| \equiv \left| \frac{\partial z}{\partial t} \right| = 1; \quad \text{for } \sigma = 0, \pi \quad (2.50)$$

Since these two domains (characterized by $z' = 0$ and $z' \neq 0$) behave so differently, we are going to discuss their contributions to energy and momentum separately. We use an index g (like glue) for the interior and an index q (like quark) for the endpoints. The energy and momentum of an inner piece of string of length dl are (using eqs. (2.48,49)):

$$dE_g = \kappa dt; \quad dP_g = 0 \quad (2.51)$$

whereas we get from eqs. (2.48,50) for an endpoint, during a time step dt with a corresponding movement dz of the endpoint, the following change of energy and momentum:

$$dE_q = \kappa s dt; \quad dP_q = \kappa s dz \quad (2.52)$$

(for $dt > 0$) where $s = +1(-1)$ means the endpoint has absorbed (emitted) a piece of string (or a piece of parameter space, to be precise). Eqs. (2.51,52) demonstrate, among other things, energy conservation: the energy dE_g gained by an endpoint by absorbing a piece of string is equal to the energy loss $-dE_g$ of the string due to its contraction. It is also easy to see from eq. (2.52) that the two endpoints change momentum in an opposite way: $dP_q = -dP_q$, guaranteeing momentum conservation. We are now going to integrate

Fig. 2.1. A string with its directrix and antirectrix. The directrix segment DAE defines the string piece AB and the antirectrix FCG the string piece CB.

Fig. 2.2. Space-time picture of a "yo-yo" string with period $2P/\kappa$.

eq. (2.52). Let us consider one "basic cell" $OAC'B$ in fig. 2.2. The polygon OBC' is half a period of the directrix; instead of the other half we consider the corresponding half period of the antidirectrix OAC' (which is equivalent). So $OAC'B$ defines the string completely. We use for the left end (OBC') the index q , for the right end the index q' . At the turning points the momenta vanish

$$P_q(A) = P_{q'}(B) = 0 \quad (2.53)$$

and since this implies that at these points the parameter space specifying the endpoints consists of just one point (0 and π respectively), eq. (2.53) also requires the energy to be zero

$$E_q(A) = E_{q'}(B) = 0. \quad (2.54)$$

Now we can easily integrate eq. (2.52) backwards to point O to obtain the initial energy and momentum (we use $t_O = z_O = 0$):

$$\begin{aligned} E_q(O) &= \kappa t_A; & P_q(O) &= \kappa z_A \\ E_{q'}(O) &= \kappa t_B; & P_{q'}(O) &= \kappa z_B \end{aligned} \quad (2.55)$$

(Using light-cone coordinates $x^\pm = t \pm z$ and $p^\pm = E \pm P$ we get

$$p_q^+(O) = \kappa x^+(A); \quad p_{q'}^-(O) = \kappa x^-(B) \quad (2.56)$$

together with $p_q^-(O) = 0$ and $p_{q'}^+(O) = 0$. So eqs. (2.56) provide a simple relation between initial momenta and the length of directrix pieces, or in other words, we have a mapping "momentum space" to "real space" via

$$\Delta p = \kappa \Delta x. \quad (2.57)$$

2.5 String breaking

We do not know within our classical treatment where a string breaks, but once we know the breakpoint, we know how to proceed. As for the action we assume locality. If a breaking occurs at $x(t, \sigma)$, we have to make sure that for the future as well as the past we have periodic (anti-) directrices, and the directrices for future and past have to match properly in the present. The only way to do so is to periodically continue (independently) the directrix corresponding to one string piece and the antidirectrix corresponding to the other string piece into the future. This fully determines the time evolution of either string piece also for all the future (till the next break at least).

Let us now discuss these "cutting rules" for a yo-yo string (see fig. 2.3). Without interaction the string stretches between directrix $(t, y(t))$ and antidirectrix $(t, \bar{y}(t)) = (t, x(t, \pi))$. Let the point $B = (t, x(t, \sigma))$ be a breakpoint on the string at time t , dividing the string

Fig. 2.3. Breaking of a "yo-yo" string. The new directrices are constructed according to the rules of classical string dynamics: we first determine the segments of the (anti)directrix corresponding to the string pieces AB and CB ; these segments are then periodically continued into the future.

Fig. 2.4. Relation between (anti)directrix segments and parton momenta. In the string model the energy discontinuities (at the endpoints in the case of a yo-yo) can be identified with partons!

into two segments AB and BC with $A = (t, x(t, 0))$ and $C = (t, x(t, \pi))$. The directrix and antidirectrix corresponding to these segments are DEF with

$$\begin{aligned} D &= (t - \sigma, y(t - \sigma)) \\ F &= (t + \sigma, y(t + \sigma)) \end{aligned} \quad (2.58)$$

and GHI with

$$\begin{aligned} G &= (t - (\pi - \sigma), \bar{y}(t - (\pi - \sigma))) \\ I &= (t + (\pi - \sigma), \bar{y}(t + (\pi - \sigma))) \end{aligned} \quad (2.59)$$

Using

$$\bar{y}(t) = y(t - \pi) + \frac{P}{\kappa} \quad (2.60)$$

we verify easily that, after the appropriate shift, the segments DEF and GHI provide a full period of the unperturbed string. As discussed earlier, we obtain the directrices of the two segments after the break by continuation of DEF ($\rightarrow DEFS \dots$) and of GHI ($\rightarrow GHIN \dots$). The corresponding antidirectrices can be easily constructed from the relation between directrix y and antidirectrix $\bar{y}$:

$$\bar{y}(t) = \frac{1}{2}(y(t + \pi) + y(t - \pi)), \quad (2.61)$$

so we get $BKM \dots$ and $BQT \dots$. We realize the identities

$$\|BJ\| = \|JK\|; \quad \|HJ\| = \|JI\| \quad (2.62)$$

and

$$\|BP\| = \|PQ\|; \quad \|EP\| = \|PF\|, \quad (2.63)$$

which provide a very simple procedure for actually constructing the new directrices in numerical applications.

2.6 A string fragmentation model (AMOR)

We discussed in the last chapters string dynamics, including the case of a breaking string. For a simple one-dimensional yo-yo string this is illustrated in fig. 2.3. We have not yet specified any law determining where the string breaks. This clearly goes beyond any classical treatment. However we can restrict the variety of possible breaking laws by requiring certain properties. In the same way as for the string action S (see section 2.1) it can be shown that the simplest local, covariant and gauge invariant breaking law can be written as

$$dP(\tau, \sigma) \sim \sqrt{-\det g} \, d\tau d\sigma \quad (2.64)$$

with dP being the probability for a break at $x(\tau, \sigma)$, and g being the metric (eq. (2.3)). This means that the breaking probability is proportional to the corresponding area on the string surface: $dP \sim d^2A$ or

$$dP = (1 - P) \alpha d^2A \quad (2.65)$$

with the "break probability" α as a parameter. This is the fragmentation law first suggested by Artru and Mennessier [2.3] and later also used by other authors [2.4]. It is so appealing because it is not just a good guess but rather a strict consequence of requiring very plausible properties: locality, covariance and gauge invariance. Another nice feature is that there is only one parameter (α) which should be the same for processes so different as for example diquark fragmentation into baryons or heavy quark fragmentation into heavy mesons.

For the following we restrict ourselves to yo-yo strings, which form a closed group among all possible strings, in the sense that a yo-yo breaks into two yo-yo's again. In fig. 2.4 we show an "elementary cell" of a half period directrix OAC ("quark" q_0) and the corresponding antidirectrix $OB'C$ ("antiquark" $\bar{q}_0$). Because of eq. (2.57) the coordinates $x_0^+ = t_A + z_A$ and $x_0^- = t_B - z_B$ are related to the initial momenta of quark (p_0^+) and antiquark (p_0^-) via

$$\|OA\| = x_0^+ = \frac{p_0^+}{\kappa}; \quad \|OB\| = x_0^- = \frac{p_0^-}{\kappa}. \quad (2.66)$$

Let us consider a break-up at D into a "quark" ($DIJ \dots$) and an "antiquark" ($DEF \dots$). We may define "break-up momenta" $b_0^\pm$ via

$$\begin{aligned} \|OU\| &= \frac{b_0^+}{\kappa} \\ \|OV\| &= \frac{b_0^-}{\kappa} \end{aligned} \quad (2.67)$$

With $p_i^\pm$ being the parton momenta of the right substring at E we find

$$\begin{aligned} \|EF\| &= \frac{p_1^+}{\kappa} = \frac{p_0^+ - b_0^+}{\kappa} \\ \|EG\| &= \frac{p_1^-}{\kappa} = \frac{b_0^-}{\kappa} \end{aligned} \quad (2.68)$$

A corresponding formula holds for the other substring. The area of absolute past with respect to the break-up point D is given as

$$A \equiv \frac{1}{\kappa^2} A = \frac{1}{\kappa^2} b_0^+ b_0^-. \quad (2.69)$$

All points D having the same value of A lie on a hyperbola in space-time, given by

$$(t + z)(t - z) = A. \quad (2.70)$$

As the other variable to fix D completely we choose the space-time rapidity

$$\eta \equiv \frac{1}{2} \ln \frac{t + z}{t - z} = \frac{1}{2} \ln \frac{b_0^+}{b_0^-}. \quad (2.71)$$

Using these variables A and η eq. (2.65) becomes

$$dP = (1 - P) \frac{\alpha}{\kappa^2} dA d\eta \quad (2.72)$$

leading to

$$dP(\mathcal{A}) = \alpha_0 e^{-\alpha_0 \mathcal{A}} d\mathcal{A} \quad (2.73)$$

We are now in a position to define exactly, step by step, how we proceed to fragment a yo-yo string into two substrings. The proton content is completely arbitrary, we treat $q\bar{q}$ -strings in the same way as q - $\bar{q}$ strings, or even more complicated structures are possible ($\bar{q}$ - $qqqq$, $\bar{q}\bar{q}$ - $qqqq$ etc.). In order to fix the break point D we first determine $\mathcal{A} = \kappa^2 A$ via integrating and inverting eq. (2.73):

$$\mathcal{A} = -\frac{1}{\alpha_0} \ln r \quad (2.74)$$

with $r \in [0, 1]$ being a random number. Before fixing D completely by determining η we have to be more specific about the break-up. We create a $q\bar{q}$ - $\bar{q}\bar{q}$ pair with probability $P_{d\bar{q}}$ (fit parameter) and a q - $\bar{q}$ with $(1 - P_{d\bar{q}})$. Concerning flavour we create a strange quark with probability $P_{s\bar{u}}$ (fit parameter) and u as well $\bar{d}$ quarks with $(1 - P_{s\bar{u}})/2$. We then look into a resonance table (see VENUS 3 writeup) to determine for each substring the minimum mass $m_{\min}$ for the corresponding parton content. So the minimum mass for a $u\bar{d}$ system is the π^+ mass and so on. Suggested by the uncertainty principle we generate transverse momenta $\vec{p}_t$ and $-\vec{p}_t$ for the two partons at D , according to an exponential distribution

$$f(p_t) \sim p_t \exp \left[-\frac{p_t}{2 < p_t >} \right] \quad (2.75)$$

with a fit parameter $< p_t >$ to be chosen in the order of the inverse proton size. Taking this value of p_t together with the minimum mass $m_{\min}$ we get a minimum transverse mass $\mu_{\min}$ for each substring:

$$\mu_{\min} = \sqrt{m_{\min}^2 + p_t^2}. \quad (2.76)$$

Using for the transverse masses $\mu_{\pm}$ of the two substrings

$$\begin{aligned} \mu_+^2 &= p_0^+ b_0^- - \mathcal{A} \\ \mu_-^2 &= p_0^- b_0^+ - \mathcal{A} \end{aligned} \quad (2.77)$$

and using

$$\begin{aligned} b_0^+ &= \sqrt{\mathcal{A}} e^{\eta} \\ b_0^- &= \sqrt{\mathcal{A}} e^{-\eta}, \end{aligned} \quad (2.78)$$

we see that the requirement of minimum transverse masses restricts the rapidity η to be

$$\eta_- < \eta < \eta_+ \quad (2.79)$$

with

$$\eta_{\pm} = \ln \frac{\sqrt{\mathcal{A}} p_0^{\pm}}{\mu_{\pm \min}^2 + \mathcal{A}}; \quad \eta_- = \ln \frac{\mu_{- \min}^2 + \mathcal{A}}{\sqrt{\mathcal{A}} p_0^-}. \quad (2.80)$$

For $\eta_+ < \eta_-$ there is no solution, the string cannot be broken. Otherwise we determine the rapidity according to a constant distribution between η_- and η_+ :

$$\eta = \eta_- + r(\eta_+ - \eta_-) \quad (2.81)$$

with a random number $r \in [0, 1]$. From eq. (2.78) we see that the breakpoint is now fully determined.

