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The Super W_∞ algebra

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ABSTRACT

We construct a higher-spin $N = 2$ superalgebra whose bosonic sector is $W_\infty \oplus W_{1+\infty}$. It can be realised in terms of bilinear currents involving free complex bosonic and fermionic fields. We also discuss the spectral flow, and various truncations.

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There has been considerable recent interest in finding higher-spin extensions of the Virasoro algebra. These include the non-linear W_N algebras, which contain generators with all integer conformal spins $2 \leq s \leq N$ [1,2], and certain supersymmetric extensions [1,3]. A particular $N \rightarrow \infty$ limit of W_N , denoted by w_∞ , was described in [4], and its supersymmetric extensions were obtained in [5,6]. The latter algebras are all linear; however, the price that one pays for this simplification is that unlike the finite- N W_N algebras, which have central terms for generators of all conformal spins, w_∞ and its super-extensions admit central terms only in the (super-) Virasoro subsector. More recently, a new algebra called W_∞ was constructed [7,8], which arises as a more subtle $N \rightarrow \infty$ limit of W_N . Remarkably, W_∞ admits central terms for all conformal spins, despite the fact that it is a linear algebra. The w_∞ algebra can be obtained as a contraction of W_∞ , and one can view it as the classical limit. There is also another higher-spin algebra, called $W_{1+\infty}$, with generators having all integer conformal spins $s \geq 1$ [9].

Attempts to supersymmetrise W_∞ or $W_{1+\infty}$ indicated that the problem is a more difficult one than in the case of w_∞ . In this paper we present a solution to this problem. Our approach is based on the observation that the $W_{1+\infty}$ algebra can be realised in terms of bilinears of a free-fermion field ψ , whilst W_∞ can be realised in terms of bilinears of a free complex scalar field φ . The free-fermion construction was essentially sketched some time ago in ref. [10], and subsequently proposed in the context of W_∞ -type algebras [11,12], while the scalar-field realisation of W_∞ has recently been described explicitly in ref. [13]. To obtain a supersymmetric higher-spin algebra, we make use of both the free fermion and the free complex scalar to construct three different kinds of bilinear operators, which will necessarily yield a supersymmetric algebra. The bosonic component is the direct sum $W_\infty \oplus W_{1+\infty}$, where the two components are generated by the $\varphi\varphi$ and $\psi\psi$ operators, respectively, whilst the fermionic superpartners correspond to the mixed bilinear operators $\varphi\psi$. In the rest of this paper we describe the construction of the various bilinear operators, and then derive the explicit form of the resulting super- W_∞ algebra.

The starting point for our construction is a complex scalar field $\varphi(z)$, whose current we write as $j = \partial\varphi$, and a complex fermionic field $\psi(z)$, with two-point functions given by

$$\psi^*(z)\psi(w) \sim \frac{1}{z-w}, \quad j^*(z)j(w) \sim -\frac{1}{(z-w)^2} \quad (1)$$

with all other independent contractions vanishing. As is well known, the bilinear operator $- : j^* j :$ gives a spin-2 current that generates the Virasoro algebra. It was shown in [13] that if one defines the following currents $V^i(z)$ with conformal spin $s = i+2$ ($i \geq 0$),

$$V^i(z) = -\frac{2^{i-1}(i+2)!}{(2i+1)!} q^i \sum_{k=0}^i (-1)^k \frac{1}{i+1} \binom{i+1}{k} \binom{i+1}{k+1} : \partial^{i-k} j^* \partial^k j(z) :, \quad (2)$$

then these generate the W_∞ algebra introduced in [7,8], with central charge $c = 2$. The total number of derivatives in (2) is determined by the conformal spin. The relative sizes of the coefficients in the sum (2) are completely determined by the requirement that the central terms in the algebra should arise only between generators of the same conformal spin, while the overall normalisation of $V^i(z)$ is chosen to agree with the conventions of [8].

The parameter q is an arbitrary real constant; all non-zero values yield isomorphic algebras. As we shall see later, taking an appropriate $q \rightarrow 0$ limit gives the contraction to w_∞ .

