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CHIRAL SYMMETRY AS AN EXPERIMENTAL SCIENCE*

JOHN F. DONOGHUE[†]

CERN - Geneva

ABSTRACT

This is an introduction to the phenomenological aspects of the theory of chiral symmetry. It is meant to give an overview of the main ideas and directions of the field, without too much formalism.

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† Permanent address: Department of Physics and Astronomy, University of Massachusetts, Amherst, MA 01003, USA

1. INTRODUCTION

The common impression of chiral symmetry is that it is:

- a) incomprehensible,
- b) a theorist's theory,
- c) used mainly in the study of superstrings and/or
- d) just another model.

It is likely true that the concepts of chiral effective Lagrangians are understood by more superstring theorists than those physicists using QCD. However, this is a shame as the foundation of these ideas is in the phenomenology of the strong interactions. In reality, chiral symmetry is

- a) a true/direct consequence of QCD,
- b) a rigorous way to calculate in the very low energy region
- c) a rich phenomenological theory and
- d) subtle and fun.

In these lectures, I wish to present an introduction to the subject, hopefully one where the essential ideas are not buried by too much formalism. Some of the ideas and language are not familiar, as they have not become part of the standard graduate studies training. These then require some examination. Overall the goal is to describe how chiral symmetry is used in phenomenological applications, and to point to possible future theoretical and experimental directions.

The study of QCD can be divided into three regions of energy. We are all aware of the applications of perturbative QCD to the high energy region. As one comes down in energy, we enter a region where perturbative QCD is no longer applicable. Here we do not know how to calculate (apart from lattice computer work), and we are reduced to making models (quark models, pole models, Skyrme model, ...) which we hope capture some of the physics of QCD. However, these

are not controlled approximations in that there is no well-defined way to calculate a next approximation or to put error bars on the predictions. However, at very low energies (say, $E \lesssim 1/2$ GeV) we again enter a region where a controlled approximation is possible, using symmetry methods. It is this very low-energy region which we are concerned with here.

An analogy with the calculational procedure of perturbative QCD will shed light on the similar procedure in the very low-energy region. At high energies, hard scattering processes are known in an expansion in $\alpha_s \sim 1/\ln E$. However, there remains some dependence on (presently) non-calculable "soft" physics such as structure functions, fragmentation functions, $p_\perp$ distributions, etc., as well as the intrinsic parameter Λ_{QCD} . These must be determined phenomenologically. The content of high energy Λ_{QCD} is then relationships between scattering and decay processes, parametrized by the empirical values of Λ_{QCD} , structure functions, etc. At low energies, we will see that very soft processes are highly constrained by the symmetry. There remains some dependence on (presently) non-calculable hard physics, contained in F_π and the various low energy constants. These must be determined phenomenologically. The content of very low energy QCD is then relationships between scattering and decay processes, parametrized by the empirical values of F_π , lower energy constants, etc.

2. WHAT IS CHIRAL SYMMETRY?

First, let us describe the essential physics without mathematics. Pretend that the u, d, s quarks are massless. For each quark, there is a left-handed helicity state (spin antiparallel to the momentum) and a right-handed helicity state. QCD interactions are the same for left and right helicity and do not flip helicity. Under these conditions left-handed massless particles will always stay left handed and right handed will stay right handed. We have two separate worlds, a LH world and a RH world. Since each flavour is massless and has the same QCD coupling, there exists a separate flavour $SU(3)$ invariance in each world. We can make an

$SU(3)$ rotation on the left-handed quarks without influencing the right-handed ones. The overall invariance is then called $SU(3)_L \times SU(3)_R$. This is the chiral symmetry.