This classical picture should be appropriate as long as the two substrings have a large mass. Whenever a small string mass occurs (say below 2 GeV) clearly quantum effects become important, most easily seen by the fact that a string with low mass is a hadron, and hadrons have discrete masses. Since we cannot deal with quantum strings properly, we try to correct for the quantum effect, in our classical model. First of all we introduce a cut-off parameter $m_{\text{clu}} = m_{\min} + m_0$ which prevents strings with mass below m_{clu} from splitting further via the string fragmentation procedure. Such clusters are treated differently as discussed in the following. We are going to treat “exotic” quark configurations (like $\bar{q}q\bar{q}q$) later; first we only consider clusters which are — concerning quark and antiquark content — hadrons. We circumvent for all but the lowest hadron states the problem of discrete mass: we allow the resonances to be off-mass-shell. For each quark configuration ($u\bar{d}, u\bar{s}, \dots$) we have a table of numbers $m_1 < m_2 < \dots$ where the interval $[m_i, m_{i+1}]$ specifies the mass range for a certain resonance. This feature of resonances with continuous mass simplifies the fragmentation procedure; there is no correction needed to the breaking procedure as described above. However stable hadrons ($\Gamma \ll 1 \text{ GeV}$) have to have discrete masses, so whenever a string breaks into two pieces with at least one of these being a stable hadron (its mass falling into the lowest mass interval) we apply a correction procedure: we construct a new breakpoint D . We recall that D is specified by two parameters: the area $\mathcal{A} = b_0^+ b_0^-$ and the rapidity $\eta = \frac{1}{2} \ln(b_0^+/b_0^-)$. Since we do not want to modify the area law eq. (2.73), we are going to modify η and leave $\mathcal{A}$ fixed, if one stable hadron is involved (two hadrons will be treated later). So when the right substring is a stable hadron, with a required mass m we determine η to be (see eq. (2.80)):

$$\eta = \ln \frac{\sqrt{\mathcal{A}} p_0^+}{m^2 + p_t^2 + \mathcal{A}}; \quad (2.82)$$

if the hadron is left, we use

$$\eta = \ln \frac{m^2 + p_t^2 + \mathcal{A}}{\sqrt{\mathcal{A}} p_0^-}. \quad (2.83)$$

This guarantees the correct mass. If both substrings are stable hadrons, with masses m_+ and m_- , we have to redetermine both parameters, $\mathcal{A}$ and η , by solving the two equations (using $\mu_{\pm} = \sqrt{m_{\pm}^2 + p_t^2}$ and eqs. (2.77,78)):

$$\begin{aligned} \mu_+^2 &= p_0^+ \sqrt{\mathcal{A}} e^{-\eta} - \mathcal{A} \\ \mu_-^2 &= p_0^- \sqrt{\mathcal{A}} e^{\eta} - \mathcal{A} \end{aligned} \quad (2.84)$$

which leads to

$$A = \frac{1}{2}(p_0^+ p_0^- - \mu_-^2 - \mu_+^2) - \sqrt{\frac{1}{4}(p_0^+ p_0^- - \mu_-^2 - \mu_+^2)^2 - \mu_-^2 \mu_+^2}. \quad (2.85)$$

The other parameter η is determined from eq. (2.84). Because the mass of the hadron D is in the vicinity of the old breakpoint, so it is really a correction in the sense of a small modification.

We want to stress that the break-up of a string into two substrings occurs completely arbitrarily in the sense that each of the substrings may be a stable hadron, a resonance or a high mass string. This is the major difference from the Lund model, where one fragment has to be a hadron with a discrete mass. So in our model we have a "tree structure": a string decays into two substrings, each substring may then decay into two subsubstrings and so on. The Lund model has a "salami structure": a hadron is chopped off at the end, and then another one from the remaining string and so on.

The last step of the fragmentation procedure is resonance decay: all the primary hadrons (from string break-up) decay (if they are unstable) according to standard branching ratios. The off-shellness of resonances poses no difficulties. One has only to consider that some of the partial decays cannot occur because the energy is not available; we simply discard such decay modes. For details of the decay procedure see Ref. [2.5].

Exotic clusters are treated in a procedure called "cluster decay", to be distinguished from ordinary "resonance decay" as well as from "string splitting". Consider a cluster C of n quarks and m antiquarks with $n - m$ being an integer multiple of 3

$$C = (q_1 q_2 \dots q_n \bar{q}_1 \bar{q}_2 \dots \bar{q}_m) \quad (2.86)$$

where $q_i, \bar{q}_i$ are quark flavours ($u, d, s, \dots$). The probabilities to choose randomly a baryon (B), an antibaryon ($\bar{B}$), or a meson (M) are

$$P(B) = \frac{1}{N} \binom{n}{3}; \dots P(\bar{B}) = \frac{1}{N} \binom{m}{3}; \quad P(M) = \frac{1}{N} nm \quad (2.87)$$

with

$$N = \binom{n}{3} + \binom{m}{3} + nm. \quad (2.88)$$

According to these probabilities we are randomly selecting a hadron H , and then performing the decay

$$C \rightarrow H + C' \quad (2.89)$$

With m_C being the cluster mass and m_H and $m_{C'}$ being the masses of the two decay products, we find the available centre-of-mass momentum to be

$$p_{cm} = \frac{1}{2m_C} \sqrt{(m_C^2 - m_H^2 - m_{C'}^2)^2 - (2m_H m_{C'})^2}. \quad (2.90)$$

The momenta of the two decay products in the cm system of the cluster C are then $\pm p_{cm} \hat{u}$, with a random unit vector $\hat{u} \in S_2$. The mass m_H is a discrete hadron mass, $m_{C'}$ is either a hadron mass or — if C' is still an exotic state — a mass chosen randomly in the possible mass range. In the latter case, the cluster decay procedure is repeated for C' :

$$C' \rightarrow H' + C'' \quad (2.91)$$

and so on, till we are left just with ordinary hadrons.

Before coming to applications, we want to state some general remarks about our fragmentation model. A major motivation for keeping resonances off-shell is the fact that this fragmentation model was mainly constructed to be used in a model for hadronic interactions (VENUS 3). In particular for such reactions, interactions of produced resonances of the type

$$R_1 + R_2 \rightarrow R \rightarrow R_3 + R_4 + \dots \quad (2.92)$$

may occur, i.e. the two resonances fuse into a highly excited resonance R before decaying again. Even if R_1 and R_2 were on-mass-shell, the fused object R will in general not be, so one has to deal with off-shell resonances anyhow. If only discrete masses are considered, one has to correct again and again, which makes the whole approach questionable. Interactions of the type (2.92) are included in the model, but are, however, completely negligible for all examples to be discussed in section 2.9, therefore we do not discuss rescattering here.

Also with view to applications for hadronic collisions we included "exotic clusters". Such objects are already needed for deep inelastic lepton-nucleon scattering: in the simplest case the vector boson couples to one of the valence quarks of the nucleon to produce a diquark-quark string (see fig. 2.7). However it also happens that the boson kicks off a sea quark, leaving back in the remaining nucleon the corresponding antiquark, and thus we get a $q\text{-}qqq\bar{q}$ string. After fragmentation we might be left with an exotic $qqqqq$ system. Considering rescattering of resonances, we have to deal with exotics as well: by fusing, for example, a uuu and a $\bar{u}\bar{u}\bar{u}$ we get the five quark state $uuu\bar{u}\bar{u}\bar{u}$. We refer to the fragmentation model introduced in this section as AMOR (Artru-Mennessier Off-shell-Resonance model)

2.7 Other fragmentation models

In this chapter we would like to comment on the relation of AMOR to other fragmentation models. AMOR, as well as some other models [2.4] based on the Artru-Mennessier model [2.3], takes the string picture seriously and provides a covariant, gauge invariant (= reparametrization invariant) energy and momentum conserving string breaking procedure. On the other extreme there is the Field-Feynman model [2.7]. Instead of strings here one considers two independent partons moving in opposite directions, inspired by the experimentally observed jet-structure of produced particles. The model is not covariant and of course not gauge invariant. And since the two jets are independent, energy and momentum are not conserved. The Lund model [2.8] is somewhat in between. It is similar to the Field-Feynman model in the sense that again two partons are considered, and one fragmentation step amounts to forming a hadron which contains one of the partons.

However the two partons are linked by a colour field, which makes it possible to achieve energy and momentum conservation as well as covariance.

Whereas AMOR allows a string to split into two substrings with arbitrary masses (string $\rightarrow$ string + string), the Lund model requires one of the substrings to be an on-shell hadron (string $\rightarrow$ string + hadron). In both approaches the iterative procedure terminates whenever the string masses are below some cut-off. Those strings are identified with stable hadrons or known resonances. In our model we do not force a resonance to have its mean mass. This concept has two major advantages: in the first place correction procedures usually necessary to force particles on their mass shell are reduced to a minimum, and secondly final-state interactions (e.g. resonance + resonance $\rightarrow$ resonance) in the case of multi-string fragmentation (heavy ion collisions) can be implemented quite naturally.

Unlike some other fragmentation models [2.4, 2.8, 2.9, 2.10, 2.11, 2.12], our model does not include perturbative parton showers, since we are not interested in the fragmentation involving very high mass partons created in hard scattering processes. We are mainly interested in strings formed in soft hadronic collisions which might be considered to be unexcited yo-yo strings, without any gluon kinks.

2.8 Comparison with data

We applied AMOR (also referred to as VENUS 3, since it is part of the VENUS 3 model for nuclear collisions) to calculate particle production in e^+e^- , νp and μp reactions (figs. 2.5-7). An e^+e^- event $\{e^+e^-\}$ is related to quark-antiquark string fragmentation as a simple superposition (see fig. 2.5):

$$\{e^+e^-\} = \sum_i \frac{1}{e_i^2} \sum_j e_j^2 \{q_i - \bar{q}_j\} \quad (2.93)$$

where the weights are determined from the quark charges. The e^+e^- energy determines which flavours have to be considered. In the following figures we use the convention: data are dots, model results are histograms. In fig. 2.8 we consider inclusive spectra for e^+e^- at 14 GeV: data and model agree almost perfectly for the distributions of transverse momentum p_T of charged particles, rapidity y of charged particles and the energy fraction x of photons. The transverse momentum shows essentially the exponential behaviour of the input distribution eq. (2.75), however the correct concave shape of the photon x -distribution is quite remarkable. Photon spectra are strongly affected by η resonance decay, so we may consider this agreement as some support of our low mass cluster treatment (concerning the production of flavour neutral resonances, see [2.5]). The deviation from an ideal plateau in the rapidity distribution in fig. 2.8, namely the shallow peak at large $|y|$, is due to the decay of heavy mesons: there are plenty of c - $\bar{c}$ strings, forming ηc and $\bar{\eta} c$ mesons on the outside of the string, therefore we get a contribution at large rapidities. In figs. 2.9,10 we show the same distributions, however for higher energies (22 and 34 GeV). The photon energy fraction distributions are energy independent (scaling), and data and model agree nicely. The transverse momentum distribution in the model is almost energy

Fig. 2.5. String formation in e^+e^- annihilation.

Fig. 2.6. String formation in νp scattering.

Fig. 2.7. String formation in μp scattering.

Fig. 2.8. Inclusive spectra for e^+e^- annihilation at 14 GeV: transverse momentum (upper plot) and rapidity distributions (middle plot) of charged particles, and energy fraction distributions of photons (lower plot). The data (dots) are from ref. [2.15].

Fig. 2.9. Same as fig. 2.8, but 22 GeV.

Fig. 2.10. Same as fig. 2.8, but 34 GeV.

independent, whereas the data show an increase of the p_t tails with energy. This effect is well known to be due to a perturbative parton cascade, not included in our model. By looking at the rapidity distributions we see that these missing high p_t particles are located at central rapidities. The tails of the rapidity distributions are always reproduced properly.

We are now turning to antineutrino proton scattering: $\bar{\nu} + p \rightarrow \mu^+ + \text{"string"}^*$, see fig. 2.6. Considering only the contributions with $\Delta s = 0$, we have two possibilities: The W^- boson couples to a u quark which transforms into a d quark, forming a d - u string together with the remaining diquark $p - u = ud$ (see fig. 2.6); the other case occurs when the W^- couples to a $\bar{d}$ quark from the sea, to be transformed into a $\bar{u}$ quark, forming correspondingly a $\bar{u}$ - udd string. For the experiments we are studying, the kinematic cuts are such that the latter contribution can be neglected, so $\bar{\nu}p$ scattering is simply d - u string fragmentation:

$$\{\bar{\nu}p\} = \{d-u\} \quad (2.94)$$

In fig. 2.11 we show distributions of the longitudinal momentum fraction x for $\bar{\nu}p$ scattering at 6.2 GeV (d - u string fragmentation with a string mass of 6.2 GeV). Momentum fraction refers to the maximum possible momentum of a produced particle in the string cm system, along the longitudinal axis (experimentally the string momentum can be reconstructed since in the reaction $\bar{\nu} + p \rightarrow \mu^+ + \text{"string"}$ the momenta of $\bar{\nu}$ and μ^+ are known). The data can be at least as equally well reproduced as with the former VENUS fragmentation routine (Field-Feynman procedure, see [2.13]), although now much fewer parameters enter. The distributions in fig. 2.11 are only affected by one parameter: the string breaking probability α_0 in eq. (2.73). Since on the diquark side (negative x) the first particle has to contain the diquark, being a baryon usually, the pions are at least second in the chain, and therefore most likely slower than the baryon. On the other side, where the quark fragments (positive x), a pion is usually first and fastest. For these reasons we get the forward-backward asymmetry for the π^- , seen in fig. 2.11. The dashed line contains final state interactions, not showing any effect.

