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SUPERSYMMETRY BREAKING FROM DUALITY INVARIANT GAUGINO CONDENSATION

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ABSTRACT

It is known that the formation of gaugino condensates can be a source of supersymmetry breaking in string theory. We study the constraints imposed by target space modular invariance on the formation of such condensates. We find that the dependence of the vacuum energy on the moduli of the internal variety is such that the theory is forced to be compactified. The radius of compactification is of the order of the string scale and in the process target space duality is spontaneously broken.

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One of the major open problems in four-dimensional string theories is the breaking of space-time ($N = 1$) supersymmetry. Because of phenomenological reasons it is desirable that space-time supersymmetry is broken spontaneously below the Planck scale, and the most promising scenario realizing this requirement is the mechanism of gaugino condensation in the so-called hidden sector [1-6]. Gaugino condensation cannot, up to now, be directly analyzed in string theory; however there are strong arguments that it actually occurs at the level of the low-energy effective field theory. More recently it was shown [7], [8], [9] that the effective supergravity action following from string compactification on orbifolds or even Calabi-Yau manifolds is severely constrained by an underlying string symmetry, the so-called target space modular invariance. The target space modular group $PSL(2, \mathbb{Z})$ acts on the complex scalar T as

$$T \rightarrow \frac{aT - ib}{icT + d}, \quad a, b, c, d \in \mathbb{Z}, \quad ad - bc = 1, \quad (1)$$

where $\langle T \rangle$ is the background modulus associated to the overall scale of the internal six-dimensional space on which the string is compactified. Specifically, $T = R^2 + iB$ with R being the "radius" of the internal space and B an internal axion. The target space modular transformations contain the well-known duality transformation $R \rightarrow 1/R$ [10] as well as discrete shifts of the axionic background B , and the T moduli space has to be restricted to the fundamental region $SU(1, 1)/(U(1) \times PSL(2, \mathbb{Z}))$.

Although duality respectively target space modular invariance is only shown [11] to be an unbroken symmetry at any order of string perturbation theory, one also expects that non-perturbative string effects respect these discrete symmetries. Adopting this point of view, the effective action describing the spontaneous supersymmetry breaking via non-perturbative effects, like gaugino condensates must be also invariant under the modular transformation on T . In fact it was shown in [7] that the non-perturbative, purely T -dependent effective superpotential must be a modular form like

$$W(T) \sim \eta(T)^{-6} \quad (2)$$

where $\eta(T) = q^{1/24} \prod_n (1 - q^n)$ is the well known Dedekind function, $q \equiv e^{-2\pi T}$, and the resulting gravitino mass is of the form $m_{3/2}^2 \sim \frac{1}{(T+\bar{T})^3 |W(T)|^2}$. We will

show that gaugino condensation provides a natural dynamical realization for exactly this kind of effective actions. Using the relation $W \sim \langle \lambda \lambda \rangle \sim \exp(\frac{3}{2b_0} f(T))$, this could be seen as if the gauge kinetic function of the $N = 1$ supergravity action is of the form $f(T) = -b_0 \log[\eta(T)^4]$. Interestingly enough, this kind of expression for the gauge kinetic function was recently derived from a direct one-loop string calculation [12].

The form of the effective $N = 1$ supergravity action including gauge and matter fields is specified [13] by two functions $G(\phi, \phi^*) = K(\phi, \phi^*) + \log |W(\phi)|^2$ and $f(\phi)$. $K(\phi, \phi^*)$ is the Kähler potential and $W(\phi)$ the superpotential, where ϕ denotes all chiral matter fields. We will concentrate in the following on the two chiral fields $\phi = S, T$. Here $\text{Re} S$ corresponds to the space-time dilaton and $\text{Im} S$ is the space-time axion. At string tree level the Kähler potential for these fields looks like [14],

$$K = -\log(S + S^*) - 3\log(T + T^*). \quad (3)$$

The requirement of modular invariance of the effective supergravity action forces G to be a modular invariant function which implies that the superpotential must transform with modular weight -3 [7]

$$W \rightarrow e^{i\alpha(a,b,c,d)} (icT + d)^{-3} W, \quad (4)$$

where $\alpha(a, b, c, d)$ is a field-independent phase.