We can construct another sequence of higher-spin operators, $\tilde{V}^i$, with conformal spins $s = i+2$ ($i \geq -1$), from bilinear operators in the fermionic field ψ in the following way:

$$\tilde{V}^i(z) = \frac{2^{i-1}(i+1)!}{(2i+1)!} q^{i+1} \sum_{k=0}^{i+1} (-1)^k \binom{i+1}{k} : \partial^{i+1-k} \psi^* \partial^k \psi(z) : \quad (3)$$

Using (1) we find that the operator-product expansion for these currents coincides with that of the $W_{1+\infty}$ algebra given in [9], with central charge $c = 1$. Again, the coefficients in the sum (3) are completely determined by the requirement that the central terms should be diagonal in conformal spin.

If we consider the two constructions together, it is clear that $V^i(z)$ and $\tilde{V}^i(z)$ generate the bosonic sector of a super-algebra, whose fermionic generators are given by bilinears built from j and ψ . We find that the appropriate bilinears are:

$$G^\alpha(z) = \frac{2^\alpha(\alpha+1)!}{(2\alpha+1)!} q^\alpha \sum_{k=0}^\alpha (-1)^k \binom{\alpha}{k} \binom{\alpha+1}{k} : \partial^{\alpha-k} j^* \partial^k \psi(z) :, \quad (4)$$

and $\tilde{G}^\alpha(z)$, which is defined as the expression (4) with $j^*(z)$ replaced by $j(z)$, $\psi(z)$ replaced by $\psi^*(z)$, and for convenience an extra overall factor of $(-1)^\alpha$. The generators $G^\alpha(z)$ have conformal spin $s = \alpha + \frac{3}{2}$ ($\alpha \geq 0$). The algebra that we obtain, which we shall refer to as “super- W_∞ ”, possesses $\hat{N} = 2$ supersymmetry. It is straightforward to see that the operator product of two G ’s or two $\tilde{G}$ ’s is non-singular, while $\tilde{G}^\alpha$ with G^β yields a sum of V and $\tilde{V}$ terms, plus in general central terms. The coefficients in (4) have been chosen so that these central terms vanish unless $\alpha = \beta$. One proves that the $\tilde{V}$ and $\tilde{G}$ central terms are diagonal for the coefficients given in (3) and (4) by expanding the identity $\frac{d}{dz} \frac{d}{dz} (1-x)^n|_{x=1} = 0$ for $m < n$, paralleling the argument given in [13] for the central terms in the VV sector.

In order to present the operator-product expansions in a relatively concise form, it is useful to introduce the following quantities:

$$M_\ell^{ij}(m, n) \equiv \sum_{k=0}^{\ell+1} (-1)^k \binom{\ell+1}{k} (2i - \ell + 2) [2j + 2 - k] \ell_{i+1-k} m^{\ell+1-k} n^k, \quad (5)$$

which correspond to just the terms of total degree $(\ell+1)$ in m and n in the polynomials $N_\ell^{ij}(m, n)$ given by (19) below. We then define the operators

$$F_\ell^{ij} \equiv M_\ell^{ij}(\partial_z, \partial_w) \frac{1}{(z-w)}, \quad (6)$$

where the differential operators act both on the $1/(z-w)$ and on functions to the right. The first few products of fermion operators then give:

$$\{\tilde{G}_r^\alpha, G_s^\beta\} = \sum_{\ell=0}^{\infty} q^\ell \left(b_{\ell}^{\alpha\beta}(r, s) V_{r+s}^{\alpha+\beta-\ell} + \tilde{b}_{\ell}^{\alpha\beta}(r, s) \tilde{V}_{r+s}^{\alpha+\beta-\ell} \right) + q^{2\alpha} \tilde{c}_\alpha(r) \delta_{r+s,0}, \quad (11)$$

$$[V_m^i, G_r^\alpha] = \sum_{\ell=0}^{\infty} q^{\ell-1} a_{\ell}^{i\alpha}(m, r) G_{m+r}^{\alpha+i-\ell+1}, \quad (12)$$

$$[\tilde{V}_m^i, G_r^\alpha] = \sum_{\ell=0}^{\infty} q^{\ell-1} \tilde{a}_{\ell}^{i\alpha}(m, r) G_{m+r}^{\alpha+i-\ell+1}, \quad (13)$$

$$[V_m^i, \tilde{G}_r^\alpha] = \sum_{\ell=0}^{\infty} q^{\ell-1} (-1)^{\ell+1} a_{\ell}^{i\alpha}(m, r) \tilde{G}_{m+r}^{\alpha+i-\ell+1}, \quad (14)$$

$$[\tilde{V}_m^i, \tilde{G}_r^\alpha] = \sum_{\ell=0}^{\infty} q^{\ell-1} (-1)^{\ell+1} \tilde{a}_{\ell}^{i\alpha}(m, r) \tilde{G}_{m+r}^{\alpha+i-\ell+1}. \quad (15)$$