We finally turn to muon proton scattering ($\mu + p \rightarrow \mu' + \text{"string"}^*$, see fig. 2.7). The intermediate boson couples to either a quark or an antiquark, forming a $q(p-q)$ or $\bar{q}(p-\bar{q})$ string. The probability of a certain quark (antiquark) flavour i being involved is [2.14]:

$$P_i = N \int dx_B dy \frac{2\pi\alpha^2}{(-q^2)^2} s x_B (1 + (1-y)^2) e_i^2 q_i(x_B) \quad (2.95)$$

where the usual variables are used:

$$x_B \equiv \frac{-q^2}{2pq}; \quad y \equiv \frac{pq}{pk} \quad (2.96)$$

with k, q and p being the four-momenta of the incoming muon, the photon and the proton; $q_i(x_B)$ is the parton momentum distribution function, e_i is the parton charge, and N is the normalization. The integration of eq. (2.95) has to be performed over the appropriate acceptance area in the x_B - y plane, in order to compare with a specific experiment. Since the quark flavour i in eq. (2.95) may refer to valence quarks, sea quarks or sea antiquarks,

Fig. 2.11. Longitudinal momentum distribution for π^+ (upper plot) and π^- (lower plot) for $\bar{\nu}p$ scattering at 6.2 GeV (string mass). The dashed curve is a calculation including string-string interactions (no effect). The data (dots) are from ref. [2.16].

string	P [%]
$u-ud$	67.66
$d-uu$	8.46
$u-\bar{u}ud$	8.80
$d-\bar{d}ud$	2.20
$s-\bar{s}ud$	0.94
$\bar{u}-uud$	8.80
$\bar{d}-duu$	2.20
$\bar{s}-sud$	0.94

Table 2.1: The weights of individual string contributions for the μp reactions discussed in this section.

we get contributions from $q-q\bar{q}$, $q-q\bar{q}q$ and $\bar{q}-q\bar{q}q$ strings. The weights P used for the following figures, are given in table 2.1. For qualitative arguments it is important to notice that quark-diquark ($q-q\bar{q}$) strings are dominant, and among those $u-ud$ strings are most likely. We first consider μp scattering at 11.4 GeV. In fig. 2.12 we show longitudinal momentum distributions of pions and kaons. Again we see the strong forward-backward asymmetry for π^+ and K^+ , since for $u-ud$ strings only at the forward end of the string, π^+ and K^+ can be produced at the end. The pion distributions are steeper than the kaon distributions which is obvious after writing eq. (2.72) for leading particles as

$$dP(x, m^2) = \frac{\alpha_0}{\Delta\eta} \exp \left[-\alpha_0 m^2 \frac{1-x}{x} \right] \frac{dx}{x} dm^2 \quad (2.97)$$

where x is the momentum fraction and m the mass of the particle. We see from eq. (2.97) that heavier particles have flatter distributions. In fig. 2.13 momentum fraction distributions of baryons are shown: protons, antiprotons, lambdas and antilambdas. We again observe forward-backward asymmetries, which may be summarized as follows: the more the quark content of a produced hadron and a string end differ, the faster the distribution approaches zero for $|x| \rightarrow 1$. The reason is that for large differences in the quark content, many other particles in the chain of produced hadrons are closer to the string end. Since the diquark on the minus side differs by just one quark from a proton or a lambda, these distributions are rather flat for $x < 0$. The quark on the other side differs by two quarks from p and Λ , so the distributions fall faster towards $x \rightarrow 1$. The difference between the forward quark and $\bar{p}$ or $\bar{\Lambda}$ is 4 quarks, so the distributions fall faster than for p and Λ for $x \rightarrow 1$. The largest difference in quark content is, however, observed for the diquark side ($x < 0$) and for $\bar{p}, \bar{\Lambda}$ production: the difference of 5 quarks results in a very fast drop for $x \rightarrow -1$. In the former VENUS fragmentation (Field-Feynman) these quark counting arguments were used as an input, by using different splitting functions for different particle species, depending on the difference in quark content. Now we get this behaviour for free,

Fig. 2.12. Longitudinal momentum distributions of pions and kaons for μp scattering at 11.4 GeV (string energy). The data (dots) are from ref. [2.17].

Fig. 2.13. Longitudinal momentum distributions of protons, antiprotons, lambdas and antilambdas for μp scattering at 11.4 GeV (string energy). The data (dots) are from ref. [2.17].

Fig. 2.14. Rapidity distributions of π^- for μp scattering at 7.0 GeV, 12.1 GeV, 15.7 GeV and 18.7 GeV (string energy). The data (dots) are from ref. [2.18].

Fig. 2.15. Rapidity distributions of π^+ for μp scattering at 7.0 GeV, 12.1 GeV, 15.7 GeV and 18.7 GeV (string energy). The data (dots) are from ref. [2.18].

just from the relation between the quark content and the number of steps from an endpoint to produce a particle. To avoid confusion: the fragmentation procedure in VENUS 3 breaks the string at arbitrary points, not successively from an endpoint. Nevertheless we can analyse results by considering a chain of produced particles from one string end to the other.

An alternative variable, stretching the central region and compressing the fragmentation region, is the rapidity

$$y = \frac{1}{2} \ln \frac{E + p_L}{E - p_L} \quad (2.98)$$

where E is the energy and p_L the longitudinal momentum of a produced particle. In figs. 2.14,15 we display rapidity distributions of π^+ and π^- in μp reactions at string energies of 7 GeV, 12.1 GeV, 15.7 GeV and 18.7 GeV. Whereas the height of the distributions ($y(0)$) remains almost invariant with energy, the width increases clearly. So we observe a rapidity plateau, expected from Lorentz invariance. There is more π^+ than π^- production, since there are more u than d quarks in the string.

2.9 References

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CHAPTER 3

STRING FRAGMENTATION IN A MEDIUM

In this chapter we introduce a model for a string fragmenting in a "medium". The medium consists of hadronic objects, like fragments from other strings or simply nucleons. We describe the model quite generally, although in this chapter we only apply it for a nuclear medium, to investigate lepton-nucleus scattering. The model is an ingredient of VENUS 3 — a string model for hadronic collisions — and is therefore sometimes referred to as VENUS 3 as well.

Let us briefly discuss the main ideas behind the model. At the formation point a string is very short, most of the string energy being concentrated at the endpoints. Given this initial condition the string will — according to the classical evolution equations — become eventually very long. However this does not really happen, since the string is going to break after a typical time of $1 \text{ fm}/c$. This means that the length of a string segment will in general not exceed 1 fm , which is of the order of a nucleon size. Therefore concerning reinteraction of string segments among each other and with spectator nucleons, we consider all these objects as pointlike. Thus we have just trajectories of string segments (rather than surfaces) and define to have an interaction whenever two trajectories come closer than a certain distance r_0 . How do we realize an interaction? Whereas "String Flip" is the dominant mechanism at high energies, low energy scattering occurs predominantly via fusion. We therefore assume that reinteraction of segments occurs exclusively via fusion, the fused object decaying again after the appropriate time (if it does not interact before).

As mentioned already the model accounts for interactions among string segments and also for interactions of such segments with spectator nucleons. Whereas in hadron-nucleus and nucleus-nucleus collisions both aspects of the model are equally important, in lepton-nucleus scattering we essentially only have to deal with segment-spectator interactions (see discussion below). Therefore we restrict ourselves in this chapter to lepton-nucleus scattering as a first test of the medium fragmentation model, before proceeding towards hadronic interactions.

According to the parton model, a high energy lepton interacts with a nucleon as follows: via boson exchange a considerable amount of momentum is transferred to one of the quarks of the nucleon. The latter one is henceforth transformed into an object consisting of a quark and a diquark with a large relative momentum: a string (see fig. 3.1). Therefore lepton-nucleus scattering allows us to study the fragmentation of diquark-quark ($qq-q$) strings in vacuum. For lepton-nucleus scattering the initial part of the interaction is as discussed above: due to the very small cross section the lepton

Fig. 3.1. Lepton-nucleon scattering in the Parton Model: the intermediate boson B transfers momentum to a quark, thus forming a quark-diquark string.

Fig. 3.2. In lepton-nucleus scattering a string is formed inside the nucleus. The kicked-off quark moves forward, the diquark is close (in momentum) to the spectator nucleons.

will interact with just one nucleon, by transferring momentum to a quark, resulting in a quark-diquark string (see fig. 3.2). But since the string has been formed somewhere inside the nucleus, the space-time evolution of the string will be different from the vacuum case. The whole string or later string fragments may interact with spectator nucleons. So lepton-nucleus scattering provides the unique opportunity to study space-time evolution of strings inside a nuclear medium.

In section 3.1 we introduce the model for the fragmentation of strings in a medium (interacting strings), in section 3.2 we discuss the parton model of neutrino-nucleus scattering, and in chapter 3.3 we apply the model to calculate particle production in neutrino-neon scattering, and compare with data.

3.1 Model for string fragmentation in a medium

In this section we introduce a model for the fragmentation of a string in a medium of pointlike hadronic objects. These objects may be nucleons in a nucleus or string segments from the fragmentation of other strings. We first fragment a string S_0 according to the vacuum fragmentation procedure described earlier, see fig. 3.3. The polygons $O_0 R_0 U_0$ and $O_0 L_0 U_0$ are a half period of directrix and antirectrix ("elementary cell"). The breakpoint B_0 defines the elementary cells $O_1 R_0 U_1 B_0$ and $O_2 B_0 U_2 L_0$ for the two substrings S_1 and S_2 . Any further breaks have to occur inside these cells since the absolute past of B_0 (inside of $O_0 O_1 B_0 O_2$) is excluded from further breaks, otherwise B_0 would not be possible! The absolute future (inside of $B_0 U_1 U_0 U_2$) is of course also excluded. In the next step, the substrings S_1 and S_2 take the role of the original one. The breakpoints B_1 and B_2 (see fig. 3.3(b)) determine the string decays $S_1 \rightarrow S_{11} + S_{12}$ and $S_2 \rightarrow S_{21} + S_{22}$. The procedure is continued till all substrings S_α have a mass smaller than some cut-off.

From the example in fig. 3.3 it becomes clear that the sequence of breakpoints is in general not time-ordered in a certain Lorentz frame: the second-generation breakpoint B_1 happens to be earlier than the first-generation break at B_0 . This is a consequence of our covariant fragmentation scheme. At this point we want to give up covariance and choose an appropriate Lorentz frame. In this frame we determine the real time sequence of string breakups:

$$(B_{\alpha_1})_0 < (B_{\alpha_2})_0 < (B_{\alpha_3})_0 < \dots, \quad (3.1)$$

where the index zero ($_0$) indicates the time component. In fig. 3.3 we have for example $(B_1)_0 < (B_0)_0 < (B_2)_0$. Keeping the sequence eq. (3.1), we check at these times $(B_{\alpha_i})_0$ whether a break between two consecutive string segments occurred later; if so, the break is considered (at this time) non-existent and the two string pieces are joined again. In this way we obtain a time-ordered decay, the first break occurring at $t = (B_{\alpha_1})_0$:

$$S_0 \rightarrow \tilde{S}_1 + \tilde{S}_2, \quad (3.2)$$

the next one at $t = (B_{\alpha_2})_0$, depending whether $\tilde{S}_1$ or $\tilde{S}_2$ decays next as

$$\tilde{S}_1 \rightarrow \tilde{S}_{11} + \tilde{S}_{12} \quad \text{or} \quad \tilde{S}_2 \rightarrow \tilde{S}_{21} + \tilde{S}_{22} \quad (3.3)$$

Fig. 3.3. Consecutive string breaking: the first break occurs at B_0 (a), the substrings then break at B_1 and B_2 (b). In this example the second break at B_1 occurs in our reference system earlier than the first break at B_0 . In (c) we reconstruct time ordered decays.

and so on. In the example of fig. 3.3 we have at $t = (B_1)_0$

$$\tilde{S}_0 \rightarrow \tilde{S}_1 + \tilde{S}_2 \equiv S_{11} + \{S_{12} + S_{21} + S_{22}\} \quad (3.4)$$

then at $t = (B_0)_0$

$$\tilde{S}_2 \equiv \{S_{12} + S_{21} + S_{22}\} \rightarrow \tilde{S}_{21} + \tilde{S}_{22} \equiv S_{12} + \{S_{21} + S_{22}\} \quad (3.5)$$

and finally at $t = (B_2)_0$

$$\tilde{S}_{22} \equiv \{S_{21} + S_{22}\} \rightarrow \tilde{S}_{221} + \tilde{S}_{222} \equiv S_{21} + S_{22}. \quad (3.6)$$

We construct geometrically, as shown in fig. 3.3(c), the "origins" $\tilde{O}_\alpha$ for the string segments $\tilde{S}_\alpha$. Since we keep track of all parton momenta, we can easily determine the momenta $\tilde{p}_\alpha$ of the string segments. We define a track of a segment to be a straight line, going through the origin, with the direction given by the velocity of the segment:

$$(x_\alpha(t))_i = (\tilde{O}_\alpha)_i + \frac{(\tilde{p}_\alpha)_i}{(\tilde{p}_\alpha)_0} [t - (\tilde{O}_\alpha)_0] \quad (3.7)$$

with $i = 1, 2, 3$ being the space components, $i = 0$ being the time component. This track corresponds approximately to the diagonal $\tilde{O}_\alpha \tilde{U}_\alpha$ of the elementary cell, only approximately because of the transverse momenta involved, which are however small. We have to consider of course that the string segment $\tilde{S}_\alpha$ lives on this line only for a time interval $t_\alpha^1 < t < t_\alpha^2$, limited by the breaktime t_α^1 where this segment occurred first, and the breaktime t_α^2 where $\tilde{S}_\alpha$ further decays. The actual track

$$\{(x_\alpha(t))_i; t_\alpha^1 < t < t_\alpha^2\} \quad (3.8)$$

in general does not contain the origin $\tilde{O}_\alpha$.