Now consider adding a common mass. It is clear that we cannot maintain the separate L and R invariances. If one has a massive left-handed particle, one can always boost to the rest frame and then boost to a frame moving with other directions, such that the LH particle is now right handed. Kinematically, at the very least, the LH and RH worlds become related, and one no longer has separate $SU(3)$ invariances. However, one still has an invariance under common $L + R$ rotations, with a symmetry called $SU(3)_{L+R} = SU(3)_V$ (V is for "vectorial"). However, if the mass is small, the original $SU(3)_L \times SU(3)_R$ symmetry could also be an approximate symmetry, and the mass could be treated as a perturbation.

The real world is obtained by allowing m_u, m_d and m_s to all be different. This breaks the $SU(3)_V$ symmetry. However, to the extent that Δm is small, the $SU(3)_V$ symmetry could be considered as an approximate symmetry, and the mass differences could be treated as perturbations.

To see this same result more precisely, consider the field

$$\psi = \begin{pmatrix} u \\ d \\ s \end{pmatrix} \quad (1)$$

and the projection operators

$$\begin{aligned} \Gamma_L &= \frac{1}{2}(1 + \gamma_5) \\ \Gamma_R &= \frac{1}{2}(1 - \gamma_5) \\ \Gamma_L^2 &= \Gamma_L; \quad \Gamma_R^2 = \Gamma_R; \quad \Gamma_L \Gamma_R = 0 \\ I &= \Gamma_L + \Gamma_R \end{aligned} \quad (2)$$

such that

$$\begin{aligned} \psi_L &= \Gamma_L \psi \\ \psi_R &= \Gamma_R \psi \\ \psi &= \psi_L + \psi_R \end{aligned} \quad (3)$$

For massless particles these project out the helicity of the particle. For $m \neq 0$, Γ_L and Γ_R are still projection operators but do not yield exactly the helicity. For this reason, a new name is introduced and ψ_L and ψ_R are said to have left and right "chirality" (chiral being related to the Greek for "hand"). The Dirac Lagrangian can be rewritten using these fields

$$\begin{aligned} \mathcal{L} &= \bar{\psi}(i\not{D} - m)\psi \\ &= \bar{\psi}_L i\not{D}\psi_L + \bar{\psi}_R i\not{D}\psi_R + \bar{\psi}_L m\psi_R + \bar{\psi}_R m\psi_L \end{aligned} \quad (4)$$

Here we see that if $m = 0$, there is a decoupling of left and right, as advertized above, and that the QCD Lagrangian is invariant under

$$\begin{aligned} \psi_L &\rightarrow L\psi_L \\ \psi_R &\rightarrow R\psi_R \end{aligned} \quad (5)$$

with L and R being $SU(3)$ transformations, with in general $L \neq R$. The presence of a mass removes the separate invariances.

Given this symmetry, Noether's theorem says that we have a set of 16 conserved currents and charges. The eight vector currents and charges are the usual flavour $SU(3)_V$ ones: it is the 8 axial charges which are "new". The normal expectation is that symmetries require particles to appear in multiplets. If there exists some single particle state with

$$H|P\rangle = E_P|P\rangle \quad (6)$$

then a symmetry transformation will yield a state with the same energy, i.e. for

an axial charge Q_5^j

$$H(e^{iQ_5^j}|P\rangle) = e^{iQ_5^j} H|P\rangle = E_P(e^{iQ_5^j}|P\rangle) \quad (7)$$

since conservation of the charge requires $[H, Q_5^j] = 0$. However, this does not seem to be a property of the real world; as the proton (for example) does not live in a multiplet with partners of the opposite parity. This occurs because the symmetry is "dynamically broken" or "dynamically hidden".

3. SYMMETRY BECOMES DYNAMICS

The hardest thing to understand about chiral symmetry is why it makes predictions in the first place. How can a symmetry predict a dynamical scattering amplitude, such as that for $\pi\pi$ scattering? The answer involves an understanding of "dynamical symmetry breaking" [1]. This terminology is misleading as it reinforces the mistaken impression that "if the symmetry is broken, it does not exist anymore and I can ignore the subject". The phrase would be better called "dynamical symmetry hiding" in that the symmetry, although still valid, appears to be hidden when one looks at the spectrum. Symmetry predictions still exist.

