Archives TH5704

CC

UICB PTH 90/9
LBL 28811
March 28, 1990

TeV Dark Matter*

SAVAS DIMOPOULOS and NIKOLAOS TERRADIS¹

CERN, Theory Division,

CH-1211, Geneva 23

and

Department of Physics,

Boston University, Boston, Massachusetts 02215

RAHIM ESMAILZADEH

Center for Particle Astrophysics,

University of California, Berkeley, California 94720

LAWRENCE J. HALL

Department of Physics,

University of California,

and

Theoretical Physics Group, Physics Division

Lawrence Berkeley Laboratory,

1 Cyclotron Road, Berkeley, California 94720

Submitted to Nuclear Physics B

* This work was supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098 and in part by the National Science Foundation under grant PHY85-15857.
¹ On leave from Department of Physics, Stanford University, Stanford, CA 94305.

CERN LIBRARIES, GENEVA

CM-P00062450

CERN-PRE 90-004
C1

ABSTRACT

The minimal standard model has no candidates for non-baryonic dark matter of the universe. We add a single $SU(3) \times SU(2) \times U(1)$ fermionic multiplet to this model and compute the mass necessary for it to close the universe as a function of its gauge quantum numbers. The simplest dark matter candidate, not already excluded by existing dark matter searches, is the neutral component of a zero hypercharge Dirac triplet. It couples to nuclei via two W exchange, with an effective coupling three orders of magnitude smaller than ordinary weak interactions. Its charged partners are a few hundred MeV heavier and can be produced at SSC.

1. Introduction

The presence of substantial amounts of dark matter in the universe has important implications for particle physics. A central question is whether the standard model needs extending, and if so how extensive the changes need be. It may be possible that within the standard model the universe can be critically closed by baryons. However, this either requires non-standard nucleosynthesis,^[1] or formation of black holes during the QCD phase transition.^[2] For many years the simplest extension of the standard model which led to an $\Omega = 1$ universe involved the introduction of a fourth generation neutrino with a mass of a few GeV.^[3] This had the advantage, over other possibilities such as the lightest superpartner, that the mass was tightly constrained. Recent measurements of the Z^0 width at SLC^[4] and LEP^[5] have ruled out this possibility.^[6] However, it has been suggested that a neutrino with mass around a TeV would close the universe.^[7] The electroweak cross-section responsible for the annihilation of neutrinos and anti-neutrinos has the proper value for closure for neutrino masses of a few GeV; the cross-section is too large for larger masses which means that the neutrinos do not close the universe. However, since the cross-section eventually drops like m^{-2} for particles heavier than the W , it is expected that for neutrino masses of the order of a few TeV, one can have closure again.

This has led us to explore simple non-supersymmetric versions of the standard model with mass of the dark matter particle larger than the LEP lower bound of around 40 GeV. The mass for which one obtains closure density for the universe depends on the annihilation cross section of the heavy new particle into lighter ones. Therefore, we must classify the possible models according to the $SU(3) \times SU(2) \times U(1)$ quantum numbers of the left-handed and right-handed members of the extended sector. More specifically, we introduce a new multiplet which belongs to a representation of the electroweak group. We also examine the possibility that the new particles carry $SU(3)$ charges; however, since the experimental bounds on strongly interacting dark matter are very stringent and effectively exclude this

possibility, we discuss explicitly only the case of the fundamental 3-dimensional representation of $SU(3)$. A good dark matter candidate cannot carry electrical charge.^[8] Therefore, we must ensure that at least one member of the multiplet has no electric charge. Moreover, the neutral member of the multiplet must be the lightest one. We call this lightest neutral member, the neutrino.

