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Bounds on ordinary-exotic fermion mixing from LEP-1

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Abstract

We show that recent measurements of the partial widths of the Z boson at LEP significantly improve previous existing bounds on the mixing between ordinary fermions and possible heavy fermions with exotic $SU(2) \times U(1)$ assignments, especially for the μ and τ leptons and for the b quark. We show that in some extensions of the standard model with an additional $U(1)$ factor (as in E_6 models that we analyse in some detail), the effects of a $Z - Z'$ mixing can produce similar effects to those due to fermion mixing and then both should be taken into account. We constrain s_f^2 , the square of the mixing between ordinary and exotic fermions, to $s_\tau^2 < 0.025$, $s_b^2 < 0.077$ and $s_\mu^2 < 0.024$, improving the previous bounds by almost a factor 10 for the τ lepton, by a factor 5 for the b quark and by a factor 2 for the μ .

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One of the striking results of the first few months of running of the LEP-1 and SLC machines is that no new particles have been produced. The bounds on the masses of many new particles that are predicted by a large class of models (SUSY, Composites, GUTs,...) are already near the kinematic limit accessible with these two machines and thus it seems that the search for direct evidence of new physics must be delayed to the time when a larger centre-of-mass energy will be available.

On the other hand, the experimental data agree moderately well with the Standard Model (SM) and, even if for the moment there is no clear evidence of deviations from the theoretical predictions, it is possible that in the near future measurements at the Z -peak might reveal the existence of physics beyond the SM through indirect effects.

The increase in statistics, together with a better understanding of the systematic errors, will provide us with a set of high precision measurements that will be quite effective in the search for tiny new effects or, at least, to put stricter bounds on the relevant parameters that are generally introduced in any extension of the electroweak theory.

For example, deviations from the SM predictions are expected if gauge groups larger than $G_{SM} = SU(2)_L \times U(1)_Y [\times SU(3)_C]$ underlie the standard electroweak theory and in particular, these deviations could be due to a mixing among the standard fermions and new exotic ones (that often occur in models with enlarged gauge groups), as well as to a mixing of the standard Z_0 with additional neutral vector bosons. Both these effects will modify the fermion couplings to the gauge bosons, and most of the quantities that are measurable at the Z -peak are particularly effective for detecting possible deviations from the standard neutral current couplings.

As far as the mixing among the gauge bosons is concerned, we will assume that only one new neutral Z_1 mixes appreciably with the Z_0 , and then we are led to investigate the phenomenological consequences of an effective gauge group $G_{SM} \times U(1)'$. Since the direct product structure leaves the $U(1)'$ fermion quantum numbers, as well as the g' coupling constant completely arbitrary, a second assumption has to be made in order to obtain predictions: namely that our effective low energy gauge group originates from a *simple* group G_S , broken by some mechanism at a higher

energy scale. Then, since $U(1)'$ belongs to the Cartan subalgebra of G_S , only a few choices for the quantum numbers of the particles present in the model will be allowed, and the possible range for the value of the coupling constant g' will also be constrained.

For the sake of definiteness we will carry out our investigation in the frame of a class of E_6 models. The consequences of the presence of a new Z_1 of E_6 origin on Z -resonance physics has been deeply investigated by many authors [1]. However, to consider the modifications of the Z -couplings due to a $Z_0 - Z_1$ mixing alone is not totally consistent in these models, since similar effects can arise also from fermion mixing. In particular, since each fermion generation is assigned to a **27** representation of E_6 , besides the 15 standard fields 12 additional 'exotic' particles per generation are predicted to exist. These are: a weak doublet of leptons (N, E^-) with its charged conjugate doublet (E^+, N^c), a colour triplet weak singlet quark h of charge $-1/3$ together with h^c , and two neutral singlets ν^c and S . In general a mixing among particles that have the same quantum numbers under the unbroken $U(1)_Q \times SU(3)_C$ group will be allowed, modifying the standard couplings of the fermions.