The gauge part of the effective action is determined by the gauge kinetic function f

$$e^{-1} \mathcal{L}_{eff} = -\frac{1}{4} \text{Re} f F_{\mu\nu} F_{\mu\nu} + \frac{1}{4} i \text{Im} f F_{\mu\nu} \tilde{F}_{\mu\nu}. \quad (5)$$

Since we know from string theory [15] that the gauge fields do not transform under modular transformations it follows that f (a priori $\text{Im} f$ may change by a constant term) must be a modular invariant function of S and T . The gauge coupling constant is just determined by the vev of the gauge kinetic function, $1/g_4^2 = \langle \text{Re} f \rangle$, and the field theory θ -parameter is given by $\text{Im} f$.

At string tree level, the modular (duality) invariance of f is easily understood, remembering the relation between the four-dimensional gauge coupling constant,

the ten-dimensional gauge coupling constant g_{10} and the volume of the internal compact space

$$g_4^2 = \frac{g_{10}^2}{R^6}. \quad (6)$$

As shown in [11] the ten-dimensional gauge coupling constant must transform like $g_{10} \rightarrow \frac{g_{10}}{R^3}$ under $R \rightarrow \frac{1}{R}$ in order to obtain a duality invariant string partition function. It follows that g_4 is duality invariant. In fact, at string tree level g_4 is related to the vacuum expectation value (vev) of the four-dimensional dilaton field S : $f = S$ [14]. We conclude that S is invariant under target space modular transformations which is consistent with the transformation behaviour of the corresponding string vertex operator.*

Next consider the one-loop (genus one) contribution to the gauge coupling constant, i.e. the effective four-dimensional scale-dependent running coupling constant. Stringy renormalization effects were considered in [16], [17], [18], [19] and the running of the coupling constant in string theory is due to infrared divergences. The one-loop renormalization group equation has the form

$$\frac{1}{g_4^2(\Lambda)} = \frac{1}{g_4^2(\mu)} + 2b_0 \log\left(\frac{\mu}{\Lambda}\right). \quad (7)$$

b_0 is the one-loop renormalization group parameter ($b_0 < 0$ for asymptotic freedom) and $\Lambda < \mu$, where μ is the renormalization point below which the effective four-dimensional field theory is valid. This scale is set by the unification mass [4], [18], $\mu = [(T + T^*)(S + S^*)]^{-1/2}$. Thus the running coupling constant $g_4^2(\Lambda)$ depends on the background radius. Demanding $g_4^2(\Lambda)$ to be a modular invariant quantity, $g_4^{-2}(\mu)$ cannot simply be given by $S + S^*$, but has to get a T -dependent one-loop contribution:

$$\frac{1}{g_4^2(\Lambda)} = \frac{1}{2}(S + S^*) + \Delta(T, T^*) + 2b_0 \log\left(\frac{\mu}{\Lambda}\right). \quad (8)$$

$\Delta(T, T^*)$ can be viewed [16] as the threshold effect due to the one-loop integration over heavy string modes. In the large radius limit of string compactification,

* It is also straightforward to see that the fields S and T , as defined in [14], transform in the appropriate form. This can be seen by using the known transformation properties of the original fields ϕ and σ of [14].

the one-loop contribution to the gauge kinetic function was computed [20] via reduction of ten-dimensional Wess-Zumino terms in the ten-dimensional effective action: $f(T) \rightarrow T$, $\Delta(T, T^*) \rightarrow T + T^*$ for $R^2 \rightarrow \infty$. Note that in this large radius limit the gauge part of the effective action is invariant under the continuous Peccei-Quinn symmetry $\text{Im}T \rightarrow \text{Im}T + \text{const}$. Now we seek a modular function $\Delta(T, T^*)$ which exactly reproduces this asymptotic behaviour and has no additional singularities inside the fundamental region of the T -moduli space. A possible solution for $g_4^2(\Lambda)$ is given by:

$$\frac{1}{g_4^2(\Lambda)} = \frac{1}{2}(S + S^*) - b_0 \log |\eta(T)|^4 + 2b_0 \log\left(\frac{\mu}{\Lambda}\right). \quad (9)$$

