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3D MAXWELL-CHERN-SIMONS SPONTANEOUS SYMMETRY BREAKING

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ABSTRACT

We study the spontaneous symmetry breaking of the 3D Maxwell-Chern-Simons (MCS) system. This action has one transverse, massive, parity sensitive degree of freedom. We show that after symmetry breaking, this excitation remains massive (although with an increased value) while the other transverse, parity sensitive, vector-like degree of freedom of the system (initially non-excited) acquires mass. These two masses are always different. Covariant, light front and canonical analysis of the massive MCS system are given as well as the two coupled reduced actions. The quartic behaviour of the system propagator stems in this coupled structure.

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Due to a multiplicity of interesting features, 3D-vector Chern-Simons (VCS) and tensor Chern-Simons theories [1] have been widely studied in the last years. They present the possibility of charged particles with real spin [2] obeying what is called θ -statistics (anions). More recently, models having VCS-terms have been proposed for understanding, at the level of effective theories, high T_c superconductivity [3] and planar antiferromagnets [4]. This was due to the suggestion of Polyakov and co-workers [5] that Abelian CS theories might model the long wavelength physics of higher temperature superconducting oxides.

It seems natural to wonder what happens for these VCS theories concerning the spontaneous symmetry breaking mechanism. In that direction, recently Dezer and Yang [6] pointed out that, starting from the more economical model consisting of the VCS term plus charged scalars with a Higgs potential, spontaneous symmetry breaking leads to the presence of a mass term in the vector sector which completes the pure VCS term to obtain the self-dual (massive) action. The Goldstone boson is eaten by the (locally) 0-degree of freedom statistical CS vector field and emerges as a massive spin-one parity-sensitive excitation.

On the other hand, even when the bare action is the pure VCS-term, Semenov et al. [7] have shown that a Maxwell term is radiatively generated in the effective action. Conversely if fermions are coupled to 3D-Maxwell field, they induce the VCS-parity breaking term in the effective action [8]. Therefore it seems to us worthwhile analyzing symmetry breaking for the MCS system.

This was considered some time ago by Pisarski and Rao and by Paul and Khare [9] who discovered the propagation of two distinct masses which satisfy a quartic equation.

This mechanism depends on the structure of the (naive) massive MCS system (where massive means the presence of the standard mass term). The dynamical analysis of the massive MCS system is the central aim of this letter.

We shall see the physical reasons why the quartic equation which determines the two vector masses factorizes in two quadratic expressions, each providing one of the two physical values of the masses. We shall also show both at the level of the light-front approach and of the canonical dynamical analysis that the two massive excitations remain coupled giving rise to the presence of a quartic propagator - and that indeed the naive massive MCS system propagates

two transverse massive excitations. It does not contain longitudinal excitations.

The first-order unbroken $U(1)$ initial action is

$$S = \langle p_r p_r^\dagger - p_r D_r \varphi - p_r^\dagger D_r^\dagger \varphi^\dagger \rangle - (h|\phi|^2 + u^2) (|\phi|^2 - v^2)^2 - \frac{1}{2} \langle f_r^2 \rangle + \langle f_r \epsilon^{rmn} \partial_m a_n \rangle - \frac{1}{2} \mu \langle a_r \epsilon^{rmn} \partial_m a_n \rangle \quad (1)$$

where p_r , $p_r^\dagger$, φ , $\varphi^\dagger$, f_r , a_m are independent variables. Under an infinitesimal Abelian transformation of parameter χ , $\delta p_r = -i\chi p_r$, $\delta \varphi = i + i\chi \varphi$, f_r is invariant and $\delta a_r = e^{-1} \partial_r \chi$, $D_r \varphi = (\partial_r - ie a_r) \varphi$. In 3D, bosons (fields obeying second-order differential equations) have dimension $M^{\frac{1}{2}}$. The Abelian coupling constant $e \sim M^{\frac{1}{2}}$, $D_r \varphi = (\partial_r - ie a_r) \varphi$ transforms covariantly like $p_r^\dagger$, u , $v \sim M^{\frac{1}{2}}$, h is dimensionless.