Bounds on the mixing of a Z_1 of E_6 origin with the standard neutral boson have been derived in ref. [2]. The results of that analysis constrain the mixing to $\tan \Theta_{mix} \lesssim 0.22$ for a Z_1 almost decoupled from neutrinos, and to a much lower value ($\tan \Theta_{mix} \lesssim 0.05$) in the other cases. However, the analysis in [2] does not take into account the possibility of fermion mixing, so that a combined analysis of these two effects should turn out in slightly worse bounds than the ones quoted.

On the other hand, the implications of the fermion mixing alone in a very large variety of observables have been used to constrain the mixing angles between ordinary and exotic fermions [3], resulting in $s^2 \equiv \sin^2 \xi \lesssim 0.030 - 0.050$ for the first generation and for ν_μ , $s_\mu^2 < 0.055$ while the bounds for the fermions in the third generation and for the second generation quarks are much worse. For instance, for the $b - h_b$ mixing the bound is $s_b^2 < 0.43$ and for the $\tau - E_\tau$ mixing it is $s_\tau^2 < 0.22$ (all at 90% c.l.).

Although these mixings can in principle vanish, there are good reasons to believe that they are non-zero. In fact, it has recently been shown, using cosmological and astrophysical arguments, together with experimental bounds from heavy isotope searches, that new charged leptons [4] and new coloured particles [5] cannot be stable.

Clearly, the mixing of the exotic particles with the ordinary ones provides a natural channel for their decay.

It is our purpose here to show that the present LEP results on partial widths of the Z boson already improve the previously mentioned bounds on s_α^2 by a factor of 5, taking into account also the possible effects of a $Z_0 - Z_1$ mixing, while for s_τ^2 the bound is improved almost by a factor 10 and this last result is essentially model independent.

In particular, it is important to constrain the b and τ mixings, not only because they are poorly bounded at present, but also because they are theoretically expected to be the largest ones since, if the masses arise from a seesaw mechanism, one has

$$\begin{aligned} \sin^2 \xi &\simeq (m/M) & \text{linear - seesaw} \\ \sin^2 \xi &\simeq (m/M)^2 & \text{quadratic - seesaw} \end{aligned} \quad (1)$$

where m and M are the light and heavy fermion masses respectively, and for a large class of models one typically expects that the mixings will fall within the range suggested in (1). This argument also leads us to expect tiny mixings in the first two generations ($\lesssim 10^{-3}$), since unsuccessful searches for exotic particles that couple to the Z -boson suggest $M > M_Z/2$. We will concentrate on the consequences of the mixing between the ordinary and exotic charged leptons and between h_i and b quarks. A general analysis of the effects of lepton mixing in the charged and neutral sectors, of the quark mixing and $Z_0 - Z_1$ mixing in several quantities measurable at LEP (partial widths and asymmetries) will appear elsewhere [6].

The exceptional group E_6 [7] is one of the most interesting candidates as a unifying group, and the reason for this is at least twofold: first, it contains as subgroups the symmetry groups of the most popular grand unified and left-right symmetric theories, like *e.g.* $SO(10)$, $SU(5)$, $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ and second, it is the only phenomenologically acceptable group that can arise from ten-dimensional superstring theories after Calabi-Yau compactification down to the 4 physical dimensions [8]. A nice feature of the E_6 algebra is that the embedding of the colour and weak isospin subgroup $SU(2)_L \times SU(3)_C$ is unique, but clearly in going from rank 6 down to rank 3, three Cartan generators are left, and it follows that the identification of the hypercharge axis is not unique. Here we will consider the embedding of $\mathcal{G}_{SM}$ in E_6 through the maximal subalgebras chain:

$$E_6 \longrightarrow U(1)_\psi \times SO(10) \xrightarrow{\quad} U(1)_\chi \times SU(5) \xrightarrow{\quad} \mathcal{G}_{SM} \quad (2)$$