Another issue which is crucial for our results is the mechanism through which the neutrino obtains its mass. If it gets its mass through the usual breaking of the $SU(2)$ group, its coupling to the Higgs particle must be strong. In other words the theory becomes strongly interacting and, contrary to recent claims,^[9] one is unable to compute the annihilation cross section for the neutrino and therefore find the proper mass for closure. Griest and Kamionkowski^[10] have shown that unitarity of the annihilation cross-section limits the mass to be less than 340 TeV, even if the Higgs annihilation channel is strongly coupled. To overcome this difficulty, we consider the case of a Dirac neutrino, whose left and right handed components belong to the same representation of the electroweak group. In this case, we can introduce a bare mass, while the theory remains perturbative, allowing us to compute the annihilation cross section and therefore the freeze out abundance accurately. More generally stated, we consider the minimal lagrangian of the Standard Model, with a new electroweak vector multiplet added. The theory is anomaly free and does not require additional particle states. We assume that the lightest member of the multiplet is stable due to a global symmetry, which is the fermion number of the new multiplet.

A possible loophole to these arguments would involve very precise inflation of an initially overcritical density down to the critical value. This scenario, which we call "just so inflation", necessitates an extreme fine tuning of the amount of inflation and is not considered here.

The paper is organized as follows. In section 2, we consider the case a doublet. We find the annihilation rate for particles and anti-particles and compute the freeze out abundance. We also calculate the mass splitting between the charged

and neutral members of the doublet, due to radiative corrections after the electroweak symmetry breakdown. Finally, we obtain the upper bound on the mass of the neutrino for not overclosing the universe. In section 3, we generalize the above results to the case of a strongly interacting particle in the fundamental representation of $SU(3)$. In section 5, we review experimental bounds on weakly and strongly interacting dark matter. We also list the models which survive as dark matter candidates. Finally, we present our conclusions in section 6.

2. The Case of a Doublet

The Lagrangian of our model is

$$L = L_{SM} + L_{gauge}^R + M_R R \bar{R} \quad (2.1)$$

L_{SM} is the Lagrangian of the minimal standard model with three generations of quarks and leptons and a Higgs doublet. L_{gauge}^R includes the gauge interactions of a new multiplet which we shall denote by $R(C, W, Y)$. C , W and Y represent the transformation properties of this multiplet under $SU(3)$ of color, weak $SU(2)$ and $U(1)$ of hypercharge. We shall consider color singlets ($C = 1$) and triplets ($C = 3$) only. However, we shall study general representations of $SU(2)$ and $U(1)$; with the constraint that after the electroweak symmetry is broken, the lightest member of the multiplet is neutral. We have also imposed a global $U(1)_R$ symmetry, which prevents interactions of the new multiplet with the standard three generations and the Higgs. The bare mass term is allowed by the above symmetries.

The present abundance of the lightest member of the multiplet is determined through the usual Lee-Weinberg analysis.^[9] One only needs to compute the total annihilation cross section for $R\bar{R}$. We are interested in particles with masses in the TeV range, which allows us to carry out our computations using the unbroken Lagrangian of the electroweak group. Even in the case when the electroweak phase transition takes place before the freeze-out process is complete, the induced masses of the annihilation products are negligible compared to the mass of the lightest member. We can also treat all the members of the multiplet as mass degenerate states, since as we shall show later, the radiative mass corrections due to the electroweak breaking are small compared to the freeze-out temperature.

Let us consider the case of an $SU(2)$ doublet, which is a color singlet. We shall list all the possible annihilation cross sections, averaged over the (mass degenerate) members of the doublet. This effectively means that we calculate the average probability of any two members of the doublet to annihilate into the specified final

products. The cross section for $R\bar{R}$ annihilation to two $SU(2)$ gauge bosons is

$$\begin{aligned} \sigma_{R\bar{R}-2A} = & \frac{\pi\alpha_s^2}{8s\beta} \left(3\left(\frac{2}{3} + \frac{1}{3}(1-\beta^2)\right) \right. \\ & + \frac{9}{8} \left[\frac{4+2(1-\beta^2)}{\beta} \ln\left(\frac{1+\beta}{1-\beta}\right) - 4(1+(1-\beta^2)) \right] \\ & \left. + \frac{3}{2} \left[\frac{2(1-\beta^2)}{\beta} \ln\left(\frac{1+\beta}{1-\beta}\right) - 4(1+(1-\beta^2)) - \frac{3}{8} 2\beta(1-\beta^2) \ln\left(\frac{1+\beta}{1-\beta}\right) \right] \right) \end{aligned} \quad (2.2)$$

