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**POLARIZED ELECTROPRODUCTION AND
THE SPIN OF THE QUARKS INSIDE THE PROTON**

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1. INTRODUCTION - SUMMARY OF THE DATA

Deep inelastic leptonproduction has played a fundamental rôle in the development of the QCD-improved parton model. This set of processes is important because of its simplicity. The processes are initiated by leptons (with no strong interactions) and are totally inclusive in the hadronic final state. As is also the case for the total hadronic e^+e^- cross-section at large centre-of-mass energy Q , these properties make a very clean theoretical approach possible for leptonproduction in the deep inelastic region. But leptonproduction has a much richer structure than the hadronic e^+e^- cross-section. First, there are a number of structure functions for each process and several processes are induced by different lepton beams. Then, the structure functions depend on two variables, the squared four-momentum transfer at the lepton vertex $q^2 = -Q^2 < 0$ and the Bjorken variable $x = \frac{Q^2}{2(pq)}$ with p_μ being the nucleon target four-momentum ($0 \leq x \leq 1$). Thus while the hadronic e^+e^- cross-section is one single function of Q^2 the leptonproduction structure functions are several functions of both x and Q^2 ; a much wider theoretical laboratory. Over the years the experimental study of unpolarized deep inelastic scattering has led to the determination of the different parton densities in the nucleon and of their Q^2 evolution in good agreement with the parton model and QCD.

Recently the EMC Collaboration at CERN has published [1] very interesting new data on the deep inelastic scattering of polarized muons on polarized protons. Together with previous data from SLAC [2], these results allow for a reasonably accurate determination of the polarized proton structure function in the range $0.01 \leq x \leq 0.7$. These data have suscitated a particular interest because they appear to imply that the

total helicity carried by quarks and antiquarks in the proton is compatible with zero. Since we pretend to well understand deep inelastic scattering, the question is whether experiment is not providing us with an important message of theoretical significance.

These lectures are intended to provide a review of the present status of this issue [3]. We will discuss what really is the problem and some theoretical ideas that have been proposed for its solution. We shall see that the subject is quite interesting, rich of non-trivial aspects and of intriguing connections between perturbative and non-perturbative sectors of QCD, still not completely elucidated as the lively ongoing debate is demonstrating.

2. THE DATA

The quantity which is measured is the asymmetry A defined by:

$$A = \frac{d\sigma_{\uparrow\downarrow} - d\sigma_{\uparrow\uparrow}}{d\sigma_{\uparrow\downarrow} + d\sigma_{\uparrow\uparrow}} \quad (2.1)$$

where the difference in the numerator is between the cross-sections from left-handed muons on a proton at rest, with its spin along the direction of the μ beam and opposite to it. By parity it would be the same (since Z exchange is negligible at present Q^2 values) to measure the difference between the cross-sections from left- and right-handed muons on a proton with spin in the μ direction. We refer mostly to this configuration in the following. Neglecting small terms (down by powers of the lepton energy) the polarized proton structure function $g_1(x, Q^2)$ is obtained from A by the relation

$$g_1(x, Q^2) = \frac{AF_2(x, Q^2)}{2Dx(1 + R(x, Q^2))} = \frac{AF_1(x, Q^2)}{D} \quad (2.2)$$

where D is a known kinematic factor and $R = F_L/F_T$, i.e. the ratio of the longitudinal and transverse structure functions ($F_T \simeq 2F_1$, $F_L = F_2/x - 2F_1$). It is important to note that in order to extract g_1 from A one must know F_1 or F_2 on protons. There is a well-known discrepancy between the values of F_2 on protons at small x measured by the EMC and the BCDMS (the EMC value is smaller by 10%-15% than that of BCDMS at $x \simeq 0.1 - 0.3$). This ambiguity is taken into account by averaging over different determinations of F_2 and including the dispersion in the error quoted (see Table 9 of Ref. [1]).

The results on g_1^p obtained by the EMC and at SLAC are plotted in Fig. 1.

The corresponding value for the integral $\int_{x_m}^1 g_1^p(x, Q^2)dx$ as a function of x_m is plotted in Fig. 2.

- Figure 1 -

The first moment of g_1^P , when EMC and SLAC data are combined and the statistical and systematic errors are added in quadrature, is obtained to be:

$$\int_0^1 g_1^P(x, Q^2) dx = 0.126 \pm 0.018 \quad (2.3)$$

The average value of Q^2 for the EMC (SLAC) data is $\langle Q^2 \rangle = 10.7 \text{ GeV}^2$ ($< Q^2 > \sim 5 \text{ GeV}^2$). The actual value of Q^2 is different at each value of x . For the EMC data it increases with x from $Q^2 \simeq 3.5 \text{ GeV}^2$ for $x \simeq 0.01 - 0.02$ up to $Q^2 \simeq 29.5 \text{ GeV}^2$ for $x \simeq 0.40 - 0.70$. The SLAC and EMC data can be combined because within the accuracy of the data there is no visible Q^2 dependence of $A_1 = A/D$ at all measured values of x , as is seen from Fig. 3.

This is an important experimental fact and we shall come back to it in the following. The result quoted in Eq. (2.3) is derived by assuming a smooth extrapolation at

- Figure 2 -

unmeasured values of x for $x \lesssim 0.01$. Some authors [4] have questioned the validity of this extrapolation which is based on conventional Regge behaviour. However, there is no *a priori* theoretical reason to suspect that the polarized structure function is special at small x . Thus we assume in the following that $x \simeq 0.01$ is small enough and that approximate conventional Regge behaviour is legitimate.

3. NAIVE PARTON MODEL AND THE CONSTITUENTS OF THE PROTON

In the naive parton model we have:

$$g_1^P(x) = \frac{1}{2} \sum_i e_i^2 \delta q^i(x) = \frac{1}{2} \sum_i e_i^2 [q_+^i(x) - q_-^i(x)] \quad (3.1)$$

where $q_{\pm}^i$ are the densities of quarks with helicity $\pm$ in the proton with helicity $+$, e_i is the corresponding electric charge and the sum runs over all excited flavours of quarks

and antiquarks. We understand the appearance of the differences $q_4^+ - q_4^-$ by going in the Breit frame for the muons, where the space-like virtual photon carries no energy. In this frame (as a consequence of the vector coupling of the photon) the momentum and the spin of the muon are flipped and the differences are directly passed to the photon. This is seen in Fig. 4.

- Figure 4 -

The outgoing muon has exactly opposite momentum and spin with respect to the incoming muon. The conservation of angular momentum along the common direction of flight for the muons implies that the spin of the photon is in the direction of the initial muon. At the quark vertex exactly the same situation is reproduced provided that one can neglect the intrinsic transverse momentum of the quarks inside the target. Then only a quark with the same helicity as the photon can conserve angular momentum. In conclusion, starting from a left-handed (right-handed) muon we count left-handed (right-handed) quarks or antiquarks in the proton. Thus, Eq. (3.1) is reproduced (the factor of $\frac{1}{2}$ is a matter of normalization for g_1^p). Note, for comparison, that the unpolarized structure function F_1 is given by

$$F_1(x) \simeq \frac{1}{2} \sum_i e_i^2 q^i(x) = \frac{1}{2} \sum_i e_i^2 [q_4^+(x) + q_4^-(x)] \quad (3.2)$$

The unpolarized parton densities $q^i(x)$ are the sum of the corresponding densities with helicities + and -.

