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FUSION RULE AUTOMORPHISMS FROM INTEGER SPIN SIMPLE CURRENTS

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ABSTRACT

A new class of modular invariant partition functions associated with simple currents is discussed. These partition functions correspond to fusion rule automorphisms, but unlike the class that is already known, they are generated by currents of integral rather than fractional spin.

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The set of modular invariant partition functions for given rational conformal field theories often has a rich, complicated and surprising structure. A subclass of modular invariant partition functions that should show a more regular behavior is formed by the simple current invariants. This class can be described as follows. Many conformal field theories have primary fields that have unique fusion rules with any other fields. The complete set of such fields ('simple currents') forms a discrete $Z_{N_1} \times \dots \times Z_{N_l}$ group under fusion. We have called this group the center of the conformal field theory [1]. All other primary fields are organized into orbits generated by the currents. Two fields belong to the same orbit if they can be mapped to each other by the action of a simple current under fusion.

In [1] we have proposed a method for obtaining off-diagonal modular invariant partition functions from simple currents. The simple currents are used formally as twist fields in an orbifold-like procedure. The resulting partition functions can be written in a very simple and general form [2]. In this way one associates with every simple current J an off-diagonal partition function given by a matrix $M(J)$ that connects the holomorphic and anti-holomorphic characters. The set of modular invariant partition functions can be generalized further by considering products $M(J_1) \dots M(J_k)$, each of which defines a valid modular invariant partition function [3] (of course these products are not all different).

The partition functions obtained in this way have the property that $M_{ab} = 0$ if a and b lie on different orbits, i.e. if b is not connected to a by the action of one or more simple currents under fusion. It is now natural to ask whether the converse is also true, namely whether this procedure yields all the modular invariant partition functions that have matrix elements only among fields on the same orbit. In this paper we will show that the answer to this question is negative. This requires a more precise definition of the term 'simple current invariant'. From now on we will call a modular invariant partition function a *simple current invariant* if it has the property that M_{ab} is non-zero only if a and b are related by simple currents.

A general feature of all solutions obtained by the method of Ref. [1] is that integer spin currents yield extensions of the chiral algebra, whereas fractional spin currents yield fusion rule automorphisms. If both types of currents are involved one gets more complicated

* If the order N of J is even the conformal weight h_J of the current must satisfy the level-matching condition $Nh_J = 0 \pmod{1}$. The partition functions obtained with currents J_1 and J_2 are the same if $J_1 = (J_2)^n$ and $J_2 = (J_1)^m$ for some integers n and m .

† This question was raised by D. Altschuler (private communication).

invariants of mixed type. The class of modular invariants we will obtain in this paper appears for certain systems of purely integer spin currents, but the invariants correspond to fusion rule automorphisms, and there is no extension of the chiral algebra.

The new invariants discussed here appear for centers that consist of more than one simple factor. The results presented in Ref. [1] include this possibility in principle, but it will be necessary to develop the formalism a bit further. Most of the discussion is valid for currents of arbitrary spin, and we will focus on integer spin systems only at the end.

Consider a rational, unitary conformal field theory with modular transformation matrices S and T that have the usual properties. Suppose that among the primary fields of this theory there is a set of simple currents. We can always choose a basis of independent currents $J_1, \dots, J_l$ with $(J_i)^{N_i} = 1$ (where N_i is the smallest positive integer with that property). These currents generate a center $Z_{N_1} \times \dots \times Z_{N_l}$. There are many ways to choose the basic currents, and, depending on the choice, different values may be obtained for the orders N_i and for the number of basic currents l , due to isomorphisms such as $Z_3 \times Z_5 \sim Z_{15}$. It is always possible to choose the basic currents so that $N_i = p_i^{n_i}$, $n_i \in \mathbb{Z}$ and p_i prime. This still does not fix the basis uniquely, but it does completely determine the N_i 's. The advantage of this basis is that different primes do not mix, as we will see.

