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THE $U(1)$ GOLDBERGER-TREIMAN RELATION AND THE TWO COMPONENTS OF THE PROTON "SPIN"

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Abstract

We present a simple and rigorous derivation of a recently proposed generalisation of the Goldberger-Treiman relation, and discuss its renormalisation group properties. We then show how to define separately isospin-conserving quark and gluon components of the proton "spin", which evolve precisely as in the QCD parton model.

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1. Introduction

One of the most celebrated successes of current algebra and PCAC is the Goldberger-Treiman (GT) relation amongst the isotriplet axial-vector coupling of the nucleon, its mass, the pion decay constant and the pion nucleon coupling,

$$g_A(0) = \frac{F_\pi}{M_N} g_{\pi NN}, \quad (1.1)$$

which holds to within a few per cent (in our normalisations, $g_A(0) \simeq 1.25$, $F_\pi \simeq 93 \text{ MeV}$ and $g_{\pi NN} \simeq 13.5$).

Interest in a $U(1)$ version of the GT relation has been recently aroused by some puzzling EMC data[1] indicating an unexpectedly small flavour-singlet axial-vector nucleon coupling (improperly called “spin”).

Indeed, modulo several assumptions (for a review, see ref.[2]), the EMC result can be converted into a statement about the flavour-singlet analogue of the l.h.s. of eq.(1.1), viz.

$$G_A(0)|_{q^2 \simeq 11 G^2 v^2} \simeq 0.12 \pm 0.24, \quad (1.2)$$

as opposed to a naive quark model expectation of around 0.65. Notice that, unlike eq.(1.1), the renormalisation scale of the non-conserved singlet axial-vector current has to be specified in eq.(1.2). In the following, we shall denote this renormalisation scale by q and the momentum transfer (which is precisely zero in eqs.(1.1), (1.2)) by k .

At first sight, a $U(1)$ GT relation might be expected to lead to an “explanation” of the EMC result as due to a particular suppression of the η' coupling to the nucleon. More simply, it could lead to new predictions once the EMC result is assumed.

Hatsuda[3] has obtained an approximate $U(1)$ GT relation using the large N_C effective action for QCD[4]. Subsequently, one of us[5] has argued that there exists a $U(1)$ version of the GT relation which is *exact* in the chiral limit for any number of colours and flavours. This latter equation, called the TGV relation in the following, involves the coupling of the nucleon to a state which is *not* an eigenstate of the full QCD Hamiltonian – viz. the Nambu-Goldstone (NG) boson occurring in the OZI limit of QCD. This mixes with gluonic states via OZI-violating interactions and acquires a mass through the now familiar ghost mechanism[6,4].

References [3,5] (see also [7]) expose an unfortunate circumstance. If one tries to define a quark and a gluon component of the proton “spin” (ie. the axial-vector matrix element measured by EMC) via, respectively, the matrix elements of the soft and anomalous divergences of the axial-vector current, one finds $O((m_u + m_d)/(m_u + m_d))$ isospin violations (in the gluon component, for example) making such a definition useless in practice. This kind of isospin violation, well known for some time in other matrix elements[8], disappears in the matrix elements of the currents.

These results have raised doubts as to the possibility of defining meaningful quark and gluon components of the proton “spin”, in apparent agreement with the observation[9] that the axial-vector current is the only dimension-three gauge-invariant operator in this channel and thus cannot mix under renormalisation.

This tentative conclusion appears, however, to clash with parton model ideas which seem to allow[10,11] a definition of separately isospin-conserving quark and gluon components (possibly to be observed independently in other hard processes). The extra component may provide an understanding[11] of why the naive quark model fails in the flavour singlet channel, without however explaining the smallness of $G_A(0)$.

The TGV relation has been challenged recently[12] by the suggestion that a certain form factor had been neglected leading to an incorrect identification of the l.h.s. of the TGV equation. Some objections were also raised in ref.[12] about the consistency of the TGV equation with renormalisation group (RG) constraints.