For g_1 on protons and on neutrons at values of Q^2 where heavy quarks in the nucleon are not excited, we have

$$g_1^p = \frac{1}{2} \left[\frac{4}{9} \delta u + \frac{1}{9} \delta d + \frac{1}{9} \delta s \right] \quad (3.3)$$

- Figure 3 -

$$g_1^n = \frac{1}{2} \left[\frac{4}{9} \delta d + \frac{1}{9} \delta u + \frac{1}{9} \delta s \right] \quad (3.4)$$

For economy of notation, we have included in δq^i both quark and antiquark terms. For example:

$$\delta u(x) \equiv u_+(x) - u_-(x) + \bar{u}_+(x) - \bar{u}_-(x) \quad (3.5)$$

We also denote by Δq the first moment of δq . For example:

$$\Delta u = \int_0^1 dx \delta u(x) \quad (3.6)$$

so that:

$$\int_0^1 dx g_1^p(x) \equiv M_1^p = \frac{1}{2} \left[\frac{4}{9} \Delta u + \frac{1}{9} \Delta d + \frac{1}{9} \Delta s \right] \quad (3.7)$$

$$\int_0^1 dx g_1^n(x) \equiv M_1^n = \frac{1}{2} \left[\frac{4}{9} \Delta d + \frac{1}{9} \Delta u + \frac{1}{9} \Delta s \right] \quad (3.8)$$

Δq is the total number of right-handed minus left-handed quarks and antiquarks in a proton with spin up. While the number density is (the fourth component of) a vector current, number times spin leads to the axial current: $q_R \gamma_\mu q_R - q_L \gamma_\mu q_L = q \gamma_\mu \gamma_5 q$ where $q_{R,L} = \frac{1 \pm \gamma_5}{2}$. The relation is of the form

$$\Delta q \, s^\mu = \langle p, s = +\frac{1}{2} | \bar{q} \gamma^\mu \gamma_5 q | p, s = +\frac{1}{2} \rangle \quad (3.9)$$

where s^μ is the polarization four-vector.

Note that for unpolarized densities $\int_0^1 dx (q - \bar{q})$ is a charge, while for polarized densities it is $\int_0^1 dx (q_+ + \bar{q}_+ - q_- - \bar{q}_-)$ (i.e. the sum of q and $\bar{q}$) which is connected to the axial current, because of the opposite charge-conjugation properties of γ_μ and $\gamma_\mu \gamma_5$.

The connection with the axial current implies for example that

$$\Delta u - \Delta d = \frac{g_A}{g_V} = F + D \quad (3.10)$$

where $g_A/g_V = 1.254 \pm 0.006$ is the measured axial-vector isovector coupling of the nucleon, and D, F are the two $SU(3)_{flavour}$ reduced matrix elements for the octet axial

current sandwiched between baryon octet states. Note the Bjorken sum rule [5]:

$$\int_0^1 dx [g_1^p(x) - g_1^n(x)] = \frac{1}{6} (\Delta u - \Delta d) = \frac{1}{6} g_A/g_V \quad (3.11)$$

This sum rule is only mildly modified by QCD corrections that, at order α_s , lead to:

$$\begin{aligned} \int_0^1 dx [g_1^p(x, Q^2) - g_1^n(x, Q^2)] &= \frac{1}{6} (\Delta u - \Delta d) \left(1 - \frac{\alpha_s(Q)}{\pi}\right) \\ &= \frac{1}{6} \frac{g_A}{g_V} \left(1 - \frac{\alpha_s(Q)}{\pi}\right) \end{aligned} \quad (3.12)$$

Similarly, one has

$$\Delta u + \Delta d - 2\Delta s = 3F - D \quad (3.13)$$

for the isoscalar combination of axial currents that transform as the hypercharge under $SU(3)_{flavour}$. From a fit to hyperon decays one has found [6]

$$\begin{aligned} F &= 0.477 \pm 0.011 \\ D &= 0.755 \pm 0.011 \end{aligned} \quad (3.14)$$

Note that $F/D = 0.632 \pm 0.024$. Equations (3.10) and (3.13) give two inputs for the three quantities $\Delta u, \Delta d, \Delta s$. Clearly if we add the EMC/SLAC result all three of them can be determined. By also including the leading QCD corrections (for $N_f = 3$ where N_f is the number of excited flavours), we have:

$$\begin{aligned} \int_0^1 dx g_1^p(x, Q^2) &= \frac{1}{12} \left\{ (\Delta u - \Delta d) + \frac{1}{3} (\Delta u + \Delta d - 2\Delta s) \cdot \left(1 - \frac{\alpha_s(Q)}{\pi}\right) \right. \\ &\quad \left. + \frac{4}{3} (\Delta u + \Delta d + \Delta s) \left(1 - \frac{\alpha_s(Q)}{3\pi}\right) \right\} \end{aligned} \quad (3.15)$$

At relevant values of Q^2 these QCD corrections are irrelevant within the present accuracy. Solving for $\Delta u, \Delta d$ and Δs one finds [1, 2]

$$\Delta \Sigma \equiv \Delta u + \Delta d + \Delta s = 0.12 \pm 0.17 \quad (3.16)$$

and

$$\begin{aligned} \Delta u &= 0.78 \pm 0.06 \\ \Delta d &= -0.47 \pm 0.06 \\ \Delta s &= -0.19 \pm 0.06 \end{aligned} \quad (3.17)$$

Equation (3.16) (where we also introduced the notation $\Delta \Sigma$) is the by now famous result that the total helicity carried by all quarks and antiquarks in a polarized proton is small and actually compatible with zero.

It is important at this point to clearly state what the problem really is. The question has to do with the relation of the EMC result with the constituent quark model of the proton. In this model the spin of the proton is carried by the three constituent quarks. Should one abandon this familiar and intuitive picture? The answer is a qualified no. The EMC result refers to parton quarks and not to constituent quarks. It is perfectly possible to imagine that the three constituents carry all of the proton spin. Yet, if we could do polarized electron scattering on a polarized constituent quark, we would find, according to the EMC, that the spin of the constituent quark is not carried by the parton quarks inside the constituent. In a model where the proton is made up of three constituent quarks (two U 's and one D), each with a structure in terms of partons, one obtains quite generally:

$$\begin{aligned}\Delta u - \Delta d &= (\Delta U - \Delta D)(\Delta u - \Delta d)_U \\ \Delta u + \Delta d - 2\Delta s &= (\Delta U + \Delta D)(\Delta u + \Delta d - 2\Delta s)_U \\ \Delta u + \Delta d + \Delta s &= (\Delta U + \Delta D)(\Delta u + \Delta d + \Delta s)_U\end{aligned}\quad (3.18)$$

where Δq_U is the total spin carried by the parton quarks q inside the constituent quark U . Only U and D constituents are assumed to be in the proton and Δq_D does not explicitly appear because of the isospin symmetry relations $\Delta u_D = \Delta d_U$, $\Delta d_D = \Delta u_U$, $\Delta s_D = \Delta s_U$. The simplest constituent model, not necessarily exact, is the $SU(6)$ model where the octet and decuplet baryons belong to the 56-dimensional representation of the group. In this model $\Delta U = 4/3$, $\Delta D = -1/3$. The $SU(6)$ value of g_A/g_V is too large ($g_A/g_V = \Delta U - \Delta D = 5/3$), while the F/D ratio is nearly correct ($F/D = 2/3$). Going back to Eq. (3.18), we see that if $\Delta U - \Delta D$ is given by $SU(6)$, then $(\Delta u - \Delta d)_U \sim 0.75$, i.e. the g_A/g_V of the constituent quark U is not 1 but ~ 0.75 . Similarly $\Delta U + \Delta D = 1$ in $SU(6)$, but $\Delta u + \Delta d$ is not necessarily 1 in the physical proton because of the extra factor $(\Delta u + \Delta d)_U$. Thus the EMC result is not directly in contradiction with the constituent quark model. Similarly, the problem is not so much the fact that Δs is found to be rather large and negative, in contradiction with the intuitive expectation by Ellis and Jaffe [7] that $\Delta s \sim 0$. Note that if one assumes that $\Delta s = 0$, then from $\Delta u - \Delta d = g_A/g_V = F + D$ and $\Delta \Sigma = \Delta u + \Delta d = 3F - D$ one obtains

$$\begin{aligned}\Delta u_{EJ} &= \frac{1}{2}g_A/g_V \left[1 + \frac{3F/D - 1}{F/D + 1} \right] \simeq 0.97 \\ \Delta d_{EJ} &= \frac{1}{2}g_A/g_V \left[\frac{3F/D - 1}{F/D + 1} - 1 \right] \simeq -0.28\end{aligned}\quad (3.19)$$

so that $\Delta \Sigma_{EJ} \simeq 0.69$ and $\int_0^1 g_1^p(x)dx \simeq 0.20$. The real problem is to understand the dynamical reason for the large difference between constituent quarks and parton quarks. In fact in the unpolarized case the difference between constituent and parton

quarks can be well understood, at least at a qualitative level, by extrapolating the perturbative QCD evolution down to small values of the energy scale Q [8]. In fact at $Q^2 = 1 - 2 \text{ GeV}^2$, i.e. at the border of the perturbative region, one observes relative proportions and x distributions for valence, gluon and sea densities which are in semi-quantitative but significant agreement with the expectation derived from three valence quarks at some low value of Q^2 and a reasonable extrapolation of the perturbative QCD evolution outside its domain of validity. The same approach applied to the polarized case appears to lead to a contradiction with the EMC result because the spin carried by each type of quark and antiquark is conserved by leading-order QCD evolution [9, 10]. Thus one cannot in a similar way understand the drastic difference between constituent quarks, which carry all of the proton spin in naive $SU(6)$, or at least around 70% of it, and parton quarks, which, according to the conclusion derived from the EMC experiment and the naive parton model, give a nearly vanishing contribution to it.