Each primary field a has a set of charges $Q_i(a)$ with respect to the currents. These charges are quantized in units of $1/N_i$. It is convenient to define a lattice Λ consisting of the points $(i_1 N_1, \dots, i_l N_l)$, $i_i \in \mathbb{Z}$. The complete set of distinct currents consists of all combinations $J_1^{a_1}, \dots, J_l^{a_l}$, $\vec{a} \in \Lambda$. In the following we will denote such a combination of currents simply as $[\vec{a}]$. Furthermore we denote by $[\vec{a}]_a$ the field obtained by acting with $[\vec{a}]$ on the field a . The allowed charges lie on the dual Λ^* of Λ . Since the charges are defined modulo integers, the set of distinct ones is $\Lambda^*/\mathbb{Z}^l$.

The relative monodromies of the currents give additional structure to the discrete group generated by the fusion rules. This structure can be parametrized by means of the current-current charges

$$Q_i(J_j) \equiv \frac{\tilde{r}_{ij}}{N_i} = Q_j(J_i) = \frac{\tilde{r}_{ji}}{N_j},$$

where $\tilde{r}_{ij}$ is defined modulo N_i . Note that currents with orders that are relative prime cannot have a charge with respect to each other. As explained in Ref. [1], the parameter $\tilde{r}_{ii}$ determines the conformal weight of the current J_i up to integers if N_i is odd, and up to

half-integers if N_i is even. To deal with the latter case, it is convenient to double the range of $\tilde{r}_i$ for N_i even. Using Ref. [1] we then obtain the following formula for the conformal weight of a current combination $[\vec{\alpha}]$

$$h([\vec{\alpha}]) = \frac{1}{2} \sum_i \tilde{r}_i \alpha_i - \frac{1}{2} \alpha_i R_{ij} \alpha_j \pmod{1}, \quad (1)$$

where R_{ij} is a symmetric matrix,

$$R_{ij} = \frac{r_{ij}}{N_i} = \frac{r_{ji}}{N_j},$$

where r_{ij} is defined modulo N_i if $i \neq j$ or if N_i is odd, and modulo $2N_i$ if N_i is even and $i = j$. Furthermore $\tilde{r}_{ij} = r_{ij} \pmod{N_i}$, so that as far as the monodromy charges are concerned there is no difference between r and $\tilde{r}$. Note that the first term in Eq. (1) can be eliminated if N_i is odd, since r_{ii} can always be chosen even in that case.

The matrix elements of S in a theory with simple currents satisfy the following useful relations [2], [4]:

$$S_{b,[\vec{\alpha}]} = e^{2\pi i \vec{\alpha} \cdot \vec{Q}(b)} S_{ba}. \quad (2)$$

It is also possible to apply simple currents to the index b . In that case one obtains,

$$S_{[\vec{\beta}]b,[\vec{\alpha}]} = e^{2\pi i (\vec{\alpha} \cdot \vec{Q}(b) + \vec{\beta} \cdot \vec{Q}(a) + \alpha_i R_{ij} \beta_j)} S_{ba}. \quad (3)$$

For most purposes the simpler form Eq. (2) turns out to be sufficient.

Suppose now that one has a matrix M_{ab} with the following properties

1. $[M, S] = [M, T] = 0$
2. $M_{ab} \in \mathbb{Z}$ and $M_{ab} \geq 0$
3. $M_{00} = 1$
4. $M_{0b} = 0$ if $b \neq 0$

5. $M_{ab} = 0$ if a and b are on different orbits.

Using points 1-4 one can prove that M_{ab} defines a permutation of the fields, i.e. that its matrix elements are 0 or 1, and every row and column contains precisely one 1. This can be proved as follows*. From 1. we see that $\sum_b M_{0b} S_{b0} = \sum_b S_{0b} M_{b0}$. Since $S_{0b} = S_{b0} > 0$, point 4. implies that also $M_{b0} = 0$ if $b \neq 0$. Consider now the matrix MM^T . This matrix satisfies points 1-4, and furthermore it is symmetric. Now consider $(MM^T)^2$. This matrix also satisfies 1-4, and it has non-vanishing diagonal elements, except if MM^T has rows that vanish completely. Such rows form an invariant subspace of $(MM^T)^2$ with eigenvalue zero. Finally, consider the matrix $D = [(MM^T)^2 - 1]^2$. This matrix is modular invariant, has non-negative matrix elements, but obviously $D_{00} = 0$. On the other hand, $D_{00} = \sum_a S_{0a} D_{aa} S_{a0}$, with $S_{0a} > 0$ in a unitary CFT. Hence $D = 0$, and $(MM^T)^2 = 1$. Then MM^T is a symmetric orthogonal matrix with integral entries, which can only be a permutation of order 2. It follows that M is a permutation as well (but not necessarily of order 2). Using Verlinde's formula [6] and the relation $SM = MS$ it is not hard to show that such a permutation is an automorphism of the fusion algebra [5].