So we have the following situation: between the time $(O_0)_0$ where the string was formed and the time $(B_\alpha)_0$ of the first break, we have just one track, the track $\tilde{x}_0(t)$ of the whole string. After the first break, till the next one at $(B_\alpha)_0$, we have two tracks corresponding to the two substrings $\tilde{S}_1$ and $\tilde{S}_2$, and so on. The final string segments have no upper limits (so far, till we consider resonance decay); they follow their track till $t = \infty$.

As the next step we have to consider resonance decay (a detailed description is given in [3.1], see also section 2.6), which also limits the lifetime of final string segments. Knowing the mass and quark content of a segment, we identify it with a resonance according to our resonance table. Also from the table we obtain the lifetime τ_R of the resonance R , which therefore lives on the straight line trajectory between the formation time t_R and $t_R + \gamma\tau_R$, with the Lorentz dilation factor γ . At the latter time we decay the resonance according to a standard decay table (see [3.1]), thus at that point the resonance track splits into two (or more) daughter tracks and so on.

We have thus a complete space-time evolution of the string and its decay products from the string formation time till infinity. This is an improvement on earlier space-time

studies [3.2], where we only considered resonances, but not, however, their "parent" string segments. So in a way particles were created at some stage from "nothing", whereas now we keep track of energy and momentum from the beginning till the end.

All this exercise of constructing time-ordered string decays is only necessary if we consider interactions at an early stage, before the last breakup. For interactions of segments it is obviously important which event happens earlier than some other event (we mean events of two segments coming close to each other and interacting). We are now considering how string fragmentation as described above is modified in a medium of pointlike hadronic objects H_β , the latter ones moving on tracks $\tilde{x}_\beta(t)$. Whenever at some time t such a track comes close to the track $\tilde{x}_\alpha(t)$ of a string segment $\tilde{S}_\alpha$, i.e.

$$|\tilde{x}_\alpha(t) - \tilde{x}_\beta(t)| \leq r_0, \quad (3.9)$$

then these two objects interact. The two are most likely close in momentum space (for the cases where we want to apply this model: lepton-nucleon or nucleus-nucleus scattering), so for simplicity we assume all interactions to result in fusion of the two objects:

$$\tilde{S}_\alpha + H_\beta \rightarrow R \quad (3.10)$$

where the quark content and momenta of R are just the sums of these quantities of the two ingredients. According to its mass and quark content this object R is identified with the help of a resonance table, its lifetime τ_R is determined, and after the time interval $\gamma\tau_R$ this object R decays again into resonances or hadrons:

$$R \rightarrow R_1 + R_2 + \dots \quad (3.11)$$

In fact we perform this procedure of checking the distance and fusing (if necessary) for all string segments, decay products, "medium" particles, results of fusions (eq. (3.10)), or decay products of such fused objects (eq. (3.11)), so we include all particles present at a certain time. Of course if a segment $\tilde{S}_\alpha$ is involved in an interaction eq. (3.10) at a time t , all the decay products of $\tilde{S}_\alpha$, which would otherwise occur later, are non-existent! Technically speaking, all the breaks of the vacuum string are first done, even if some of them later turn out to be irrelevant, due to a fusion process eq. (3.10) of some parent string segment.

Theoretically there is the possibility that string segments from one single string reinteract and fuse; however, this happens so rarely that this process is irrelevant. The dominant reinteraction process for string fragmentation in a nucleus (lepton-nucleus scattering) is the following: low mass hadrons from string decay, which are close in momentum space to the spectator nucleons, interact with these nucleons, fuse into moderately excited resonances, and decay again into two or three hadrons, the knocked-off baryon being slow in the nuclear rest frame. In this way the multiplicity is not drastically increased, apart from the additional baryons, which without interactions would remain spectators.

3.2 Parton model of neutrino-nucleus scattering

This section provides the link between string fragmentation in a nucleus — for which we constructed a model — and lepton-nucleus scattering — where data exist. Let us first consider lepton-nucleon scattering (see fig. 3.1). We use the following variables: $k \equiv (k_0, \vec{k})$ and $k' \equiv (k'_0, \vec{k}')$ are the momenta of the incoming and the outgoing leptons, $q \equiv k - k'$ is the momentum of the exchanged boson, we use as usual $Q^2 \equiv -q^2$ for the virtual mass of the boson, θ and ϕ are the polar and azimuthal angles of the scattered lepton relative to the beam axis in the nucleon rest frame and W^2 is the square of the “string mass”, i.e. the mass of “boson + nucleon”. For the experiment which we are going to consider [3.3] we know average values for k_0 , Q^2 , and W^2 . From momentum conservation at the boson-nucleon vertex we have (with m_N being the nucleon mass)

$$-Q^2 + m_N^2 + 2m_N(k_0 - k'_0) = W^2 \quad (3.12)$$

from which we obtain the energy k'_0 of the scattered lepton. In the nucleon rest frame we have

$$q = \begin{pmatrix} k_0 - k'_0 \\ -k'_0 \sin \theta \sin \phi \\ -k'_0 \sin \theta \cos \phi \\ k_0 - k'_0 \cos \theta \end{pmatrix} \quad (3.13)$$

with θ to be obtained from

$$Q^2 = 2k_0 k'_0 (1 - \cos \theta). \quad (3.14)$$

The string momentum p is the sum of the boson and nucleon momentum, so in the nucleon rest system we have

$$p = q + (m_N, 0, 0, 0). \quad (3.15)$$

We use the string cm system as the reference system where the fragmentation procedure as described in the last chapter is being performed. We boost the target nucleus into this system via

$$\begin{aligned} x'_0 &= \beta_0 x_0 + \sum_{i=1}^3 \beta_i x_i \\ x'_j &= \beta_j x_0 + x_j + \frac{\beta_j}{1 + \beta_0} \sum_{i=1}^3 \beta_i x_i \end{aligned} \quad (3.16)$$

with

$$\beta_\mu = \frac{p_\mu}{\sqrt{p^2}}, \quad \mu = 0, 1, 2, 3 \quad (3.17)$$

being the string velocity.

To provide some numbers: for the neutrino-neon experiment with which we want to compare [3.3], we have an incident neutrino energy of 43 GeV, a momentum transfer Q^2 of 5.4 GeV² and a string mass squared W^2 of 27 GeV². From eq. (3.12) we obtain

current	p-string	P [%]	n-string	P [%]
$d \rightarrow u$	$u-uu$	27.37	$u-ud$	58.21
$d \rightarrow c$	$c-uc$	1.44	$c-ud$	3.06
$s \rightarrow u$	$u-\bar{s}ud$	0.13	$u-\bar{s}udd$	0.15
$s \rightarrow c$	$c-\bar{s}ud$	2.43	$c-\bar{s}udd$	2.88
$\bar{u} \rightarrow \bar{d}$	$\bar{d}-uud$	1.88	$\bar{d}-uudd$	2.23
$\bar{u} \rightarrow \bar{s}$	$\bar{s}-uud$	0.10	$\bar{s}-uudd$	0.12

Table 3.1: The weights of individual string contributions for the ν -Ne reactions discussed in this section. $p(n)$ -string refers to a proton(neutron) being the target.

an energy transfer $q_0 = k_0 - k'_0$ of 16.76 GeV and the scattering angle from eq. (3.14) turns out to be 3.96°. So the W-boson moves with almost half of the incident neutrino energy almost in beam (=forward) direction. Since this momentum is transferred to one quark of the hit nucleon (see fig. 3.1), this quark consequently moves with a large momentum forward (see fig. 3.2). After the Lorentz boost eq. (3.16), in the string frame, the nucleons move almost along the string axis in backward direction (in the diquark direction). We expect therefore rescattering effects in the backward region (diquark fragmentation region). This will be investigated in the next chapter.

Finally we have to specify the flavour content of strings in charged current neutrino-nucleus scattering. Using the variables

$$x_B \equiv \frac{Q^2}{2m_N(k_0 - k'_0)}, \quad y \equiv \frac{k_0 - k'_0}{k_0}, \quad (3.18)$$

the probability $P(q_\alpha \rightarrow q_\beta)$ for a certain current $\langle q_\beta | J | q_\alpha \rangle$ is, if q_α and q_β are quarks, given as

$$P(q_\alpha \rightarrow q_\beta) = N c(q_\alpha \rightarrow q_\beta) \int dx_B dy \frac{G^2}{\pi} s_{xB} q_\alpha(x_B), \quad (3.19)$$

for antiquarks q_α and q_β as

$$P(q_\alpha \rightarrow q_\beta) = N c(q_\alpha \rightarrow q_\beta) \int dx_B dy \frac{G^2}{\pi} s_{xB} (1-y)^2 q_\alpha(x_B), \quad (3.20)$$

where N is a normalization constant, $q_\alpha(x)$ are the (anti)quark momentum distribution functions, and $c(q_\alpha \rightarrow q_\beta)$ represents Cabibbo mixing, the non-zero coefficients being

$$c(q_\alpha \rightarrow q_\beta) = \begin{cases} \cos^2 \theta_C & \text{for } d \rightarrow u, \bar{u} \rightarrow \bar{d} \\ \sin^2 \theta_C & \text{for } d \rightarrow c, \bar{c} \rightarrow \bar{d} \\ \sin^2 \theta_C & \text{for } s \rightarrow u, \bar{u} \rightarrow \bar{s} \\ \cos^2 \theta_C & \text{for } s \rightarrow c, \bar{c} \rightarrow \bar{s} \end{cases} \quad (3.21)$$

with the Cabibbo angle θ_C . To each current ($q_\alpha \rightarrow q_\beta$) there corresponds a string with q_β and ($N_0 - q_\alpha$) at the ends, where N_0 is the nucleon involved. So for example a current $d \rightarrow u$ in neutrino-proton scattering provides a u - u string. Since there are no $\bar{c}$ quark in a nucleon, only six of the eight possibilities of eq. (3.21) are realized. The numerical values for the corresponding weights, calculated according to eqs. (3.19,20,21), taking equal probability for neutron and proton targets (symmetric nuclei), are given in table 3.1. For qualitative arguments later we note that we have more than 50 % u - d strings, and more than 25 % u - u strings; everything else has small weight.

3.3 Comparison with data

In this section we apply our model for medium-string-fragmentation to calculate inclusive particle spectra for neutrino-neon scattering, and compare with data. Since our fragmentation model is part of the more general string model VENUS 3 for hadronic scattering, we refer to all calculations as VENUS 3. We generate strings with mass 4.5 GeV according to table 3.1, to compare with the data of [3.3]. Rescattering is crucially affected by the interaction radius r_0 in eq. (3.9). In the following we are comparing two scenarios (using $r_i = 0.7$ fm for baryons and $r_i = 0.35$ fm for mesons, $i=1,2$):

$$\begin{aligned} r_0 &= 0 & \longleftrightarrow & \text{rescattering turned off} \\ r_0 &= r_1 + r_2 & \longleftrightarrow & \text{rescattering} \end{aligned}$$

where our default values may not be the final answer; it is really a best guess rather than an optimized parameter. In the following figures we shall use dashed lines if using $r_0 = 0$ and solid lines if using $r_0 = r_1 + r_2$, points represent data.

We first consider distributions of the longitudinal momentum fraction $x = p_l/p_l^{\max}$, where p_l is the longitudinal momentum of a produced particle in the string frame and $p_l^{\max}$ is the maximum possible momentum. Positive x is defined to be the direction of the (anti)quark kicked off by the intermediate boson, negative x is the direction of the remainder, which is most likely a diquark (see table 3.1). As discussed in the last chapter, the target spectator nucleons reside (in momentum) close to the "diquark", therefore we expect effects at $x < 0$.

In fig. 3.4 we display x -distributions of pions with (solid) and without (dashed) final state interactions. Let us first discuss the π^- distribution. For $x > 0$ we see almost no difference between the solid and dashed histogram, there is no rescattering effect — as we expected — since $x > 0$ corresponds to the forward side of the string, where the fragmentation occurs outside the nucleus. However, since the diquark end of the string fragments well inside the nucleus, we see a huge effect for $x < 0$: in the tails the two histograms differ by one order of magnitude. So there are considerably more π^- produced in the diquark fragmentation region, if rescattering is considered. The overall multiplicity however changes only slightly, since this quantity is dominated by the production in the central region, around $x = 0$ (note the logarithmic scale in fig. 3.4). The VENUS results with interaction reproduce the π^- data over the full x range.

Fig. 3.4. Longitudinal momentum distributions of π^- (upper plot) π^+ (middle) and K_s (lower) in ν -Ne scattering. The points are data, the solid histograms are VENUS results with final state interactions, for the dashed ones interactions are turned off.