In this paper we shall show that:

- (i) the TGV relation can be derived rigorously (and simply) using an effective action originally introduced by Zumino[13]; as a result, its precise meaning is clarified;
- (ii) the relation holds away from the chiral limit, modulo usual off-shell extrapolations;
- (iii) the RG properties of the TGV relation are indeed precisely as required;
- (iv) as a result of (i) and (iii), there exists a natural way of splitting the matrix element measured by EMC into “quark” and “gluon” components which (a) are separately isospin- (or $SU(3)$ -) conserving, and (b) evolve in q^2 precisely as required in the QCD parton model. Finally, a couple of mechanisms are suggested to explain why, in the context of the TGV relation, the two components might tend to cancel each other.

2. GT relations and the Zumino effective action

We begin by introducing an appropriate effective action following the method described in Zumino’s 1970 lectures[13]. Consider first the generating functional of connected Green’s functions $W(S_a, V_\mu)$ defined by

$$\exp W(S_a, V_\mu) = \langle \exp(S_a \Phi_a + V_\mu J_{\mu 5}) \rangle, \quad (2.1)$$

where repeated indices also imply a space-time integration. We have distinguished sources S_a for the composite fields Φ_a and the source V_μ for the gauge-invariant $U(1)$ axial-vector current $J_{\mu 5}$,

$$J_{\mu 5} = \sum_f \bar{q}_f \gamma_\mu \gamma_5 q_f. \quad (2.2)$$

(We shall not need to introduce the conserved, gauge-variant current in our discussion.) The set of fields Φ_a which are relevant here is

$$\Phi_a = (\phi^i, \phi_s^i, Q, N, \bar{N})$$

$$\text{with } \phi^i = \sum_{f,f'} \bar{q}_f \lambda_{ff'}^i q_{f'}, \quad \phi_s^i = \sum_{f,f'} \bar{q}_f \lambda_{ff'}^i \gamma_5 q_{f'}, \quad Q = \frac{\alpha_s}{8\pi} \text{tr}(F\bar{F}), \quad (2.3)$$

where λ^i ($i \neq 0$) are $SU(N_F)$ generators, and $\lambda^0 = \frac{1}{\sqrt{2N_F}} \mathbf{1}$. N ($\bar{N}$) is an interpolating field for the nucleon (antinucleon). For the moment, we consider the regularised, bare operators, leaving the subtleties of renormalisation to section 3.

The peculiarity of the Zumino effective action, which makes it well suited for our purposes (see the discussion later in this section), is that it is defined as the Legendre transform of W *only with respect to the fields* and their sources but *not with respect to the currents*. In our context,

$$\Gamma_Z(\Phi_a^c, V_\mu) = W(S_a, V_\mu) - \Phi_a^c S_a, \quad (2.5)$$

where as usual the “classical” fields Φ_a^c (simply denoted by Φ_a hereafter) are defined by

$$\Phi_a = \Phi_a(S, V) = \frac{\delta W}{\delta S_a}. \quad (2.6)$$

The following relations follow from eqs.(2.5), (2.6):

$$\frac{\delta \Gamma_Z}{\delta \Phi_a} = -S_a, \quad \frac{\delta W}{\delta V_\mu} = \frac{\delta \Gamma_Z}{\delta V_\mu}. \quad (2.7)$$

In massless QCD the anomalous chiral Ward-Takahashi (WT) identities for W have the simple form

$$\partial_\mu \frac{\delta W}{\delta V_\mu} - 2N_F \frac{\delta W}{\delta \theta} + \delta_5 S_a \frac{\delta W}{\delta S_a} = 0, \quad (2.8)$$

where δ_5 are the infinitesimal tree-level chiral variations, and $\theta \equiv S_Q$. For Γ_Z (simply denoted by Γ in the following) they become

$$\partial_\mu \frac{\delta \Gamma}{\delta V_\mu} - 2N_F Q - \delta_5 \Phi_a \frac{\delta \Gamma}{\delta \Phi_a} = 0, \quad (2.9)$$

where $\delta_5 Q = 0$, $\delta_5 \phi_5^i = -2\phi^i$ and $\delta_5 \phi^i = 2\phi_5^i$.