where $\beta = v_{rel}/2$. The cross section for annihilation to two $U(1)$ gauge bosons is

$$\begin{aligned} \sigma_{R\bar{R}-2B} = & \frac{\pi\alpha_s^2}{64s\beta} \tan^4\theta_w \left\{ \frac{4+2(1-\beta^2)}{\beta} \ln\left(\frac{1+\beta}{1-\beta}\right) \right. \\ & \left. - 4(1+(1-\beta^2)) + 2\beta(1-\beta^2) \ln\left(\frac{1+\beta}{1-\beta}\right) \right\} \end{aligned} \quad (2.3)$$

The cross section for annihilation to a $U(1)$ gauge boson and an $SU(2)$ gauge boson is

$$\begin{aligned} \sigma_{R\bar{R}-AB} = & \frac{3\pi\alpha_s^2}{32s\beta} \tan^2\theta_w \left\{ \frac{4+2(1-\beta^2)}{\beta} \ln\left(\frac{1+\beta}{1-\beta}\right) \right. \\ & \left. - 4(1+(1-\beta^2)) + 2\beta(1-\beta^2) \ln\left(\frac{1+\beta}{1-\beta}\right) \right\} \end{aligned} \quad (2.4)$$

The cross section to light fermions is

$$\sigma_{R\bar{R}-\psi\bar{\psi}} = \frac{\pi\alpha_s^2}{24s^2\beta} (36 + 20\tan^4\theta_w)(s + 2M_R^2). \quad (2.5)$$

Finally, the annihilation to two Higgs is

$$\sigma_{R\bar{R}-HH} = \frac{\pi\alpha_s^2}{48s^2\beta} (3 + \tan^4\theta_w)(s + 2M_R^2). \quad (2.6)$$

Since freeze out happens at temperatures around $M_R/25$, the particles are non-relativistic, and we can neglect their kinetic energy. Therefore, in the approxima-

tion $s = 4M_R^2$ we have the following result for the total annihilation cross section

$$\sigma_{R\bar{R}-anything} v = \frac{2\pi\alpha_s^2}{M_R^2} \left(\frac{81}{128} + \frac{3\tan^2\theta_w}{32} + \frac{43\tan^4\theta_w}{128} \right). \quad (2.7)$$

At the TeV scale the values of α_2 and $\sin^2\theta_w$ are given as

$$\sin^2\theta_w = 0.24 \quad (2.8)$$

$$\alpha_2 = \frac{1}{30}. \quad (2.9)$$

After the electroweak breaking occurs, the mass degeneracy of the two members of the doublet is broken and the radiatively induced mass difference can be computed by considering the loop diagrams of figure 1. The contributions from the W loops cancel and the relevant interaction terms in the lagrangian are

$$\begin{aligned} L_{NC} = & -g\sin\theta_w \bar{L}\gamma_\mu L A^\mu + \frac{g}{2\cos\theta_w} \bar{N}\gamma_\mu N Z^\mu \\ & + \frac{g}{2\cos\theta_w} (-1 + 2\sin^2\theta_w) \bar{L}\gamma_\mu L Z^\mu, \end{aligned} \quad (2.10)$$

where we have denoted the charged and neutral members of the doublet by L and N respectively.

A simple calculation gives for the mass difference

$$\delta m = \frac{\alpha_s^2}{2\pi} M_R \sin^2\theta_w \int_0^1 dx (2-x) \ln|1 + \frac{xm_R^2}{(1-x)^2 M_R^2}|. \quad (2.11)$$

In figure 2, we plot the values of the mass difference as a function of the bare mass. For the case of a doublet, the neutral member is the lightest.