The true vacuum lies at $\varphi_0 = v$, so we shift from $\varphi \rightarrow \varphi^{New} = \varphi - v$. It is also convenient to shift from p_r , $p_r^\dagger \rightarrow p_r^{New} + ie v a_r$, $p_r^{New} - ie v a_r$. In terms of these new variables, S becomes

$$S = \langle p_r p_r^\dagger - p_r D_r \varphi - p_r^\dagger D_r^\dagger \varphi^\dagger + ie v a_r (D_r^\dagger \varphi^\dagger - D_r \varphi) - [h|\varphi|^2 + h v (\varphi + \varphi^\dagger) + h v^2 + u^2][|\varphi|^2 + v(\varphi + \varphi^\dagger)]^2 \rangle + S_{mass} \quad (2)$$

where

$$S_{mass} = -\frac{1}{2} \langle f_r^2 \rangle + \langle f_r \epsilon^{rmn} \partial_m a_n \rangle - \frac{1}{2} \mu \langle a_r \epsilon^{rmn} \partial_m a_n \rangle - \langle e^2 v^2 a_r^2 \rangle \quad (3)$$

is what we call (naive) massive MCS action. The scalar sector of action (3) is the standard one, it only propagates the real scalar excitation $\sim \varphi + \varphi^\dagger$. The other possible scalar excitation $\sim \varphi - \varphi^\dagger$ does not exist anymore.

In 4D, (3) does not contain the CS term and what is left represents the propagation of the three components of a massive spin-1 boson. Here we initially have a vector-like, parity sensitive excitation of mass μ .

We want to examine what are the dynamical consequences of the presence of the mass term $\sim a_r^2$ (due to symmetry breaking).

First we do a covariant treatment. Variations of f_r lead to

$$f_r = \epsilon_r^{mn} \partial_m a_n \quad (4)$$

while varying a_r gives the basic propagating equation

$$E_r^{mn} := \epsilon^{mnar} \partial_a f_r - \mu \epsilon^{mnar} \partial_a a_r - 2e^2 v^2 a^{rn} = 0, \quad (5)$$

$\eta = (-, +, +)$ and $\epsilon^{012} = +1 = \epsilon^{12}$. By taking the divergence we learn that a_r is 3D-transverse: $\partial_r a_r = 0$. This indicates that, at most, its transverse part (two degrees of freedom) will propagate, as we will show in a minute.

We use the 3D-projectors of [10]. In the case of a_r , since we have already seen it is transverse, it can be decomposed as follows:

$$a_r = a_r^+ + a_r^-; \quad a_r^\pm := 2^{-1}(\delta_r^s \pm \epsilon_r^{3s} \rho_s) a_s, \quad (6)$$

$$\rho_r := \Pi^{-\frac{1}{2}} \partial_r, \quad \rho_r a_r = 0. \quad (7)$$

Insertion of (4) into (5) and substitution of $a_r = a_r^+ + a_r^-$ in it yields

$$E_r^{mn} := \Pi(a_m^+ + a_m^-) - \mu \Pi^{\frac{1}{2}}(a_m^+ - a_m^-) - 2e^2 v^2(a_m^+ + a_m^-) = 0. \quad (8)$$

Taking the $+$, $-$ projections leads to the two equations which govern the evolution of the two independent transverse degrees of freedom a_m^+ , a_m^- :

$$\Pi a_m^+ - \mu \Pi^{\frac{1}{2}} a_m^+ - 2e^2 v^2 a_m^+ = 0, \quad (9)$$

$$\Pi a_m^- + \mu \Pi^{\frac{1}{2}} a_m^- - 2e^2 v^2 a_m^- = 0. \quad (10)$$

The equations providing the value of the two masses, one for a_m^+ and the other for a_m^- are,

respectively,

$$X^2 \mp \mu X - 2e^2 v^2 = 0. \quad (11)$$

Their solutions are:

$$X_\pm = 2^{-1} \mu [(1 + 8e^2 v^2 \mu^{-2})^{\frac{1}{2}} \pm 1]. \quad (12)$$

This formula shows that the dominant presence, concerning mass spectrum, is the CS-term and not the Maxwell one. Incidentally, if in Eqs. (11) one throws away the Maxwell contribution $\sim \Pi$, one then has the self-dual system [11], Eq. (11) loses X^2 and then the unique mass is given by $X = \mu^{-1} 2e^2 v^2 \approx X_-$. The value X_+ is due to the presence of the Maxwell terms and to its connection with the CS term. It cannot be obtained solely with CS and naive mass terms (self-dual model). The mathematical drawback of this covariant treatment stems in the non-locality of $\Pi^{-\frac{1}{2}}$. In the first of Ref. [9] it has been shown that Eq. (5) factorizes into the product of two self-dual operators $(\delta_r^s - (x^+)^{-1} \epsilon_r^{3s} \partial_s) (\delta_r^s + (x^-)^{-1} \epsilon_r^{3s} \partial_s)$. Unlike projectors defined in (7), self-dual operators are only on-shell projectors. Now we go to the light-front co-ordinates, where the treatment is essentially local. We will see that the two physical excitations remain coupled in the reduced action, this being the reason for having a quartic equation (the product of Eq. (11) + times Eq. (11) -) for the mass spectrum. We define $x^\pm := 2^{-\frac{1}{2}}(x^0 \mp x^3)$, x^1 remains unchanged, $-\eta^{+-} = +1 = \eta^{11}$ for arbitrary two vectors $a_r, b_r = a_+ b^+ + a_- b^- + a_1 b^1 = -a_+ b_- - a_- b_+ + a_1 b_1$, $\epsilon^{+1-} = +1$. Our "time" is x^+ . We start from the first-order action (3) and write it as a functional of the six light-front components f_r, a_m . It has the aspect $(\partial_+, f =: \dot{f}, \partial_- f =: f')$