We are now in a position to derive the usual GT relation for the idealised case of a non-anomalous $U(1)$ current. For simplicity, take also $N_F = 1$. (The generalisation to the realistic non-singlet, non-anomalous current is trivial.) In this case the anomaly term involving Q is absent in eq.(2.9). Differentiating with respect to N and $\bar{N}$ and setting all fields to their VEV's gives

$$\partial_\mu \Gamma_{V_\mu NN} = -2\langle \phi \rangle \Gamma_{\phi NN}, \quad (2.10)$$

suffixes on Γ or W denoting the corresponding differentiation. The l.h.s. of eq.(2.10) gets contributions only from the pseudovector form factor, since the dangerous $k = 0$ poles in the pseudoscalar form factor are absent in the IPI functional Γ . Re-expressing the r.h.s. in terms of the decay constant and nucleon coupling of the canonically normalised “pion” of this idealised scenario, we find the GT relation

$$G_A(0) = \frac{1}{2} \frac{F_\pi}{M_N} \Gamma_{\pi NN}. \quad (2.11)$$

The familiar result

$$g_A = \frac{F_\pi}{M_N} g_{\pi NN}, \quad (2.12)$$

for the isotriplet current and physical pion is an immediate generalisation.

What happens in the real case of an anomalous $U(1)$ current? Proceeding as above, still for massless QCD, we find

$$\begin{aligned} M_N G_A(0) &= - \sum_i \langle \phi^i \rangle \Gamma_{\phi_i NN} \\ &= - \langle \phi \rangle \Gamma_{\phi NN}, \end{aligned} \quad (2.13)$$

since the anomaly term does not contribute upon differentiation w.r.t. N and $\bar{N}$. From here on, we use the simple notations ϕ , ϕ_5 for the singlet fields ϕ^0 , ϕ_5^0 , so that in eq.(2.13) $\langle \phi \rangle = \sqrt{\frac{N_F}{2}} \langle \bar{q} q \rangle$ (no sum).

This is precisely the TGV relation (cf. eq.(21) of ref.[5]) which was previously found by a more pedestrian method. The present derivation, besides being simpler and more rigorous, also clarifies the exact meaning of “the coupling $G(\eta_1^{(0)} NN)$ of the would-be NG boson to the nucleon” introduced in ref.[5]. This quantity is now seen to be the derivative w.r.t. ϕ_5 of the Zumino action *at fixed* Q . As we shall see below, where we re-express the r.h.s. of eq.(2.13) in analogy to eq.(2.11), this is *not* simply the coupling of the physical η' , even neglecting the extrapolation from zero momentum to the physical mass.

At this point we can discuss the motivations for introducing the Zumino action. First we should emphasise that this is an effective action in the sense of a generating functional for vertices which are IPI with respect to the specified fields. No approximation is involved – in particular, it is *not* merely a low energy effective Lagrangian. Now, any definition of an effective action involves a choice of which fields should be included in the Legendre transform. A good choice involves a judgment of the relevant physical degrees of freedom in the situation considered, and the fields with respect to which the vertices should be IPI. Three choices are reasonable here. We could define an effective action $\tilde{\Gamma}$ by transforming with respect to all the sources in eq.(2.1), including that for the current. However, it is not difficult to see that this only relates the EMC matrix element to a vertex $\tilde{\Gamma}_{J_{as} NN}$, a near tautology conveying little physical insight. At the other extreme, we could omit the Legendre transform with respect to the sources for both the current and the anomalous divergence, leaving ϕ_5^i as the only pseudoscalar field in $\tilde{\Gamma}$. In this case, using the WT identity (2.8) expressed in terms of Γ , the EMC matrix element is seen to depend not only on the proper vertex $\tilde{\Gamma}_{\phi_5 NN}$ but also on the mixed derivative $\tilde{\Gamma}_{\theta NN}$. What this is telling us is simply that we are trying to work with too few pseudoscalar degrees of freedom. Clearly the correct choice for our purposes, which provides the maximum physical insight, is the Zumino action Γ_Z , where we transform *with respect to the sources for all fields except the current itself*.

