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AXIONIC DOMAIN WALL PRODUCTION DURING INFLATION

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ABSTRACT

It is shown that quantum fluctuations of the axion field produced during inflation may lead to formation of exponentially big axionic domain walls even in the simplest ($N = 1$) axion models. Unlike the axionic walls previously discussed in the literature, the walls produced during inflation are exponentially (or even infinitely) large. Similar walls appear also in other theories of pseudo-Goldstone bosons. Possible cosmological consequences of our results are discussed.

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For a long time it was believed that the only density perturbations produced during inflation are adiabatic perturbations with a scale-independent (Harrison-Zeldovich) spectrum [1]. Even though it was clear that some other types of perturbations, like isothermal ones, can also be produced, it was usually believed that they are negligibly small [2]. However, a more detailed investigation of this question has shown that isothermal perturbations produced during inflation can also be large [3-6]. A simplest example of such a theory is the theory of axions, where the spectrum of isothermal perturbations proves to be not scale-independent but rather a power-law one due to the periodicity of the effective potential $U(a)$ with respect to the axion field a [3, 5, 6]. The purpose of the present paper is to show that the same reason (periodicity of the effective potential $U(a)$) leads also to formation of exponentially big axionic domain walls in the inflationary universe. The effect to be discussed is quite general and may occur in many other theories of pseudo-Goldstone bosons. Depending on the parameters of such theories, these domain walls may either lead to disastrous cosmological consequences ruling out the corresponding theories, or may play an important role in the theory of the galaxy formation.

First of all we must specify our assumptions about the behaviour of the axion fields during inflation and after it. One possibility is that the Peccei-Quinn symmetry breaking, after which the (Goldstone) axion fields appear, occurs after inflation. However, most of the axion models of that type are ruled out by the cosmological considerations related to the axion domain wall problem [7] and a very narrow "axion window" between the astrophysical constraint on the axion mass $m_a < 10^{-2}$ eV [8] and the cosmological constraint $m_a > 10^{-5}$ eV [9] (or maybe even $m_a > 10^{-3}$ eV [10]). Another possibility is that the Peccei-Quinn symmetry breaking occurred during inflation, and at the late stages of inflation, at which density inhomogeneities responsible for the galaxy formation were produced, the axions were massless Goldstone bosons. In this case there are no cosmological bounds on the axion mass m_a [11], and until now it was believed that the domain wall problem is also resolved by inflation. (We will show that this is not always the case.) Isothermal perturbations are produced during inflation just in the models of this type. For all these reasons in the present paper we will assume that during the last stages of inflation the Peccei-Quinn symmetry was broken.

To describe such a symmetry breaking one may consider a simple model of two scalar fields, the inflaton field φ and the complex Peccei-Quinn field Φ , with the effective potential

$$V(\varphi, \Phi) = V(\varphi) - m_\Phi^2 \Phi^* \Phi + \lambda_\Phi (\Phi^* \Phi)^2 + \nu \varphi^2 \Phi^* \Phi + \zeta R \Phi^* \Phi + V_\Phi, \quad (1)$$

where $V(\varphi)$ is the effective potential of the inflaton field; in the simplest case

$$V(\varphi) = \frac{m_\varphi^2}{2} \varphi^2$$

and R is the curvature scalar, which is given by

$$R = 12H^2 \sim \frac{32\pi V(\varphi)}{M_p^2}$$

during inflation with a Hubble constant H .

Let us consider first the simplest case $\xi = 0$, $|\nu| \ll m_\Phi^2/M_p^2$. According to the chaotic inflation scenario, at the beginning of inflation the inflaton field φ is very big. At that time, according to eq. (1), the effective mass squared of the field Φ is positive for all Φ . When the value of the field φ becomes smaller than critical, $\varphi < \varphi_c = \frac{m_\Phi^2}{\sqrt{\lambda_\Phi}}$, the phase transition occurs and the field Φ after the spontaneous symmetry breaking can be represented in the form

$$\Phi(x) = \Phi_0 \exp\left(\frac{ia(x)}{\sqrt{2}\Phi_0}\right), \quad (2)$$

where $\Phi_0 = \frac{m_\Phi^2}{\sqrt{\lambda_\Phi}}$ for $\varphi < \varphi_c$. It is important that $\varphi_c \gg M_p$ for $\nu \ll m_\Phi^2/M_p^2$, so that Φ_0 does not change during the last part of inflation, when perturbations responsible for the large-scale structure of the observable part of the universe were formed.