The generic case of hidden symmetry occurs when the Lagrangian is invariant under the symmetry, but when there exists a continuous family of ground state solutions, which are related to each other by the symmetry but which are not individually invariant under the symmetry. The classic example is that of magnetic domains. The magnet Hamiltonian is rotationally invariant, but the possible ground states consist of configurations with all the spins lined up in the same direction. The overall direction can be arbitrary (this is the continuous family of solutions), but each ground state is not invariant. Any of these solutions could be "the" ground state, and one is chosen. (We assume here that it fills all of space). Within this ground state one does not see the full rotational invariance; it is hidden.

Fig. 1: A spin wave in a magnet system

An immediate consequence of this situation is that there will be massless particles in the theory. This is Goldstone's theorem, and it is not hard to see conceptually. Let us define the ground state to have $E = 0$. Then there clearly must be other states with $E = 0$. These are the partners of the vacuum state, all of which have the same energy because of the original rotational invariance of the Lagrangian (Hamiltonian). In quantum mechanics, all states are quantized and, apart from the vacuum itself, must be described by particles. To have $E = 0$, these particles must have $m = 0$. In the magnet example, the excitations are spin waves where the spins vary in a wave-like pattern, Fig. 1. This spin wave in general carries energy. However, as the wavelength gets large, the wave consists of just rotating the spins as a whole over a large region. As $\lambda \rightarrow \infty$, it is a rotation of the whole system of spins. However, by the rotational invariance, the rotation of the whole system does not require any energy. The energy therefore falls with λ for large λ , $E \sim c/\lambda \sim cp$, i.e. it corresponds to the massless particle. Here we see physically that a symmetry transformation corresponds to the excitation of a massless, zero energy Goldstone boson.

The situation with the chiral symmetry of QCD is similar. In the limit of massless quarks, there would be different possible vacua, all equivalent. It is not as easy to picture these as it was in the case of the spin vector of the magnet. My own mental picture imagines the vacuum to be a complex soup of virtual quark pairs. In the symmetry limit, there could be any combination of LH and RH quarks, as all their QCD interactions are the same. The continuous family of

vacua then corresponds to differing LH/RH compositions of the vacuum. In any case, the lack of invariance of the true chosen vacuum can be seen by considering matrix elements like

$$(0|\bar{\psi}\psi|0) = (0|\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L|0) \quad (8)$$

Since the operator is not invariant under the symmetry, the matrix element must vanish if the vacuum is invariant. Theoretical studies of this operator indicate that it is non-zero if the symmetry is hidden. Given this limit the π, K, η would be the massless Goldstone bosons.

We now come to the question of the nature of the predictions. Usual symmetries like isospin require that when one makes a symmetry transformation on a state (*i.e.* the proton), one obtains another state in the same multiplet (*i.e.* the neutron). The couplings of the neutron and proton are then related by the symmetry. For a hidden symmetry, one does not have the usual multiplet structure. The partner of the proton under an axial transformation is the state composed of a proton plus a zero energy pion. These have the same energy, and there was nothing in the general quantum mechanical argument of Eq. (7) that required that we allow only single particle states. More generally, if $|\beta\rangle$ is an arbitrary state, then the symmetry relates

$$|\beta\rangle \longleftrightarrow |\beta + \pi(p_\mu = 0)\rangle \quad (9)$$

and their couplings are related. This is manifest in the "soft pion theorem" of chiral symmetry [2]

$$\lim_{p_\pi \rightarrow 0} \langle \beta | \pi^i(p) | \theta \rangle \langle \alpha | = -\frac{i}{F_\pi} \langle \beta | [Q^i, \theta] | \alpha \rangle \quad (10)$$

where β and α are arbitrary states, θ is some operator and $F_\pi = 93$ MeV is the pion decay constant.