Finally, let us add quark masses. The WT identity now becomes

$$\partial_\mu \frac{\delta \Gamma}{\delta V_\mu} - 2N_F Q - 2m_i \phi_5^i - \delta_5 \Phi_a \frac{\delta \Gamma}{\delta \Phi_a} = 0. \quad (2.14)$$

Since again the extra terms do not contribute after differentiation w.r.t. N and $\tilde{N}$, the TGV relation (2.13) remains unchanged. Just as for the usual GT relation for a massive pion, the dynamical approximation is not so much in the GT relation itself as in the extrapolation from $k = 0$ to the physical pion mass. Here, the extrapolation is from $k = 0$ to the “OZI mass” of the would-be singlet pseudo-Goldstone boson, approx. $2/3m_K^2 \simeq 0.16\text{GeV}^2$.

3. Renormalisation group consistency of the TGV relation

We now turn our attention to the RG consistency of the TGV relation (2.13). The aim of this exercise is twofold:

- (i) we would like to understand how the r.h.s. of the TGV relation can scale under the renormalisation group with the anomalous dimension appropriate to $J_{\mu 5}$. This involves understanding how the RG works in detail for the Zumino effective action, which turns out to be quite subtle.
- (ii) the behaviour of the TGV relation under the RG should suggest ways to split the total proton “spin” into quark and gluon components which, following parton model ideas [10,11], should evolve in q^2 in a precise way dictated by their RG properties.

In this section we shall briefly outline point (i), leaving a detailed discussion to a future publication [14]. The reader interested in the physical application (ii) of our results may jump immediately to section 4.

The considerations made in the previous section referred to the regularised, bare operators. When renormalisation is performed, care has to be taken of the fact that the operators Q and $\partial^\mu J_{\mu 5}$ mix, while ϕ_S^i and ϕ^i renormalise multiplicatively¹ (expressions without the suffix B hereafter refer to the renormalised quantities) as follows:

$$\begin{aligned}\phi_S^i &= Z_\phi \phi_B^i, & \phi_S^i &= Z_\phi \phi_{SB}^i, \\ J_{\mu 5} &= Z J_{\mu 5B}, & 2N_F Q &= 2N_F Q_B - (1 - Z) \partial^\mu J_{\mu 5B},\end{aligned}\quad (3.1)$$

corresponding to the fact that the combination $\partial^\mu J_{\mu 5} - 2N_F Q$ is not renormalised.

The reason why the RG equations are non-trivial is that Γ treats two fields which mix under renormalisation, viz. $J_{\mu 5}$ and Q , asymmetrically, the Legendre transform only being performed on Q .

The RG equation for W , written in terms of renormalised sources, is standard. In our specific case it reads,

$$\mathcal{D}W - \gamma \left(V_\mu \frac{\delta W}{\delta V_\mu} + \frac{1}{2N_F} \theta \partial_\mu \frac{\delta W}{\delta V_\mu} \right) - \gamma_\phi \sum_i \left(S_\mu^i \frac{\delta W}{\delta S_\mu^i} + S^i \frac{\delta W}{\delta S^i} \right) = 0,$$

¹ Strictly speaking, there are extra infinities occurring in matrix elements of products of composite operators. These require further subtractions beyond the conventional renormalisations. A formalism for dealing with such divergences in the functional formalism used here has recently been developed (ref.[15]). Since one can show that these extra complications are inessential for the problem discussed here, we shall simply ignore these terms in the following. For a complete discussion, see ref [14].

$$\mathcal{D} = \mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g}, \quad \gamma = \mu \frac{d}{d\mu} \log Z, \quad \gamma_\phi = \mu \frac{d}{d\mu} \log Z_\phi, \quad (3.2)$$

and is linear in W . For the renormalised Γ (defined by eq.(2.5) with renormalised sources and fields) the RG equation instead becomes non-linear, and, after some work, can be shown to take the form

$$\mathcal{D}\Gamma - \gamma \left(V_\mu \frac{\delta \Gamma}{\delta V_\mu} - \frac{1}{2N_F} \theta \partial_\mu \frac{\delta \Gamma}{\delta V_\mu} \right) + \gamma_\phi \sum_i \left(\phi_S^i \frac{\delta \Gamma}{\delta \phi_S^i} + \phi^i \frac{\delta \Gamma}{\delta \phi^i} \right) = 0. \quad (3.3)$$