Using the known modular transformation properties of $\eta(T)$, $\eta(T) \rightarrow (iT + d)^{1/2} e^{i\alpha(a,b,c,d)} \eta(T)$ (α is a field-independent phase) under $PSL(2, \mathbb{Z})$, it is easy to show that $g_4^2(\Lambda)$ is indeed modular invariant and has the correct asymptotic limit since $\eta(T) \rightarrow e^{-T}$ for $R^2 \rightarrow \infty$. It is remarkable that this general result was recently found [12] performing a direct computation of the one-loop gauge coupling constant in some orbifold compactifications. The modular invariant gauge coupling constant and thus the modular invariant one-loop effective action are no longer invariant under continuous shifts in $\text{Im}T$ but only under discrete transformations $T \rightarrow T + in$, $n \in \mathbb{Z}$. In this sense, the one-loop effective string action still exhibits a discrete no-scale structure. (This one-loop breakdown of the continuous Peccei-Quinn symmetry is analogous to the string tree level breaking [8] of the continuous $SU(1, 1)$ isometries of the Kähler potential eq.(3) by T -dependent Yukawa couplings among twisted matter fields in orbifold compactifications.)

Eq.(9) immediately determines the scale Λ_C where the gauge coupling constant becomes strong:

$$\Lambda_C = \mu \exp\left(\frac{1}{2b_0 g_4^2(\mu)}\right) = \frac{\exp\left(\frac{S+S^*}{2b_0}\right)}{|\eta(T)|^2 (T + T^*)^{1/2} (S + S^*)^{1/2}}. \quad (10)$$

Note that $\frac{1}{g_4^2(\mu)} = \frac{1}{2}(S + S^*) - b_0 \log |\eta(T)|^4$ is not modular invariant. However one can obtain a modular invariant function $g_4^2(\mu')$ by simply absorbing the renormalization scale, i.e. considering a new T -independent renormalization point

μ' :

$$g_4^{-2}(\mu') = \frac{1}{2}(S + S^*) - b_0 \log(|\eta(T)|^4(T + T^*)). \quad (11)$$

Conversely one can define a modular invariant renormalization point $(\mu'')^2 = \frac{1}{|\eta(T)|^4(T + T^*)(S + S^*)}$ with $g_4^{-2}(\mu'') = \frac{1}{2}(S + S^*)$. This point of view seems rather sensible since the effective four-dimensional field theory should be valid for scales smaller than the lightest string excitation, which is a momentum mode for large radii but a winding mode for small radii.* A similar argument is given in [12] in order to reabsorb the $(T + T^*)$ -factor into the definition of the unification mass.

Using $g_4^{-2} = \text{Ref}$, we find from equation (11)

$$f(\mu', S, T) = S - b_0 \log[\eta(T)^4(T + T^*)\delta]. \quad (12)$$

Here δ is a phase factor to ensure the proper modular behaviour of $f(\mu', S, T)$. Notice that $f(\mu', S, T)$ is non-holomorphic. On the other hand $f(\mu, S, T) = S - 4b_0 \log \eta(T)$ is holomorphic but non-modular invariant.

The impossibility of writing the modular invariant one-loop gauge coupling constant $g_4^{-2}(\mu')$ as the sum of a holomorphic plus an anti-holomorphic function with the only singularity at $R^2 = \infty$ is analogous to the so-called Quillen's holomorphic anomaly. To understand this consider

$$\partial_T g_4^{-2}(\mu') = \frac{b_0}{2\pi} \hat{G}_2(T, \bar{T}). \quad (13)$$

In general, the Eisenstein functions

$$G_{2k}(T) = \sum_{m,n \in \mathbb{Z}} (imT + n)^{-2k} \quad (14)$$

are holomorphic functions of modular weight $2k$ for $k > 1$. For $k = 1$ however, the sum does not converge and the modular invariant regularization procedure, which defines $\hat{G}_2(T, T^*)$, destroys holomorphicity. However $\hat{G}_2$ simply transforms

* The absorption of the T dependence into the definition of the renormalization point will obviously be modified from higher loop corrections to the running coupling constant.

with modular weight two. On the other hand, using a holomorphicity preserving regulator leads to an alternative definition $G_2(T) = \frac{2\pi}{b_0} \partial_T g_4^{-2}(\mu) = \hat{G}_2(T, T^*) + \frac{\pi}{R^2 \bar{T}}$ which transforms under $PSL(2, \mathbb{Z})$ as $G_2(T) \rightarrow (icT + d)^2 G_2(T) - 2\pi ic(icT + d)$. This transformation behaviour of the one-loop gauge coupling constant is in total analogy to the modular properties of G : the superpotential is holomorphic, but non-modular invariant, whereas G has to be modular invariant but non-holomorphic due to the contribution of the Kähler potential.