The end result is that symmetry has led to non-trivial dynamics. It relates processes with differing numbers of pions, such as

$$\begin{aligned} K &\rightarrow 3\pi \iff K \rightarrow 2\pi \\ K &\rightarrow \pi\pi e\nu \iff K \rightarrow \pi e\nu \\ \pi^0 &\rightarrow \gamma\gamma \iff \gamma \rightarrow 3\pi \\ K &\rightarrow \pi\pi e\nu \iff \pi\pi \rightarrow \pi\pi \end{aligned} \quad (11)$$

Sometimes the predictions are very surprising, as in Weinberg's absolute prediction of $\pi\pi \rightarrow \pi\pi$ amplitudes. If one thinks of what a mess it would be to try to calculate these processes directly from QCD, the simplicity and power of the chiral predictions are truly remarkable.

4. HISTORY AND SOCIOLOGY

Much work has been done on the subject of chiral symmetry. This section contains some comments that may help one to understand the general patterns in the literature of the subject.

The subject of chiral symmetries started in the 1960s. There was an initial discovery period, where the general ideas were invented and explored. The issues were the idea of hidden symmetry, the patterns of symmetry breaking, the representation dependency, the relation to field theory, etc. There was also a period of phenomenology [2]. The idea was still a hypothesis, and one tested it by applying it to physics and seeing if it accurately represented reality. Kaon decay relations, the Goldberger-Treiman relations and Weinberg's $\pi\pi$ scattering amplitudes were explored at this time and experimental studies also addressed their issues. The theoretical techniques generally involved the soft-pion theorem plus current algebra, although effective Lagrangian methods were also developed. The issues and notations of this period are often obscure to present day readers. One can obtain a good flavour of this period from S. Coleman's Erice lectures on the subject [1].

The 1970s were a quiet period for the subject, both theoretically and experimentally, as most effort was devoted to the development of gauge theory and the standard model. However, there gradually developed a general awareness that the standard model implied the validity of chiral symmetry and that the idea was no longer a hypothesis but was obligatory.

One might refer to the 1980s as the modern theoretical period, as the theorists rediscovered chiral symmetry as an active field. Many of us date this new period from a characteristically clear paper of Weinberg [3] which is recommended reading for anyone interested in the subject. In this period, there are a variety of purposes for which chiral symmetry is used, ranging over QCD, CP violation, WW scattering, strings, etc. The language here uses effective Lagrangians, and the state of the art involves "next-to-leading order" calculations. The theoretical machinery has been explored again at a new level. In phenomenological applications, one has had to reanalyze old experiments, as this style of low energy experiments is no longer fashionable.

There is some hope that the 1990s could become the modern experimental period. There is a wide variety of high intensity, low energy machines planned such as $\pi, K, \eta, \phi, B, \dots$ factories or $e, \bar{p}, \gamma, \dots$ machines. Experimentalists (including many coming from nuclear physics) want to study QCD. The framework now exists, and theorists are interested. It requires some work to be aware of the issues and language of chiral symmetry and very low energy QCD, but the potential now exists for fruitful interplay between theory and experiment.

There is also an important cautionary note that must be added concerning the varieties of chiral symmetry. There are many different ways in which one can use the ideas of chiral symmetry, and one must carefully distinguish among them. A special place is held by what is now called chiral perturbation theory, which is the prime focus of these lectures. This uses chiral symmetry in its fullest generality, so that the predictions are those of the symmetry alone without additional dynamical assumptions. It is phenomenological in character. This use is the only one which is a rigorous controlled approximation. However, it is often somewhat