This equation, plus the chiral WT identity (2.9) for Γ , are the two basic ingredients for investigating the RG behaviour of the TGV relation. Actually, we shall prove RG consistency not only for the zero-momentum TGV relation (2.13), but for its non-forward generalisation,

$$-ik^\mu \langle N | J_{5\mu} | N \rangle = 2N_F \left(\sum_i W_{\theta S_i^*} \Gamma_{\phi_i^* N N} + W_{\theta\theta} \Gamma_{\theta N N} \right), \quad (3.4)$$

which can easily be derived from eq.(2.9) and the definition of the Zumino action.

It is straightforward to extract the RG behaviour of the 3-point vertices occurring in the above relation, and to find for Γ_{QNN} , $\Gamma_{\phi_i^* NN}$,

$$\hat{\mathcal{D}}\Gamma_{QNN} = \gamma(\Gamma_{QV} \Gamma_{QNN} - \sum_j 2\langle \phi^j \rangle \Gamma_{QQ} \Gamma_{\phi_j^* NN}), \quad (3.5)$$

$$\hat{\mathcal{D}}\Gamma_{\phi_i^* NN} = \gamma(\Gamma_{\phi_i^* V} \Gamma_{QNN} - \sum_j 2\langle \phi^j \rangle \Gamma_{\phi_i^* Q} \Gamma_{\phi_j^* NN}) - \gamma_\phi \Gamma_{\phi_i^* NN},$$

where

$$\hat{\mathcal{D}} = \mathcal{D} + \gamma_\phi \sum_i \phi^i \frac{\delta}{\delta \phi^i}, \quad (3.6)$$

and we use the shorthand notation $\Gamma_{QV} = \partial_\mu \Gamma_{QV_\mu}$, etc.

The remaining quantities entering the non-forward TGV relation are the propagators $W_{\theta\theta}$ and $W_{\theta S_i^*}$. A direct calculation shows that their RGE's are

$$\begin{aligned}\hat{\mathcal{D}}W_{\theta\theta} &= 2\gamma W_{\theta\theta}, \\ \hat{\mathcal{D}}W_{\theta S_i^*} &= (\gamma + \gamma_\phi) W_{\theta S_i^*} - 2\gamma \langle \phi^i \rangle,\end{aligned}\quad (3.7)$$

where one has to keep in mind that here $\mathcal{D}$ is a differential operator acting at fixed Φ_a .

After all these preliminaries, we can finally prove our claim by writing

$$\begin{aligned}\hat{\mathcal{D}}(W_{\theta\theta} \Gamma_{QNN} + \sum_i W_{\theta S_i^*} \Gamma_{\phi_i^* NN}) \\ = \gamma(2W_{\theta\theta} + W_{\theta V}) \Gamma_{QNN} - \gamma \sum_i (W_{V S_i^*} - 2\langle \phi^i \rangle) \Gamma_{\phi_i^* NN} \\ = \gamma(W_{\theta\theta} \Gamma_{QNN} + \sum_i W_{\theta S_i^*} \Gamma_{\phi_i^* NN}).\end{aligned}\quad (3.8)$$

Thus the r.h.s. of eq.(3.4) satisfies precisely the RGE expected for the l.h.s. from the multiplicative renormalisation of the gauge invariant current $J_{\mu 5}$.

Although the forward TGV relation is of course a special case, it is perhaps useful to mention that its RG consistency can be shown more directly from the zero momentum limit of eq.(3.5). After noting that

$$\Gamma_{\phi_3^* V} = 0, \quad \sum_i 2\langle\phi^i\rangle\Gamma_{\phi_3^* Q} = -1, \quad \text{at } k = 0, \quad (3.9)$$

$$\text{the RGE} \quad \hat{\mathcal{D}} \sum_i \langle\phi^i\rangle\Gamma_{\phi_3^* NN} = \gamma \sum_i \langle\phi^i\rangle\Gamma_{\phi_3^* NN} \quad (3.10)$$

follows, allowing the TGV identification with $-M_N G_A(0)$. This concludes our check of RG behaviour.