The field $a(x)$ in eq. (2) is the massless Goldstone scalar field with vanishing effective potential, $U(a) = 0$. The main difference between this theory and the theory of the axion field $a(x)$ is that the axion field at the late stages of the evolution of the universe, when the temperature of the universe after reheating drops down to less than $T_c = O(100)MeV$, acquires a non-vanishing potential energy density

$$U(a) = Cm_\pi^4 \left(1 - \cos \frac{Na}{f_a}\right). \quad (3)$$

Here $C = O(1)$, $f_a = \sqrt{2}\Phi_0$, and N is an integer that depends on the detailed structure of the theory; in the simplest case $N = 1$. The field $a(x)$ is a pseudo-Goldstone field with the standard kinetic term and the effective mass $m_a \sim \frac{m_\pi^2}{f_a}$. Sometimes instead of the field $a(x)$ we will use the angle variable $\theta(x) = \frac{a}{f_a}$.

After the Peccei-Quinn phase transition various non-trivial large-scale structures may appear. For example, in the theory with $N > 1$ the effective potential (3) has N different minima of the same depth. After the symmetry breaking the Goldstone field $a(x)$ may take different values in different parts of the universe. As a result, at low temperature, when the effective potential $U(a)$ acquires N different minima, the universe becomes divided into many domains corresponding to each of these minima [7]. In the non-inflationary cosmology this causes the domain wall problem mentioned above. However, if the Peccei-Quinn phase transition occurs at the early stages of inflation, the walls produced due to this phase transition (or, more exactly, the regions with $\theta = \frac{(2n+1)\pi}{N}$, which form the walls when the temperature becomes small enough) are inflated out of the observable part of the universe.

In the simplest theory with $N = 1$, which we will discuss in the main part of this paper, the domain wall problem does not exist at the classical level, even when there is no inflation. The domain walls do appear after the phase transition, at the surfaces $\theta = (2n + 1)\pi$ (for example, the walls between the regions with $\theta < \pi$ and the regions with $\theta > \pi$), but typically they are bounded by the axionic strings, the lines with $\Phi = 0$. Some number of closed walls also may be formed, but the very fact that the wall can end up with a string leads to the exponential suppression of the number of large closed (as well as bounded) axionic walls. In the non-inflationary cosmology a typical size of a domain wall in the $N = 1$ theory is very small; these walls rapidly oscillate, self-intersect, very soon radiate all their energy away and do not lead to any considerable cosmological effects. In the inflationary cosmology these walls may lead to important (probably disastrous) consequences only if they were produced late enough; otherwise they are inflated away, just as in the case $N > 1$.

The situation proves to be much more complicated with long-wave quantum fluctuations of the axion field generated during inflation taken into account. According to [12], inflation leads to a production of long-wave fluctuations of all scalar fields with masses $m \ll H$ and with wavelengths bigger than H^{-1} , where H is the Hubble parameter. Fluctuations of massless fields $a(x)$ produced during a typical time interval H^{-1} at the stage of inflation do not oscillate and do not change their initial mean amplitude

$$\delta a = \frac{H}{2\pi}, \quad (4)$$

whereas their wavelength exponentially grows together with the growth of the size of the universe. This leads to a formation of a classical inhomogeneous scalar field $a(x)$ with the spectrum of inhomogeneities $\delta a \sim H/2\pi$ which does not depend on the logarithm of momentum k and with dispersion squared

$$\sigma^2 \equiv \langle (\delta a)^2 \rangle \simeq \frac{H^2}{4\pi^2} \int_0^H d \ln k. \quad (5)$$

The infrared divergence in eq. (5) disappears if one takes into account initial conditions at the beginning of inflation [12]. After inflation the upper limit of the integral in eq. (5) changes, not only due to the overall expansion of the universe, but also due to the rapid decrease of the amplitude of those perturbations, which after inflation have a wavelength smaller than the horizon, $l \sim k^{-1}, H^{-1}$. Such perturbations oscillate and their amplitude decreases rapidly, whereas the long-wave perturbations remain frozen until the temperature drops down to $T_c = O(100) \text{ MeV}$. This introduces an ultraviolet cut-off to the spectrum of axionic perturbations, which at the present time corresponds to the wavelength $l_c \sim k_c^{-1} \sim 10^{17} \text{ cm}$. (An exact number depends on the actual value of T_c , which, in its turn, depends on the value of the axion mass m_a .)