The supersymmetry breaking scale $(S + S^*)\Lambda_C^3$ (see eq.(10)) is identical to the gravitino mass $e^{G/2}$ provided that the superpotential is given by

$$W = \frac{e^{\frac{3}{2}S}}{\eta(T)^6}. \quad (15)$$

Thus the T -dependence of the effective superpotential generated by the gaugino condensate is exactly like that obtained in ref.[7], as already mentioned in the introduction. This can also be seen in an even more direct way by computing the vev of the gaugino condensate:

$$\langle \lambda \lambda \rangle = \mu^3 e^{\frac{3}{2}S} f(\mu', S, T) = \frac{e^{\frac{3}{2}S}}{\eta(T)^6 (T + T^*)^{3/2} (S + S^*)^{3/2} \delta^{3/2}}. \quad (16)$$

Since the gaugino condensate is the scalar component of a chiral superfield its vev may be viewed as an additional contribution to the superpotential. However, to see this one has to redefine the gaugino fields in the following way [21], [22]: $\lambda' = \lambda e^{-K/4} \left(\frac{W^*}{W} \right)^{1/4}$. (Here W is the complete superpotential containing in general also all charged matter fields.) Then the superpotential generated by the gaugino condensate is given by $W = (S + S^*)(\lambda' \lambda') = e^{\frac{3}{2}S} f(\mu', S, T)$ which exactly coincides with (15). Holomorphicity of W suggests identifying $\delta = \left(\frac{W}{W^*} \right)^{1/3}$. Therefore the change from a modular invariant, but non-holomorphic to a non-modular invariant, but holomorphic gauge kinetic function by a change of the renormalization point μ is equivalent to a field redefinition of the gaugino fields.

Local supersymmetry is spontaneously broken if the auxiliary fields

$$h^i = \exp\left(\frac{G}{2}\right) G^i = |W| \exp\left(\frac{K}{2}\right) \left[K^i + \frac{W^i}{W}\right] \quad (17)$$

get non-vanishing vacuum expectation values. Thus we have to study the asso-

ciated scalar potential and see if there are local minima with non-vanishing h^i . The scalar potential has the following form [13]:

$$V = |h^S|^2 G_{SS}^{-1} + |h^T|^2 G_{TT}^{-1} - 3e^G. \quad (18)$$

Our approach is going to be to some extent similar to the one of ref.[4]: we will consider the tree-level $N = 1$ supergravity Lagrangian and describe the effect of the formation of a gaugino condensate by an effective superpotential. This is certainly a naive approach since once the gauge interactions become strong close to the Λ scale, complicated dynamics including bound states should appear [23]. However our main concern here will be the dynamics associated to the moduli field T and not the one associated to the gauge dilaton S . From this point of view the general conclusions we obtain will not depend on the particular description chosen to simulate the gaugino condensation dynamics. We will, however, briefly mention below what changes if e.g., one just directly substitutes the gaugino bilinear term in the Lagrangian of ref. [13] by the corresponding condensate. Let us then consider an effective superpotential like the one of eq. (15). One can consider a slightly more general situation by including, in the spirit of refs. [3],[4], an additional S -independent piece $W_c = c/\eta(T)^6$ which is a modular invariant generalization of the constant superpotential considered in [4]. Our superpotential looks like

$$W = (c + h_a e^{\frac{3S}{2\Lambda}}) \eta(T)^{-6} \equiv \Omega(S) \eta(T)^{-6} \quad (19)$$

where the index a is counting the number of gaugino condensates [24]. In addition, the superpotential of the matter fields should also be present but we will assume, to simplify the discussion, that the vev's of possible matter fields are zero. Under these conditions the scalar potential derived from eq.(18), (3) yields

$$V = \frac{1}{S_R T_R^3 |\eta(T)|^{12}} \{ |S_R \Omega_S - \Omega|^2 + 3|\Omega|^2 \left(\frac{T_R^2}{4\pi^2} |\hat{G}_2|^2 - 1 \right) \} \quad (20)$$

where $S_R = 2\text{Re}S$, $T_R = 2\text{Re}T$ and $\Omega_S = \partial\Omega/\partial S$. Here the function $\hat{G}_2$ is the

weight two Eisenstein function $\hat{G}_2 = G_2 - 2\pi/T_R$ and is given by the expansion

$$G_2(T) = \frac{\pi^2}{3} - 8\pi^2 \sum_{n=1}^{\infty} \sigma_1(n) e^{-2\pi n T} \quad (21)$$

where $\sigma_1(n)$ is the sum of the divisors of n . The function $\hat{G}_2$ appears in eq.(20) due to the property

$$G_2 = -\frac{4\pi}{\eta(T)} \frac{\partial \eta(T)}{\partial T}. \quad (22)$$

Notice that the scalar potential is explicitly modular invariant.