Fig. 3.5. Rapidity distributions of π^- (upper plot) π^+ (middle) and K^0 (lower) in ν -Ne scattering. The points are data, the solid histograms are VENUS results with final state interactions, for the dashed ones interactions are turned off.

47

Concerning the π^+ distributions the situation is quite similar: taking into account final state interactions considerably improves the agreement between calculation and data. However there still seem to be considerably more particles in the data. Since this excess is also observed for $x \geq 0$, it cannot be due to a wrong interaction model. Because we can reproduce lepton-nucleon data from other experiments which seems to support our vacuum fragmentation model, we are tempted to blame particle misidentification for the difference.

In the lower plot of fig. 3.4 we investigate kaon production. Also for kaons (K_s) we observe something similar as for pions: rescattering has no effect in forward direction, but increases particle production in the backward tails. However the latter effect is smaller than for pions.

In fig. 3.5 we show rapidity distributions corresponding to the x -distributions discussed above. We observe the same features as discussed before: a satisfactory improvement in the backward hemisphere by considering final state interactions, a smaller effect for kaons than for pions, an excess of π^+ in the data.

3.4 References

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particular concerning the "language" usually used to introduce them. Therefore we try in this chapter to find a common language — the string language introduced in chapter 2 — to learn about real differences between the models.

We were discussing classical string dynamics in chapter 2 in great detail. A disadvantage of such a complete description is that important statements may be overseen by the reader, so we are going to repeat (without reference) the crucial results, to be relevant for the forthcoming discussion.

(1) A string is completely characterized by the trajectory of one endpoint (directrix). This trajectory is periodic, so even one period of the directrix defines the string for ever. Correspondingly the string is completely given by less than a period of the directrix, however supplemented by the complementary part of the antidirectrix (trajectory of the other endpoint). The simplest possible (1-dimensional) so called yo-yo string is correspondingly completely defined by an "elementary rectangle" $OACB$ (in space-time), see fig. 4.1, with OAC and OBC representing a half period of directrix and antidirectrix respectively.

(2) The trajectories in fig. 4.1 are parallel to the light-cone directions, so it is convenient to use light-cone coordinates $x^\pm = t \pm x_3$ and momenta $p^\pm = E \pm p_3$. The endpoints moving along OA and OB can be identified with partons (massless relativistic point particles), starting at O with momenta p^+ and p^- respectively. These momenta are related to the width and length of the rectangle in fig. 4.1 via

$$\|OA\| = x^+ = \frac{p^+}{\kappa}; \quad \|OB\| = x^- = \frac{p^-}{\kappa}. \quad (4.1)$$

So we have a mapping from momentum to real space and vice versa via $\Delta p = \kappa \Delta x$, and therefore the elementary rectangle $OACB$ in fig 4.1 has a meaning both in space-time and in momentum space (often we do not have to specify what space we are working in).

(3) The area S of the rectangle is proportional to the string mass:

$$S = \frac{p^+ p^-}{\kappa^2} = \frac{E^2 - p_3^2}{\kappa^2} = \frac{m^2}{\kappa^2} \quad (4.2)$$

This relation also implies that for a "fast" string with $p^+ \gg m$ we obtain a very small p^- i.e. $p^- \ll m$, so the rectangle looks almost linear, as demonstrated in fig. 4.2a (we consider a fast forward-moving string here). In the limit of $p^+ \rightarrow \infty$ (or equivalently $m \rightarrow 0$), the rectangle degenerates into a line, being thus identical to the trajectory of a point particle (see fig. 4.2b).

(4) A break of a yo-yo string into two substrings is completely determined by specifying a breakpoint D inside the rectangle $OACB$ (see fig. 4.3). The

CHAPTER 4

SOFT HADRON-HADRON INTERACTIONS

Soft interaction refers to little transfer of transverse momentum (few hundred MeV). Whereas hard processes (large momentum transfer) can be treated in perturbation theory, there is presently no method to calculate soft processes, which requires models. We consider in this chapter models which assume that a hadron-hadron collision at ultra-relativistic energies results in two (or more) strings which are (approximately) aligned along the beam axis. The string evolution and fragmentation is then performed as described in the last chapters. We introduce and compare in section 4.1 three different methods to form strings in hadron-hadron collisions. In section 4.2 we discuss the string model VENUS which employs one of these methods ("Colour Exchange"), where strings are formed as a result of rearrangement of colour singlet structures due to the interaction. The VENUS model has been tested against many experimental data concerning very different observables. Due to time and space limitations we only consider one application here: we explain, why a string model like VENUS obeys "KNO scaling" which is an experimentally verified scaling law for multiplicity distributions. We show that KNO scaling occurs due to a scaling law for string mass distributions, the latter scaling law being a consequence of the parton structure of the nucleon.

4.1 String formation procedures

It is generally assumed that a high energy interaction of two hadrons results in (at least) two strings which are oriented along the beam axis. Only a small transverse momentum component is allowed, of order of r_N^{-1} with r_N being the size of the nucleon. We restrict ourselves first to the case of exactly two strings. To be more precise: by string formation we mean the production of strings with large mass (being much larger than the mass of the original hadrons). In this sense we would also talk about string formation in a model where the incident hadrons are already considered as strings — low mass strings however.

In this section we want to introduce and compare three different models for string formation: (a) "String Rearrangement" [2.3, 4.1, 4.2] which seems to be the natural method to form massive strings in a model where all objects — in particular the incident hadrons — are considered as ideal classical strings; (b) "String Flip" or "Colour Exchange" which is the method used in the VENUS model [4.3], to be described in detail later; (c) "Longitudinal Excitation" which is the method adopted by the Lund group [4.4]. These models differ considerably concerning the ideas behind them and in

Fig. 4.1. An "elementary rectangle" characterizing a yo-yo string. OAC and OBG are the trajectories of the endpoints, and the string stretches in between. These trajectories may be periodically continued into future or past.

Fig. 4.2. A yo-yo string with small mass (a) and zero mass (b).

Fig. 4.3. String breaking at D . The hatched areas S_1 and S_2 represent the two substrings, the areas F and P are absolute future and absolute past of the breakpoint D .

two "subrectangles" for the two substrings are constructed as shown in fig. 4.3. So the original rectangle S is divided into four rectangles: S_1 and S_2 representing the two substrings, the absolute past P and the absolute future F . The relative size of these areas measures the relation of potential (=mass) and kinetic energy production. If for example a break occurs early, close to O , the areas S_1 and S_2 are small, F however is large. This means that the two strings were formed with a small mass but with a large relative kinetic energy.

We are now in a position to introduce the different string formation procedures and discuss differences and similarities between them.

"String Rearrangement"

We already discussed string breaking, i.e. the division of a string S into two substrings S_1 and S_2 :

$$S \rightarrow S_1 + S_2 \quad (4.1)$$

which provides the mechanism of particle production from a massive string. The inverse process, namely fusion of two strings S_1 and S_2 into one

$$S_1 + S_2 \rightarrow S \quad (4.2)$$

provides a possible interaction mechanism of two strings. Fusion requires however that the two strings meet exactly at their endpoints which from a purely geometrical point of view is not very likely. More frequent is the case that the two strings meet at interior points, as indicated in fig 4.4. A possible local interaction would be a combination of breaking and fusion: each of the two strings S_1 and S_2 breaks at the interaction point into two substrings, i.e. S_{11} , S_{12} and S_{21} , S_{22} (see fig 4.4). Then two substrings from different original strings fuse together, say S_{11} and S_{22} and correspondingly S_{12} and S_{21} . The whole process (breaking and subsequent fusion)

$$S_1 + S_2 \rightarrow S_{11} + S_{12} + S_{21} + S_{22} \rightarrow S'_{11} + S'_{22} \quad (4.3)$$

with

$$S'_{11} = S_{11} + S_{22}; \quad S'_{22} = S_{12} + S_{21} \quad (4.4)$$

is called "Rearrangement" (see [2.3, 4.1, 4.2]) and sketched in fig 4.4.

From now on we restrict ourselves to yo-yo strings, which are fully defined by one rectangle in t - x_3 space (t : time, x_3 : longitudinal space coordinate). We assume the two incident strings to be aligned, otherwise we would no longer have yo-yo's after the "Rearrangement" process. We furthermore assume the two strings to have small masses and large momenta (in opposite directions in the string-string centre-of-mass system. The situation is then as shown in fig. 4.5a: the forward string is represented by a rectangle S_2 elongated along the x_+ axis and the rectangle S_1 representing the backward moving string is elongated along the x_- axis. The first stage of the interaction — string breakup — is indicated by the hatched areas S_{11} and S_{12} representing the breakup of

Fig. 4.4. "Rearrangement" interaction: two strings approach each other (a), touch at an internal point with the effect of cutting each string into two substrings (b), and fusing the pieces from different original strings together (c).

Fig. 4.5. "Rearrangement" interaction for yo-yo strings: breaking (cutting) procedure (a) and fusion (b).

Fig. 4.6. "Rearrangement" interaction for massless yo-yo strings: breaking (cutting) procedure (a) and fusion (b). Compare with fig. 4.5 for the case of finite masses.

S_1 and correspondingly S_{21} and S_{22} representing the breakup of S_2 . The space-time diagram of fusion is just the same as for breaking, so we construct from the rectangles S_{11} and S_{22} a greater one S'_1 as shown in fig. 4.5b. In the same way we construct S'_2 from S_{12} and S_{21} . Since we fuse the strings at their turning points, the fused strings are simple yo-yo's again, which is not guaranteed for fusion of two yo-yo strings, even if they are aligned.

An important feature of the above "Rearrangement" process can be seen from fig. 4.5: although we started with two strings (S_1 and S_2) with small masses, the resulting strings (S'_1 and S'_2) may have a very large mass. The areas representing S'_1 and S'_2 are in particular large when the original strings break up somewhere in the middle (concerning the elongated axis); this is realized in fig. 4.5.

Let us consider the limit of massless incident strings. As seen from fig. 4.6 the rectangles degenerate to lines, and the string breaking procedure amounts to dividing each line into two segments. We may now characterize the substrings (segments) simply by their lengths relative to the length of the corresponding original string, so we introduce fractions x_1 and $1 - x_1$ for S_{11} and S_{12} and fractions x_2 and $1 - x_2$ for S_{21} and S_{22} . For simplicity of notation we work in the S_1 - S_2 centre-of-mass system, so the lengths corresponding to these two strings are the same, and they may be rescaled to 1, so we only have to care about fractions for the following discussion. The second stage of the interaction — fusion — amounts now simply to constructing a rectangle of lengths $1 - x_1$ and x_2 for S'_1 and correspondingly a rectangle of lengths $1 - x_2$ and x_1 for S'_2 , as shown in fig. 4.6. In this figure we see that even having incident strings with zero mass produces in general strings with large mass (provided of course the momenta of the incident strings were large). So "Rearrangement" provides the possibility to transform relative kinetic energy into string mass ("potential energy" in the sense that this energy might be used for particle production).

"String Flip" or "Colour Exchange"

The interaction principle to be discussed in the following was introduced in connection with the Dual Parton Model [4.5]. For pedagogical reasons we use a somewhat different approach, motivated by studies of the Strong Coupling QCD expansion [4.6]. Let us for simplicity first consider interactions between two mesons with large relative momentum ($\gg m_\pi$). The situation before the collision is shown in fig. 4.7a (lhs): the projectile quark q_p and antiquark $\bar{q}_p$ are linked together by a string; a corresponding picture applies for the target meson. We are not considering gluons explicitly; they are replaced by strings, since we think that soft processes are represented better by an effective theory of (anti)quarks and strings rather than (anti)quarks and gluons. On a lattice, a link (or string) U between two neighbouring lattice sites is related to the gluon field A via [4.6]

$$U = \exp\left\{iag\frac{\lambda^a}{2}A^a\right\}, \quad (4.5)$$

and the theory is then formulated in terms of U rather than A , so the lattice world is another example where (anti)quarks and strings rather than (anti)quarks and gluons are the basic constituents.

Let us specify the meson-meson interaction: we assume that "some effective string interaction" acts in such a way that as the final result the two strings are "flipped", i.e. after the interaction two strings link the (former) projectile quark q_p and the (former) target antiquark $\bar{q}_t$ and correspondingly $\bar{q}_p$ and q_t , see fig. 4.7a (rhs). On the lattice the plaquette operator $\text{tr} U U^\dagger U U^\dagger$ provides such a transition [4.6]. This "String Flip" interaction may be formally represented as

$$(q_p q_p) + (\bar{q}_t q_t) \rightarrow (\bar{q}_p q_t) + (q_t q_p) \quad (4.6)$$

where a bracket () indicates that the objects inside are linked by a string, i.e. they form a colour singlet. Since the rhs and lhs of eq. (4.6) differ by the exchange

$$q_p \leftrightarrow q_t \quad (4.7)$$

the interaction eq. (4.6) is also referred to as "Colour Exchange" (between the quarks q_p and q_t). It is not a particle or momentum exchange, just colour exchange: since eq. (4.6) represents the colour singlet structure, all the momenta of the partons are completely unchanged. We know that hadron-hadron interactions are mostly "soft", little transverse momentum is transferred (few hundred MeV); "String Flip" or "Colour Exchange" provides an interaction mechanism which is also "soft" longitudinally, since no longitudinal momentum is transferred at all (rearrangement of strings does the job), so the picture seems to be consistent.