As long as the temperature of the universe is larger than this mass difference, weak interactions keep the number densities of the charged and neutral member

equal. Once the temperature falls below this mass difference, the heavier charged member decays into the lighter neutral one with the decay rate

$$\Gamma = \frac{G_F^2}{15\pi^3} (\delta m)^5. \quad (2.12)$$

Therefore, in calculating the freeze-out abundance, we need not worry about decay of the heavier member of the doublet, even in the case that freeze-out occurs after the electroweak phase transition.

The calculation of the freeze out abundance has been carried out by many authors. We must generalize this formalism to our case of several mass degenerate states participating at freeze out. If the abundances of different members of a multiplet are grossly different, then one is required to solve the full set of coupled equations governing their evolution. On the other hand, since weak interactions maintain an equality among the abundances of the various members, one can solve a single differential equation for the total abundance of all the members, using an averaged annihilation cross section. As in the case of freeze-out of a single particle, this equation has the form:

$$\dot{f} \propto \sigma v > (f^2 - f_o^2), \quad (2.13)$$

where f is the total abundance of all the members of the multiplet divided by the entropy density, the derivative is taken with respect to the temperature, $< \sigma v >$ is the averaged annihilation cross section, and f_o is the total abundance in thermal (and chemical) equilibrium.

The freeze-out temperature T_f is given by

$$x_f = \ln B - \frac{1}{2} \ln \ln B \quad (2.14)$$

where $x_f = M_R/T_f$, $B = d \times 0.076 \times M_\mu M_R < \sigma v > / \sqrt{g_*}$, g_* is the effective number of degrees of freedom ($g_* = 107$) and d is the number of members of the

multiplet ($d = 2$). For $M_R \sim T_{\text{TeV}}$ and $< \sigma v >$ in equation (2.7), we get $x_f \sim 25$. The ratio of the present average energy density of R 's to the closure density is

$$\Omega_R h_0^2 = \frac{1.07 \times 10^9 x_f \text{GeV}^{-1}}{g_*^{1/2} M_\mu < \sigma v >} \approx \frac{3 \times 10^{-27} \text{cm}^3 \text{s}^{-1}}{< \sigma v >}, \quad (2.15)$$

which depends on d through x_f only. Substituting for the annihilation cross section, one can obtain an upper bound for the mass of the lightest member

$$\frac{M_R}{T_{\text{TeV}}} = 4.4 \times \sqrt{\Omega_R h_0^2}, \quad (2.16)$$

for the case of the doublet. In section 5, we shall show that this candidate is excluded by the existing dark matter searches.

3. The General Multiplet

The results of the previous chapter, can be generalized to particles belonging to any representation of the electroweak group. In the appendix we list the annihilation channels and cross sections for particles in the d dimensional representation of $SU(2)$. We still have to make certain that this d dimensional representation contains a neutral member which becomes the lightest member after the electroweak phase transition. This means that there exists a member of the multiplet with isospin $I_3 = Y/2$, where Y is the hypercharge of the multiplet.

The mass difference δm , between a charged member of the multiplet and the neutral one is determined by Feynman loops analogous to those depicted in figure 1. The relevant interaction term in the lagrangian for a member ℓ with charge Q of the multiplet with hypercharge Y , is

$$g \bar{\ell} \gamma^\mu \ell \left\{ \frac{1}{\cos \theta_w} [Q \cos^2 \theta_w - \frac{Y}{2}] Z_\mu + Q \sin \theta_w A_\mu \right\}. \quad (3.1)$$

A calculation similar to the case of a doublet in chapter 2, gives

$$\frac{\delta m}{M_{\text{TeV}}} \simeq 350 \times Q^2 + 170 \times (Y - Q)Q. \quad (3.2)$$

For a doublet ($Y = -1, 1$), or a triplet ($Y = -2, 0, 2$), the neutral member is

the lightest. For higher representations this is not always true. For example, in a $d = 5$ representation, with hypercharge of (-2) , the member with charge $+1$ is the lightest. Also in a $d = 4$ representation, for some values of Y and Q , δm is almost equal to zero and its sign is determined by higher loop corrections.