The most general form for $U(1)'$ compatible with (2) will then be a linear combination of the $U(1)_\psi$ and $U(1)_\chi$ generators that we will parametrize in terms of an angle α . Correspondingly, the couplings of the fermions to the Z' boson will depend on both the ψ and χ quantum numbers through the combination ($c_\alpha = \cos \alpha$, $s_\alpha = \sin \alpha$):

$$Q' = c_\alpha Q_\psi + s_\alpha Q_\chi. \quad (3)$$

For the left-handed fermions belonging to the 27 fundamental representation of E_6 , the values of the Abelian Q_ψ and Q_χ charges are listed in Tab. I.

	S_L	$(\frac{E^+}{N_{S^+}})_L$	h_L	$(\frac{N_{S^-}}{E^-})_L$	h_L^c	ν_L^c	$(\frac{\nu}{e^-})_L$	d_L^c	e_L^c	u_L^c	$(\frac{u}{d})_L$
$6\sqrt{\frac{2}{5}}Q_\psi$	4							-2			1
$6\sqrt{\frac{2}{3}}Q_\chi$	0		2		-2	-5	3			-1	

Table I

Quantum numbers for the left-handed fermions of the fundamental 27 representation of E_6 . Abelian charges are normalized to the hypercharge axis according to: $\sum_{f=1}^{27} (Q')^2 = \sum_{f=1}^{27} (\frac{Y'}{2})^2 = 5$.

The multiplicative factors have been chosen in order to have the same normalization for the three Abelian axes: $\text{Tr } Q_\psi^2 = \text{Tr } Q_\chi^2 = \text{Tr } (\frac{Y'}{2})^2$, so that at the unification scale the same coupling constant g_Y is associated to both Y and Q'

charges. Possible deviations that could arise at the 100 GeV scale, as a consequence of a different running of the couplings, can be taken into account by writing:

$$g' = \kappa \frac{e}{c_w} \quad (4)$$

where the SM relation $e = g_V c_w$ has been used.

We will denote with Z_1 the $U(1)$ vector boson gauge eigenstate that in general has a non-diagonal mass matrix with the standard Z_0 . The mass eigenstates Z and Z' are related to Z_0 and Z_1 through an orthogonal transformation, parametrized in terms of a mixing angle Θ ,

$$\begin{pmatrix} Z \\ Z' \end{pmatrix} = \begin{pmatrix} \cos \Theta & \sin \Theta \\ -\sin \Theta & \cos \Theta \end{pmatrix} \begin{pmatrix} Z_0 \\ Z_1 \end{pmatrix}. \quad (5)$$

Then, the physical Z boson couples to fermions via the effective Lagrangian:

$$\mathcal{L}_{NC}(Z) = - \left(\frac{G_\mu \cos^2 \Theta M_Z^2}{2\sqrt{2}} \right)^{1/2} \sum_f \bar{\psi}_f^\alpha \gamma_\mu [\bar{v}_f - \bar{a}_f \gamma_5] \psi_f^\alpha \cdot Z^\mu \quad (6)$$

where the superscript “ α ” reminds us that for the moment we are considering unmixed fermions. In (6) the effective couplings $\bar{v}_f$ and $\bar{a}_f$ correspond to the SM couplings v_f and a_f shifted by a quantity proportional to the $Z_0 - Z_1$ mixing and dependent on the Q' fermion quantum numbers:

$$\bar{v}_f = v_f + s_w \bar{t}_\Theta v'_f \quad \bar{a}_f = a_f + s_w \bar{t}_\Theta a'_f \quad (7)$$

with

$$\begin{aligned} v_f &= 2T_3^f - 4Q^f s_w^2 = 2T_3^f - Q^f(v+1) & a_f &= 2T_3^f \\ v'_f &= 2s_\alpha(Q_\chi^f - Q_\chi^{f^c}) & a'_f &= 4c_\alpha Q_\psi^f + 2s_\alpha(Q_\chi^f + Q_\chi^{f^c}) \end{aligned} \quad (8) \quad (9)$$