limited in scope because of the need to determine the low-energy constants phenomenologically. A second form is what I call models of chiral symmetry. These are often motivated by the desire to remove some of the limitations of strict chiral perturbation theory. For example, dispersion relations may be used to extend the predictions to higher energy, or models for the low energy constants may be used when these are not known from experiment. These attempts are only as good as the physics which one puts into the model, and one can be led astray. Thus, these give up the rigour in hopes of being more widely useful. More comments are made on these in Section 7. Finally, there are some uses of effective chiral Lagrangians outside the range of validity of the energy expansion, *i.e.* when the energy is large. In this case, there is no perturbative expansion, and one can have 100% corrections, and we have no *a priori* clue as to the validity of predictions. A prime example of this class is the Skyrme model. This model uses a chiral Lagrangian with two terms, [the meaning of this will be clearer below], and the condition for forming a soliton requires that the two terms contribute equally to the energy function. This means that higher order terms could also contribute equally, and that many results can be changed drastically by the inclusion of more terms in the chiral Lagrangians. The Skyrme model is quite interesting, but it must be remembered that it is just a model. The uninitiated reader will often assume that all papers that use the words "chiral symmetry" are on the same footing, and then will be confused when not all agree. If you are aware of these distinctions, it is not hard to classify the framework of each paper and to sort out the differences.

5. AN EXAMPLE OF EFFECTIVE LAGRANGIAN

The language of the field now uses non-linear effective Lagrangians as the basis of application of chiral symmetry [4,5]. Before presenting this language in general, it is useful to have a "hands-on" example, where all of the manipulations can be easily and explicitly seen. For this purpose, the linear sigma model is useful. It is defined by a Lagrangian involving a scalar field σ , the π 's, and a "nucleon" doublet

$$\psi = \begin{pmatrix} p \\ n \end{pmatrix} \quad (12)$$

with the form

$$\mathcal{L} = \bar{\psi} i \not{\partial} \psi + 1/2[(\partial_\mu \sigma)^2] + (\partial_\mu \vec{\pi})^2 - g \bar{\psi}(\sigma + i \vec{\tau} \cdot \vec{\pi} \gamma_5) \psi + \frac{\mu^2}{2}(\sigma^2 + \vec{\pi}^2) - \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2)^2 \quad (13)$$

This has a $SU(2)_L \times SU(2)_R$ invariance where the fields transform as

$$\begin{aligned} \psi_L &\rightarrow \psi'_L = L \psi_L \\ \psi_R &\rightarrow \psi'_R = R \psi_R \\ \sigma + i \vec{\tau} \cdot \vec{\pi} &\rightarrow \sigma' + i \vec{\tau}' \cdot \vec{\pi}' = L(\sigma + i \vec{\tau} \cdot \vec{\pi}) R^\dagger \end{aligned} \quad (14)$$

with L, R being $SU(2)$ transformation matrices. This is a conventional renormalizable field theory. Apart from the Yukawa couplings, it is just like the Higgs sector of the standard model. The usual way to solve it is to minimize the energy to find the ground state at $\langle \sigma \rangle = v$, with $v = \sqrt{\mu^2/\lambda}$, then expand about this state using $\sigma = v + \tilde{\sigma}$. When one does this, the Lagrangian reads

$$\begin{aligned} \mathcal{L} = & \bar{\psi}(i \not{\partial} - gv)\psi + 1/2[(\partial_\mu \tilde{\sigma})^2 - 2\mu^2 \tilde{\sigma}^2] + 1/2(\partial_\mu \vec{\pi})^2 - g \bar{\psi}(\tilde{\sigma} + i \vec{\tau} \cdot \vec{\pi} \gamma_5) \psi \\ & - \lambda v \tilde{\sigma}(\tilde{\sigma}^2 + \vec{\pi}^2) - \frac{\lambda}{4}(\tilde{\sigma}^2 + \vec{\pi}^2)^2 \end{aligned} \quad (15)$$

This now describes massive σ and "nucleons", with

$$\begin{aligned} m_P &= gv \\ m_\sigma &= \sqrt{2}\mu \end{aligned} \quad (16)$$

and massless pions. The pion decay constant turns out to be $F_\pi = v$.