The "Colour Exchange" ("String Flip") principle can equally well be applied to baryon-baryon scattering, just the antiquark in the meson has to be replaced by a diquark, see fig 4.7b. Here the string which keeps the diquark together is not involved in the interaction. Formally the baryon-baryon interaction may be written as

$$(q_p q_p q_p) + (q_t q_t q_t) \rightarrow (q_p q_p q_t) + (q_t q_t q_p) \quad (4.8)$$

Here the last of the projectile and target quarks have been exchanged.

Another useful representation of the "Colour Exchange" process is the quarkline diagram, shown in fig. 4.8a: quarks (dots) surrounded by closed lines are meant to be coupled to colour singlets. Due to a colour exchange (arrow), singlet structures are changed, leading to singlets consisting of a quark and a diquark from different original baryons. We also indicate the momentum balance in fig. 4.8a: in their centre-of-mass system the baryons initially have momenta $+P$ and $-P$. These momenta are shared between a quark and a diquark, so the quarks have momenta $x_1 P$ and $-x_2 P$ and the diquarks $(1-x_1)P$ and $-(1-x_2)P$, with x_1, x_2 being numbers between 0 and 1. The new singlet structures (qq -strings) are now composed each of a diquark and quark moving in opposite directions — having momenta $(1-x_1)P$ and $-x_2 P$ for one string and $-(1-x_2)P$ and $x_1 P$ for the other.

We are now going to plot the situation from fig. 4.8a in a momentum space diagram. Since for the moment we ignore transverse momenta, we consider the p_0 - p_3 plane (p_0 : energy, p_3 : longitudinal momentum) or equivalently the p^+ - p^- plane ($p^\pm = p_0 \pm p_3$). Neglecting baryon masses we get for the forward-moving baryon $p^+ = 2P$, $p^- = 0$ —

Fig. 4.7. The "String Flip" or "Colour Exchange" principle for meson-meson (a) and baryon-baryon (b) scattering. The momenta of all partons are unchanged, just the strings linking them together are modified ("flipped").

Fig. 4.8. (a) Quark line diagram of "String Flip" or "Colour Exchange" (c.e.) interaction for baryon-baryon scattering. Due to the interaction, partons from different incident baryons are grouped together to singlets (closed lines). Also indicated is how the partons (quarks and diquarks) share the initial momenta $\pm P$. (b) Representation in momentum space — renormalized to $P = 1$ — of the situation in (a): the lhs represents the incident baryons and how their momenta are shared by the quark and diquark; the rhs represents the putting together of partons from different original baryons.

represented by the dashed line parallel to the p^+ axis in fig 4.8b. The backward-moving baryon fulfils $p^+ = 0$, $p^- = 2P^-$ — represented by the solid line parallel to the p^- axis in fig. 4.8b. The momentum sharing between quark and diquark as discussed in the last paragraph is indicated by splitting the two lines into two segments each, with fractions x_1 , $(1 - x_1)$ and x_2 , $(1 - x_2)$. Putting together diquarks and quarks with momentum fractions $(1 - x_1)$, x_2 and $(1 - x_2)$, x_1 is indicated on the rhs of fig. 4.8b. Comparing with fig. 4.6 we realize that the two diagrams are completely identical, the only difference being that fig. 4.6 represents space-time ($x_0 - x_3$) whereas fig. 4.8b represents momentum space ($p_0 - p_3$). However, we mentioned earlier that because of the mapping $\Delta p = \kappa \Delta x$ it does not matter which space we are considering, in particular since we are using renormalized lengths and momenta. This means that “Colour Exchange” is identical to “Rearrangement”, however giving structure to the endpoints: we have diquarks and quarks at the endpoints.

We have not yet specified in either case the momentum (or length) sharing distribution $f(x_1, x_2)$. In the VENUS model — which uses the “Colour Exchange” mechanism — we use

$$f(x_1, x_2) = f(x_1)f(x_2); \quad f(x) = \frac{g(x)}{x^\alpha} \quad (4.9)$$

with $\alpha > 0$ and $0 < g(0) < \infty$. Different options for f are possible which may lead to quite different predictions concerning distributions of measurable quantities.

“Longitudinal Excitation”

“Longitudinal Excitation” [4.4] means the following: the interaction between two hadrons with large relative momentum results in a transfer of longitudinal momentum between the hadrons such that the result is two longitudinally excited objects (strings), see fig. 4.9a,b. Each string contains exactly the quarks of one of the incident hadrons, so the singlet structure of the hadrons is not changed like for “Colour Exchange” interaction, see fig. 4.9c,d.

For the following we restrict ourselves to yo-yo strings (in this section we are in general not so much interested in giving a complete description of the model, but rather want to understand the basic ideas). The interaction is defined in momentum space. Let us again consider two incident baryons in their centre-of-mass system, having momenta $\pm P$, and correspondingly the light-cone momenta $p^+ = 2P$, $p^- = 0$ (dashed line in fig. 4.10a) and $p^+ = 0$, $p^- = 2P$ (solid line in fig. 4.10a). The prescription for “Longitudinal Excitation” can be formulated as follows (see fig. 4.10): divide the lines into two segments each with fractions x_1 , $1 - x_1$ and x_2 , $1 - x_2$ according to some distribution function $f(x_1, x_2)$. Construct rectangles (representing the new strings) with lengths $1 - x_1, x_2$ and $1 - x_2, x_1$ (see fig. 4.10b). This looks like the “Rearrangement” (fig. 4.6) or “Colour Exchange” (fig. 4.8b) diagram, but there is a substantial difference (see fig. 4.10b): the right string consists of only dashed parton lines, the left one only of solid ones. This indicates: all the partons (quarks and diquarks) of each string are coming from one original baryon. This implies that the energy in the (dashed) segment of length x_2 in fig. 4.10a is used to bend one end of the solid line representing the other baryon (fig. 4.10c). This is obviously a very violent process in momentum space, contrary

Fig. 4.9. “Longitudinal Excitation” (a,b) requires all partons in a string to originate from one baryon, whereas “Colour Exchange” (c,d) provides strings with partons from different baryons.

Fig. 4.10. Momentum space picture of a “Longitudinal Excitation” (again original momenta are renormalized to one): the incident momenta are split into two parts (a); one momentum share is then transferred to the other string and vice versa (b), so the straight line representing an incident baryon is bent by a massive momentum transfer acting on one end of the string (c).

to “Colour Exchange” where no longitudinal momentum is transferred. Although the physical picture of the two approaches is very different, they are quite similar since the momentum space diagrams (figs. 4.8a, 4.10ab) are so similar; a measurable difference is provided only by the different flavour content of the strings.

A substantial difference occurs of course through the fixing of the splitting distributions $f(x_1, x_2)$, which in [4.4] is given as

$$f(x_1, x_2) = f(x_1)f(x_2); \quad f(x) = \frac{1}{x} \quad (4.10)$$

One has to keep in mind that not only the choice of f is important but also the cut-off procedure to avoid small x values.

4.2 The string model VENUS

The string model VENUS employs the “Colour Exchange” principle to form strings in energetic hadron-hadron collisions. We are only going to sketch the model in the following; we do not intend to give a complete description, the interested reader is referred to [4.3]. The dominant process for baryon-baryon collisions in VENUS is the “Colour Exchange” (“String Flip”) process eq. (4.8)

$$(q_p q_p q_p) + (q_t q_t) \rightarrow (q_p q_p q_t) + (q_t q_t q_p) \quad (4.11)$$

where a string flip between the projectile proton ($q_p q_p q_p$) and the target proton ($q_t q_t q_t$) (left-hand side of eq. (4.11)) results in two diquark quark strings (right-hand side of eq. (4.11)) with a large relative momentum between the quark q and the diquark qq . Since a nucleon may have a more complicated structure, like $(qqq)(q\bar{q})$, in VENUS also “diffractive scattering”

$$(q_p q_p q_p)(q_p \bar{q}_p) + (q_t q_t q_t) \rightarrow (q_p q_p q_p) + (q_t \bar{q}_p) + (q_t q_t q_p) \quad (4.12)$$

contributes, where the “surviving” nucleon ($q_p q_p q_p$) keeps a large fraction of the initial momentum, thus giving rise to a diffractive peak at large momenta.

Altogether we have (per def.) four “One-Colour Exchange” contributions (involving colour exchange between quarks):

$$\begin{aligned} (a) \quad & (q_p q_p q_p) + (q_t q_t q_t) \rightarrow (q_p q_p q_t) + (q_t q_t q_p) \\ (b) \quad & (q_p q_p q_p)(q_p \bar{q}_p) + (q_t q_t q_t) \rightarrow (q_p q_p q_p) + (q_t \bar{q}_p) + (q_t q_t q_p) \\ (c) \quad & (q_p q_p q_p) + (q_t q_t q_t)(q_t \bar{q}_t) \rightarrow (q_p q_p q_t) + (q_t \bar{q}_t) + (q_t q_t q_t) \\ (d) \quad & (q_p q_p q_p)(q_p \bar{q}_p) + (q_t q_t q_t)(q_t \bar{q}_t) \rightarrow (q_p q_p q_p) + (q_t \bar{q}_p) + (q_t q_t q_t) + (q_p \bar{q}_t). \end{aligned} \quad (4.13)$$

These basic VENUS “One-Colour Exchange” contributions are referred to as (a) non-diffractive (b,c) single diffractive and (d) double Pomeron exchange (contribution (a)

having by far the largest weight). There are in addition four corresponding contributions for “One-Colour Exchange” between antiquarks ([4.7]):

$$\begin{aligned} (a) \quad & (q_p q_p q_p \bar{q}_p) + (q_t q_t q_t \bar{q}_t) \rightarrow (q_p q_p q_p \bar{q}_t) + (q_t q_t q_t \bar{q}_p) \\ (b) \quad & (q_p q_p q_p)(q_p \bar{q}_p) + (q_t q_t q_t)(q_t \bar{q}_t) \rightarrow (q_p q_p q_p) + (q_t \bar{q}_t) + (q_t q_t q_t \bar{q}_p) \\ (c) \quad & (q_p q_p q_p \bar{q}_p) + (q_t q_t q_t)(q_t \bar{q}_t) \rightarrow (q_p q_p q_p q_p \bar{q}_p) + (q_t q_t q_t) + (q_t \bar{q}_p) \\ (d) \quad & (q_p q_p q_p)(q_p \bar{q}_p) + (q_t q_t q_t)(q_t \bar{q}_t) \rightarrow (q_p q_p q_p) + (q_t \bar{q}_t) + (q_t q_t q_t) + (q_t \bar{q}_p). \end{aligned} \quad (4.14)$$

Once considering a complex nucleon structure, consistency requires us to take into account also “Multiple-Colour Exchange”. In VENUS, for example, the following “Double-Colour Exchange” occurs:

$$(q_p q_p q_p)(q_p \bar{q}_p) + (q_t q_t q_t)(q_t \bar{q}_t) \rightarrow (q_p q_p q_t) + (q_t \bar{q}_p) + (q_p \bar{q}_t) + (q_t q_t q_p), \quad (4.15)$$

leading to four strings rather than the two strings as in the lowest order contribution eq. (4.11). As shown in [4.3] one can set up a consistent model by expanding the whole contribution σ as

$$\sigma = \sum_n w_n \sigma_n \quad (4.16)$$

where σ_n are contributions with n colour exchanges, w_n being the corresponding weight. These weights w_n are assumed to be of the form

$$w_n \sim c^n \quad (4.17)$$

with some constant c .

4.3 KNO scaling

In a string model like VENUS the multiplicity distribution $P(n)$ for pp scattering is given as a superposition $\int dm W(m) P(m, n)$ of contributions for a given string mass m . We demonstrate that KNO scaling, an experimentally well established scaling law for multiplicity distributions at different energies, follows as a consequence of a scaling property for the string mass distribution $W(m)$, thus relating the KNO scaling function ψ to the quark structure functions, which determine $W(m)$.

As predicted by Koba, Nielsen and Olesen [4.8], multiplicity distributions $P(n)$ in high energy hadron-hadron collisions obey a scaling law (KNO scaling)

$$\langle n \rangle > P(n) = \psi\left(\frac{n}{\langle n \rangle}\right) \quad (4.18)$$

with an energy independent function ψ , which implies energy independent moments

$$\frac{\langle n^k \rangle}{\langle n \rangle^k} = \psi_k = \int dz z^k \psi(z). \quad (4.19)$$

Although recent collider experiments [4.9] indicate violations of KNO scaling, it is nevertheless very important to understand why this scaling law is approximately valid up to ISR energies [4.10]. KNO scaling can be derived as an asymptotic form of negative binomial (NB) distributions

$$P(n, \bar{n}, k) = \frac{k(k+1) \dots (k+n-1)}{n!} \frac{\bar{n}^n k^k}{(\bar{n}+k)^{n+k}}, \quad (4.20)$$

which were found to fit experimental multiplicity distributions for various pseudorapidity intervals from ISR energies up to collider energies [4.11]. Negative binomial distributions have been proposed from stochastic cell models [4.12, 4.13] or as a consequence of the recurrence relation [4.14]

$$(n+1)P(n+1) = g(n)P(n) \quad (4.21)$$

with a linear function $g(n)$

$$g(n) = \frac{\bar{n}}{\bar{n}+k}(k+n) \quad (4.22)$$

which might be realized by a cascade process, for example [4.14]. Also parton branching models [4.15] lead to NB distributions and KNO scaling in a certain limit.