where T_3^f is the left-handed fermion weak isospin, Q^f the electric charge, $s_w^2 = \sin^2 \vartheta_w$ with ϑ_w the weak mixing angle and $v = 4s_w^2 - 1$ is the charged lepton vector coupling. Since v is a small quantity that can be used as an expansion parameter for truncating expressions ($v \simeq -0.08$), it is useful to express all the fermions couplings as a function of v , as we have done in (8). In eq. (7), the ratio $g'/g_V = \kappa$ has been absorbed by

rescaling the mixing angle: $\bar{t}_\Theta = \kappa \cdot \tan \Theta$. On resonance, the shifts (9) of the standard couplings are by far the most important effects of the $Z_0 - Z_1$ mixing. $Z - Z'$ interference, as well as Z' exchange diagrams, are suppressed at least by a factor $\Gamma_Z \Gamma_{Z'}/M_Z M_{Z'}$ and can then be safely neglected. We will discuss later the effects of the shift on the physical Z mass due to the mixing.

The next step is to allow for a mixing among the standard and exotic fermions that will further modify the couplings in eq. (7). We note that since the electromagnetic (and colour) quantum numbers of the exotic quarks and leptons are the same as those of the ordinary ones (as it must be, since otherwise no mixing would be allowed), the electromagnetic current is unchanged. Moreover, table I shows that the $SU_L(2)$ transformation properties of the right-handed $Q = -1/3$ quarks and left-handed leptons also coincide with the ordinary ones, so that only the couplings of left-handed down-type quarks (weak isospin doublet) and right-handed ordinary leptons (weak singlets) will be modified by the mixing since their heavy partners are respectively singlets and doublets of weak isospin. Following ref. [3] we introduce two vectors for the ordinary and exotic left- and right-handed weak eigenstates $\psi_{L(R)}^\alpha = (\psi_{\alpha d}^\alpha, \psi_{\alpha x}^\alpha)^T$, and two other vectors for the light (i.e., standard) and heavy mass eigenstates $\psi_{L(R)} = (\psi_l, \psi_h)^T$, where for example for the down-type light quarks $\psi_l = (d, s, b)^T$. The weak and mass eigenstates are related by unitary transformations

$$\psi_L^\alpha = U_L \psi_L; \quad \psi_R^\alpha = U_R \psi_R \quad (10)$$

with

$$U_{L(R)} = \begin{pmatrix} A & E \\ F & G \end{pmatrix}_{L(R)} \quad (11)$$

and from the unitarity of U

$$A^\dagger A + F^\dagger F = A^\dagger A + E^\dagger E = I \quad (12)$$

The 3×3 matrices E and F describe the mixing between the light and heavy states. The part of the weak neutral current that gets modified by the mixing can be written:

$$\begin{aligned} \frac{1}{4} J_\mu^Z &\sim \sum_f \bar{\psi}_f^\alpha [t_3 I_L^f P_L + t_3 I_R^f P_R - Q s_w^2 I] \psi_f^\alpha \\ &= \sum_f \bar{\psi}_f [t_3 U_L^{\dagger f} I_L^f U_L^f P_L + t_3 U_R^{\dagger f} I_R^f U_R^f P_R - Q s_w^2 I] \psi_f \end{aligned} \quad (13)$$

where the sum involves only the $Q = -1/3$ quarks $q = d, h$ and the charged leptons $\ell = e, E$, $P_{L(R)} = \frac{1}{2}(1 \mp \gamma_5)$ and

$$I_L^q = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}; \quad I_R^q = \begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix} \quad (14)$$

while

$$I_R^d = I_L^e = I = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix} \quad (15)$$