The calculation of the upper bound on the mass of the lightest member of a d dimensional multiplet is analogous to the case of a doublet. For multiplets with small hypercharge, the annihilation cross section is dominated by hypercharge independent contributions. In figure 3, we plot this upper bound on the mass, M_Ω , for different values of d .

4. The Strongly Interacting Case

The next possibility we are going to discuss is the case of a color triplet particle. The annihilation cross section is dominated by strong color interactions. There are two possible channels, namely into two light fermions or two gluons. The corresponding cross sections are

$$\sigma_{RR-\psi\psi} = \frac{16\pi\alpha_3^2}{9s^2\beta}(s + 2M_R^2), \quad (4.1)$$

and

$$\begin{aligned} \sigma_{RR-gg} = & \frac{\pi\alpha_3^2}{18s\beta} \left\{ 12\left(\frac{2}{3} + \frac{1}{3}(1-\beta^2)\right) \right. \\ & + \frac{16}{3} \left[\frac{4+2(1-\beta^2)}{\beta} \ln\left(\frac{1+\beta}{1-\beta}\right) - 4(1+(1-\beta^2)) \right] \\ & + 6 \left[\frac{2(1-\beta^2)}{\beta} \ln\left(\frac{1+\beta}{1-\beta}\right) - 4(1+(1-\beta^2)) \right] \\ & \left. - \frac{2}{3} 2\beta(1-\beta^2) \ln\left(\frac{1+\beta}{1-\beta}\right) \right\}. \end{aligned} \quad (4.2)$$

In the limit $s = 4M_R^2$, the total annihilation cross section becomes

$$\sigma_{RR-\text{anything}}^v = \frac{43}{27} \frac{\pi\alpha_3^2}{M_R^2}, \quad (4.3)$$

where $\alpha_3 \sim 1/13$ at the TeV scale. Substituting the above result in equation (2.15),

with $x_f = 27$, gives an upper bound on the mass of

$$\frac{M_R}{\text{TeV}} = 10.7 \sqrt{\Omega h_0^2}. \quad (4.4)$$

5. Experimental Bounds

One basic idea of dark matter detection, is to detect the energy deposited by nuclei recoiling from elastic scattering with halo particles. Recent experiments have put stringent constraints on the strength of interactions between dark matter particles and some heavy nuclei, such as Silicon and Germanium. Goodman and Willett^[19] have calculated the coherent elastic scattering cross section between a weakly interacting particle and a nucleus. In the mass range of interest to us, this cross section is mass independent. For Germanium detectors, the relevant cross section is

$$\sigma \simeq 8 \times 10^{-32} Y^2 \text{ cm}^2. \quad (5.1)$$

Any cross section larger than

$$\sigma_{Ge} \simeq 3 \times 10^{-32} \left(\frac{M}{10 \text{ TeV}} \right) \text{ cm}^2$$

is excluded.^[19] Therefore, for non-zero values of hypercharge, particles with masses up to $M_{Ge} = 25 Y^2 \text{ TeV}$ are ruled out as comprising the halo of our galaxy.^[19] This excludes all models with multiplets of dimension ten and less, non-zero hypercharge and stable neutral lightest member. In models with zero hypercharge, the neutral member does not have neutral current interactions, and hence, is very difficult to detect. Such models are not constrained by the experimental data. The representation with the smallest dimension, satisfying all of the above constraints is a triplet with hypercharge zero. The upper bound on the mass of the neutral member is $6.6 \sqrt{\Omega h_0^2} \text{ TeV}$. This neutrino interacts with Germanium nuclei via two

W-exchange. The effective coupling is estimated to be two orders of magnitude smaller than ordinary weak interactions, proceeding through Z^0 exchange. Thus, the predicted cross section with Ge is at least four orders of magnitude smaller than the upper bound set by the experiment of reference 13.