We conclude this section by mentioning another way of rewriting the forward TGV relation which (i) exhibits clearly the RG properties of the relation and (ii) allows (*a priori*) for another mechanism of suppressing $G_A(0)$ (see section 5).

Let us introduce a canonically normalised flavour singlet field

$$\eta_0 = b\phi_5, \quad (3.11)$$

in such a way that

$$\frac{\delta^2 \Gamma}{\delta \eta_0 \delta \eta_0} = -k^2. \quad (3.12)$$

It is not hard to prove that the TGV relation written in terms of the coupling of this normalised field takes the form

$$2M_N G_A(0) = (\chi'_t(0))^{\frac{1}{2}} g_{\eta_0 NN}, \quad (3.13)$$

where

$$\chi'_t(0) = \frac{\partial}{\partial k^2} \int dx e^{ikx} \langle Q(x) Q(0) \rangle|_{k^2=0} \quad (3.14)$$

is related to the first moment of the topological susceptibility in QCD with massless quarks ($\chi_t(0) = 0$ in the chiral limit). It is easy to check that $g_{\eta_0 NN}$ is now RG-invariant and that the whole scale dependence is accounted for by $(\chi'_t(0))^{\frac{1}{2}}$. A possible explanation for the smallness of $G_A(0)$ based on that of $\chi'_t(0)$ rather than of $g_{\eta_0 NN}$ will be discussed (and unfortunately discarded) in section 5.

4. The two components of the proton "spin"

According to QCD parton model ideas[10,11], the quark component of the proton spin should be RG-invariant and thus q -independent. In searching for a split of the total "spin", one is thus naturally led to look for some RG-invariant object. One such object is, of course, the coupling of the physical η' to the nucleon, but we can certainly add to it some other RG-invariant quantity and include it in the quark component at the expense of the gluon component. Nevertheless, we shall be able to fix this ambiguity by requiring that the gluon component contains only the gluonic vertex Γ_{QNN} .

From the RG results of section 3, we can easily prove that such a decomposition can only be made at $k = 0$, which is the case of interest for deep inelastic scattering. We therefore restrict ourselves immediately to this case.

The physical η' is identified through the diagonalisation of the propagator matrix. It is easy to see that, in the chiral limit, this is achieved by introducing

$$\eta' = c\phi_5, \quad g = Q + d\phi_5, \quad (4.1)$$

where $d = -W_{\theta S_5}/W_{S_5 S_5}$. Requiring a canonically normalised η' propagator,

$$\frac{\delta^2 \Gamma}{\delta \eta' \delta \eta'} = -(k^2 - m_{\eta'}^2), \quad (4.2)$$

fixes

$$c = \frac{F_{\eta'}}{W_{V S_5}} \frac{k^2}{(k^2 - m_{\eta'}^2)}, \quad (4.3)$$

where $\langle 0|J_{\mu 5}|\eta'\rangle \equiv \frac{\delta^2 \Gamma}{\delta V_\mu \delta \eta'} = ik_\mu F_{\eta'}$.

It is now simple to rewrite the TGV relation in terms of the couplings of η' and Q to the nucleon,

$$2M_N G_A(0) = 2N_F \left(-W_{\theta S_{\eta'}} \Gamma_{\eta' NN} + \frac{W_{\theta S_{\eta'}}^2}{W_{S_{\eta'} S_{\eta'}}} \Gamma_{QNN} \right). \quad (4.4)$$

Let us now analyse the RGE's satisfied by the two terms appearing on the r.h.s. of eq.(4.4). For the first, we obtain simply

$$\hat{\mathcal{D}} W_{\theta S_{\eta'}} \Gamma_{\eta' NN} = 0, \quad (4.5)$$

while for the second,

$$\hat{\mathcal{D}} \frac{W_{\theta S_{\eta'}}^2}{W_{S_{\eta'} S_{\eta'}}} \Gamma_{QNN} = \gamma \left(-W_{\theta S_{\eta'}} \Gamma_{\eta' NN} + \frac{W_{\theta S_{\eta'}}^2}{W_{S_{\eta'} S_{\eta'}}} \Gamma_{QNN} \right), \quad (4.6)$$

and, of course, they sum to give the correct RG dependence of $G_A(0)$.