Using (13) and the unitarity relations (12) it is easy to see how the couplings of the light states d and ℓ are further modified with respect to eq. (7):

$$\begin{aligned} \tilde{v}_d \rightarrow \hat{v}_d &= \tilde{v}_d - 2t_3(F_L^{d1} F_L^d) & \tilde{a}_d \rightarrow \hat{a}_d &= \tilde{a}_d - 2t_3(F_L^{d1} F_L^d) \\ \tilde{v}_\ell \rightarrow \hat{a}_\ell &= \tilde{v}_\ell + 2t_3(F_R^{\ell1} F_R^\ell) & \tilde{a}_\ell \rightarrow \hat{a}_\ell &= \tilde{a}_\ell - 2t_3(F_R^{\ell1} F_R^\ell) \end{aligned} \quad (16)$$

The matrices $F^\dagger F$ are in principle 3×3 non-diagonal matrices that describe inter-generational mixing too. However, the off-diagonal terms that would induce flavour changing neutral currents at the tree level are severely constrained by experiments [3]. We will assume that these terms are negligibly small so that the light-heavy mixing occurs essentially between particles belonging to the same generation. We will then parametrize:

$$\text{diag}(F_{R(L)}^\dagger F_{R(L)}) = ((s_1^{R(L)})^2, (s_2^{R(L)})^2, (s_3^{R(L)})^2) \quad (17)$$

with $s_i^2 = \sin^2 \xi_i$.

Clearly, in our procedure to define the effective coupling of the fermions to the Z -boson, second-order effects proportional to $\hat{\ell}_e \cdot \sin^2 \xi$ have been neglected. In the following we will consider as 'first-order terms' the following set of small parameters: $\hat{\ell}_e$, $\sin^2 \xi$ and v , and we will neglect terms involving products or higher powers of them.

Before beginning the analysis of the bounds that can be derived from the measurements of the partial widths $Z \rightarrow b\bar{b}$ and $Z \rightarrow \ell^+ \ell^-$, we want to discuss an indirect effect of the fermion mixing that will enter as a theoretical uncertainty in any prediction for electroweak quantities. The set of electroweak parameters that is known with the best experimental accuracy is α , M_Z and G_μ (the Fermi constant measured in μ decay). In particular, G_μ is introduced to replace the W mass, whose

experimental value is still affected by a large error. In order to do this, one uses the relation:

$$\frac{G_\mu}{\sqrt{2}} = \frac{\pi\alpha}{M_W^2 \sin^2 \theta_W (1 - \Delta r)} \quad (18)$$

where Δr is a radiative correction that (taking into account only the leading contributions) can be written as $\Delta r \simeq \Delta\alpha - (c_w^2/s_w^2)\Delta\rho$. Here, the effect of $\Delta\alpha$ ($\simeq 0.06$) is to renormalize the electromagnetic charge to the scale M_Z

$$\alpha(M_Z^2) = \frac{\alpha(0)}{(1 - \Delta\alpha)} \quad (19)$$

while $\Delta\rho$ [9] contains potentially large corrections that in the SM are essentially due to the top-bottom mass splitting, but that in general can arise from a mass difference between the components of any additional isodoublet of fermion [10] or scalar [11] particles that is present in the model. In a theory that allows for a $Z_0 - Z_1$ mixing, it is possible to take into account this effect by replacing $M_Z^2 \rightarrow \rho_{\text{mix}} M_Z^2$ since ρ_{mix} enters any expression in the same way as a $\rho_0 \neq 1$ generated by non-standard Higgses [1,12]. In such a theory, a possible (and useful) definition of the Weinberg angle is $c_w^2 = M_W^2/(\rho_{\text{mix}} M_Z^2)$. Allowing now also for a fermion mixing, eq. (18) will be modified into

$$\frac{G_\mu}{\sqrt{2}} = \frac{\pi\alpha(M_Z^2)}{M_W^2 (1 - \frac{M_W^2}{\rho M_Z^2})} (1 - \Delta s) \quad (20)$$