In the case of a strongly interacting particle, we found the closure mass to be $\sim 11 \text{ TeV}$. During the QCD phase transition, such a particle would form heavy pion or baryon states with ordinary quarks. Even if such bound states are neutral, so that the limits on charged dark matter do not apply, their strong interactions with matter would make them unlikely dark matter candidates. In particular, a silicon semiconductor detector near the top of the atmosphere was used to search for strongly interacting particles.^[14] This experiment has excluded all masses between 2 GeV and 10^2 TeV . However, a window in this mass range might still be open for strongly interacting dark matter.^[15]

6. Conclusion

In this paper we have examined simple non-supersymmetric extensions of the standard model, which could lead to an $\Omega = 1$ universe and thus solve the dark matter problem. We have classified the possible models according to the $SU(3) \times SU(2) \times U(1)$ quantum numbers of the new particles. In order to perform analytic calculations, it was necessary to consider a theory which would remain perturbative for large masses of the members of the new sector. This has led us to the addition of an anomaly free vector multiplet of heavy Dirac fermions. Giving these fermions only ordinary gauge interactions has allowed us to compute their closure mass as a function of their gauge quantum numbers. Our main conclusions are:

I) The majority of models are excluded as dark matter candidates, even if a cosmic asymmetry is allowed. For masses beneath 40 GeV , they contribute too much to the Z^0 width. From 40 GeV to some mass $M_{Ge}(Y)$ in the multi-TeV region, they are excluded by direct searches for dark matter in the halo. Above some mass $M_{\Omega}(d, Y)$, they overclose the universe. A major part of this paper had to do with

calculating $M_{\Omega}(d, Y)$ and showing that it is less than $M_{Ge}(Y)$, thus restricting the allowed window.

II) Models with $Y = 0$ are not excluded, since in such cases the dark matter particle does not couple to the Z^0 . The simplest example is the weak triplet (R^+, R^0, R^-) . In the absence of a cosmic asymmetry, the lightest member R^0 , can be the dark matter only if it has a mass of $6.6 \sqrt{\Omega h^2} \text{ TeV}$. The almost degenerate members of this triplet can all be produced at SSC or LHC.

Acknowledgements: S.D. acknowledges support from the NSF under Grant PHY 86/17280. R.E. acknowledges support from the NSF Office of Science and Technology Centers, under cooperative agreement AST-8809616. L.H. is partially supported by a Sloan Fellowship and by a Presidential Young Investigator Award. N.T. acknowledges support from the DOE under the grant number DE-FG03-88ER40472, and the NSF under Grant PHY 88/01426.

APPENDIX

In this appendix, we list the annihilation cross sections for a general multiplet. The notation is identical to the case of a doublet discussed in the text.

$$\begin{aligned}\sigma_{RR-2A} &= \frac{\pi\alpha_s^2}{2d^2s\beta} \left\{ A_{ss} \left(\frac{2}{3} + \frac{1}{3}(1-\beta^2) \right) \right. \\ &\quad + A_{uu} \left[\frac{4+2(1-\beta^2)}{\beta} \ln \left(\frac{1+\beta}{1-\beta} \right) - 4(1+(1-\beta^2)) \right] \\ &\quad + A_{us} \left[\frac{2(1-\beta^2)}{\beta} \ln \left(\frac{1+\beta}{1-\beta} \right) - 4(1+(1-\beta^2)) \right] \\ &\quad \left. + A_{tu} 2\beta(1-\beta^2) \ln \left(\frac{1+\beta}{1-\beta} \right) \right\},\end{aligned}$$

where

$$\begin{aligned}A_{ss} &= \frac{(d^2-1)d}{2} \\ A_{uu} &= \frac{(d^2-1)^2d}{16} \\ A_{us} &= \frac{4}{(d^2-1)d} \\ A_{tu} &= \frac{(d^2-1)d(d^2-5)}{16}.\end{aligned}$$

$$\begin{aligned}\sigma_{RR-2B} &= \frac{\pi\alpha_s^2}{32d^2s\beta} \tan^4\theta_w \left\{ A_{uu} \left[\frac{4+2(1-\beta^2)}{\beta} \ln \left(\frac{1+\beta}{1-\beta} \right) \right. \right. \\ &\quad \left. \left. - 4(1+(1-\beta^2)) \right] + A_{tu} [2\beta(1-\beta^2) \ln \left(\frac{1+\beta}{1-\beta} \right)] \right\},\end{aligned}$$