We find that the structure constants are given by

$$\begin{aligned} g_{2\ell}^{ij}(m, n) &= \frac{1}{2(2\ell+1)!} \phi_{2\ell}^{ij}(0, 0) N_{2\ell}^{ij}(m, n), \\ \tilde{g}_{2\ell}^{ij}(m, n) &= \frac{1}{2(2\ell+1)!} \phi_{2\ell}^{ij}(0, -\frac{1}{2}) N_{2\ell}^{ij}(m, n), \\ b_{\ell}^{\alpha\beta}(r, s) &= \frac{1}{\ell!} B_{\ell}^{\alpha\beta} N_{\ell-1}^{\alpha-\frac{1}{2}, \beta-\frac{1}{2}}(r, s), \\ \tilde{b}_{\ell}^{\alpha\beta}(r, s) &= \frac{(-1)^\ell}{\ell!} \tilde{B}_{\ell}^{\alpha\beta} N_{\ell-1}^{\alpha-\frac{1}{2}, \beta-\frac{1}{2}}(r, s), \\ a_{\ell}^{i\alpha}(m, r) &= \frac{(-1)^\ell}{4(\ell+1)!} A_{\ell}^{i\alpha} N_{\ell-1}^{i, \alpha-\frac{1}{2}}(m, r), \\ \tilde{a}_{\ell}^{i\alpha}(m, r) &= \frac{1}{4(\ell+1)!} \tilde{A}_{\ell}^{i\alpha} N_{\ell-1}^{i, \alpha-\frac{1}{2}}(m, r), \end{aligned} \quad (16)$$

where

$$\begin{aligned} B_{\ell}^{\alpha\beta} &= -(2\alpha+2\beta+2-2\ell) \phi_{\ell}^{\alpha\beta}(\frac{1}{2}, -\frac{1}{4}) + (2\alpha+2\beta+3-2\ell) \phi_{\ell+1}^{\alpha\beta}(\frac{1}{2}, -\frac{1}{4}), \\ \tilde{B}_{\ell}^{\alpha\beta} &= (2\alpha+2\beta+4-2\ell) \phi_{\ell}^{\alpha\beta}(\frac{1}{2}, -\frac{1}{4}) - (2\alpha+2\beta+3-2\ell) \phi_{\ell+1}^{\alpha\beta}(\frac{1}{2}, -\frac{1}{4}), \\ A_{\ell}^{i\alpha} &= (i+1-\ell) \phi_{\ell}^{i\alpha}(0, 0) - (i+2) \phi_{\ell}^{i\alpha}(0, -\frac{1}{2}), \\ \tilde{A}_{\ell}^{i\alpha} &= (i+1) \phi_{\ell}^{i\alpha}(0, 0) - (i-\ell) \phi_{\ell}^{i\alpha}(0, -\frac{1}{2}). \end{aligned} \quad (17)$$

The functions $\phi_{\ell}^{ij}(\rho, \sigma)$ are particular examples of Saalschützian ${}_4F_3(1)$ generalized hypergeometric functions [8], as follows:

$$\phi_{\ell}^{ij}(\rho, \sigma) = {}_4F_3 \left[\begin{matrix} -\frac{1}{2} - \rho - 2\sigma, \frac{3}{2} - \rho + 2\sigma, -\frac{\ell}{2} + \rho - \frac{1}{2}, -\frac{\ell}{2} + \rho \\ -i - \frac{1}{2}, -j - \frac{1}{2}, i + j - \ell + \frac{5}{2} \end{matrix} ; 1 \right]. \quad (18)$$