It is possible at this point to identify

$$\Delta q = -\frac{1}{M_N} N_F W_{\theta S_{\eta'}} \Gamma_{\eta' NN}, \quad \Delta \Gamma = -\frac{1}{M_N} \frac{W_{\theta S_{\eta'}}^2}{W_{S_{\eta'} S_{\eta'}}} \Gamma_{QNN}, \quad (4.7)$$

with the quark and gluon components of the QCD parton model, where[10,11]

$$G_A(0) = \Delta q - N_F \Delta \Gamma. \quad (4.8)$$

We emphasise again that this decomposition is only unique if the further assumption is made that the gluon component contains only the 1PI coupling of $F\bar{F}$ to the nucleon. It is not clear to us whether a similar assumption can fix the analogous ambiguity occurring in the parton model approach[11].

Eqs.(4.7) can be further simplified to read

$$2M_N \Delta q = F_{g_r NN}, \quad 2M_N \Delta \Gamma = \frac{1}{2N_F^2} F^2 m_{\eta'}^2 g_{QNN}, \quad (4.9)$$

where, under some standard smoothness assumptions, we can estimate F to be $\sim \sqrt{2N_F} F_\pi$, allowing in principle both components to be determined from a combination of the EMC result and $g_{r NN}$.

The case of small quark masses can easily be studied. Unlike the situation encountered in refs [3,5,7], Δq and $\Delta \Gamma$ are now perfectly well behaved under isospin or $SU(3)$ owing to the fact that the problematic poles near $k = 0$ are by definition absent in Γ .

5. Conclusions

In conclusion, our results appear to reconcile a correct RG treatment of the anomaly with the possibility of defining well-behaved quark and gluon components of the proton “spin”. This is achieved by introducing the proper vertices generated by the effective action, rather than working directly with matrix elements of local operators such as $\bar{q}\gamma_5 q$ or $F\bar{F}$. The apparent impossibility of obtaining sensible quark and gluon components from local operators is probably related to the fact that only one gauge-invariant, dimension-three local operator with the correct quantum numbers exists for each quark flavour.

Our analysis of the TGV relation has the merit of connecting the EMC matrix element to something simple, viz. the coupling of the proton to the OZI NG boson, a gluon-uncontaminated object which could probe interesting properties of the nucleon. We emphasise again that it is the OZI NG boson η_0 , and not the physical η' , which enters in the relation. The η' is a mixture of two components, a gluonic part which is neutral under chirality, and the chirally non-trivial η_0 . It is reasonable, therefore, that the axial-vector matrix element measured by EMC should be simply related to the η_0 rather than the η' .

In this light, we suspect that the Skyrme “explanation” of the EMC result[16] could follow, at all N_C , from the fact that the Skyrme nucleon lives entirely in the $SU(3)$ sector of $U(3)$. It would then be very natural for it to couple only weakly to the η_0 , while it would be difficult to imagine that it decouples from a complicated, massive object like the physical η' . The cancellation between the two components of the “spin” would therefore not be accidental.

Another possible way of explaining eq.(1.2) would be to argue that the quantity $\chi'_t(0)$ of eq.(3.14) is unexpectedly small. With some smoothness assumption, one can relate $\chi'_t(0)$ to the topological susceptibility $\chi_t(0)_{YM}$ of the quarkless theory, a quantity measured on the lattice[17] and typically found to be small compared to the phenomenological value of $(180 MeV)^4$. (For the $U(1)$ problem this is not too serious, since the discrepancy can be attributed to the extrapolation to the η' mass.) One could therefore imagine relating the smallness of $G_A(0)$ to that of $\chi'_t(0)$ or $\chi_t(0)_{YM}$. The problem with this scenario, however, is that $\chi'_t(0)$ also controls the η' decay constant $F_{\eta'}$, which enters into the calculation of $\eta' \rightarrow \gamma\gamma$. Since this process is well explained[18] with an unsuppressed η' decay constant, it is rather unlikely that this explanation of the EMC result could be reconciled with $\eta' \rightarrow \gamma\gamma$.