An explanation based on a possible rapid Q^2 dependence [11] (the first moment of the polarized proton structure function is known to change sign [12] at $Q^2 = 0$ owing to the Drell-Hearn sum rule [13]) appears to be drastically limited by experiment. In fact data at different Q^2 exist, down to Q^2 as low as $Q^2 = 0.5 \text{ GeV}^2$ [14] and no appreciable Q^2 dependence is visible, as already mentioned, so that one can at most try to take advantage of the error bars. Indeed it is found that the asymmetry becomes strongly negative at the N^* resonance [14]. Thus one can understand the validity of the Drell-Hearn sum rule as an effect of the dominance of the N^* resonance at $Q^2 = 0$. However, the corresponding fluctuation of the asymmetry is very local. Note that in Ref. [14] it is found that even at $Q^2 = 0.5 \text{ GeV}^2$ the scaling curve is a good average (in the Bloom-Gilman sense [15]) of the fluctuating asymmetry measured in the resonance region. Also note that the N^* resonance is far outside the x range measured by the EMC near $x = 1$. Thus, only some small part of the deviation from the Ellis-Jaffe prediction can be attributed to this effect, while most of the discrepancy remains to be explained.

A possible solution to this problem has been indicated in Refs. [16, 17] (although in Ref. [16] the quantitative consequences of the basic idea were not correctly developed). It was shown there that, because of the axial anomaly [18], the gluon contribution to the first moment of g_1^p is not suppressed by a power of the QCD running coupling α_s , evaluated at a large scale. As a consequence, the EMC result can after all be consistent with a large quark-spin component. What the experiment shows is that the matrix element between polarized protons of the flavour-singlet axial current is nearly zero. But, owing to the axial anomaly, an *a priori* non-negligible gluon contribution is present in the singlet axial current matrix element, so that in principle the vanishing

of the quark-spin term no longer follows from the EMC result. Note that this mechanism, while perhaps reconciling a large quark-spin contribution with the data, does not explain why the singlet axial current matrix element is nearly vanishing. It was argued in Ref. [19] that in the large N_c limit and for massless quarks the singlet axial current matrix element might indeed be suppressed. This result could be an important complementary input to the understanding of the problem. It would be very important to prove this statement in a model-independent way, since the argument of Ref. [19] is formulated on the basis of the Skyrme model [20].

4. THE ANOMALOUS GLUON COMPONENT

In this section we summarize the results obtained in Refs. [17, 21] and discuss a number of important points. We denote by $M_1^S(Q^2)$ the singlet component of the first moment of the polarized proton structure function g_1^p :

$$M_1^S(Q^2) \equiv \frac{2}{<e^2>} \int_0^1 dx g_1^p(x, Q^2)|_{\text{singlet}} \quad (4.1)$$

We consider the polarized quark and gluon densities $\delta q(x, Q^2)$ [see Eqs. (3.1), (3.5)], $\delta g(x, Q^2)$, and their first moments $\Delta q(Q^2)$, $\Delta g(Q^2)$. In particular:

$$\Delta g(Q^2) \equiv \int_0^1 dx \delta g(x, Q^2) \quad (4.2)$$

with

$$\delta g(x, Q) \equiv g_+(x, Q) - g_-(x, Q) \quad (4.3)$$

where $g_{\pm}$ are the gluon densities with helicity ± 1 in a polarized proton with helicity $+\frac{1}{2}$.

The first step in the argument is to notice that $\Delta g(Q^2)$ increases logarithmically with Q^2 , as a consequence of the well-known QCD evolution equations for polarized quark and gluon distributions. This logarithmic increase exactly compensates the decrease of $\alpha_s(Q^2)$ with Q^2 . Thus both the quantities $\Delta \Sigma$ and $\Delta \Gamma = \alpha_s/2\pi \Delta g$ are conserved if all effects of order α_s^2 are neglected in $Qd\Delta\Sigma/dQ$ and $Qd\Delta\Gamma/dQ$. This implies that if a contribution of Δg to M_1^S is induced at order α_s , the gluon component will be proportional to $\Delta \Gamma$ and will not decouple at large Q^2 .

We now discuss in some detail the QCD parton model formalism in this somewhat special case. In lowest order the QCD evolution equations for the first moment of polarized densities lead with no ambiguity to the results:

$$\begin{aligned} \frac{d}{dt} \begin{pmatrix} \Delta \Sigma \\ \Delta g \end{pmatrix}_t &= \frac{\alpha_t}{2\pi} \begin{pmatrix} \gamma_{qq}^{(1)} & \gamma_{qg}^{(1)} \\ \gamma_{gq}^{(1)} & \gamma_{gg}^{(1)} \end{pmatrix} \begin{pmatrix} \Delta \Sigma \\ \Delta g \end{pmatrix}_t + 0(\alpha_t^2) \\ &= \frac{\alpha_t}{2\pi} \begin{pmatrix} 0 & 0 \\ \frac{3}{2}C_F & \beta_0 \end{pmatrix} \begin{pmatrix} \Delta \Sigma \\ \Delta g \end{pmatrix}_t + 0(\alpha_t^2) \end{aligned} \quad (4.4)$$

where $t = \ln Q^2/\mu^2$ (with μ being a reference scale), $\alpha_t \equiv \alpha_s(t)$, $\Delta \Sigma_t \equiv \Delta \Sigma(t)$, $\Delta g_t \equiv \Delta g(t)$, $C_F = \frac{4}{3}$ and β_0 is defined by

$$\frac{d}{dt} \frac{\alpha_t}{2\pi} = -\beta_0 \left(\frac{\alpha_t}{2\pi} \right)^2 - \beta_1 \left(\frac{\alpha_t}{2\pi} \right)^3 + 0(\alpha_t^4) \quad (4.5)$$

The form of Eq. (4.4) and in particular the fact that $\gamma_{gg}^{(1)} = \beta_0$ suggest the change of variables $(\Delta \Sigma, \Delta g) \rightarrow (\Delta \Sigma, \Delta \Gamma = \frac{\alpha}{2\pi} \Delta g)$ or

$$D' \equiv \begin{pmatrix} \Delta \Sigma \\ \Delta \Gamma \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \frac{\alpha}{2\pi} \end{pmatrix} \begin{pmatrix} \Delta \Sigma \\ \Delta g \end{pmatrix} \equiv M(\alpha) D \quad (4.6)$$

The general solution of the evolution equations for moments has the form:

$$D(\alpha_t) = E(\alpha_t, \alpha) D(\alpha) \quad (4.7)$$

with

$$E(\alpha_t, \alpha) = T \exp \int_0^t \gamma(t') dt' \quad (4.8)$$

and $\alpha \equiv \alpha_s(t=0)$. From Eqs. (4.6) and (4.7) it follows that:

$$D'(\alpha_t) = M(\alpha_t) E(\alpha_t, \alpha) M^{-1}(\alpha) D'(\alpha) \quad (4.9)$$

and

$$\frac{d}{dt} D'(\alpha_t) = \left[\frac{dM}{dt}(\alpha_t) M^{-1}(\alpha_t) + M(\alpha_t) \gamma(\alpha_t) M^{-1}(\alpha_t) \right] D'(\alpha_t) \quad (4.10)$$

where

$$\gamma(\alpha) = \frac{\alpha}{2\pi} \gamma^{(1)} + \left(\frac{\alpha}{2\pi} \right)^2 \gamma^{(2)} + \dots \quad (4.11)$$

is the ‘‘anomalous dimension’’ matrix for $D = \begin{pmatrix} \Delta \Sigma \\ \Delta g \end{pmatrix}$ expanded to one and two loops.