with

$$\rho \equiv \rho_{\text{mix}} (1 + \Delta\rho)$$

and where

$$1 - \Delta s \equiv c_\mu^e c_\mu^\ell c_{\nu_\mu}^\ell \simeq 1 - \frac{1}{2}(s_\mu^{\ell^2} + s_\nu^{\ell^2} + s_e^{\ell^2} + s_\nu^{\ell^2}) \quad (21)$$

takes into account the effect of fermion mixing in μ -decay [3]. Two remarks are in order: first, both a $Z_0 - Z_1$ mixing and a heavy top produce positive deviations of ρ from the SM tree level value $\rho = 1$. Since the theoretical upper bound on m_t comes from measurements of the ρ parameter, in general allowing the top mass to vary in the range $80 \text{ GeV} < m_t < 230 \text{ GeV}$ automatically takes into account the uncertainty related to a $Z_0 - Z_1$ mixing. This is not the case for the partial decay width into b -quarks, since this quantity receives an additional m_t -dependent contribution from the $Zb\bar{b}$ vertex correction that also involves the top mass and that almost cancels

against the $\Delta\rho^{\text{top}}$ correction [13]. As a result, $\Gamma_{b\bar{b}}$ turns out to be nearly insensitive to the value of the top mass. Thus, in this particular case the uncertainty coming from $\Delta\rho_{\text{mix}} \equiv \rho_{\text{mix}} - 1$ must be separately taken into account. A second point that we need to discuss is the effect of e , ν_e , μ and ν_μ mixing in the measured Fermi constant. We can include this effect simply by replacing $G_\mu \rightarrow G_\mu(1 + \Delta s)$ in all the expressions, but in so doing Δs will induce an additional theoretical uncertainty. However, we expect this correction to be quite small since the mixings involved in μ -decay should be negligible ($\Delta s < 10^{-3}$ according to eq. (1)). In the forthcoming expressions we will keep trace of both these effects but, as we will see, they will not affect very much our numerical analysis since the overall error is largely dominated by the experimental uncertainty.

From the Lagrangian (6), after the replacements $\bar{v} \rightarrow \hat{v}$ and $\hat{a} \rightarrow \hat{a}$, we can write the tree-level expressions for the partial widths $Z \rightarrow f\bar{f}$ as:

$$\Gamma_{ff} = \frac{\sqrt{2}\rho_{\text{mix}}G_\mu(1 + \Delta s)M_Z^3}{48\pi}(\hat{v}_f^2 + \hat{a}_f^2) \quad (22)$$

Then, from the expression for the effective neutral couplings (7-9) and (16) we obtain for the partial decay width into b -quarks:

$$\Gamma_{b\bar{b}} = \Gamma_{b\bar{b}}^{\text{SM}} \left[1 + \frac{19}{13}(\Delta\rho_{\text{mix}} + \Delta s) - \frac{3}{13}i_\Theta \left(\sqrt{10}c_\alpha - \sqrt{\frac{2}{3}}s_\alpha \right) - \frac{30}{13}(s_b^t)^2 \right] \quad (23)$$

with, at the tree level

$$\Gamma_{b\bar{b}}^{\text{SM}} = \frac{\sqrt{2}G_\mu M_Z^3(13 - 4v)}{48\pi} \quad (24)$$

Including the 1-loop electroweak and QCD corrections we have in the frame of the SM¹

$$\Gamma_{b\bar{b}}^{\text{SM}} = 377 \cdot (1 \pm 0.012)\text{MeV} \quad (25)$$

where the theoretical uncertainty corresponds to the variation of the top mass, Higgs mass and $\alpha_s(M_Z^2)$ in the ranges

$$80 \text{ GeV} < m_t < 230 \text{ GeV}; \quad 25 \text{ GeV} < M_H < 1 \text{ TeV}; \quad 0.10 < \alpha_s < 0.14 \quad (26)$$

¹ We thank W. Hollik for providing us with the Fortran program WIDTH that we have used to compute (25)