The quantity which will be most important for us is the contribution $\sigma_k^2 \equiv \sigma_l^2$ to the dispersion squared σ^2 from the fluctuations with momenta from $k = l^{-1}$ to k_c . It is given by

$$\sigma_k^2 \equiv \sigma_l^2 = \frac{H^2}{4\pi^2} \int_k^{k_c} d \ln k = \frac{H^2}{4\pi^2} \ln \frac{k_c}{k} = \frac{H^2}{4\pi^2} \ln \frac{l}{l_c}. \quad (6)$$

To give a particular example, the value of σ_l corresponding to $l \sim l_H \sim 10^{28} \text{ cm}$ (l_H is the size of the observable part of the universe) is given by

$$\sigma_H \sim 0.8H, \quad (7)$$

and the value corresponding to the galaxy scale, $l_g \sim 10^{22} \text{ cm}$, is given by

$$\sigma_g \sim 0.5H. \quad (8)$$

(To be more precise, one should note that the values of $H(\phi)$ at the time when the perturbations were formed on a scale l_H differs somewhat from its value at a later time, when perturbations on a scale l_g were produced. However, typically this difference may only decrease the value of H in eq. (8) by about 20% or so [6].)

This means that the field $a(x)$ on a scale $l \sim l_H$ will have a Gaussian distribution of inhomogeneities,

$$P(a) \sim \exp \left(-\frac{(a(x) - a_l)^2}{2\sigma_l^2} \right), \quad (9)$$

where a_l is the mean value of the field $a(x)$ in the domain of a size l (in general, a_l depends on a choice of a particular domain). In terms of the angle variable θ the corresponding distribution is

$$P(\theta) \sim \exp \left(-\frac{(\theta(x) - \theta_l)^2 f_a^2}{2\sigma_l^2} \right) = \exp \left(-\frac{(\theta(x) - \theta_l)^2 f_a^2}{2C^2(l)H^2} \right), \quad (10)$$

where $C(l) = (2\pi)^{-1} \ln^{1/2}(l/l_c) \sim 0.8$ for $l = l_H$; $C(l) \sim 0.5$ for $l = l_g$. This means that the dispersion $\sigma_\theta(l)$ of the angle variable θ is given by

$$\sigma_\theta(l) = \frac{C(l)H}{f_a}. \quad (11)$$

The simplest case is $\sigma_\theta(l) \ll \pi$. In this case perturbations of the axion field are very small and the corresponding perturbations of density generated during inflation can be described by a simple equation [2-6]

$$\frac{\delta \rho}{\rho} \sim \frac{U'(\theta)\delta\theta}{\rho}, \quad (12)$$

which for a near the minimum of $U(a) \equiv U(\theta)$ gives $\frac{\delta \rho}{\rho} \sim \frac{2i\theta}{\theta} \cdot \Omega_\theta$, where Ω_θ is the relative contribution of the axion field to the energy density of the universe; during inflation $\Omega_\theta = \frac{U(\theta)}{\rho}$. This gives

$$\frac{\delta \rho}{\rho} \sim \frac{H}{\pi f_a} \cdot \Omega_\theta \quad (13)$$

for the typical initial value $\theta = O(1)$ of the field θ .

In the opposite case, $\sigma_\theta(l) \gg \pi$, the situation becomes much more interesting and non-trivial. The field θ with an almost equal probability takes all values on the circle $-\pi < \theta < \pi$. Adding any long-wave perturbation of the field θ only rotates the previous distribution of the axion field practically without any modification of the resulting distribution of energy density. This leads to the power-law suppression of the long-wave isothermal density perturbations [3, 5, 6],

$$\frac{\delta \rho}{\rho} \sim C(\theta_l) \frac{H}{2\pi f_a} \left(\frac{l}{l_c} \right)^{-\beta} \Omega_\theta, \quad (14)$$

where $\beta = \frac{H^2}{8\pi^2 f_a^2}$; $C(\theta) = O(1)$ for all θ not close to $(2n+1)\pi$. (In the corresponding equation in [5, 6] the ultraviolet cut-off of the axion perturbation spectrum was assumed to occur at a somewhat smaller value of l_c .)