The foregoing argument holds independently of the presence of simple currents. Now suppose that also point 5 holds. Then for every field a there is precisely one other field $[\vec{\mu}(a)]a$ so that $M_{a,[\vec{\mu}(a)]} = 1$, and also precisely one field $[\vec{\nu}(a)]a$ so that $M_{[\vec{\nu}(a)]a} = 1$. Of course the functions $\vec{\mu}$ and $\vec{\nu}$ are related to each other, and they are identical if the automorphism is of order 2. Using $MS = SM$ and (2) we find then

$$M_{a,[\vec{\mu}(a)]a} e^{2\pi i \vec{\mu}(a) \cdot \vec{Q}(b)} \vec{Q}(b) S_{ab} = S_{ab} e^{2\pi i \vec{\nu}(b) \cdot \vec{Q}(a)} M_{[\vec{\nu}(b)]b}.$$

Since the matrix elements of M that appear are equal to 1, we can divide them out. If furthermore $S_{ab} \neq 0$ we can divide it out as well, and we get the condition

$$\vec{\mu}(a) \cdot \vec{Q}(b) = \vec{\nu}(b) \cdot \vec{Q}(a) \pmod{1}, \text{ if } S_{ab} \neq 0. \quad (4)$$

Although matrix elements of S might vanish sporadically, there are restrictions due to unitarity of S . Consider for a given field a the set of fields b such that $S_{ab} \neq 0$. The charges

* This fact also follows from the work of Moore and Seiberg [5]. The proof given here is more elementary.

of the fields b and their linear combinations span a certain sublattice $\Lambda^*(a)$ of the lattice Λ^* . Since $\Lambda^*(a) \supset \mathbb{Z}^l$, its dual satisfies $\Lambda(a) \subset \mathbb{Z}^l$. Furthermore, if $\Lambda^*(a)$ has fewer lattice points than Λ^* , $\Lambda(a)$ has more lattice points than Λ . Hence there must exist a vector $\vec{\alpha} \in \mathbb{Z}^l$, $\vec{\alpha} \notin \Lambda$, so that $\vec{\alpha} \cdot \vec{Q}(b) = 0 \pmod 1$ for all $Q(b)$ with $S_{ab} \neq 0$. But then, according to Eq. (2), $S_{[a]b} = S_{ab}$ for all b . This is in conflict with unitarity of S unless $[\vec{\alpha}]a = a$, i.e. a is a fixed point of the non-trivial current $[\vec{\alpha}]$. Conversely, if a field a is a fixed point of some current α , it cannot have matrix elements with any field b with $\vec{\alpha} \cdot \vec{Q}(b) \neq 0 \pmod 1$.

This can be summarized as follows. For each field a we can define a lattice $\Lambda(a)$, consisting of all integer vectors $\vec{l}$ so that $[\vec{l}]a = a$. The foregoing argument shows then that the charges of the fields b with which a has non-vanishing matrix elements S_{ab} must span $\Lambda^*(a)$. Obviously $\Lambda(a) = \Lambda$ for all fields a that are not fixed points of any of the currents, and $\Lambda(a) \supset \Lambda$ for any a .

Unfortunately this is not quite sufficient for our purpose, and we will have to make an additional assumption regarding the presence of vanishing matrix elements of S . Suppose that two fields b and b' have the same charges, $\vec{Q}(b) = \vec{Q}(b')$. Define the lattice $\Lambda(b, b') = \Lambda(b) \cap \Lambda(b')$. Then according to (4) $[\vec{v}(b) - \vec{v}(b')] \cdot \vec{Q}(a) = 0$ for all a to which both b and b' are connected by a non-vanishing matrix element of S . The extra assumption we need to make is that the charges of this set of fields a span $\Lambda(b, b')$. In that case it follows that $\vec{v}(b) = \vec{v}(b') \pmod{\Lambda(b, b')}$. A similar result holds for $\vec{\mu}$.