With a little algebra, from Eqs. (4.10) and (4.11), we obtain

$$\frac{d}{dt} \begin{pmatrix} \Delta\Sigma \\ \Delta\Gamma \end{pmatrix}_t = \begin{pmatrix} \alpha_t \\ 2\pi \end{pmatrix}^2 \begin{pmatrix} \gamma_{q\bar{q}}^{(2)} & \gamma_{q\bar{q}}^{(3)} \\ \gamma_{g\bar{g}}^{(1)} & \gamma_{g\bar{g}}^{(2)} - \beta_1 \end{pmatrix} \begin{pmatrix} \Delta\Sigma \\ \Delta\Gamma \end{pmatrix}_t + 0(\alpha_t^3) \quad (4.12)$$

Here the explicit form of the matrix $\gamma^{(1)}$ given in Eq. (4.4) was taken into account and also the fact that $\gamma_{g\bar{g}}^{(2)}$ must be zero for consistency. Equation (4.12) has been introduced and discussed in Ref. [17] where the vanishing of $\gamma_{g\bar{g}}^{(2)}$ was proved in the massless theory.

An approximate solution of the evolution equation, Eq. (4.12), is given by:

$$D'(\alpha_t) \equiv \begin{pmatrix} \Delta\Sigma \\ \Delta\Gamma \end{pmatrix}_t = \left\{ 1 + \frac{\alpha - \alpha_t}{2\pi \beta_0} \begin{pmatrix} \gamma_{q\bar{q}}^{(2)} & \gamma_{q\bar{q}}^{(3)} \\ \gamma_{g\bar{g}}^{(1)} & \gamma_{g\bar{g}}^{(2)} - \beta_1 \end{pmatrix} \right\} \begin{pmatrix} \Delta\Sigma \\ \Delta\Gamma \end{pmatrix}_{t=0} \quad (4.13)$$

For the physical quantity of interest $M_1^S(Q)$ defined in Eq. (4.1), one has:

$$M_1^S(Q) = c(\alpha_t) D(\alpha_t) = c'(\alpha_t) D'(\alpha_t) \quad (4.14)$$

with

$$\begin{aligned} c'(\alpha) &= c(\alpha) M^{-1}(\alpha) \\ &= \left(1 + \frac{\alpha}{2\pi} c\Sigma + \dots, \frac{\alpha}{2\pi} c + \left(\frac{\alpha}{2\pi} \right)^2 c\Gamma + \dots \right) \cdot \begin{pmatrix} 1 & 0 \\ 0 & \left(\frac{\alpha}{2\pi} \right)^{-1} \end{pmatrix} \\ &= \left(1 + \frac{\alpha}{2\pi} c\Sigma + \dots, c + \frac{\alpha}{2\pi} c\Gamma + \dots \right) \end{aligned} \quad (4.15)$$

By combining Eqs. (4.13) - (4.15), on the one hand, one obtains the result:

$$\begin{aligned} M_1^S(Q) &= \left[1 + \frac{\alpha}{2\pi} c\Sigma + \frac{\alpha - \alpha_t}{2\pi \beta_0} (\gamma_{q\bar{q}}^{(2)} + c\gamma_{g\bar{g}}^{(1)} - \beta_0 c\Sigma) \right] \Delta\Sigma_{t=0} \\ &\quad + \left[c + \frac{\alpha}{2\pi} c\Gamma + \frac{\alpha - \alpha_t}{2\pi \beta_0} (\gamma_{q\bar{q}}^{(3)} + c(\gamma_{g\bar{g}}^{(2)} - \beta_1) - \beta_0 c\Gamma) \right] \Delta\Gamma_{t=0} \end{aligned} \quad (4.16)$$

On the other hand, the light-cone formalism, in terms of the single operator j_μ^S , leads [22-24] to the corresponding expression:

$$M_1^S(Q) = \left[1 + \frac{\alpha}{2\pi} c_j + \frac{\alpha - \alpha_t}{2\pi \beta_0} (\gamma_j^{(2)} - \beta_0 c_j) \right] < j^S >_{t=0} \quad (4.17)$$

By comparison one obtains:

$$\begin{aligned} c_j &= c\Sigma = c\Gamma/c \\ \gamma_j^{(2)} &= \gamma_{q\bar{q}}^{(2)} + c\gamma_{g\bar{g}}^{(1)} = \gamma_{q\bar{q}}^{(3)}/c + \gamma_{g\bar{g}}^{(2)} - \beta_1 \\ &< j^S >_t = (\Delta\Sigma + c \Delta\Gamma)_t \end{aligned} \quad (4.18)$$

Finally one can write:

$$M_1^S(Q) = \left[1 + \frac{\alpha}{2\pi} c\Sigma + \frac{\alpha - \alpha_t}{2\pi \beta_0} (\gamma_{q\bar{q}}^{(2)} + c\gamma_{g\bar{g}}^{(1)} - \beta_0 c\Sigma) \right] (\Delta\Sigma + c\Delta\Gamma)_{t=0} \quad (4.19)$$

This is an important result which makes the relation explicit between the operator formalism and the parton method.

Both the approximate Q independence of $\Delta\Gamma$ and the value of its coefficient in the expression of M_1^S can be derived from the operator product expansion and the known value of the axial anomaly. There is only one single gauge-invariant operator of dimension three with the appropriate quantum numbers to contribute to M_1^S , i.e. the flavour-singlet axial vector current j_μ^S :

$$j_\mu^S = \sum_{i=1}^{N_f} \bar{q}_i \gamma_\mu \gamma_5 q_i \quad (4.20)$$

Thus M_1^S does indeed measure the diagonal matrix element between polarized proton states of the flavour-singlet axial current.

Even for massless quarks j_μ^S is not conserved because of the anomaly:

$$\partial^\mu j_\mu^S = N_f \frac{\alpha_s}{2\pi} \text{Tr}(F_{\mu\nu} \tilde{F}^{\mu\nu}) = N_f \partial^\mu k_\mu \quad (4.21)$$

where

$$\tilde{F}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}, \quad F_{\mu\nu} = \sum_{i=1}^8 F_{\mu\nu}^i t^i, \quad \text{Tr}(t^i t^j) = \frac{1}{2} \delta^{ij} \quad (4.22)$$

From Eq. (4.21), it follows that the operator $j_\mu^S - N_f k_\mu$ is conserved, for massless quarks, with

$$k_\mu = \frac{\alpha}{2\pi} \epsilon_{\mu\nu\lambda\sigma} \text{Tr}[A^\nu (\tilde{F}^{\lambda\sigma} - \frac{2}{3} A^\lambda A^\sigma)]$$

The operator k_μ is gauge-dependent. Thus it can neither appear in the operator product expansion of physical currents, nor can it mix with j_μ^S . However, the diagonal matrix elements of k_μ are indeed gauge-invariant (at least for gauge transformations that do not change the winding number, which are those relevant to the perturbative case), in that the gauge-dependent part of k_μ can be expressed as a four-divergence. As the parton model is formulated in terms of diagonal matrix elements between quarks and gluons, the operator k_μ can be useful in understanding the relation between the operator product expansion formalism and the parton language.

Schematically, the operator product expansion at the tip of the light cone, relevant for the first moment, reads:

$$JJ \simeq a j^5 \quad (4.23)$$

where a is the coefficient function and all indices have been suppressed for simplicity.

By taking matrix elements between polarized protons one obtains (at the scale $Q = \mu$):

$$\begin{aligned} M_1^S &\sim \int dx e^{iqx} \langle p | J(x) J(0) | p \rangle \\ &\sim a[\Delta\Sigma < q | j^5 | q > + \Delta g < g | j^5 | g >] \\ &\sim a'[\Delta\Sigma + \frac{< g | j^5 | g >}{< q | j^5 | q >} \Delta g] \end{aligned} \quad (4.24)$$

where a' is a reduced coefficient which is 1 in lowest order. Owing to the anomaly, $< g | j_\mu^5 | g >$ does not vanish at order α_s . In order to compute the coefficient of Δg in Eq. (4.24), we observe that the operator $j_\mu^5 - N_f k_\mu$, in the limit of massless quarks, is conserved and has vanishing diagonal matrix elements between one-gluon states:

$$< g | j_\mu^5 - N_f k_\mu | g > = 0 \quad (4.25)$$

Similarly the operator k_μ has vanishing diagonal matrix elements between quarks:

$$< q | N_f k_\mu | q > = 0 \quad (4.26)$$

Equations (4.25) and (4.26) are valid at order α_s by construction and can be used to specify at all orders the definition of the quark and gluon first moments, which becomes ambiguous beyond the leading approximation. That is, in parton language, the conserved quantity corresponding to $j_\mu^5 - N_f k_\mu$ is identified with $\Delta\Sigma$ in the sense that the anomalous dimensions, not the quantities, are the same. The light-cone expansion can be written in the form:

$$JJ \simeq a j^5 \simeq a[(j^5 - N_f k) + N_f k] \quad (4.27)$$

and

$$\begin{aligned} M_1^S &\sim a[\Delta\Sigma < q | (j^5 - N_f k) | q > + \Delta g < g | N_f k | g >] \\ &\sim a'[\Delta\Sigma + \frac{< g | N_f k | g >}{< q | j^5 | q >} \Delta g] \end{aligned} \quad (4.28)$$

As a consequence one direct method of obtaining the finite coefficient c of the gluon in M_1^S , already indicated in Ref. [17], is to use the explicit forms of j^5 and k and compute the ratio $< g | N_f k | g > / < q | j^5 | q >$. In this way one does not need to perform a relatively complicated calculation, introduce regulators, etc., because the result of the anomaly-loop diagram is already contained in the expression of k .