$$\begin{aligned} S = & \{f_- \dot{a}_1 - 2^{-1} f_1^2\} + \{f_+ [f_- - a_1' + a_{-1}]\} \\ & + \{a_+ [\mu a_1' + f_{-1} - f_1' - \mu a_{-1} + 2e^2 v^2 a_-]\} \\ & + \{a_- [f_1 - \mu a_1]\} - e^2 v^2 \{a_1^2\} \end{aligned} \quad (13)$$

We realize that f_+ and a_+ are Lagrange multipliers associated with two constraints. The first of them, $\delta S / \delta f_+$, is an algebraic one, provides the value of $f_- = a_1' - a_{-1}$. The second one, $\delta S / \delta a_+$ is the generalization to this system of the Gauss constraint. Its rôle is to determine

$f_1 = f_1(a_-, a_1)$ through one integration $\sim (\partial_-)^{-1}$. From the reading of action (13) it seems that a_1 will be one of the two dynamical variables of the system. The above analysis tells us that the unconstrained action will be a functional of two variables a_-, a_1 but this does not ensure that these variables are light-front canonical. In fact they are not. To solve the Gauss constraint $\delta S/\delta a_+ = 0$ we introduce $A := \partial_-^{-1} a_-$. Then f_1 turns out to be

$$f_1 = (\partial_1 + \mu)a_1 - (\partial_1 + \mu)\partial_1 A + 2e^2 v^2 A \quad (14)$$

where $f_- = a_1' - a_{-,1}$ has been used. Insertion of these values of f_-, f_1 into action (13) will give a reduced, unconstrained action. However, it does not have a canonical form. It is convenient to introduce a new variable $b := a_1 - \partial_1 A = \partial_-^{-1} f_-$, and then redefine $A \rightarrow \sqrt{2} ev A$. The reduced action becomes

$$S_{\text{unconstr.}} = (b' + A'A) - 2^{-1} \left((\partial_1 b)^2 + (\mu^2 + 2e^2 v^2) b^2 + (\partial_1 A)^2 + 2e^2 v^2 A^2 + 2\sqrt{2} \mu ev bA \right). \quad (15)$$

Its $q'q'$ part appears in canonical form, and the Hamiltonian, while legal, contains one term which couples the two dynamical variables. As we can see, the quartic structure of the propagator of the system stems in the presence of this term. The Hamiltonian of Eq. (15) is simple enough to permit to see that it is positive semi-definite (by realizing that $\mu^2 b^2 + 2e^2 v^2 A^2 + 2\sqrt{2} \mu ev bA = (\mu b + \sqrt{2} ev A)^2$). Varying b, A in action (15) gives the evolution of this coupled system

$$[\Pi - (\mu^2 + 2e^2 v^2)b - \sqrt{2} ev \mu A = 0, \quad (16)$$

$$[\Pi - 2e^2 v^2]A - \sqrt{2} ev \mu b = 0. \quad (17)$$

Both b, A satisfy the quartic equation

$$[\Pi^2 - (\mu^2 + 4e^2 v^2)\Pi + 4e^4 v^4](b, A) = 0 \quad (18)$$

This is the equation found in Ref. [9]. It is straightforward to check that this quartic operator is the product of the quadratic operators appearing in Eqs. (9) and (10).