The main difference between eq. (20) and the potential in ref.[4] is the overall $|\eta|^{-12}$ factor and the second term inside the curly brackets. The latter makes the potential non-positive definite.

Without any further analysis one can already draw some general conclusions from the form of the potential. In the decompactification limit $T_R \rightarrow \infty$ (and its dual $T_R \rightarrow 0$), the potential diverges $V \rightarrow \infty$ since the product $T_R^3 |\eta|^{12}$ vanishes exponentially at both points and $\hat{G}_2 \rightarrow \pi^2/3$ as $T_R \rightarrow \infty$. Thus, if gaugino condensation takes place the theory is forced to be compactified. This is an important difference with previous analyses, and is rather generic and independent of the description used to estimate the gaugino condensate.

Another generic fact is that the orbifold points in moduli space $T = 1$, $T = e^{\frac{2\pi i}{3}}$ (the self-dual points) are necessarily extrema of the potential [7]. This is because $\partial V/\partial T$ transforms with modular weight two and has no singularities inside the fundamental domain. One can also see this fact explicitly by computing $\partial V/\partial T$:

$$\begin{aligned} \frac{\partial V}{\partial T} = & \frac{3}{2\pi S_R T_R^3 |\eta|^{12}} \{ \hat{G}_2 [|S_R \Omega_S - \Omega|^2 + 3|\Omega|^2 \left(\frac{T_R^2}{4\pi^2} |\hat{G}_2|^2 - 1 \right)] \right. \\ & \left. + \frac{T_R}{2\pi} |\Omega|^2 [2|\hat{G}_2|^2 + T_R \hat{G}_2^* \hat{G}_{2T} + T_R \hat{G}_{2T}^* \hat{G}_2] \right\} = 0. \end{aligned} \quad (23)$$

The function $\hat{G}_2$ is known to have its only zeros at $T = 1$, $T = e^{\frac{2\pi i}{3}}$ and thus these self-dual points are necessarily extrema. At these points the auxiliary field $h_T = \exp(G/2) G_T$ also vanishes for the same reason that $\partial V/\partial T$ does, and hence supersymmetry remains unbroken (or at least this sector of the theory

does not break it). There are however generically other extrema of the scalar potential. In fact a numerical analysis of the T -dependent potential shows that typically the self-dual points correspond to local maxima or saddle points whereas there are other nearby extrema corresponding to local minima. At these minima supersymmetry is broken.

In order to study the extrema of the scalar potential in the full S - T space one has, in addition to eq.(23), the constraint

$$\frac{\partial V}{\partial S} = \frac{1}{S_R T_R^3 |\eta|^{12}} \{ S_R^2 \Omega_{SS} (S_R \Omega_S - \Omega)^* + \Omega^* (S_R \Omega_S - \Omega) \left[\frac{3 T_R^2}{4 \pi^2} |\hat{G}_2|^2 - 2 \right] \} = 0. \quad (24)$$

Notice that there is always an extremum at

$$S_R \Omega_S - \Omega = 0. \quad (25)$$

In fact one can check numerically that this extremum is always a minimum (for weak gauge coupling) as long as $c \neq 0$. Eq.(25) also corresponds to the local minimum with zero cosmological constant found originally in ref. [4]. In the present case, however, the cosmological constant is going to be different from zero at this level due to the existence of the second piece in eq.(20). On the other hand, this second piece is essential since it fixes the value of T_R and forces the theory to be compactified. The full potential can be written

$$\hat{V} = \frac{1}{S_R T_R^3 |\eta|^{12}} |S_R \Omega_S - \Omega|^2 + \frac{3 |\Omega|^2}{S_R} \hat{V}(T, T^*) \quad (26)$$

where $\hat{V}(T, T^*)$ is given by

$$\hat{V}(T, T^*) = \frac{1}{T_R^3 |\eta|^{12}} \left[\frac{T_R^2}{4 \pi^2} |\hat{G}_2|^2 - 1 \right]. \quad (27)$$