This assumption does not follow from the result obtained above. Indeed, it is in principle possible that the set of primary fields consists of several mutually disconnected components. For example, in a system with a spin- $\frac{1}{2}$ current one might have two kinds of Ramond fields that are not transformed into each other by S . In that case there will be four invariants (instead of the usual two) since one may twist both Ramond systems independently. In general, if the assumption is violated (i.e. if S has matrix elements that vanish in a certain systematic way) there are fewer modular invariance constraints, and hence there might be more invariants. Since we do not know any example of such a system, we will not pursue this further, and we will assume from now on that all matrix elements of S are non-vanishing, except when they are required to vanish because of fixed points. A few 'accidental' zeroes will not affect the argument.

If the fields b and b' are not fixed points of any of the currents, $\Lambda(b) = \Lambda(b') = \Lambda(b, b') = \Lambda$. Then the foregoing argument shows that $\vec{\mu}(b)$ and $\vec{v}(b)$ are determined completely by

the charge of b . By a very similar argument it can be shown see that the dependence of $\vec{\mu}$ and $\vec{v}$ on the charges is linear. Therefore we know $\vec{\mu}$ and $\vec{v}$ completely (i.e. mod Λ) if we know them on a set of fields whose charges are linearly independent and which are not fixed points. Such fields exist in almost all conformal field theories. Integer spin simple currents can have fixed points only on fields with vanishing charge. Because of unitarity of S (see the argument given above) there must exist fields with fractional charge, and therefore there must exist fields that are not fixed points. Fractional spin currents cannot have fixed points, except when their spin is half-integer. There are indeed half-integer spin systems in which no complete set of non-fixed-point fields with charges spanning Λ^* exists; an example is the Ising model, in which the only half-integer charge field is a fixed point. In such theories $\vec{\mu}$ and $\vec{v}$ are not fully determined, because there are several choices that lead to the same automorphism even though they are not identical modulo Λ . However, in any case the set of simple current invariants of such 'degenerate' examples is determined completely by the set of invariants of a generic theory with a 'complete' set of non-fixed-point fields. Therefore we can concentrate on such generic theories from now on, and ignore fixed points.

Thus the automorphism $\vec{\mu}$ can be specified completely by fixing it for a set of basic charges, $\vec{e}_i = (0, \dots, 0, 1/N_i, 0, \dots, 0)$. If we denote the action of the automorphism on these charges as $\vec{\mu}_i$ and $\vec{v}_i$, then we get the following answer for an arbitrary field a .

$$\vec{Q}(a) = \sum_i q_i \vec{e}_i \rightarrow \vec{\mu}(a) = \sum_i q_i \vec{\mu}_i; \quad \vec{v}(a) = \sum_i q_i \vec{v}_i. \quad (5)$$

There are several restrictions on $\vec{\mu}$ and $\vec{v}$. First of all condition (4) for S -invariance leads to

$$\frac{\mu_{ij}}{N_j} = \frac{\nu_{ji}}{N_i}, \quad (6)$$

where $\mu_{ij}(\nu_{ji})$ is the j^{th} component of $\vec{\mu}_i(\vec{v}_i)$. The second requirement is invariance under T . Using Ref. [1] it is not hard to show that

$$h([\vec{\alpha}]a) = h(a) + h([\vec{\alpha}]) - \vec{\alpha} \cdot \vec{Q}(a) \pmod 1. \quad (7)$$

Applied to the non-vanishing matrix elements of M , given by Eq. (5) this yields

$$\frac{1}{2} \sum_{ij} q_i (\mu R \mu^T)_{ij} q_j + \sum_{ij} q_i \frac{\mu_{ij}}{N_j} q_j - \frac{1}{2} \sum_{ij} q_i \mu_{ij} r_{ij} = 0 \pmod{1}, \quad (8)$$

and the same with ν_{ij} instead of μ_{ij} . Finally there is a consistency condition to be satisfied since μ and ν should define the same automorphism. Because $M[\vec{\nu}(a)]_{a,a} = 1$, it must clearly be true that

$$\vec{\mu}([\vec{\nu}(a)]a) = -\vec{\nu}(a) \pmod{\Lambda},$$

and the same with $\vec{\mu}$ and $\vec{\nu}$ interchanged. This yields

$$\frac{\nu_{lm}}{N_m} + \frac{\mu_{lm}}{N_m} + \sum_{k,p} \frac{\nu_{lk} \tau_{pk} \mu_{pm}}{N_m} = 0 \pmod{1}.$$