Finally, let us consider the 2+1 decomposition, $x^\tau = (x^0, x^i)$ $i = 1, 2$ and see what happens. Our starting point is, again, action (3). The 2+1 decomposition yields

$$S = 2^{-1} \left\{ f_0^2 - f_i^2 \right\} - e^2 v^2 (a_i^2 - a_0^2) + \langle f_0 \epsilon^{ij} \partial_i a_j \rangle \\ \langle a_0 \epsilon^{ij} \partial_i (f_j - \mu a_j) \rangle + 2^{-1} \mu \langle a_i \epsilon^{ij} \partial_0 a_j \rangle - \langle f_i \epsilon^{ij} \partial_0 a_j \rangle. \quad (19)$$

It is useful to introduce a T+L decomposition and projectors in the 2D pure space

$$\rho := (-\Delta)^{\frac{1}{2}}, \quad \rho_i := \bar{\rho}^{-1} \partial_i (\rho_i \rho_i = -1), \quad v_i = \rho_i v_L + \epsilon_{ij} \rho_j v_T, \quad (20)$$

$$\epsilon^{ij} \partial_i a_j = \rho a_T, \quad \langle v_i \epsilon^{ij} a_j \rangle = \langle v_T a_L - v_L a_T \rangle \quad (21)$$

Reading Eq. (19) one observes the presence of two quadratic constraints, one constituted by the terms containing a_0 and the other one formed by the terms depending upon f_0 . Their respective solutions are:

$$a_0 = 2^{-1} e^{-2} v^{-2} \rho (\mu a_T - f_T), \quad f_0 = -\rho a_T \quad (22)$$

Then we insert these values of a_0, f_0 into action (19) and we shift f_L, f_T to canonical variables $f_L \rightarrow \tilde{f}_L := f_L, f_T \rightarrow \tilde{f}_T := f^T - \mu a^T$. Omitting tildes, the unconstrained, canonical form of action (19) turns out to be

$$S_{\text{unconstr}} = \langle f_L \dot{a}_T - f_T \dot{a}_L \rangle - 2^{-1} \langle (\rho a_T)^2 \rangle - 4^{-1} e^{-2} v^{-2} \langle (\rho f_T)^2 \rangle \\ - \langle e^2 v^2 a_L^2 + e^2 v^2 a_T^2 \rangle - 2^{-1} \langle f_L^2 \rangle - 2^{-1} \langle (f_T + 2^{-1} \mu a_T)^2 \rangle. \quad (23)$$

As expected from the previous results, there are two canonical co-ordinates a_T, a_L whose corresponding momenta are $f_L, -f_T$. Unfortunately there is no decoupling, a non-diagonal term is present $\sim f_T a_T$. Action (23) explicitly shows that its energy is definite positive. Concerning its propagation one can derive, performing independent variations of a_T, a_L, f_T, f_L , their field

$$f_L = \dot{a}_T \quad (24)$$

$$2\dot{e}^2 v^2 a_L = \dot{f}_T \quad (25)$$

$$(2^{-1} e^{-2} v^{-2} \Delta - 1) f_T - \mu a_T = \dot{a}_L \quad (26)$$

$$-\mu f_T + (\Delta - (\mu^2 + 2\dot{e}^2 v^2)) a_T = \dot{f}_L \quad (27)$$

In this form it seems a little cumbersome to find out the propagation equation obeying a_T, a_L . However, since we know the system is relativistic and we can assume it is massive, we go to the rest frame of the excitations. Terms $\sim \Delta$ vanish from the system (24) - (27). In the rest frame both a_T, a_L have to obey the equation

$$[(\partial_0^2 + 2e^2 v^2)^2 + \mu^2 \partial_0^2] a_T, a_L = 0. \quad (28)$$

Since, in the rest frame $\partial_0^2 = -X^2$, X being the mass, it is immediate to check it that coincides with Eq. (18). Now, since the system is relativistic, we substitute $\partial_0^2 \rightarrow -\Pi$ and obtain the decoupled propagation equations for both a_L, a_T . A direct calculation from (24) - (27) will confirm this.

In conclusion, we have seen how symmetry breaking proceeds for the MCS action. Initially it has one massive excitation and, after eating the Goldstone scalar, contains two transverse massive excitations, one stemming from the effect of the (naive) mass term on the pure CS term and the other essentially due to the stability of the MCS system. CS effects dominate the system (compared with the Maxwellian ones). The covariant approach leads to a quick insight (although illegal because non-local) of what is the underlying structure of massive MCS. Light-front co-ordinates allowed to show what are the dynamical reasons behind the presence of a quartic propagator: the two physical excitations are coupled. Finally, for the sake of completeness we studied its canonical form where again it was shown that one of the two physical co-ordinates is coupled with the (non-respective) momenta through one term.

We expect to report soon on the problem of the spin and statistics behaviour of this system [12], as well as on the generalization to the non-Abelian case, where there is no self-dual action.

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