By some straightforward algebra one obtains:

$$\begin{aligned} M_1^S &\simeq a'[\Delta\Sigma - N_f \frac{\alpha_s}{2\pi} \Delta g] \\ &\simeq a'[\Delta\Sigma - N_f \Delta\Gamma] \end{aligned} \quad (4.29)$$

The two eigenvectors of the Q^2 evolution are:

$$\begin{aligned} j^5 - N_f k &\rightarrow \Delta\Sigma : Q^2 \frac{d\Sigma}{dQ^2} = 0 \\ j^5 \rightarrow X &\equiv \Delta\Sigma - N_f \Delta\Gamma : Q^2 \frac{dX}{dQ^2} = \frac{[\alpha_s(Q^2)]^2}{2\pi} \gamma_j^{(2)} X + \dots \end{aligned} \quad (4.30)$$

where

$$\gamma_j^{(2)} = -\frac{3N_f C_F}{2\pi} \quad (4.31)$$

is the two-loop anomalous dimension computed by Kodaira [23] (see also Ref. [24]). In fact the result of Ref. [23] can be used as an alternative method of deriving the contribution of $\Delta\Gamma$ to M_1^S . Thus, as discussed in Ref. [17], the value of $\gamma_j^{(2)}$ is reproduced as a product of the coefficient of $\Delta\Gamma$ in Eq. (4.29) times the $g - q$ entry of the one-loop anomalous dimension matrix: $\gamma_j^{(2)} = c\gamma_{gq}^{(1)} = -N_f \gamma_{gq}^{(1)}$. This relation corresponds to the conservation of $j^5 - N_f k$.

The gluon contribution to g_1^F can also be obtained in the parton model approach by convoluting the polarized gluon density with the polarized photon-gluon cross-section

$$\Delta\sigma = \sigma(\gamma + g_+) - \sigma(\gamma - g_+) \quad (4.32)$$

which can be evaluated in lowest order from the diagrams shown in Fig. 5.

In the massless theory one obtains:

$$\begin{aligned} d\Delta\sigma &= \frac{\alpha_s T}{2\pi} [x^2 - (1-x)^2] \left(\frac{1}{1+\cos\theta} + \frac{1}{1-\cos\theta} - 1 \right) dx d\cos\theta \\ &\simeq \frac{\alpha_s T}{2\pi} [x^2 - (1-x)^2] \left(\frac{2}{\sin^2\theta} - 1 \right) dx d\cos\theta \end{aligned} \quad (4.33)$$

where an overall dimensional factor has been dropped. In this result, $x = Q^2/(s + Q^2)$ and $\sqrt{s}$, θ are the $\gamma - g$ centre-of-mass total energy and angle. The cross-section is normalized in such a way that the factor in front of the logarithmically divergent terms $(1 \pm \cos\theta)^{-1}$ is $\Delta P_{qg} = \alpha_s T / 2\pi (x^2 - (1-x)^2)$ with $T = N_f/2$, i.e. it is precisely the polarized splitting function [9, 10]. As the first moment of ΔP_{qg} evidently vanishes there is no $\log Q^2$ term at order α_s in the contribution of Δg to M_1^S . Thus

- Figure 5 -

the corresponding finite term does not suffer from scale ambiguities and is completely specified once the definition of $\Delta\Sigma$ has been fixed. In particular we have adopted the definition of $\Delta\Sigma$ as the conserved quantity which corresponds to $j_\mu^5 - N_f k_\mu$ in the massless theory. Thus we expect the same finite term for all regularization procedures that do not contrast with this definition, for example by breaking chiral invariance. The importance of the infra-red regulator was fully discussed in the interesting work quoted in Ref. [25]. It is remarkable in this respect that one can extract the finite coefficient from $\Delta\sigma$ in Eq. (4.33), even without explicitly introducing a regularization, by simply changing variables from x to k_T [25], the transverse momentum with respect to the photon-gluon axis of the final quark or antiquark:

$$k_T^2 = \frac{Q^2}{4x} (1-x) \sin^2 \theta \quad (4.34)$$

With a little algebra, one can in fact cast $\Delta\sigma$ in the form:

$$d\Delta\sigma = \frac{\alpha_s T}{2\pi} \left[\frac{-1}{\lambda + \sin^2 \theta} + \frac{2 + 3\lambda}{(\lambda + \sin^2 \theta)^2} - \frac{2\lambda(\lambda + 2)}{(\lambda + \sin^2 \theta)^3} \right] d\lambda \, d\cos \theta \quad (4.35)$$

where $\lambda = 4k_T^2/Q^2$. At fixed λ the integration over $\cos \theta$ is now finite and leads to the result:

$$\frac{d\Delta\sigma}{d\lambda} = -\frac{\alpha_s T}{2\pi} \left\{ \frac{\lambda(\lambda + 4)}{4(\lambda + 1)^{5/2}} \ln \frac{\sqrt{\lambda + 1} + 1}{\sqrt{\lambda + 1} - 1} + \frac{2 - \lambda}{2(\lambda + 1)^2} \right\} \quad (4.36)$$

The k_T distribution of the gluon term obtained in Eq. (4.36) is interesting *per se*.

The integration over λ is convergent both at $\lambda = 0$ ($d\Delta\sigma/d\lambda = \text{const.}$) and at $\lambda \rightarrow \infty$ ($d\Delta\sigma/d\lambda \sim 1/\lambda^2$), and one obtains:

$$\int_0^\infty \frac{d\Delta\sigma}{d\lambda} d\lambda = -\frac{\alpha_s}{2\pi} 2T = -\frac{\alpha_s}{2\pi} f \quad (4.37)$$

which is equivalent to Eq. (4.29).

Alternatively one can introduce a small off-shell squared mass p^2 for the external gluon. As computed in Ref. [25] one finds (in our notation):

$$d\Delta\sigma = \frac{\alpha_s T}{2\pi} [x^2 - (1-x)^2] \times \\ \times \left\{ \frac{2}{\sin^2 \theta + \frac{4p^2}{s} x(1-x)} - \frac{8p^2}{s} \frac{x(1-x)}{(\sin^2 \theta + \frac{4p^2}{s} x(1-x))^2} - 1 \right\} dx \, d\cos \theta + \dots \quad (4.38)$$

Here the ellipsis indicates negligible contributions to $\Delta\sigma$ (of higher order in p^2/s). A simple calculation shows that, once the integration over $\cos \theta$ is performed, the only finite contribution to $\Delta\sigma$ in the limit $p^2 = 0$ arises from:

$$\Delta\sigma = \frac{\alpha_s T}{2\pi} \int_0^1 dx [x^2 - (1-x)^2] 2 \ln \frac{s}{p^2} \\ = \frac{\alpha_s T}{2\pi} \int_0^1 dx [x^2 - (1-x)^2] 2 \ln \frac{Q^2(1-x)}{p^2 x} \\ = -\frac{\alpha_s}{2\pi} 2T \quad (4.39)$$

so that the correct result is again reproduced.