We follow a different approach. KNO scaling is typical for hadronic collisions, whereas e^+e^- produces simple Poissonian multiplicity distributions [4.16]. Contrary to e^+e^- annihilation, hadronic collisions provide a superposition of strings with different masses, due to the parton structure of the nucleon. Thus, our aim will be to trace KNO scaling back to properties of string mass distributions, which amounts to relating the scaling function ψ to the quark structure functions.

As demonstrated in fig. 4.11, the VENUS model shows approximate KNO scaling, the reason for which we are going to explain in the following. According to eq. (4.16) we can write the multiplicity distribution in VENUS as

$$P(n) = \sum_i w_i P_{2i}(n), \quad (4.23)$$

$P_j(n)$ being the multiplicity distribution of events with j strings. Whereas the energies of the j strings are correlated via

$$\sum_k E_k^j = E_0, \quad (4.24)$$

the string masses are only weakly correlated, so P_j is approximately a folding of 1-string multiplicity distributions P_1 :

$$P_j = P_1 \otimes \dots \otimes P_1. \quad (4.25)$$

The P_1 in eq. (4.25) may be different from each other, since different kinds of string are involved (diquark-quark or quark-antiquark strings). Scaling of individual distributions

Fig. 4.11. Multiplicity distributions versus the scaling variable $n/\langle n \rangle$: VENUS results for 23 and 53 GeV are nearly identical and also agree with the empirical KNO function of [4.22]. Thus, the multistring model VENUS approximately shows KNO scaling!

Fig. 4.12. Mass distributions $W(m)$ versus the scaling variable m/E ($E = \sqrt{s}$ is the pp cm energy). The upper curves represent all quark-diquark strings, whereas the lower curves consider only valence quark-diquark strings. The VENUS results for 23 and 53 GeV agree more or less (string mass scaling).

P_1 leads to scaling for P_j if the ratios of individual mean values are energy independent. Since eqs. (4.23,25) indicate — provided the w_i are energy independent — that scaling for $P(n)$ can be traced back to scaling for $P_1(n)$, we restrict ourselves in the following to string multiplicities $P_1(n)$.

As promised above, we want to draw a connection between string mass scaling and KNO scaling. So, what is the mass distribution $W(m)$ of strings in our model? Consider a string stretching between partons with momenta xP and $-x'P$ respectively, P being the proton momentum in the pp cm system. Neglecting parton masses, the string mass is given as

$$m^2 = 4P^2 x x' = s x x', \quad (4.26)$$

x and x' being momentum fractions according to quark structure functions. Eq. (4.26) implies an energy independent distribution of $m/\sqrt{s}$, i.e.,

$$W(m) = \frac{1}{\sqrt{s}} \phi\left(\frac{m}{\sqrt{s}}\right), \quad (4.27)$$

with an energy independent, universal function ϕ , the form of ϕ depending on the quark structure functions. In fig. 4.12 we show the mass distribution of diquark-quark strings as obtained by the multistring model VENUS. Apart from low mass cut-off effects, the distributions are indeed energy independent, as suggested by eq. (4.27).

In the following we show that string mass scaling (eq. (4.27)) leads to KNO scaling (eq. (4.19)). In order to connect string-masses and multiplicity, we decompose the string multiplicity distribution $P_1(n)$ as

$$P_1(n) = \int dm W(m) P(m, n), \quad (4.28)$$

$P(m, n)$ being the multiplicity distribution of a string with mass m . With eq. (4.28) we obtain

$$\begin{aligned} \frac{\langle n^k \rangle}{\langle n \rangle^k} &= \frac{\int dm n^k P_1(n)}{[\int dm n P_1(n)]^k} \\ &= \frac{\int dm W(m) \int dn n^k P(m, n)}{[\int dm W(m) \int dn n P(m, n)]^k} \\ &= \frac{\int dm W(m) \langle n^k \rangle_m}{[\int dm W(m) \langle n \rangle_m]^k} \end{aligned} \quad (4.29)$$

with

$$\langle n^k \rangle_m = \int dn n^k P(m, n).$$

For Poisson distributions $P(m, n)$ we have

$$\langle n(n-1) \dots (n-k+1) \rangle_m = \langle n \rangle_m^k,$$

which leads for large $\langle n \rangle_m$ to

$$\langle n^k \rangle_m = \langle \langle n \rangle_m \rangle^k. \quad (4.30)$$

As shown in [4.16], $\langle n \rangle_m$ can be parametrized as

$$\langle n \rangle_m = \alpha m^\beta, \quad (4.31)$$

which is less popular than a $\log(m)$ fit, but nevertheless valid. Using eq. (4.30) and eq. (4.31), we obtain from eq. (4.29):

$$\frac{\langle n^k \rangle}{\langle n \rangle^k} = \frac{\int dm W(m) (\alpha m^\beta)^k}{[\int dm W(m) m^\beta]^k}. \quad (4.32)$$

Using the mass scaling law eq. (4.27), we obtain

$$\begin{aligned} \frac{\langle n^k \rangle}{\langle n \rangle^k} &= \frac{\int \frac{dm}{\sqrt{s}} \phi\left(\frac{m}{\sqrt{s}}\right) (\alpha m^\beta)^k}{\left[\int \frac{dm}{\sqrt{s}} \phi\left(\frac{m}{\sqrt{s}}\right) m^\beta\right]^k} \\ \text{or} \\ \frac{\langle n^k \rangle}{\langle n \rangle^k} &= \frac{\int dz \phi(z) z^{\beta k}}{\left[\int dz \phi(z) z^\beta\right]^k}, \end{aligned} \quad (4.33)$$

the latter one being energy independent since ϕ is energy independent. According to eq. (4.19) this means KNO scaling!

We have demonstrated that scaling of string masses as $m/\sqrt{s}$ leads to KNO scaling. String mass scaling should be obtained in all "Dual Parton-like" models, [4.17, 4.18, 4.19, 4.20, 4.21], not only in VENUS, as demonstrated.

Originally [4.8] KNO scaling was proposed as a consequence of Feynman scaling of inclusive cross sections. In the above considerations, Feynman scaling manifests itself in the fact that in the quark structure functions $q(x)$ only momentum fractions x occur. This leads to energy independent x and x' distributions in eq. (4.26) and thus to string mass scaling.

4.4 References

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CHAPTER 5

NUCLEUS-NUCLEUS SCATTERING

We demonstrate in section 5.1 that the VENUS model for hadron-hadron scattering can almost straightforwardly be extrapolated towards nucleus-nucleus collisions. The basic assumption is that a projectile nucleon moves through the target (and vice versa) on a straight line. A "Colour Exchange" occurs whenever a projectile and target nucleon come close to each other (we call these interactions "primary"). This implies that after the first interaction, a nucleon is not a nucleon any more, rather a "leading diquark" or some other coloured object (in general: nucleon minus the parton involved in the colour exchange). Technically all primary interactions are performed before fragmentation sets in. This is very reasonable at high energies (> 100 GeV incident energy), however, at considerably lower energies this concept becomes questionable. The fragmentation of strings is done according to the procedure introduced in chapter 3. That implies a full space-time treatment and taking into account "secondary interactions" between string fragments among each other and with spectators. The latter type of secondary scattering ("cascading") is studied in section 5.2.

5.1 VENUS model for nucleus-nucleus scattering

The VENUS model for NN scattering as described in the last chapter serves as a good basis to make a nearly straightforward extrapolation towards nucleus-nucleus collisions. The NN model is more than just a fit to NN data — few basic assumptions and few parameters are sufficient to reproduce a large number of experimental data. So we want to use the same concept, namely "Colour Exchange" as the source of string production, also in nucleus-nucleus collisions.

We assume the nucleons to move through the other nucleus on a straight line. Although the nature of a nucleon may change on its way through the nucleus, we assume the geometrical size to be unchanged. That means that we use a constant nucleon-nucleon cross section σ_{NN} to determine whether a projectile nucleon interacts with a target nucleon: they have to come closer than

$$r_{i,a} = \sqrt{\sigma_{NN}/\pi}. \quad (5.1)$$

Each interaction means "Colour Exchange" and string formation as in NN collisions. The "nucleon" (whatever its nature is after the first collision) is assumed to make its way through the nucleus and be outside already before it starts to hadronize into observable

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particles. This is reasonable as long as the hadronization time

$$\tau_h = \tau_0 \cosh y = \tau_0 \gamma \quad (5.2)$$

is larger than the reaction time

$$\tau_r = 2R \coth y \quad (5.3)$$

R being the nuclear radius and y being the rapidity of a projectile nucleon. A condition for the applicability of our model is therefore

$$\tau_h > \tau_r. \quad (5.4)$$

For 200 GeV incident energy per projectile nucleon we obtain $\tau_h \approx 200\tau_0$ and $\tau_r \approx 2R$, so for $\tau_0 \approx 1$ fm the condition $\tau_h > \tau_r$ is fulfilled for all nuclei. At 14.5 GeV we find $\tau_h \approx 15\tau_0$, which means the condition eq. (5.4) may be "just" fulfilled, so this energy is close to the borderline of applicability of the model.

Let us consider a projectile nucleus (mass number A) with high enough energy to fulfil the condition $\tau_h > \tau_r$ hitting a target nucleus (mass number B). Our reference system is the centre-of-mass system of one projectile and one target nucleon, further on referred to as the NN cm system, having a rapidity

$$y_{NN} = \frac{1}{2} \log \frac{E+P}{E-P} = \frac{1}{2} \log \frac{\sqrt{m^2 + P^2} + P}{m} \approx \frac{1}{2} \log \frac{2P}{m}, \quad (5.5)$$

where E, P and m are the projectile nucleons energy, momentum and mass. The NN energy in this system is

$$\sqrt{\sigma_{NN}} = \sqrt{(\sqrt{m^2 + P^2} + m)^2 - P^2} \approx \sqrt{2Pm}, \quad (5.6)$$

the last equalities being true in the ultrarelativistic limit $P \gg m$. In the numerical calculations we always use the exact formulas! For 200 AGeV incident energy, for example, we obtain from eq. (5.5) and eq. (5.6) $y_{NN} = 3.0$ and $\sqrt{\sigma_{NN}} = 19.4$ GeV; for 14.5 AGeV incident energy we get $y_{NN} = 1.7$ and $\sqrt{\sigma_{NN}} = 5.4$ GeV. From eq. (5.6) we obtain for the initial energies and momenta of the projectile nucleons in the NN cm system ($1 \leq i \leq A$)

$$E_{proj}^i = \frac{1}{2} \sqrt{\sigma_{NN}} \approx \sqrt{\frac{Pm}{2}} \\ P_{proj}^i = \sqrt{\frac{\sigma_{NN}}{4} - m^2} \approx \sqrt{\frac{Pm}{2}} \quad (5.7)$$

and of the target nucleons ($1 \leq j \leq B$)

$$E_{targ}^j = \frac{1}{2} \sqrt{\sigma_{NN}} \approx \sqrt{\frac{Pm}{2}} \\ P_{targ}^j = -\sqrt{\frac{\sigma_{NN}}{4} - m^2} \approx -\sqrt{\frac{Pm}{2}} \quad (5.8)$$

Correspondingly, the coordinates of the nucleons are labelled $x_{proj}^i, y_{proj}^i, z_{proj}^i$ with $1 \leq i \leq A$ and $x_{targ}^j, y_{targ}^j, z_{targ}^j$ with $1 \leq j \leq B$. The nucleons are distributed isomorphically according to a Woods-Saxon density distribution

$$\rho(r) = \frac{\rho_0}{1 + \exp\left[\frac{r-R_0}{a}\right]}, \quad (5.9)$$

with the parameters

$$a = 0.54 \\ \tau_0 = 1.19 A^{1/3} - 1.61 A^{-1/3}. \quad (5.10)$$

Although not very important, we take into account the nucleon's hard core

$$(x^k - x^l)^2 + (y^k - y^l)^2 + (z^k - z^l)^2 \geq (2r_c)^2, \quad (5.11)$$

with a core radius $r_c = 0.4$ fm. A nucleon is assumed to move on a straight line through the other nucleus, making an interaction whenever it comes close enough to a nucleon:

$$(x_{proj}^i + b_x - x_{targ}^j)^2 + (y_{proj}^i + b_y - y_{targ}^j)^2 \leq \frac{\sigma_{NN}}{\pi}, \quad (5.12)$$

σ_{NN} being the inelastic NN cross section (we use $\sigma_{NN} = 3.1$ fm² for 200 GeV) and (b_x, b_y) being the impact parameter. As mentioned earlier, we use a constant value of σ_{NN} , regardless of how many collisions precede the actual collision, even though a nucleon after a collision is usually not a nucleon any more.