According to eq. (14), the power-law suppression of the large-scale density perturbations may become efficient only for $H > f_a$. However, as it will be argued below, in this case eq. (14) should be modified. Moreover, at large H the leading effect is not the one discussed above, but the production of domain walls, corresponding to the surfaces at which the field θ takes values $(2n+1)\pi$.

Indeed, whereas the field θ in all other places at late stages of the evolution of the universe oscillates near the minima of $V(\theta)$ with $\theta = 2n\pi$ and rapidly loses its energy density, the surface energy density κ of the walls with $\theta = (2n+1)\pi$ does not vanish and remains rather high [7]

$$\kappa \sim m_\pi^2 f_a. \quad (15)$$

It can be easily checked that the existence of even just one such domain wall within the observable part of the universe would be in drastic contradiction with observational data. In particular, the total mass of such a wall would be bigger than the total mass of all particles in the observable part of the universe by many orders of magnitude. Meanwhile, from our results it follows that on a sufficiently big length scale $\sigma_\theta(l) = \frac{H}{2\pi f_a} \ln^{1/2} \frac{l}{l_c}$ becomes bigger than π , which means that on such a scale there will be many axionic domain walls.

However, if $H \ll f_a$, the corresponding scale is much bigger than the size of the observable part of the universe l_H . The number of axionic domain walls on a smaller scale

will be exponentially suppressed, as one can easily understand from eq. (10). It follows from the theory of random Gaussian fields that for $\sigma_\theta(l) \ll \pi$ the walls form nearly spherical bubbles, with number density [13]

$$n \sim 5 \times 10^{-3} l_c^{-3} \alpha^2 \exp(-\alpha^2/2), \quad (16)$$

where $\alpha = \frac{\pi}{\sigma_\theta(l)}$. The number of bubbles in the observable universe is therefore of order

$$N \sim 10^{30} \alpha^2 \exp(-\alpha^2/2). \quad (17)$$

It is more than 1 if $\alpha < 12$, i.e. if $H/f_a > 0.3$.

Most of such rear domain walls will look like bubbles of a size $l \sim l_c$. Each bubble would have an extremely big mass, $M \geq 10^{60} g$. At the late stages of the evolution of the universe such bubbles would collapse, oscillate and radiate their energy away. It would be very interesting to speculate about the possible significance of such bubbles for the theory of formation of the large-scale structure of the universe. One should check, however, whether these bubbles would lead to a too big anisotropy of the microwave background radiation. Moreover, it can be shown [14] that for $H/f_a > 0.3$ not only the bubbles, but just the ordinary isothermal density perturbations produced during inflation lead to an unacceptably big anisotropy of MBR unless the value of f_a is very close to its astrophysical bound $10^8 - 10^9 \text{ GeV}$ [8]. The reason is that for bigger f_a the contribution Ω_θ of axions to the total energy density of the universe becomes significant, and for $H/f_a > .3$, $f_a > 10^8 - 10^9 \text{ GeV}$ the isothermal density perturbations (13), (14) become too big. However, this is not an unavoidable problem, since Ω_θ can be made much smaller if some mechanism of the late-time entropy production can be suggested, see e.g. [15].

Previously we studied the case $H \ll f_a$. In the opposite case $H > f_a$ one could expect from eqs. (10), (17) that there should be extremely many walls in the observable part of the universe, the total mass of such walls being unacceptably large. However, at large H these equations should be somewhat modified.