As a final remark, our definition of the quark and gluon components could allow their experimental determination (modulo smoothness assumptions on pole-free quantities) from physical low energy coupling constants such as $g_{\eta' NN}$. It would be interesting to extract values for Δq and $\Delta \Gamma$ in this way, and then use them to predict the absolute magnitude of some hard cross sections of the kind discussed in the literature[11].

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References

- [1] J. Ashman et al., Phys. Lett. B206 (1988) 364; Nucl. Phys. B328 (1990) 1
- [2] Ta-Pei Cheng and Ling-Fong Li, “The proton ‘spin contents’: current status and perspectives”, preprint CMU-HEP-90-2 (1990)
- [3] T. Hatsuda, Nucl. Phys. B329 (1990) 376
- [4] C. Rosenzweig, J. Schechter and G. Trahern, Phys. Rev. D21 (1980) 3388
- [5] P. Di Vecchia and G. Veneziano, Nucl. Phys. B171 (1980) 253
- [6] E. Witten, Ann. Phys. 128 (1981) 1789
- [7] P. Nath and R. Arnowitz, Phys. Rev. D23 (1981) 473
- [8] G. Veneziano, Mod. Phys. Lett. A4 (1989) 1605
- [9] E. Witten, Nucl. Phys. B156 (1979) 269
- [10] G. Veneziano, Nucl. Phys. B159 (1979) 357; Phys. Lett. 95B (1980) 90
- [11] Ta-Pei Cheng and Ling-Fong Li, Phys. Rev. Lett. 62 (1989) 1441
- [12] H.-Y. Cheng, Phys. Lett. B219 (1989) 347
- [13] G. Lopez Castro and J. Pestieau, Phys. Lett. B222 (1989) 459
- [14] D.J. Gross, S. Treiman and F. Wilczek, Phys. Rev. D19 (1979) 2188
- [15] R.L. Jaffe and A. Manohar, MIT preprint CPT 1706 (1989)
- [16] A.V. Efremov and O.V. Teryaev, Dubna preprint E2-88-287 (1988)
- [17] G. Altarelli and G. Ross, Phys. Lett. B212 (1988) 391
- [18] R.D. Carlitz, J.C. Collins and A.H. Mueller, Phys. Lett. B214 (1988)
- [19] M. Anselmino and E. Leader, Santa Barbara preprint NSF ITP 88 142 (1988)
- [20] G. Altarelli and W.J. Stirling, Particle World 1 (1989) 40
- [21] G. Altarelli and B. Lampe, preprint CERN-TH.5645 (1990)
- [22] A.V. Efremov, J. Soffer and N.A. Tornquist, Marseille preprint CPT-90/P2303 (1989)
- [23] B. Zaimino, in “Lectures on Elementary Particles and Quantum Field Theory” Brandeis 1970, vol.2, p.441, eds. S. Deser et al., MIT Press
- [24] G.M. Shore and G. Veneziano, in preparation
- [25] H. Osborn, Nucl. Phys. B308 (1988) 629
- [26] H. Osborn and I.D. Jack, Cambridge preprint DAMTP 90-2 (1990)
- [27] G.M. Shore, unpublished
- [28] S. Brodsky, J. Ellis and M. Karliner, Phys. Lett. B206 (1988) 309
- [29] M. Campostrini, A. Di Giacomo and H. Panagopoulos, Phys. Lett. B212 (1988) 206, and refs. therein
- [30] M.S. Chanowitz, Phys. Rev. Lett. 44 (1980) 59
- [31] P. Di Vecchia, F. Nicodemi, R. Pettorino and G. Veneziano, Nucl. Phys. B181 (1981) 318