Two quantities characterizing the geometrical aspect of a nucleus-nucleus collision are the number ν of nucleon-nucleon interactions and the number ν_p of participation projectile nucleons. The distributions of these quantities show a qualitative change by going from a symmetric collision ($A \approx B$) to a very asymmetric one ($A \ll B$ or $B \ll A$). In fig. 5.1 we show ν_p distributions for oxygen-induced reactions on carbon, copper and gold targets. For the nearly symmetric system, O + C, the distribution peaks at $\nu_p = 1$, dropping continuously for increasing ν_p . One participant is only likely, when all projectile nucleons are peripheral with respect to the target — so a maximum at $\nu_p = 1$ means a wide region of peripheral collisions as a consequence of surface diffuseness. As the target size increases, a second maximum develops at $\nu_p = 16$, which means all projectile nucleons participate. This maximum is due to the fact that for a target much larger than the projectile there is a wide range of impact parameter values which lead to a "central collision" where all projectile nucleons participate. Related to this qualitative change of ν_p distribution is a qualitative change of the distribution of the collision number ν as a function of the asymmetry. As shown on the left part of fig. 5.1 (consider only the solid lines here) with increasing target size a shoulder appears corresponding to the maximum of the ν_p distribution as discussed above. Regardless of the target size, all ν distributions peak at small ν , meaning that peripheral collisions dominate. Therefore, it is rather useless to consider mean values of observables, since always a large portion of uninteresting peripheral collisions contribute — unless triggers are used which require a certain amount of interaction.

Fig. 5.1. Distributions of the number ν of nucleon-nucleon collisions (left plot, solid lines) and distribution of the number ν_p of participating projectile nucleons (right plot) for O + Au, O + Cu, O + C (from top to bottom) at 200 AGeV. The curves are displaced by a factor of 10.

Fig. 5.2. An example for a "Colour Exchange" diagram in nucleus-nucleus collisions with two projectile and two target nucleons participating.

So far we have talked about collisions between nucleons in nucleus-nucleus scattering without specifying what the interaction really looks like and what the nature of the nucleon is after one or more collisions. We are going to discuss that in the following. Let us recall the definition of an "ordinary" nucleon-nucleon interaction as given in chapter 4. A collision is realized by one or more colour exchanges between a projectile and a target (anti)quark. As a result, a certain number of $q\bar{q}$ strings emerge and, most important, one "projectile-like" and one "target-like" string, the latter ones (let's call them leading strings) carrying the baryon number and most of the energy of the reaction. A "projectile-like" string, for example, consists of a fast forward-moving nucleon remnant, tied per colour force to slowly backward-moving partons from the other nucleon. By nucleon remnant we mean a nucleon, reduced by some quarks and/or antiquarks, so in general a coloured object. Therefore, in a nucleus-nucleus collision we have to consider interactions between nucleon remnants rather than between nucleons, yet the interaction can be defined in exactly the same way: an interaction between nucleon remnants is realized by "Colour Exchange" involving quarks or antiquarks of the remnants. This definition implies that we only consider those (anti)quarks which have not been struck in an earlier collision to participate in the "Colour Exchange". As for NN collisions, we have four basic "One-Colour Exchange" contributions σ_a , σ_b , σ_c and σ_d involving quarks (eq. (4.13)), and four basic "One-Colour Exchange" contributions $\sigma_{\bar{a}}$, $\sigma_{\bar{b}}$, $\sigma_{\bar{c}}$ and $\sigma_{\bar{d}}$ involving antiquarks (eq. (4.14)) leading to the following "Multiple-Colour Exchange" interaction between projectile-nucleon remnant i and target-nucleon remnant j :

$$\sigma(i, j) = \sum_{k=1}^{\infty} w_k \left[\sum_{m \in M} \alpha_m \sigma_m(i, j) \right]^k \quad (5.13)$$

with w_k being the probability for k colour exchanges ($\sum w_k = 1$), α_m being the probability for the basic contribution σ_m ($\sum \alpha_m = 1$), and M being the set $\{a, b, c, d, \bar{a}, \bar{b}, \bar{c}, \bar{d}\}$. If i_μ and j_μ are the projectile and target nucleon involved in the μ^{th} collision of an event, the whole contribution is (using eq. (5.13)):

$$\prod_{\mu} \sigma(i_{\mu}, j_{\mu}) = \prod_{\mu} \sum_{k=1}^{\infty} w_k \left[\sum_{m \in M} \alpha_m \sigma_m(i_{\mu}, j_{\mu}) \right]^k \quad (5.14)$$

In fig. 5.2 we show as an example for three collisions ($\nu = 3$) with two projectile and two target nucleons involved, the contribution $\sigma_d(2,1) \sigma_c(1,2) \sigma_a(2,2)$. The entire nucleus-nucleus collision can be written as (see eq. (5.14))

$$\sigma_{AB} = \sum_{\{i,j\}} P(\{i,j\}) \prod_{\mu} \sum_{k=1}^{\infty} w_k \left[\sum_{m \in M} \alpha_m \sigma_m(i_{\mu}, j_{\mu}) \right]^k \quad (5.15)$$

where $P(\{i,j\})$ is the probability to have a sequence $\{i,j\} = i_1, j_1; i_2, j_2 \dots$ of nucleon-

nucleon collisions. Equation 5.15 can also be written as

$$\sigma_{AB} = \sum_{(i,j)} \sum_{k_1 k_2 \dots} \sum_{m_1^1 \dots m_{k_1}^1} \sum_{m_2^1 \dots m_{k_2}^1} \dots \left(P(\{i, j\}) \prod_{\mu} w_{k_{\mu}} \prod_{k=1}^{k_{\mu}} \alpha_{\{m_k^{\mu}\}} \right) \times \left(\prod_{\mu} \prod_{k=1}^{k_{\mu}} \sigma_{\{m_k^{\mu}\}}(i_{\mu}, j_{\mu}) \right) \quad (5.16)$$

String properties (momenta and flavour content) for the contributions

$$\prod_{\mu} \prod_{k=1}^{k_{\mu}} \sigma_{\{m_k^{\mu}\}}(i_{\mu}, j_{\mu})$$

of eq. (5.16) are calculated according to exactly the same prescription as corresponding diagrams in NN collisions, described in chapter 4. However, now, instead of the sharing functions $f(x)$ in eq. (4.9) we have to use a multi-parton-distribution function $f(\vec{x})$ which we assume to be of the form

$$f\{\dots x_n \dots \vec{x}_m \dots\} = \prod_n f_n(x_n) \prod_m \tilde{f}_m(\vec{x}_m) \theta\left(\sum_n x_n + \sum_m \vec{x}_m\right) \quad (5.17)$$

where $f_n(x)$ and $\tilde{f}_m(\vec{x})$ are the sharing functions (or single parton distribution functions) f defined earlier for the NN model in eq. (4.9).

5.2 Cascading in nuclear collisions

It is a commonly accepted picture for the initial stage of an ultrarelativistic heavy ion collision to cut the nuclei geometrically into spectator and participant regions. A small nucleus colliding centrally with a big one would drill a cylindrical hole through the latter, so that all the nucleons in the cylinder and all projectile nucleons participate in the interaction, whereas all target nucleons outside the cylinder just spectate.

In a string model (like VENUS), the participant region is the zone where strings are formed: so we have a central cylinder with longitudinal oriented strings, surrounded by the spectator nucleons outside the cylinder. The strings expand along their longitudinal axes, and break for the first time after typically 1 fm/c. Whenever a string breaks, the resulting two substrings acquire transverse momentum. The transverse momentum component allows the string segments to move outside the participant cylinder and interact with spectator nucleons. Such secondary interactions with spectators are referred to as "cascading".

The string model VENUS used the fragmentation procedure described in chapter 3, i.e. secondary interactions among produced particles as well as interactions of produced particles with spectators are included in the model. Presently there exists only

one other model [5.1] which fully includes both these aspects. Other models dealing with secondary interactions are described in refs. [5.2, 5.3]. Although interactions of produced particles among each other in particular concerning the formation of a quark gluon plasma are important, we concentrate here on the other aspect — cascading. It is the purpose of this section to apply VENUS to calculate observables which are sensitive to cascading, and compare the results with data.

The model of interacting string fragmentation has (in chapter 3) already been applied to neutrino-neon scattering which is essentially the fragmentation of a diquark-quark string inside a nucleus. So this model has been tested already, the parameters have been fixed, and it can therefore without further freedom be applied to the more complicated proton-nucleus and nucleus-nucleus collisions.

Rescattering is crucially affected by the interaction radius r_0 in eq. (3.9). In the following we are comparing two scenarios (using $r_i = 0.7$ fm for baryons and $r_i = 0.35$ fm for mesons, $i=1,2$):

$$\begin{array}{lcl} r_0 = 0 & \longleftrightarrow & \text{rescattering turned off} \\ r_0 = r_1 + r_2 & \longleftrightarrow & \text{rescattering} \end{array}$$

where our default values may not be the final answer; it is really a best guess rather than an optimized parameter. In the following figures we shall use dashed lines if using $r_0 = 0$ and solid lines if using $r_0 = r_1 + r_2$, points represent data.

In figs. 5.3,4 we show rapidity distributions of negative pions and protons in p -Ag reactions at 100 GeV (data are from [5.4]). We observe that the total number of pions remains almost unchanged by turning on interactions. However, we observe a shift from central to backward rapidity. We see a much greater effect for protons: the number of observed protons is considerably increased, and we even observe protons at negative rapidity, which is not possible in p -p collisions. Without rescattering (dashed histogram) the protons have the following origin in the model: for each collision (on the average ≈ 3) we have a diquark-quark string, the diquark facing backwards. Each of these diquarks leads after fragmentation to a baryon, so with roughly half of these being protons we expect approximately 1.5 protons in the fragmentation region ($y < 3$), which is more or less the integral of the dashed histogram (fig. 5.4). Concerning rescattering the most important process is the following: a produced particle (baryon or meson) hits a target spectator, the two hadrons fuse and decay again. This leads to the large increase of baryon number in fig. 5.4 and the shift in the pion distribution in fig. 5.3. We observe qualitatively the same behaviour for the reaction $O+Au$, see figs. 5.5,6 (data are from [5.5, 5.6]).

Another variable widely used in experiments is the transverse energy E_T which sums up weighted energies of all particles in a given rapidity interval. This quantity is essentially a convolution of multiplicity and single particle transverse momentum distribution. By comparing with the rapidity distributions in figs. 5.3-6 we try to understand qualitatively the effect of rescattering on E_T -distributions. Fig. 5.7 shows E_T -spectra for a pseudo-rapidity interval $0.6 < \eta < 2.4$ in p -Pb collisions at 200 GeV (data are from [5.7]). From our multiplicity studies we conclude that the pion multiplicity should not change much in this interval (compare fig. 5.3); however, one effect of the rescattering

Fig. 5.3. Rapidity distributions of pions for p -Ag scattering at 100 GeV. Points are data, the dashed histogram represents VENUS without rescattering, the solid histogram shows VENUS results with rescattering.

Fig. 5.4. Rapidity distributions of protons for p -Ag scattering at 100 GeV. Same conventions as in fig. 5.3.

Fig. 5.5. Pseudo-rapidity distributions of charged particles for O-Au scattering at 200 GeV. Points are data, the dashed histogram represents VENUS without rescattering, the solid histogram shows VENUS results with rescattering.

Fig. 5.6. Pseudo-rapidity distributions of protons for O-Au scattering at 200 GeV. Same conventions as in fig. 5.5.

Fig. 5.7. Transverse energy distributions for p-Pb collisions at 200 GeV for a backward rapidity interval. Points are data, the histograms represent VENUS with (solid) and without (dashed) rescattering.

Fig. 5.8. Transverse energy distributions for p-Pb collisions at 200 GeV for a central rapidity interval. Same conventions as in fig. 5.7. The data actually represent p-Au scattering.

Fig. 5.9. Transverse energy distributions for O-W collisions at 200 GeV for a backward rapidity interval. Points are data, the histograms represent VENUS with (solid) and without (dashed) rescattering.

Fig. 5.10. Transverse energy distributions for O-Au collisions at 200 GeV for a central-forward rapidity interval. Same conventions as in fig. 5.9.

process eqs. (3.10,11) is the increase of proton number, and another one the transformation of longitudinal momentum into transverse one [5.8]. Both effects lead to a broader E_t -distribution — as observed in fig. 5.7. Fig. 5.8 demonstrates that for a central rapidity interval $2.2 < \eta < 3.8$ there is almost no effect. In this interval however the multiplicity in a rescattering scenario is decreased. This is obviously just compensated by an increase of the average p_t [5.8]. Both histograms (VENUS with and without rescattering) fit the data [5.9] equally well, although for other observables there is quite a difference between the two scenarios. This shows the limited value of the E_t variable to be used to discriminate between models.

We now turn to E_t -distributions for nucleus-nucleus collisions. In fig. 5.9 we show E_t -spectra for a backward rapidity range ($-0.1 < \eta < 2.9$) for O-W scattering at 200 GeV (the data are from [5.10]). The solid histogram represents as usual full VENUS (with rescattering), we also show VENUS results without rescattering (dashed). We see the same effect as for p -A at backward rapidities: rescattering leads to wider distributions, due to more $< p_t >$ per particle and more protons. For central-forward rapidities we see again — as for p -A at central rapidities in fig. 5.8 — almost no effect due to rescattering, see fig. 5.10 (the data are from [5.11]).

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