This Lagrangian has the chiral $SU(2)$ invariance and hence is consistent with the low-energy prediction of chiral symmetry. For example, to obtain Weinberg results for $\pi\pi$ scattering, one can have both the direct π^4 coupling plus the effect of $\tilde{\sigma}$ exchange from the $\tilde{\sigma}\pi^2$ coupling. In the case of $\pi^+\pi^0$ scattering one obtains

$$\begin{aligned} \mathcal{M}(\pi^+\pi^0 \rightarrow \pi^+\pi^0) &= -2i\lambda + (-2i\lambda v^2)^2 \frac{i}{q^2 - m_\sigma^2} \\ &= -2i\lambda \left[1 + \frac{2}{q^2 - 2\lambda v^2} \right] \\ &= \frac{iq^2}{v^2} + \dots = \frac{iq^2}{F_\pi^2} + \theta(q^4) \end{aligned} \quad (17)$$

which is the Weinberg result. Note the cancellation required in order to get a low energy result which is proportional to q^2 .

One can better expose the content of the theory by using a change of variable. Let us define new scalar field "S", pion fields π' , and "nucleon" fields B , using

$$\begin{aligned} (\sigma + i \vec{\tau} \cdot \vec{\pi}) &\equiv (v + S)U \\ U &= \exp(i \frac{\vec{\tau} \cdot \vec{\pi}'}{v}) \\ &\equiv \xi \xi \\ \xi^\dagger \psi_L &\equiv B_L \\ \xi \psi_R &\equiv B_R \end{aligned} \quad (18)$$

Again this is similar to the Higgs change of variables in the standard model. One does not change the content of the theory by renaming the fields. In the new basis

the Lagrangian has the form

$$\begin{aligned} \mathcal{L} + \bar{B}(i\cancel{D} - gv)B + 1/2((\partial_\mu S)^2 - 2\mu^2 S^2) \\ - \lambda v S^3 - \frac{\lambda}{4} S^4 + \frac{v^2}{4} Tr(\partial_\mu U \partial^\mu U^\dagger) [1 + \frac{S}{v}]^2 \end{aligned} \quad (19)$$

with

$$\begin{aligned} D_\mu &= \partial_\mu + V_\mu + A_\mu \gamma_5 \\ V_\mu &= \frac{1}{2}(\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger) \\ A_\mu &= \frac{i}{2}(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger) \end{aligned} \quad (20)$$

The essential point is that, apart from the πNN couplings contained in D_μ , the interactions of pions have been compacted into a single term, the last one given above. This already has two derivatives so that it always contributes to matrix elements with a power of q^2 . The nucleons and scalar S are heavy so that we are able to drop them at very low energy. [This makes only corrections at order q^4]. The result is then a very simple form

$$\mathcal{L} = \frac{v^2}{4} Tr(\partial_\mu U \partial^\mu U^\dagger) + \dots \quad (21)$$

This is the effective low energy Lagrangian for the theory. It is very non-linear in that it contains all numbers of pion fields. However, it retains the $SU(2)_L \times SU(2)_R$ chiral symmetry, which from Eqs. (14), (18) can be seen to be

$$U \rightarrow U' = L U R^\dagger \quad (22)$$

To obtain results for $\pi\pi$ scattering, one expands this Lagrangian (dropping primes)

$$\mathcal{L} = 1/2(\partial_\mu \pi)^2 + \frac{2}{3v^2}[(\vec{\pi} \partial_\mu \vec{\pi})^2 - \vec{\pi}^2(\partial_\mu \vec{\pi})^2] + \dots \quad (23)$$

and one may directly calculate the scattering amplitude, obtaining

$$\mathcal{M}(\pi^+ \pi \rightarrow \pi^+ \pi^0) = \frac{iq^2}{v^2} \quad (24)$$

as before. Likewise any other scattering process involving more pions will agree

to order q^2 when calculated in the two approaches.