This latest expression is sufficient to discuss the minima in the T -direction. One can check that the expected extrema in the self-dual points correspond to a local

maximum in the case of $T = e^{\frac{2\pi}{3}}$ and a saddle point for $T = 1$ (maximum in the $\text{Re}T$ direction and minimum in the $\text{Im}T$ direction). The existence of nearby minima is easy to understand. Consider the extremum at $T = 1$. We know it is a maximum in the $\text{Re}T$ direction. If we keep $\text{Im}T=0$ fixed and let $T_R \rightarrow \infty$ (likewise for $\text{Re}T \rightarrow 0$) we know that $V \rightarrow +\infty$. Thus in this direction there must exist a minimum in the interval $1 < \text{Re}T < \infty$ and a "dual minimum" in the interval $0 < \text{Re}T < 1$. Those points are minima also along the $\text{Im}T$ direction so that they correspond to genuine local minima in the complete S, T space. The precise position for the field $\text{Re}S$ depends on the choice for the constant c and the beta function b_0 and is determined by eq.(25). The minimum for $\text{Re}T$ is always around $\text{Re}T \simeq 1.2$ in string units and rather independent of the rest of the fields. There is, of course, always its dual minimum at $1/\text{Re}T$. As expected from the modular invariance of the scalar potential, the latter is a periodic function of $\text{Im}T$ so that there is a whole family of local minima with the same $\text{Re}T$ but shifted $\text{Im}T = n, n \in \mathbb{Z}$. The qualitative shape of the potential in the $(\text{Re}T, \text{Im}T)$ plane is shown in the figure.

It is clear from the above discussion that the theory has to choose among an infinity of degenerate minima whose positions differ by modular transformations. Once the theory chooses one of them, target-space modular invariance is spontaneously broken. This was to be expected since the theory somehow has to fix the compactification scale and, once this is done, duality has to be spontaneously broken. As is the case for all spontaneously broken symmetries, all couplings will still respect duality and only the vacuum will be asymmetric. At this point one could make some speculations about the effects of this symmetry breaking. Thus, for example, since duality is a discrete symmetry, if there was a phase in the evolution of the universe in which the compactification radius was spontaneously chosen, "duality domain walls" could be created separating different universes (e.g. the "R-world" in which momenta are used to define a position operator, and the "1/R-world" in which winding modes play that role).

In the above study we have described the gaugino condensation through an effective superpotential. One can find similar results if instead [1] we explicitly substitute the gaugino bilinear in the supergravity action of ref. [13] by the expression eq.(16), still keeping a S -independent superpotential $W = c\eta^{-6}(T)$.

In addition we use here the tree level gauge kinetic function $f = S$, since also the Kähler potential (3) corresponds to string tree-level computations. In this case the scalar potential has the form

$$V = \frac{1}{S_R T_R^3 |\eta|^{12}} |c + h_a e^{\frac{2\pi}{3} a}|^2 + \frac{3|c|^2}{S_R} \hat{V}(T, T^*) \quad (28)$$

where $\hat{V}$ is defined in eq. (27). The only essential difference is that instead of a factor $|\Omega|^2$ multiplying $\hat{V}$, there is the constant $|c|^2$. Still the minimum of this potential is qualitatively the same, as long as $|c|$ is small enough (i.e. the weak coupling region). S is fixed to (approximately) cancel the first term and the vev for T is fixed by the same function $\hat{V}(T, T^*)$. Thus the results described above are independent of the way one describes the gaugino condensate. We also expect the qualitative results to remain valid in the effective Lagrangian approach of reference [23].