In Ref. [17] the collinear singularity was regularized by restoring the quark mass in the denominators, i.e., by replacing $(1 \pm \cos \theta)^{-1}$ by $(1 \pm \beta \cos \theta)^{-1}$, with $\beta = p/E \cong 1 - 2m^2/s = 1 - 2m^2 x/Q^2(1-x)$. This immediately leads to the correct result. This procedure was criticized in Ref. [25]. In fact, in the case of a physical massive quark, the complete expression for $d\Delta\sigma$ reads [25, 26]:

$$d\Delta\sigma = \frac{\alpha_s T}{2\pi} \times \\ \times \left\{ [x^2 - (1-x)^2] \left[\frac{2}{1 - \beta^2 \cos^2 \theta} - 1 \right] + \frac{16m^2}{s} \frac{1-x}{(1 - \beta^2 \cos^2 \theta)^2} + \dots \right\} dx \, d\cos \theta \quad (4.40)$$

where terms of higher order in m^2/s have been omitted. Actually the integral of $d\Delta\sigma$ over $\cos \theta$ and x gives a vanishing result [25] in this case. However, as discussed in

detail in Section 5, the effect of the additional double pole term is exactly cancelled by the correction that must be added in order to maintain the definition of $\Delta\Sigma$ as a conserved quantity. In fact the mass m breaks the conservation of $j_\mu^5 - N_f k_\mu$ and the double pole term exactly compensates the corresponding appearance of a non-vanishing contribution to the two-loop anomalous dimension of $j_\mu^5 - N_f k_\mu$. Thus for non-vanishing m , $\langle g|j^5|g \rangle$ differs from $\langle g|N_f k|g \rangle$ by an additional term which must be taken into account.

5. CRITICISM AND DEFENCE OF THE ANOMALOUS GLUON COMPONENT

The explanation in terms of an anomalous gluon component has been criticized on different points. In the light-cone formalism [22-24] only one operator couples to M_1^S : the singlet axial current. Thus the separation of this single contribution into a quark and a gluon part has been questioned [27]. Moreover, while the anomalous gluon term is obtained consistently in the case of massless quarks by several different methods (e.g. from the known operator form of the anomaly, or from a direct diagrammatic evaluation), it was shown in Ref. [25] that the contribution to M_1^S of the diagrams in Fig. 5 depends on the regulator. As already mentioned, it is zero for $m^2 \neq 0, p^2 = 0$ (where m is the mass of the produced quarks, and p^2 is the off-shell mass of the gluon), while it gives a finite result $c_g = -N_f \frac{g_s}{2\pi}$ for the coefficient of the gluon moment Δg in the opposite limit $m^2 = 0, p^2 \neq 0$. It has been claimed that this regulator dependence implies that the anomalous gluon component cannot be properly defined [27, 28]. Other authors [29-32] pointed out that, if care is not taken in properly separating a quark and a gluon term, instabilities might be produced when the values of the light quark masses are varied and large isospin violations can appear in each term (their sum M_1^S being stable and isospin invariant).

As is well known, the light-cone operator expansion provides a method of general validity for the study of the structure functions of deep inelastic scattering and their scaling violations in QCD. But not all questions can be answered by the light-cone approach. For example, the magnitude of $\Delta\Sigma$ or of Δs (the strange quark contribution to $\Delta\Sigma$) is not restricted by the light-cone method. Moreover this method cannot be extended to other hard processes where polarized parton densities could also be measured.

The QCD-improved parton model, based on the factorization theorem [33] derived by diagrammatic techniques, provides a generalization of the light-cone results that has been successfully applied and tested in all kinds of hard processes [34]. It is interesting

that the direct application of the standard techniques of the QCD-improved parton model to polarized leptonproduction leads to the anomalous gluon component and thus provides testable predictions for other hard processes.

In the parton approach one assumes that all quark and gluon densities can be defined starting from a sufficient number of physical hard processes. The QCD evolution equations for quark and gluon densities can be written down (for both polarized and unpolarized densities) with kernels that are, at leading order, directly obtained from the QCD vertices without reference to the particular process used to define the densities. Of course, beyond the leading order, the two-loop evolution kernels start depending to some extent on the exact definition of the parton densities. In the parton approach the primary quantities are the parton densities: $q(x, Q)$ and $g(x, Q)$ for unpolarized targets, $\delta q(x, Q)$ and $\delta g(x, Q)$ in the polarized case. The moments, which are the basic quantities for the light-cone expansion, are derived entities in the parton picture. Provided that the corresponding x -integration is convergent, any moment (even non-integer ones) can be constructed from the densities. In the singlet sector of polarized leptonproduction there are two sets of local operators in the light-cone expansion (for general n values): one set is constructed out of quark fields and their covariant derivatives and one set is made of gluon fields. However, for $n = 1$ (which corresponds to M_1^S) there is no gauge-invariant gluon operator of dimension three: in the gluon set one element is missing. This fact does not necessarily imply that the first moment of Δg cannot be defined and measured in any hard process. In the parton picture, one sees no reason why the first moment of the polarized gluon density should not be considered. While it is true that the only operator which appears in the light-cone expansion for M_1^S is j_μ^5 , the axial current, the problem remains of the relation between the operator j_μ^5 , its matrix elements and coefficient functions, and the first moments of $\Delta\Sigma(Q)$ and $\Delta g(Q)$. Usually, in similar cases, only a minor ambiguity can be expected: a quark operator corresponds to a moment of the quark density apart from a possible small correction of order $\alpha_s(Q)$ from the gluon density. The peculiarity of the present case is that $\Delta g(Q)$, the first moment of the gluon density, as computed with no ambiguity in leading order QCD, evolves as $(\alpha_s(Q))^{-1}$ so that the product $\alpha_s(Q)\Delta g(Q)$ is not necessarily small. Then either Δg identically decouples from M_1^S for whatever objective definition one takes of quark and gluon densities, or the result obtained in the light-cone method from j_μ^5 must correspond to some combination of the quark and gluon moments which have been independently defined. One finds that the latter is true in terms of a simple definition of $\Delta\Sigma$.

In order to discuss the apparent regulator dependence of the diagrams in Fig. 5, we go back to Eq. (4.17). In general only the coefficient of α_t is independent of the

regularization, i.e. $\gamma_j^{(2)} - \beta_0 c_j$. However, in the present case, owing to the different form of the colour and flavour Casimir factors, c_j and $\gamma_j^{(2)}$ or equivalently c_j and $\gamma_{qq}^{(2)} + c_j \gamma_{gg}^{(1)}$ are separately independent of the regularization:

$$\begin{aligned} c_j &= c_\Sigma = -\frac{3}{2} C_F \\ \gamma_j^{(2)} &= \gamma_{qq}^{(2)} + c_j \gamma_{gg}^{(1)} = -\frac{3}{2} C_F N_F \end{aligned} \quad (5.1)$$

As $\gamma_{gg}^{(1)}$ is unambiguously determined, one concludes that $\gamma_{qq}^{(2)}$ and c change in a related way and that there is a unique value of c that corresponds to $\gamma_{qq}^{(2)} = 0$, a necessary condition for conserved quarks.

We now consider the dependence on m , the mass of produced quarks. Here m is considered just as a regulator, because we assume that the massless theory is the relevant framework for light quarks u, d and s ($N_f = 3$). The case of heavy quarks, e.g. charmed quarks, will be considered in the next section.

For $m = 0$, as discussed in detail in Section 4, one has

$$\begin{aligned} [c]_{m=0} &= -N_f \sigma_f < j^5 >_t = (\Delta\Sigma - N_f \Delta\Gamma(t))_{m=0} \\ [\gamma_{qq}^{(2)}]_{m=0} &= 0 \end{aligned} \quad (5.2)$$

We have a different situation when the quark mass m is used as a regulator. First, as shown in Ref. [25]:

$$[c]_{m \neq 0} = 0 \text{ or } < j^5 >_t = [\Delta\Sigma(t)]_{m \neq 0} \quad (5.3)$$

Also, as explicitly computed in Ref. [35],

$$[\gamma_{qq}^{(2)}]_{m \neq 0} = -\frac{3}{2} C_F N_f \quad (5.4)$$

We see that in this case $\Delta\Sigma(t)$ is not conserved. When the mass m is introduced not only is c changed, but also $\gamma_{qq}^{(2)}$ (or in other words, the definition of $\Delta\Sigma$ as is evident from the relation $[\Delta\Sigma]_{m \neq 0} = [\Delta\Sigma]_{m=0} - N_f \Delta\Gamma$), so that the physics is unaltered.