In the case $H > f_a$ the Hubble constant is bigger than $m_\Phi \sim \sqrt{\lambda_\Phi} f_a$. In such a situation the scalar field Φ will not take its equilibrium value (2) with $\Phi_0 = \frac{\pi}{\sqrt{\lambda_\Phi}}$. Rather it will be randomly distributed near $\Phi = 0$ with the typical radius of the distribution (i.e. with dispersion) $\Phi_{rms} \sim H(\frac{3}{4\pi^2 \lambda})^{1/4}$. (This expression can be derived by stochastic methods described in [16]; alternatively one can make an estimate based on the well-known expression $\langle \Phi^2 \rangle = \frac{3H^4}{8\pi^2 m_\Phi^2}$ for the dispersion squared of the massive scalar field in the inflationary universe [12].) An investigation of the domain wall formation in such a situation is much more complicated, but apparently the main effect can be taken into account if one just substitutes to all equations an effective value $f_{rms} = \sqrt{2} \Phi_{rms} \sim H(\frac{3}{4\pi^2 \lambda})^{1/4}$ instead of f_a . This means that at $H > f_a$ the parameter H/f_a should be replaced by

(approximately) $(3\lambda_\Phi)^{1/4}$, and the domain wall density in the observable part of the universe will be suppressed by $\exp\left(-\frac{C}{\sqrt{\lambda_\Phi}}\right)$, where $C \approx 2$. Therefore at small λ_Φ the number of domain walls is exponentially suppressed, but for $\lambda_\Phi \sim 10^{-2}$ (which seems quite natural) one can get a considerable number of axionic bubbles in the observable part of the universe.

Note, however, that the effect discussed above leads to a simultaneous decrease of the parameter β in eq. (14). Therefore, as in the case $H \ll f_a$, the isothermal density perturbations produced in the case $H > f_a$ will be sufficiently small only if $f_a < 10^9 \text{ GeV}$. This apparently means that the power-law suppression of the large-scale density perturbations (14) cannot remove the constraints on the ratio between H and f_a derived in [17].

One should note, however, that though the constraints on H/f_a discussed above are rather general, they are not entirely model-independent. For example, let us consider the case $\xi = 0$, $|\nu| \gg m_\Phi^2/M_P^2$, $\nu < 0$ in eq. (1). In this case the Peccei-Quinn symmetry was broken from the very beginning, and the parameter f_a was time-dependent both during inflation and after it. During inflation

$$f_a = \sqrt{2\nu}\varphi^2/\sqrt{\lambda_\Phi} \gg \sqrt{2m_\Phi}/\sqrt{\lambda_\Phi}. \quad (18)$$

This means that during inflation the value of f_a in this model was much bigger than its present value $\sqrt{2m_\Phi}/\sqrt{\lambda_\Phi}$. Therefore the value of the H may be much smaller than f_a during inflation, but it may be much bigger than the present value of f_a . For example, for $\lambda_\Phi = 10^{-2}$, $\nu = -10^{-6}$, $m_\varphi < 10^{-7}M_P$ (which is necessary to get adiabatic density perturbations smaller than 10^{-5} [1, 16]) one has $H/f_a \sim 10^{-5}$ during inflation, whereas the present value of $f_a = \sqrt{2m_\Phi}/\sqrt{\lambda_\Phi}$ may be arbitrarily small. In any case, H at the end of inflation may be bigger than 10^{12} GeV since there is no constraint $f_a < 10^{12} \text{ GeV}$ in the axion model under consideration [11].

Another interesting possibility is that $0 < \nu \sim m_\Phi^2/M_P^2$, $m_\Phi \gg m_\varphi$. In this case the Peccei-Quinn phase transition occurs at the last stages of inflation. Before the phase transition the field Φ has a mass much bigger than H , so the long-wave perturbations of this field are not generated. This generation starts near the time of the phase transition. Soon after the phase transition the perturbations of $\theta = \frac{\varphi}{f_a}$ are very large (since f_a is very small), which may lead to generation of big large-scale isothermal density perturbations and domain walls. With a subsequent increase of f_a the perturbations of θ become small. This may result in a formation of domain walls accompanied by isothermal perturbations with a spectrum which has an infrared cut-off and a peak corresponding to the moment of the phase transition. Other interesting possibilities may arise when one takes into account the term $\xi R\Phi^*\Phi$ in eq. (1). Our main conclusion is that after 7 years of existence of axion cosmology, this branch of physics has still not been completely explored. Moreover, the effects similar to those considered above may appear in other theories of pseudo-Goldstone

fields, for example in the models which serve as a basis for the recently proposed theories of the late-time phase transitions [18]. We hope to return to a discussion of the various possibilities mentioned above in a separate publication.

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