This is, however, not an independent condition. It can be obtained by multiplying Eq. (8) by two and substituting some of the μ 's by ν 's using Eq. (6). The complete set of automorphisms thus corresponds simply to the complete set of solutions to Eq. (8).

This set of solutions is especially easy to find in the special case of purely integer spin currents. This means that the basic currents have integer spin, as well as their products. Hence the basic currents must be mutually local, and $R_{ij} = 0 \pmod{1}$. This yields an enormous simplification of Eq. (8), which now becomes

$$\frac{\mu_{ij}}{N_j} = -\frac{\mu_{ji}}{N_i} \pmod{1}.$$

This is the only condition to be satisfied, and obviously it has many solutions, at least if the center has more than one factor. The complete set of solutions corresponds to the set of integers m_{ij} , $i < j$, defined modulo the greatest common divisor G of N_i and N_j . The matrix elements of μ are then $\mu_{ij} = N_j m_{ij}/G$ and $\mu_{ji} = -N_i m_{ij}/G$, for $i < j$, $\mu_{ii} = 0$.

Examples of conformal field theories in which such invariants occur are not hard to find. Any tensor product of two or more theories with orbits of integer spin currents

has such invariant, provided that the orbit lengths have common factors. For example $SU(2)_{4k_1} \times \dots \times SU(2)_{4k_l}$ has $2^{\frac{1}{2}l(l-1)}$ such invariants (including the diagonal one) if l is the number of factors. Similarly $SU(p)_{pk_1} \times \dots \times SU(p)_{pk_l}$ has $p^{\frac{1}{2}l(l-1)}$ automorphisms of this type. These automorphisms are non-trivial, and should not be confused with interchange automorphisms that occur when some of the k_i 's have the same value. In particular the fields that are mapped to each other have, in general, different conformal weights.

Centers with more than one Abelian factor are by no means restricted to tensor products of simple Kac-Moody algebras. There are numerous examples for lattices, orbifolds or coset theories. An interesting example occurs already for simple KM algebras, namely D_n , n even, level k . The conformal weight of the three simple currents is $\frac{k}{2}$ and (twice) $\frac{kn}{8}$, and they all have integer spin if k is even and either k or n are a multiple of 4. The automorphism is quite easy to describe. The simple currents are J_v , J_s and J_c , characterized by a highest weight that is k times the fundamental vector, spinor and conjugate spinor weight. The corresponding charges are Q_v , Q_s and Q_c . The currents generate a center $Z_2 \times Z_2$ and are not independent, since $J_v J_s J_c = 1$ under fusion. Hence $Q_v + Q_s + Q_c = 0 \pmod{1}$. The primary fields can have charges $(0,0,0)$, $(\frac{1}{2}, \frac{1}{2}, 0)$, $(\frac{1}{2}, 0, \frac{1}{2})$ or $(0, \frac{1}{2}, \frac{1}{2})$. The automorphism maps fields with charges $(0,0,0)$ to themselves, and takes the other three possible charge combinations to J_c , J_s and J_v times themselves, respectively. Note that the field always has vanishing charge with respect to the current acting on it, as required by T -invariance.

The general solution for arbitrary R is more complicated. To get some insight in the answer it is instructive to solve Eq. (8) for the center $Z_2 \times Z_2$. The currents generating this center are denoted by 1 , J_1 , J_2 and $J_3 = J_1 J_2$. Since they have order two, their conformal weight is $\frac{m_i}{4}$, $i = 1, 2, 3$, where $m_1 + m_2 + m_3$ must be even. A rather elegant result is obtained if one enlarges the set of automorphisms obtained by solving Eq. (8), by multiplying them with all possible integer spin invariants. The latter correspond simply to all purely integer spin subgroups of the center. It is likely that the product of all pure automorphism with all pure integer spin invariants yields the complete set of simple current invariants. If two of the currents have quarter-integral spin, there are two distinct modular invariant partition functions of this type, namely the diagonal one and the one generated by the third current, which must have (half-)integer spin. If all three currents have (half-)integer spin there are always six modular invariants. It turns out that with one exception all these invariants are products of matrices $M(J)$, obtained by means of the method of Ref. [2]. The exception occurs for $h_1 = h_2 = h_3 = 0$, and is precisely the new type of invariant discussed above.