$$\begin{aligned} \tilde{G}^0(z) G^0(w) &\sim V^0(w) + \tilde{V}^0(w) - 2q F_0^{-\frac{1}{2}, -\frac{1}{2}} \tilde{V}^{-1}(w) + \frac{1}{(z-w)^3}, \\ \tilde{G}^0(z) G^1(w) &\sim V^1(w) + \frac{2q}{3} F_0^{-\frac{1}{2}, -\frac{1}{2}} V^0(w) \\ &\quad + \tilde{V}^1(w) - \frac{4q}{3} F_0^{-\frac{1}{2}, -\frac{1}{2}} \tilde{V}^0(w) + \frac{8q^2}{9} F_1^{-\frac{1}{2}, -\frac{1}{2}} \tilde{V}^{-1}(w), \\ \tilde{G}^1(z) G^1(w) &\sim V^2(w) + \frac{8q}{9} F_0^{\frac{1}{2}, \frac{1}{2}} V^1(w) + \frac{16q^2}{45} F_1^{\frac{1}{2}, \frac{1}{2}} V^0(w) \\ &\quad + \tilde{V}^2(w) - \frac{10q}{9} F_0^{\frac{1}{2}, \frac{1}{2}} \tilde{V}^1(w) + \frac{28q^2}{45} F_1^{\frac{1}{2}, \frac{1}{2}} \tilde{V}^0(w) - \frac{8q^3}{27} F_2^{\frac{1}{2}, \frac{1}{2}} \tilde{V}^{-1}(w) \\ &\quad + \frac{32q^2}{3(z-w)^5}. \end{aligned} \quad (7)$$

Note that in the $\tilde{G}^0 G^0$ operator product the spin-2 fields appear in the combination $V^0 + \tilde{V}^0$, corresponding to the total stress tensor for the system; the $N = 2$ superconformal algebra naturally involves this single field in the spin-2 sector. However, in the higher-spin extension we see that other combinations occur in the operator products, making it necessary to distinguish the spin-2 fields from the φ and the ψ sectors. A similar doubling also occurs for each higher spin.

Based on experience with the bosonic W_∞ and $W_{1+\infty}$ algebras [8, 9], one might hope that the coefficients appearing on the right-hand sides of the operator-product expansions can be related to certain Saalschützian ${}_4F_3$ generalised hypergeometric functions. By looking at the coefficients in the expansions (7), together with other examples including some in the Fermi-Bose sectors, we have been able to deduce general expressions for the products of generators of arbitrary spins. The complete result will be given below, after re-expressing the operator-product expansions in terms of commutators and anticommutators of the Fourier modes of the operators.

For each of the fields $V^i(z)$, $\tilde{V}^i(z)$, $G^\alpha(z)$ and $\tilde{G}^\alpha(z)$, we can define the Fourier modes by using the expansion

$$f(z) = \sum_m f_m z^{-m-s}, \quad (8)$$

for the modes f_m of a field $f(z)$ with conformal spin s . Using standard techniques, we can then rewrite the operator-product expansions into commutators and anticommutators of the Fourier modes, as follows:

$$[V_m^i, V_n^j] = \sum_{\ell=0}^{\infty} q^{2\ell} g_{2\ell}^{ij}(m, n) V_{m+n}^{i+j-2\ell} + q^{2i} c_i(m) \delta^{ij} \delta_{m+n,0}, \quad (9)$$

$$[\tilde{V}_m^i, \tilde{V}_n^j] = \sum_{\ell=0}^{\infty} q^{2\ell} \tilde{g}_{2\ell}^{ij}(m, n) \tilde{V}_{m+n}^{i+j-2\ell} + q^{2i} \tilde{c}_i(m) \delta^{ij} \delta_{m+n,0}, \quad (10)$$

Our supersymmetric construction is naturally covariant under the diagonal $SL(2, R)$ algebra generated by the $m = -1, 0, 1$ modes of $V_m + \tilde{V}_m$. This covariance dictates that the dependence of the structure constants upon the mode numbers of the generators be given essentially in terms of Clebsch-Gordan coefficients. Explicitly, this dependence is described by the functions

$$N_\ell^{ij}(m, n) = \sum_{k=0}^{\ell+1} (-1)^k \binom{\ell+1}{k} (2i+2-\ell)_k (2j+2-k)_{\ell+1-k} [i+1+m]_{\ell+1-k} [j+1+n]_k. \quad (19)$$

The Pochhammer symbols are defined as usual by $(a)_n = (a+n-1)!/(a-1)!$ and $[a]_n = a!/(a-n)!$.