According to our previous discussion we will neglect the effect of Δs in eq. (23). To estimate the uncertainty due to $\Delta\rho_{\text{mix}}$, we use again eq. (20) with the experimental value of the $W - Z$ mass ratio averaged over the UA2 [14] and CDF [15] experiments: $M_W^2/M_Z^2 = 0.775 \pm 0.007$, and $M_Z = 91.170 \pm 0.033$ from LEP [16]. Neglecting again Δs , and subtracting the contribution of an 80 GeV top quark, we obtain at 90% c.l. $\Delta\rho_{\text{mix}} < 0.007$. We note that although the experimental value of ρ obtained in this way is slightly less precise than what could be obtained from low energy neutral to charged current ratio [2], this estimation is insensitive to possible Z' exchange diagrams.

To estimate the uncertainty induced in $\Gamma_{b\bar{b}}$ by the $Z_0 - Z_1$ mixing we have evaluated the values of the corresponding term $\delta_\Theta^b = \frac{3}{13}i_\Theta \left(\sqrt{10}c_\alpha - \sqrt{\frac{2}{3}}s_\alpha \right)$ in the range experimentally allowed for Θ as a function of α that is quoted in ref. [2]. We obtain for this effect $-0.174 \leq \delta_\Theta^b \leq 0.013$. Although the bounds obtained in [2] ignored the effects of fermion mixing, we think that they should be reliable since they are derived from deep inelastic ν scattering off nucleons and from $e^+e^- \rightarrow \mu^+\mu^-$ data that involve only fermions for which mixing effects are expected to be small.

In conclusion, our numerical prediction for the partial decay width of the Z boson into b quarks is the following:

$$\Gamma_{b\bar{b}} = 377 \cdot \left(1 \pm 0.012 \begin{matrix} +0.010 & +0.013 \\ -0 & -0.174 \end{matrix} - \frac{30}{13}(s_b^t)^2 \right) \text{MeV} \quad (27)$$

where the first error comes from SM uncertainties, the second from ρ_{mix} and the third from $Z_0 - Z_1$ mixing. Since the effect of the $b - h_b$ mixing tends to decrease the decay rate, we have to compare the experimental data with the maximum allowed value of (27). Moreover, since also the effect of $Z_0 - Z_1$ mixing in this quantity turns out to be negative for almost all values of α , and the possible positive shift is bound to be quite small, the inclusion of this effect in our analysis does not change the bounds from what one would obtain assuming $\Theta = 0$. Clearly if any non-zero $Z_0 - Z_1$ mixing is experimentally found, this will generally result in a better bound on $(s_b^t)^2$.

For what concerns the partial width into τ and μ leptons, it looks convenient to get rid of the overall multiplicative coefficient in (22) by defining a quantity nor-

malized with the electron width

$$R_r \equiv \frac{\Gamma_{t+\ell^-}}{\Gamma_{e+e^-}} = \frac{\hat{v}_t^2 + \hat{a}_t^2}{\hat{v}_e^2 + \hat{a}_e^2} \quad (28)$$

for which we obtain:

$$R_t = R_t^{SM} - 2(s_t^R)^2 + 2(s_e^R)^2 \quad (29)$$

and, neglecting the tiny effect of m_t , $R_t^{SM} = 1$. Clearly the quantity R_t is exactly one even in the presence of a $Z_0 - Z_1$ mixing, since the lepton couplings to the Z_1 boson also obey universality. In fact, the bound on the quantity $(s_t^R)^2 - (s_e^R)^2$ can be effectively thought of as a bound on any source of violation of universality, of which the mixing among fermions that we are considering now is probably one of the most obvious. In comparing (29) with the experimental data we will again neglect $(s_e^R)^2$, obtaining thus an upper limit on the $\tau - E_\tau$ and $\mu - E_\mu$ mixing.