where

$$\begin{aligned}A_{uu} &= dY^2 \\ A_{tu} &= A_{uu}.\end{aligned}$$

$$\begin{aligned}\sigma_{RR-AB} &= \frac{\pi\alpha_s^2}{4d^2s\beta} \tan^2\theta_w \left\{ A_{uu} \left[\frac{4+2(1-\beta^2)}{\beta} \ln \left(\frac{1+\beta}{1-\beta} \right) \right. \right. \\ &\quad \left. \left. - 4(1+(1-\beta^2)) \right] + A_{tu} 2\beta(1-\beta^2) \ln \left(\frac{1+\beta}{1-\beta} \right) \right\},\end{aligned}$$

where

$$\begin{aligned}A_{uu} &= \frac{(d^2-1)}{4} dY^2 \\ A_{tu} &= A_{uu}.\end{aligned}$$

$$\sigma_{RR-\psi\bar{\psi}} = \frac{2\pi\alpha_s^2}{3s^2\beta} \frac{1}{4d^2} \{ 6(d^2-1)d + 10dY^2 \tan^4\theta_w \} (s + 2M_R^2).$$

$$\sigma_{RR-HH} = \frac{\pi\alpha_s^2}{3s^2\beta} \frac{1}{4d^2} \left\{ \frac{(d^2-1)d}{2} + \frac{dY^2}{2} \tan^4\theta_w \right\} (s + 2M_R^2).$$

REFERENCES

1. G. M. Fuller, G. J. Mathews and C. R. Alcock, *Phys. Rev. D* **37** (1988) 1380, S. Dimopoulos, R. Esmailzadeh, L. J. Hall and G. D. Starkman, *Phys. Rev. Lett.* **60** (1988) 7, and *Ap. J.* **330** (1988) 545,
E. D. Carlson, R. Esmailzadeh, L. J. Hall and S. D. Hsu, UCB-PTH-90/03 and LBL-28609.
2. L. J. Hall and S. Hsu, UCB-PTH-89/30.
3. B. W. Lee and S. Weinberg, *Phys. Rev. Lett.* **39** (1977) 165.
4. Mark II Collaboration (Alan J. Weinstein, et al.), CALT-68-1591, Aug. 1989.
5. OPAL Collaboration (M.Z. Akrawy, et al.), CERN-EP/89-133, Oct. 1989.
6. K. Griest and J. Silk, CIPA-TH-89-014.
7. A. D. Dolgov and Ya. B. Zeldovich, *Rev. of Mod. Phys.* **53** (1981) 1.
8. K. Olive and M. Turner, *Phys. Rev. D* **25** (1982) 213.
9. S. Dimopoulos, D. Eichler, R. Esmailzadeh and G. Starkman, IASSNS-AST-89/53, Nov. 1989,
R. S. Chivukula, A. G. Cohen, S. Dimopoulos and T. P. Walker, BUHEP-89-31, and
A. Gould, B. T. Draine, R. W. Romani and S. Nussinov, IASSNS-AST-89/35.
10. K. Enqvist, K. Kainulainen and J. Maalampi, *Nuc. Phys. B* **317** (1988) 647.
11. K. Griest and M. Kamionkowski, CIPA-TH-89-013, 1989.
12. M. Goodman and E. Wilten, *Phys. Rev. D* **31** (1985) 3059.
13. D. O. Caldwell, et al., *Phys. Rev. Lett.* **61** (1988) 510.
14. J. Rich, R. Rochia and M. Spiro, *Phys. Lett. B* **194** (1987) 173.
15. G. D. Starkman, A. Gould, R. Esmailzadeh and S. Dimopoulos, IASSNS-AST-90/1.

FIGURE CAPTIONS

- 1) Loop contributions to the mass difference between the charged and the neutral member of a doublet.
- 2) Mass difference between the charged and neutral member of a doublet as a function of the bare mass.
- 3) Upper bound on the mass of the lightest member for d dimensional multiplets, with small hypercharge.