The central extensions are given by

$$\begin{aligned} c_i(m) &= (m-i-1)(m-i) \cdots (m+i)(m+i+1) c_i, \\ \tilde{c}_i(m) &= (m-i-1)(m-i) \cdots (m+i)(m+i+1) \tilde{c}_i, \\ \tilde{c}_\alpha(r) &= (r-\alpha-\frac{1}{2})(r-\alpha+\frac{1}{2}) \cdots (r+\alpha-\frac{1}{2})(r+\alpha+\frac{1}{2}) \tilde{c}_\alpha, \end{aligned} \quad (20)$$

where the central charges are given by

$$\begin{aligned} c_i &= \frac{2^{2i-3} i! (i+2)!}{(2i+1)! (2i+3)!} c, \\ \tilde{c}_i &= \frac{2^{2i-3} ((i+1)!)^2}{(2i+1)! (2i+3)!} c, \\ \tilde{c}_\alpha &= \frac{2^{2\alpha-2} \alpha! (\alpha+1)!}{[(2\alpha+1)!]^2} c, \end{aligned} \quad (21)$$

in terms of an overall real constant c . The generators V_m^0 and $\tilde{V}_m^0$ form two commuting Virasoro algebras, with central charges given by $12c_0 = c$ and $12\tilde{c}_0 = \frac{1}{2}c$, respectively. In the free-field realisation of super- W_∞ given earlier, c takes the value 2. For the abstract algebra, the parameter c can take arbitrary values.

Although the sums in (9)-(15) run from zero to infinity, it should be emphasised that owing to remarkable properties of the structure constants, the series terminate in such a way that the algebra closes on V^i with $i \geq 0$, $\tilde{V}^i$ with $i \geq -1$, and G^α and $\tilde{G}^\alpha$ with $\alpha \geq 0$. Most of the necessary zeros in the structure constants are supplied by the three different Saalschützian ${}_4F_3$ generalised hypergeometric functions in (16)-(18). There are a few zeros supplied by the "Clebsch-Gordan" part of the structure constants. By an intriguing principle of economy, these zeros occur precisely when the hypergeometric functions fail to provide the necessary zeros.

In principle the structure constants in the algebra (9)-(15) can all be derived from the operator-product expansions of V^i , $\tilde{V}^i$, G^α and $\tilde{G}^\alpha$. The associativity of the OPE would guarantee that the algebra satisfies the Jacobi identities. In practice, however, we postulated the form (9)-(15) for the structure constants by calculating a number of examples of operator products. Therefore an explicit verification of the Jacobi identities is needed. A complete

analytic proof is not available yet; however we have performed sufficiently many non-trivial checks by algebraic computing techniques to assure us that the Jacobi identities are satisfied.

Having established the consistency of the algebra we can now recast it in the form of operator-product expansions. The procedure for doing this has been described in detail in ref. [8]. It essentially consists of replacing the "Clebsch-Gordan" part of the structure constants by the operator F_ℓ^{ij} defined in (6). As an example, the commutator (9) becomes

$$V^i(z)V^j(w) \sim -q^{2i} c_i \delta^{ij} (\partial_z)^{2i+3} \frac{1}{z-w} - \sum_\ell \frac{q^{2\ell} \phi_{2\ell}^{ij}(0,0)}{2(2\ell+1)!} F_\ell^{ij} V^{i+j-2\ell}(w). \quad (22)$$

The super- W_∞ algebra that we have constructed admits a non-trivial automorphism that generalises the spectral-flow symmetry of the $N=2$ superconformal algebra described in [14]. It can be understood most easily by considering the following transformation of the free fields φ and ψ :

$$\varphi \rightarrow \varphi, \quad \psi \rightarrow z^\eta \psi, \quad \psi^* \rightarrow z^{-\eta} \psi^*, \quad (23)$$

where η is an arbitrary real constant. One can easily see that the singular terms in the two-point functions (1) remain unchanged under this transformation, and thus it gives rise to an automorphism of the operator-product algebra of V^i , $\tilde{V}^i$, G^α and $\tilde{G}^\alpha$. The action of the transformation on the individual fields becomes quite complicated in general, although low-order examples can easily be worked out. These can then be recast in terms of the Fourier components of the fields, by using (8). A few examples are:

$$\begin{aligned} \tilde{V}_m^{-1} &\rightarrow \tilde{V}_m^{-1} - \frac{c}{8q} \eta \delta_{m,0}, \\ \tilde{V}_m^0 &\rightarrow \tilde{V}_m^0 - 4q\eta \tilde{V}_m^{-1} + \frac{c}{4} \eta^2 \delta_{m,0}, \\ \tilde{V}_m^1 &\rightarrow \tilde{V}_m^1 - 8q\eta \tilde{V}_m^0 + 16q^2 \eta^2 \tilde{V}_m^{-1} - \frac{2qc}{3} \eta^3 \delta_{m,0}, \\ G_r^0 &\rightarrow G_r^0, \\ G_r^1 &\rightarrow G_{r+\eta}^1 - \frac{8q}{3} \eta G_{r+\eta}^0. \end{aligned} \quad (24)$$

The V_m^i generators are all invariant under the transformation.

As we remarked earlier, if one sets the parameter q to zero in the W_∞ algebra, one gets the contraction to the "classical" algebra w_∞ . Similarly, setting q to zero in $W_{1+\infty}$ gives the classical algebra $w_{1+\infty}$, i.e. w_∞ with an extra spin-1 generator. In the case of the super- W_∞ algebra that we have constructed here, one cannot simply set q to zero, since the leading terms in the Fermi-Bose commutators (12)-(15) will then become infinite. It is, however, possible to take a limit by first redefining generators and then setting $q = 0$. Appropriate redefinitions are

$$V_m^{(+)} = V_m^i + \tilde{V}_m^i, \quad V_m^{(-)} = q(V_m^i - \tilde{V}_m^i). \quad (25)$$

If we now set $q = 0$, we obtain precisely the $N=2$ super- w_∞ algebra constructed in [6]. The spectral flow described above becomes equivalent to that given in [15] for the $N=2$ super- w_∞ algebra.

There is a truncation of our $N = 2$ super- W_∞ algebra to $N = 1$, which corresponds to starting with real free fields φ and ψ instead of complex ones. This picks out the V^i and $\tilde{V}^i$ with $i = 0, 2, 4, \dots$, giving the supersymmetrisation of $W_{\infty/2} \oplus W_{(1+\infty)/2}$. The fermionic generators of this “super- $W_{\infty/2}$ algebra” are given by the linear combinations $G^\alpha + (-1)^\alpha \tilde{G}^\alpha$ for $\alpha = 0, 1, 2, 3, \dots$

There is another truncation of super- W_∞ that corresponds to setting all the higher-spin generators to zero, retaining just those of the usual $N = 2$ superconformal algebra. To obtain this in its standard form, we take $L_m = V_m^0 + \tilde{V}_m^0$, $J_m = 4q\tilde{V}_m^{-1}$, $G_r^0 \rightarrow \frac{1}{\sqrt{2}}\tilde{G}_r^0$ and $c \rightarrow \frac{2}{3}c$.

In the case of the bosonic W_∞ and $W_{1+\infty}$ algebras, there are “wedge” subalgebras that correspond to restricting the Fourier-mode index m on V_m^i or $\tilde{V}_m^i$ to lie within the range $-i - 1 \leq m \leq i + 1$. The algebras generated by these subsets of generators correspond to certain $N \rightarrow \infty$ limits of $SL(N, R)$ and $GL(N, R)$ respectively [8, 9]. A similar truncation is possible for the super- W_∞ algebra constructed in this paper. In addition to the above restrictions on the Fourier-mode indices of V_m^i and $\tilde{V}_m^i$, we must also restrict the index r on G_r^α . For example, in the Neveu-Schwarz sector, we restrict r to lie in the range $-\alpha - \frac{1}{2} \leq r \leq \alpha + \frac{1}{2}$. Presumably the resulting “super-wedge” algebra is an $N \rightarrow \infty$ limit of $SL(N+1|N)$. It would be interesting to generalise the bosonic constructions of [8, 16], and build up the super wedge as an enveloping algebra of the finite-dimensional $SL(2|1) \cong OSP(2|2)$ superalgebra that lies at the bottom of the wedge. Moreover, it should be possible to extend this construction beyond the wedge and build up the entire super- W_∞ as an enveloping algebra of the $N = 2$ superconformal algebra modded out by some appropriate ideals.

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