Going back to Eqs. (4.18), we can show that actually the following relations hold:

$$\begin{aligned} \gamma_{qq}^{(3)} &= c \gamma_{qq}^{(2)} = 0 \\ \gamma_{gg}^{(2)} - \beta_1 &= c \gamma_{gg}^{(1)} \end{aligned} \quad (5.5)$$

These equations are valid in both the $m = 0$ and the $m \neq 0$ cases, with the appropriate values of c and $\gamma_{qq}^{(2)}$. For $m \neq 0$ one has $c = 0$, the Kodaira operator $< j^5 >_t$ coincides

with $\Delta\Sigma(t)$ (see the last of Eqs. (4.18) or Eq. (5.3)) and $\gamma_{gg}^{(3)}$ vanishes because $< j^5 >_t$ is multiplicatively renormalizable. In the case $m = 0$, $\gamma_{qq}^{(2)} = 0$ and $c = -N_f$. The results in Eqs. (5.5) follow from the anomalous dimension matrix for the operators j_μ^5 and k_μ defined in Eqs. (4.20) and (4.21):

$$\frac{d}{dt} \begin{pmatrix} j_\mu^5 \\ k_\mu \end{pmatrix}_t = \left(\frac{\alpha_t}{2\pi} \right)^2 \begin{pmatrix} \gamma_j^{(2)} & 0 \\ \gamma_{gg}^{(1)} & 0 \end{pmatrix} \begin{pmatrix} j_\mu^5 \\ k_\mu \end{pmatrix}_t + O(\alpha^3) \quad (5.6)$$

For $m = 0$, j_μ^5 and k_μ correspond to $\Delta\Sigma - N_f \Delta\Gamma$ and $\Delta\Gamma$ respectively (see the last of Eqs. (4.18)). By comparing Eqs. (4.12) and (5.6), the relations in Eqs. (5.5) follow. We stress that the invoked correspondence between k_μ and $\Delta\Gamma$ is limited to the statement that they have the same anomalous dimensions. The actual relation between k_μ and $\Delta\Gamma$ is discussed in Ref. [32]. Finally, note that in general from Eqs. (4.12) and (5.5) one finds:

$$\frac{d\Delta\Gamma}{dt} = \left(\frac{\alpha_t}{2\pi} \right)^2 \gamma_{gg}^{(1)} (\Delta\Sigma + c\Delta\Gamma)_t \quad (5.7)$$

We shall make use of this equation in the following.

It has been argued in Ref. [28] that the massless limit as defined here is not really orthodox. The claim is that there is no satisfactory set of regulators, in the computation of the diagrams of Fig. 5, that leads to the results given in Eqs. (5.2) valid in the massless limit. For example, using an off-shell mass p^2 for the gluon as a regulator is considered dangerous because allegedly gauge invariance is not guaranteed for Green functions. We observe that a regulator is only needed for $m = 0$ if one simultaneously considers the whole set of moments derived from the diagrams of Fig. 5. But the first moment in itself can be completely studied for $m = 0$ without introducing any regulator. We have seen in Section 4 that one possibility is to trade the integration over x for an integration over the quark transverse momentum k_T . In fact at fixed k_T , the angular integration is finite, as observed in Ref. [25] and further discussed in Ref. [21]. The resulting k_T distribution is integrable both near $k_T \simeq 0$ and at large k_T . The integral over k_T leads to the result of the massless limit. This procedure directly shows that the corresponding quark and gluon moments can be defined in terms of observable quantities. Alternatively, one can use operator methods by considering the forward matrix elements of the operators j_μ^5 and k_μ as also described in Section 4. The forward matrix elements of k_μ are gauge-invariant for ordinary gauge transformations [36]. This is sufficient to legalize the use of the forward matrix elements of k_μ in perturbative QCD for purposes of understanding the parton results. However, it has

been objected [27] that the forward matrix elements of k_μ are not invariant under topologically non-trivial gauge transformations that change the winding number. One can construct a non-local generalization of k_μ , discussed in Ref. [37], which coincides with k_μ at the perturbative level, but its forward matrix elements are invariant under all possible gauge transformations. We stress that while the consideration of the operator k_μ can be useful it is in no way necessary. What is important is that $\Delta\Sigma$ and $\Delta\Gamma$ can be related to physical processes and are useful to make predictions for other hard processes.

It has been shown in Ref. [25] that $[c]_{m \neq 0} = 0$ is obtained because of a cancellation between the contribution to the integral from the whole range of finite values of $\lambda = \frac{4k_\perp^2}{Q^2}$ and a large spike of opposite sign concentrated at $\lambda \lesssim \frac{4m^2}{Q^2}$. The physical hard gluon density can in principle be defined at each finite λ by measuring the rate of jet production at $k_T \rightarrow \infty$, $Q \rightarrow \infty$ with fixed $\lambda = \frac{4k_\perp^2}{Q^2}$. This procedure leads to a smooth distribution at small λ . The contribution of the spike at $\lambda \lesssim \frac{4m^2}{Q^2}$ has to be reabsorbed into the light-quark definition in order to make both the quark and the gluon terms smooth in the limit $\lambda \rightarrow 0$. (The case of heavy quarks with $m \gg \Lambda_{QCD}$ will be considered later.) In addition we have seen that including into the quark definition contributions from the soft k_T region is also necessary if we want our quarks to be conserved. More generally the inclusion of analogous infra-red sensitive terms into the quark definition can also be important to make $\Delta\Sigma$ and $\Delta\Gamma$ separately isospin conserved and stable under mass effects. If quarks and gluons are not appropriately defined in terms of physical quantities then isospin non-invariant terms appear both in $\Delta\Sigma$ and $\Delta\Gamma$ [29-32], while they cancel in the combination corresponding to M_1^S . These pathologies clearly show that those badly defined quark and gluon moments are not those defined in terms of physical hard processes.

It is instructive to make the relation between the massive and the massless case completely explicit. We start from the relation between the operator j_μ^5 at the scale Q and at the scale μ :

$$j_\mu^5(t) = \left[\exp \int_0^t \left(\frac{\alpha_t}{2\pi} \right) \gamma_j^{(2)} dt \right] j_\mu^5(0) \quad (5.8)$$

where higher orders in α_t are neglected in the integral. In the massive theory $c = 0$, so that $j_\mu^5(0) \Rightarrow [\Delta\Sigma(0)]_{m \neq 0}$. According to Eqs. (4.4) and (5.1), $\gamma_j^{(2)} = -\frac{3}{2} C_F N_f$ can always be written as $\gamma_j^{(2)} = -N_f \gamma_{fg}^{(1)}$. Then, Eq. (5.8) is equivalent to:

$$[\Delta\Sigma(t)]_{m \neq 0} = \left[\exp \int_0^t \left(\frac{\alpha_t}{2\pi} \right) (-N_f \gamma_{fg}^{(1)}) dt \right] [\Delta\Sigma(0)]_{m \neq 0} \quad (5.9)$$

Clearly this quark is not conserved. But we note that from Eq. (5.7) it follows that

$$\frac{d\Delta\Gamma}{dt} \simeq \left(\frac{\alpha_t}{2\pi} \right)^2 \gamma_{gq}^{(1)} [\Delta\Sigma(t)]_{m \neq 0} \quad (5.10)$$

or

$$\Delta\Gamma(t) = \left[-1 + \exp \int_0^t \left(\frac{\alpha_t}{2\pi} \right)^2 \gamma_{gq}^{(1)} dt \right] [\Delta\Sigma(0)]_{m \neq 0} + \Delta\Gamma(0) \quad (5.11)$$

As a consequence, Eq. (5.9) at two-loop accuracy can be rewritten in the form

$$\begin{aligned} [\Delta\Sigma(t)]_{m \neq 0} &= [\Delta\Sigma(0)]_{m \neq 0} + \left[-1 + \exp \int_0^t \left(\frac{\alpha_t}{2\pi} \right)^2 (-N_f \gamma_{gq}^{(1)}) \right] [\Delta\Sigma(0)]_{m \neq 0} \\ &= [\Delta\Sigma(0)]_{m \neq 0} + N_f \Delta\Gamma(0) - N_f \Delta\Gamma(t) \\ &= \Delta\Sigma - N_f \Delta\Gamma(t) \end{aligned} \quad (5.12)$$

where

$$\begin{aligned} \Delta\Sigma &= [\Delta\Sigma(0)]_{m \neq 0} + N_f \Delta\Gamma(0) \\ &= [\Delta\Sigma(t)]_{m \neq 0} + N_f \Delta\Gamma(t) \\ &\equiv \Delta\Sigma_{m=0} \end{aligned} \quad (5.13)$$

is the quark moment defined in such a way that it is evidently conserved. The idea is that the conserved $\Delta\Sigma$ is the one which should be closest to the intuition based on constituent quarks. The gluon component can in principle be measured in other hard processes.