Coming back to the superpotential description, since $h_S \propto S_R \Omega_S - \Omega = 0$ at the minima, it is the moduli T sector which is responsible for supersymmetry breaking ($h_T \neq 0$) and the gravitino mass is given by

$$m_{3/2} = e^{\frac{g}{2}} = \frac{|\Omega|}{S_R^{1/2} T_R^{3/2} |\eta|^6} = \frac{|\Omega_S|}{S_R^{-1/2} T_R^{3/2} |\eta|^6}. \quad (29)$$

The size of $m_{3/2}$ (and thus the scale of supersymmetry breaking) is governed by the size of the arbitrary constant c . In order to get the desired hierarchy, a value $|c| \sim (10^{14} \text{ GeV})^3$ is needed. This requires a certain amount of fine-tuning. Furthermore, the origin of such a term is unclear. It could originate from the vacuum expectation value of a superpotential involving “hidden sector” matter or else it could have some gravitational origin [25]. Other suggestions to replace this constant term include the possible existence of two simultaneous condensates [24], [26] with almost identical β -function. Whatever the origin of the small factor $|\Omega|$ in eq. (29) may be, we believe that the T -dependence is quite universal and essentially dictated by modular invariance, and that the qualitative results described above remain valid when these other possible complications are taken into account.

One interesting question one may ask is what the soft supersymmetry-breaking terms in the low-energy effective Lagrangian are. One of them is the gaugino mass. At the tree level it vanishes since one has $M_\lambda = f_S G_{SS}^{-1} h_S + f_T G_{TT}^{-1} h_T$ and $h_S = 0$ at the minima and $f_T = 0$ at the tree level. We know however that f gets a T -dependent piece at the one-loop level. Using eq.(12) one gets (neglecting the contribution of the matter fields to the superpotential, i.e. to the phase δ)

$$M_\lambda = f_S G_{SS}^{-1} h_S + f_T G_{TT}^{-1} h_T = f_T G_{TT}^{-1} G_T e^{G/2} = \frac{b_0 T^2}{4\pi^2} |\hat{G}_2|^2 m_{3/2} \quad (30)$$

which is of order $\sim b_0 m_{3/2}$, as expected, since it is a one-loop effect. This situation is remarkably similar to the one discussed in ref. [20]. It also implies that if we set $M_\lambda \sim 10^{2-3} \text{ GeV}$, one can have $m_{3/2} \sim 10^{4-5} \text{ GeV}$. This might be interesting in order to avoid cosmological bounds on $m_{3/2}$ from the gravitino abundance. The other two relevant renormalizable soft SUSY-breaking terms which will be induced are scalar masses $m_\phi^2 = \partial^2 V / \partial \phi \partial \phi^*|_{T=T_{\min}}$ and trilinear scalar couplings $A_{ijk} m_{3/2} = \partial V / \partial \phi_i \partial \phi_j \partial \phi_k|_{T=T_{\min}}$. These are model dependent, since in order to compute them one would have to include the matter superpotential in our previous analysis.

Possible generalizations of the previous discussion could be envisaged. The gauge kinetic function f can get corrections due to higher loops and non-perturbative effects. The same will happen with the Kähler potential, and hence other modular functions like, e.g., the absolutely modular invariant $j(T)$ function could turn out to appear in the effective action. Also, the explicit inclusion of non-vanishing matter fields and extra moduli, being a representation of an enhanced modular group would be an interesting generalization of the present work. Since the fixed points of the generalized duality transformations are usually points of enhanced gauge symmetries, if our conclusions for the field T hold generally for any other flat direction in the theory (including charged fields), this could be an indication that the preferred string vacua will have spontaneously broken duality, supersymmetry and gauge symmetries so naturally lower rank models may be selected.

Let us conclude by stating the main results presented in this letter. We have studied the constraints target-space modular invariance puts on the formation

of supersymmetry breaking gaugino condensates. We were motivated by the fact that the gaugino condensation mechanism as proposed in refs. [3],[4] is inconsistent with duality. We have presented a modular-invariant version of this mechanism which turns out to give an explicit example of a duality-invariant effective Lagrangian as suggested in [7]. Interestingly enough we find that gaugino condensation forces the theory to be compactified and the compactification scale is of the order of the string scale (but not the self-dual points). This seems to be rather independent of the detailed dynamics of gaugino condensation. Our analysis shows an overall scenario in which there is an infinite number of degenerate minima related to each other by modular transformations. The dynamical choice of one of these minima spontaneously breaks duality.

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Figure caption

The scalar potential $\hat{V}$ as a function of $(\text{Re}T, \text{Im}T)$. Notice the maxima and saddle points at the self-dual points $(\frac{\sqrt{3}}{2}, \frac{1}{2} + n)$, $(1, n)$ respectively. The minima of $\hat{V}$ are at $(\sim 1.2, n)$ and their modular transformed images which are not shown in the figure.

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