The results are summarized in the table, where the new invariant is denoted $M(J_1, J_2)$.^{*} In general the notation is not unique, since the same invariant can often be written in several different ways. Most of the conformal weights in the table occur for D_n Kac-Moody algebras for even n , for suitably chosen values of k and n .

Table of modular invariants of conformal field theories with a $\mathbb{Z}_2 \times \mathbb{Z}_2$ center.

h_1	h_2	h_3	non-diagonal modular invariants
0	0	0	$M(J_1), M(J_2), M(J_3), M(J_1)M(J_2), M(J_1, J_2)$
0	0	$\frac{1}{2}$	$M(J_1), M(J_2), M(J_3), M(J_1)M(J_2), M(J_2)M(J_1)$
$\frac{1}{2}$	$\frac{1}{2}$	0	$M(J_1), M(J_2), M(J_3), M(J_1)M(J_2), M(J_1)M(J_3)$
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$M(J_1), M(J_2), M(J_3), M(J_1)M(J_2), M(J_2)M(J_1)$
$\frac{m}{4}$	$\frac{m'}{4}$	0	$M(J_3) \quad (m, m' \text{ odd})$
$\frac{m}{4}$	$\frac{m'}{4}$	$\frac{1}{2}$	$M(J_3) \quad (m, m' \text{ odd})$

Modular invariants of D_n Kac-Moody algebras have also been obtained in Refs. [7] and [8]. The latter paper considers Wess-Zumino-Witten models on non-simply connected group manifolds. Some of the invariants in our table do not appear in Ref. [8]. In particular, only four of the six invariants are listed in Ref. [8] if n is a multiple of 4 and k is odd. On the other hand, the invariant $M(J_1, J_2)$ (occurring if k is even and nk a multiple of 8) does appear in Ref. [8]. [In section 6, formula (15), these authors give two modular invariants, related by a sign choice. The sign choice +1 corresponds to an extension of the algebra by the three currents, whereas the choice -1 corresponds to $M(J_1, J_2)$.]

An interesting observation made in Ref. [8] is that there is a linear relation among the six distinct matrices M . This turns out to be true for odd k as well. The relation is essentially the same in all cases: the sum of the three matrices $M(J_i)$ is always equal to the sum of the other three matrices. In particular $M(J_1, J_2) = M(J_1) + M(J_2) + M(J_3) - M(J_1)M(J_2) - M(J_1)M(J_3) - M(J_2)M(J_3)$. This observation does not mean that $M(J_1, J_2)$ is not really a new invariant. Although the commutant of S and T is indeed a linear space, only special points

* The precise definition is to take J_1 and J_2 as the basic currents, with $\mu_{11} = \mu_{22} = 0, \mu_{12} = \mu_{21} = 1$. It follows from the foregoing discussion for D_n that $M(J_1, J_3) = M(J_2, J_3) = M(J_1, J_2)$.

in that space (namely those with positive integer coefficients and $M_{00} = 1$) are of interest. Specifying a basis for the commutant is not sufficient to enumerate these special points.

We have shown in this paper that integral spin systems of simple currents have automorphism invariants that cannot be obtained by previous methods. Furthermore, for integer spin currents this completes the set of modular invariant partition functions of automorphism type, for automorphisms acting within simple current orbits. It should be fairly easy to extend this result to general modular invariant partition functions (i.e. including extensions of the algebra) of integral spin systems. This is a first step in the direction of a complete description of all modular invariants (as well as a proof of completeness) whose matrix elements lie on orbits of currents of arbitrary spin. As far as automorphisms are concerned, the only remaining difficulty is to find all solutions to Eq. (8) for $R \neq 0$.

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