6. HEAVY QUARK THRESHOLD

We can now consider what happens when the threshold for producing a heavy quark pair is passed. The most relevant example is the opening of the charm threshold.

We start from an indicative model where we consider the perturbative evolution with $N_f = 3$ to be valid up to $Q = Q_c$ while $N_f = 4$ is used for $Q > Q_c$, with Q_c being an appropriate scale of order m_c . For $t > t_c$ (with $t_c = \ln Q_c^2/\mu^2$), Eq. (5.12) with $N_f = 4$ can be applied to the evolution from t_c up to t and it gives:

$$\Delta\Sigma(t)_{m \neq 0} = \Delta\Sigma(t_c)_{m \neq 0} + 4\Delta\Gamma(t_c) - 4\Delta\Gamma(t) \quad (6.1)$$

In general for $\Delta\Sigma(t_c)_{m \neq 0}$ one can write the expression:

$$\Delta\Sigma(t_c)_{m \neq 0} = \Delta\Sigma_{m \neq 0} + [\Delta\Sigma - 3\Delta\Gamma(t_c)] \quad (6.2)$$

The second term in the bracket is the result that would be obtained by assuming that $\Delta\Sigma(t)_{m \neq 0}$ is continuous at $t = t_c$. $\Delta\Sigma_{m \neq 0}$ is a non-perturbative term arising from

the lowest-order diagram where the photon directly interacts with an intrinsic charm quark inside the proton. By combining Eqs. (6.1) and (6.2), one obtains:

$$\Delta\Sigma(t)_{m \neq 0} = [\Delta\Sigma - 3\Delta\Gamma(t)] + [\Delta C_{m \neq 0} + \Delta\Gamma(t_c) - \Delta\Gamma(t)] \quad (6.3)$$

The first bracket is what would be obtained in absence of the threshold, i.e. if the smooth evolution with $N_f = 3$ was followed up to t . Consequently the second term is the contribution of charm:

$$\int g_1^p dx|_{\text{above}} - \int g_1^p dx|_{\text{below}} \simeq \frac{4}{18} [\Delta C_{m \neq 0} + \Delta\Gamma(t_c) - \Delta\Gamma(t)] \quad (6.4)$$

The term $\Delta\Gamma(t_c) - \Delta\Gamma(t)$ is the contribution to the charm cross-section obtained from the diagrams of Fig. 6.

- Figure 6 -

An approximate expression for this term is given by:

$$\Delta\Gamma(t_c) - \Delta\Gamma(t) = \frac{\alpha_t - \alpha_t^{(1)}}{2\pi\beta_0} \gamma_{34}^{(1)} (\Delta\Sigma - 3\Delta\Gamma(t_c)) \quad (6.5)$$

where β_0 is computed with $N_f = 4$ and $\Delta\Sigma - 3\Delta\Gamma(t_c)$ is the nearly vanishing value measured by the EMC. When $t \rightarrow \infty$ the corresponding result is of order $\alpha(m_c)$ (and

not of order $\frac{\Lambda_{QCD}^2}{m_c^2}$ as the contribution evaluated in Ref. [27]). The coefficient of $\alpha(m_c)$ is, however, very small because of the EMC result. In Eq. (4.17), $\Delta\Gamma(t)$ is the hard component arising from large k_T , i.e., $k_T \sim 0(Q)$, while $\Delta\Gamma(t_c)$ is from $k_T \sim 0(m_c)$. Finally, $\Delta C_{m \neq 0}$ is a possible non-perturbative contribution from small $k_T \sim 0(\Lambda_{QCD})$. In Ref. [21] it was suggested that the sum $\Delta C_{m \neq 0} + \Delta\Gamma(m_c)$ should nearly vanish, so that a large variation of M_1^S , proportional to $\Delta\Gamma(t)$, would be observed at the opening of the charm threshold. We now think that a purely perturbative treatment of the heavy quark threshold should be adequate. Therefore we expect $\Delta C_{m \neq 0}$ to be of order $\frac{\Lambda_{QCD}^2}{m_c^2}$ and that the total effect of the charm threshold on M_1^S is of order $\alpha(m_c)$. The different rôle played by the heavy quark mass $m_c \gg \Lambda_{QCD}$ with respect to a light quark mass $m < \Lambda_{QCD}$ is well demonstrated by the interesting study of Ref. [38]. This clear difference, which can be seen as a manifestation of the decoupling theorem, cannot be taken (as claimed in Ref. [27]) as evidence against the point of view that the conserved $\Delta\Sigma$ for $N_f = 3$ should more directly correspond to constituent quarks than $\Delta\Sigma(t)_{m \neq 0}$.

7. CONCLUSION

Including the anomalous gluon contribution, the experimental results at $< Q^2 > = 10 \text{ GeV}^2$ can be translated into:

$$\Delta u - \Delta\Gamma = 0.78 \pm 0.06 \quad (7.1)$$

$$\Delta d - \Delta\Gamma = -0.47 \pm 0.06 \quad (7.2)$$

$$\Delta s - \Delta\Gamma = -0.19 \pm 0.06 \quad (7.3)$$

In particular

$$\Delta\Sigma - 3\Delta\Gamma \simeq 0.12 \pm 0.17 \quad (7.4)$$

In general the helicity sum rule for the proton is given by:

$$\frac{1}{2} = \frac{1}{2} \Delta\Sigma + \Delta g + L_z \quad (7.5)$$

As $\Delta\Sigma$ is conserved, $\Delta g + L_z$ must also be conserved. This implies that in general at large Q^2 both Δg and L_z grow logarithmically in a related way, so that their sum is constant. How this happens has been studied in lowest-order perturbative QCD in

Ref. [39]. When a quark emits a gluon the total helicity is clearly not conserved in the process. In fact the quark helicity does not change, while the gluon helicity is ± 1 because the gluon is massless. However, the total angular momentum is obviously conserved in the process. Thus in each individual act of emission the orbital angular momentum compensates for the helicity imbalance.

For $\Delta s = 0$, $\Delta\Sigma = \Delta u + \Delta d \simeq 3\Delta\Gamma = 0.7$ and $\Delta g + L_z = 0.15$, which implies that about 30% of the proton spin is due to gluons and to orbital angular momentum. The fraction of proton spin carried by quarks decreases rapidly for negative Δs . For example, if $\Delta s = -0.1$, then $\Delta\Sigma \simeq 3\Delta\Gamma \simeq 0.4$, $\Delta u + \Delta d \simeq 0.5$ and $\Delta g + L_z \simeq 0.3$.

In our opinion it is clear enough by now that the difference between the measured value of the singlet axial current for polarized protons and the naive expectations based on the constituent quark model is due to the presence in this channel of the axial anomaly. In the absence of the anomaly, the helicity carried by each kind of quarks would be conserved in the massless theory. We would then expect the constituent and the parton quarks to carry approximately the same amount of the proton helicity. The most conservative point of view is just [24] that in the presence of the anomaly the conservation of the singlet quark helicity is broken at the two-loop level, so that in principle the helicity of parton and constituent quarks can be different. In spite of the smallness of the corresponding effect in the perturbative region, one can still attribute the large difference that is observed to the effect of the anomaly in the non-perturbative region above and around the confinement scale. In the approach of the anomalous gluon component, one accepts the starting point that the difference is due to the anomaly and goes beyond this statement by establishing the stable connections with other hard processes. The additional input that leads to the new information is provided by the QCD-improved parton model. In this model the polarized gluon density and its moments can in principle be defined from observable hard processes (e.g. the production of jets at large k_T in deep inelastic scattering). Because of the anomaly, the evolution of the first moment of the polarized gluon density is such that the quantity $\Delta\Gamma \simeq \frac{g_A}{2} \Delta g$ is a constant in leading order. The difference between the helicity of parton and constituent quarks is attributed to this anomalous gluon component. It is a challenge for future experiments to measure the polarized gluon density and to check whether its size and z dependence are suitable [40] for an explanation of the EMC result in terms of the anomalous gluon component.

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