We now discuss the experimental data. To obtain the partial width $Z \rightarrow b\bar{b}$ the decay mode $b \rightarrow \mu$ has been used in ref. [17] (L3), while the ALEPH collaboration [18] uses both $b \rightarrow \mu$ and $b \rightarrow e$ decay modes to tag the b quark. Their results are, respectively

$$\begin{aligned} \text{Br}(b \rightarrow \mu) \Gamma_{b\bar{b}} &= 41.7 \pm 2.9(stat) \pm 3.0(syst) \text{ MeV} \\ \text{Br}(b \rightarrow \mu) \Gamma_{b\bar{b}} &= 42.6 \pm 5.0(stat) \pm 2.1(syst) \text{ MeV} \\ \text{Br}(b \rightarrow e) \Gamma_{b\bar{b}} &= 38.8 \pm 3.4(stat) \pm 1.8(syst) \text{ MeV} \end{aligned} \quad (30)$$

where the second and third results are obtained by multiplying the ALEPH results for $\text{Br}(b \rightarrow \ell) \Gamma_{b\bar{b}}/\Gamma_{had}$ by the hadronic width averaged over the four LEP experiments $\Gamma_{had} = 1788 \pm 23 \text{ MeV}$ [16]. Then we take the weighted average of the three measurements in (30), and using the value quoted in [18] for the branching ratio: $\text{Br}(b \rightarrow \ell) = 0.102 \pm 0.010$ we finally obtain

$$\Gamma_{b\bar{b}} = 399 \pm 46 \text{ MeV} \quad (31)$$

In eq. (31) the uncertainty in the branching ratios for the decay of the b quark into e and μ leptons dominates the overall error. The ALEPH [19] and OPAL [20] collaborations have published data for the $Z \rightarrow \tau^+\tau^-$ and $Z \rightarrow \mu^+\mu^-$ partial widths. The average of their measurements gives

$$\begin{aligned} \Gamma_{\tau^+\tau^-} &= 85.9 \pm 4.4 \text{ MeV} \\ \Gamma_{\mu^+\mu^-} &= 85.2 \pm 4.2 \text{ MeV} \end{aligned} \quad (32)$$

Finally, for the partial width into electrons, we use the average of the four LEP experiments [19-22]

$$\Gamma_{e^+e^-} = 81.7 \pm 1.6 \text{ MeV} \quad (33)$$

In the average eq. (33) we have assumed that the errors quoted in [19-22] are uncorrelated. We have neglected a possible correlation in the uncertainty that arises from the procedure for subtracting the t -channel Bhabha scattering from the data, since this effect cannot be larger than a relative 2% [23], and in R_t the errors that arise from the τ and μ partial widths largely dominate.

With these figures, the experimental value of our quantities at the 90 % c.l. are found to be

$$\begin{aligned} \Gamma_{b\bar{b}} &= 399 (1 \pm 0.19) \text{ MeV;} \\ R_r &= 1.051 (1 \pm 0.09) \\ R_\mu &= 1.043 (1 \pm 0.09) \end{aligned} \quad (34)$$

We note that in R_r and R_μ the error is probably overestimated since we expect that the systematic uncertainty that originates from the measurement of the luminosity should cancel in this two ratios.

From (27), (29) and (34) we get the following bounds for the b - h_b , τ - E_τ and μ - E_μ mixing angles:

$$\begin{aligned} (s_b^R)^2 &\leq 0.077 \\ (s_\tau^R)^2 &\leq 0.025 \\ (s_\mu^R)^2 &\leq 0.024 \end{aligned} \quad (35)$$

It is interesting to note that the first two bounds in eq. (35) are already comparable with the maximum values for the mixings that can be derived from eq. (1) given the present lower limits on the mass M of exotic particles.

In conclusion, we have analysed the consequences of a mixing among the ordinary fermions and new heavy exotic ones that are predicted to exist in E_8 models, centring our attention on quantities relevant for experiments at the Z peak. We have taken into account several effects that occur in this kind of theories, such as a Z_0 - Z_1 mixing that will induce deviations from the SM couplings of the fermions and will also influence the value of the ρ parameter. We have compared our results with

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