The essential point of this exercise has been to show that one can find a simple, but non-linear, effective Lagrangian that encapsulates all of the low energy predictions of a more complicated theory. The important feature is that the symmetry properties have been retained in the transition to the effective theory.

6. CHIRAL PERTURBATION THEORY

Despite my desire to keep these lectures as non-formal as possible, I have to introduce the language which theorists use in discussing chiral predictions. This involves a quick tour through the formalism of chiral perturbation theory. The method uses effective Lagrangians, a specific example of which was given in the previous section. The general idea is that if a prediction is to follow from a symmetry alone, one may use an effective theory which shares the same full symmetry properties as the original theory. The symmetry relations will be the same for each theory.

The general procedure of chiral perturbation theory is as follows:

- 1) Write out the most general effective Lagrangian consistent with the symmetry properties of the interaction.
- 2) Calculate all possible diagrams.
- 3) Renormalize all of the parameters in the effective Lagrangian from experiment.
- 4) Apply at low and moderate energies.

This sounds like a daunting program. In practice, it is made manageable by the expansion in energy. Let us now look at these ingredients.

In order to write the most general Lagrangian one may use the matrix field

U introduced above with the following symmetry

$$U = \exp \left(i \frac{\lambda^A \phi^A}{F} \right) \quad (25)$$

$$U \rightarrow LUR^\dagger$$

where in $SU(2)$, $\lambda^A \rightarrow \tau^k$, $A = 1, 2, 3$, while in $SU(3)$, λ^A are the 8 Gell-Mann matrices. The field ϕ^A is either simply the pions in $SU(2)$ or the full pseudoscalar octet in chiral $SU(3)$. One wants all possible terms invariant under this symmetry. There is no term with zero derivatives since

$$\text{Tr}(UU^\dagger) = 2 = \text{constant}$$

while with two or more derivatives, we have terms like

$$\begin{aligned} &\text{Tr}(\partial_\mu U \partial^\mu U^\dagger) \\ &\text{Tr}(\partial_\mu U \partial^\mu U^\dagger \partial_\nu U \partial^\nu U^\dagger) \\ &\text{Tr} \partial_\mu U \partial_\nu U^\dagger \partial^\mu U \partial^\nu U^\dagger \end{aligned} \quad (26)$$

The most general Lagrangian will contain all of these terms

$$\mathcal{L} = A \text{Tr}(\partial_\mu U \partial^\mu U^\dagger) + \alpha_1 \text{Tr}(\partial_\mu U \partial^\mu U^\dagger \partial_\nu U \partial^\nu U^\dagger) + \alpha_2 \text{Tr}(\partial_\mu U \partial_\nu U^\dagger \partial^\mu U \partial^\nu U^\dagger) + \dots \quad (27)$$

where we are temporarily neglecting quark masses. There is an infinite number of terms.

The most important ingredient is the idea of the energy expansions. When we take matrix elements, the derivatives turn into factors of the pions momentum. Then, schematically we have a matrix element

$$\mathcal{M} = A q^2 + \alpha_1 q^4 + \alpha_2 q^4 + \dots \quad (28)$$

However, at low enough energies $\alpha_i q^4 \ll A q^2$. Similarly, yet higher order terms at order (q^6) are yet smaller. Out of the infinite number of terms we need only one

if we work at low enough energy, i.e.

$$\begin{aligned} \mathcal{L} &= \frac{F^2}{4} \text{Tr}(\partial_\mu U \partial^\mu U^\dagger) + \dots \\ &= 1/2 (\partial_\mu \vec{\pi} \partial^\mu \vec{\pi}) + \dots \end{aligned} \quad (29)$$

where the normalization has been chosen to correctly normalize the kinetic energy term in the second line. Consideration of the axial current allows the identification $F = F_\pi = 93 \text{ MeV}$. This term (plus the effect of quark masses, once properly included) reproduces the soft pion results of the 1960's. At somewhat higher energies there will be corrections to these results, parametrized by the low energy constants α_1, α_2 , etc. At moderate energies, these corrections are under control in that each subsequent order is smaller. The perturbative parameter is q^2/Λ^2 where Λ is some mass scale. [In practice $\Lambda \sim m_\rho$ or 1 GeV.] At high energy, all terms in the energy expansion become equal and there is no possibility of prediction. Working to first non-leading order, i.e. order (q^4), is the present state of the art.

One must calculate all diagrams for full generality. This includes loop diagrams. One might worry about loop diagrams as the Lagrangian is formally non-renormalizable. However, the solution is simple: renormalization can be carried out order by order in the energy expansion. One-loop diagrams with an order (q^2) Lagrangian produce divergences at order (q^4). These can be absorbed into renormalized coefficients at order (q^4). Higher numbers of loops would renormalize higher order coefficients. While to work to all orders in the energy and loop expansions would require an infinite number of renormalizations (this is the sense in which it is non-renormalizable), at low energies one need only consider a finite number of these because of the finite number of relevant terms.

The summary of the method to order q^4 is then a subset of the rules given above:

- 1) Write out the most general Lagrangian at order q^2 and q^4 , $\mathcal{L}_2$ and $\mathcal{L}_4$. This has been done by Gasser and Leutwyler [6] and will be displayed below.
- 2) Calculate tree-diagrams with $\mathcal{L}_2$ and $\mathcal{L}_4$ plus one-loop diagrams with $\mathcal{L}_2$.

3) Renormalize the parameters, using experiment.

4) Apply at low and moderate energy.

The Lagrangian at order q^2 including quark masses is known from the 60's:

$$\mathcal{L}_2 = \frac{F^2}{4} \text{Tr}(D_\mu U D^\mu U^\dagger) + \frac{F^2}{2} B_0 \text{Tr}(mU + U^\dagger m) \quad (30)$$

where m is the quark mass matrix and B_0 is a constant. In $SU(2)$ the general form at order (q^4) is [6]

$$\begin{aligned} \mathcal{L}_4 = & \bar{L}_1 \text{Tr}(D_\mu U D_\nu U^\dagger D^\mu U D^\nu U^\dagger - D_\mu U D^\mu U^\dagger D_\nu U D^\nu U^\dagger) \\ & + \bar{L}_2 [\text{Tr}(D_\mu U D^\mu U^\dagger)]^2 \\ & + 2B_0 L_4 \text{Tr}(D_\mu U D^\mu U^\dagger) \text{Tr}(mU + U^\dagger m) \\ & + 4B_0^2 L_2 [\text{Tr}(mU + U^\dagger m)]^2 \\ & + 4B_0^2 L_1 [\text{Tr}(mU - U^\dagger m)]^2 \\ & - iL_1 \text{Tr}(L_{\mu\nu} D^\mu U D^\nu U^\dagger + R_{\mu\nu} D^\mu U^\dagger D^\nu U) \\ & + L_{10} \text{Tr}(L_{\mu\nu} R^{\mu\nu} U^\dagger) \end{aligned} \quad (31)$$

Here D_μ is a covariant derivative containing photon and/or W fields, and $L_{\mu\nu}$ and $R_{\mu\nu}$ are the appropriate left/right field strength tensors

$$\begin{aligned} D_\mu U &= \partial_\mu U - i\ell_\mu + iU\tau_\mu \\ L_{\mu\nu} &= \partial_\mu \ell_\nu - \partial_\nu \ell_\mu + [\ell_\mu, \ell_\nu] \end{aligned} \quad (32)$$

The L_i are the low-energy constants. In $SU(3)$, there are three more terms (hence the numbering scheme). The precise form of this is not important to anyone except those doing the calculations. The important thing to take away from this discussion is that one parametrizes the possible behaviour of all processes to order q^4 in terms of a few low-energy constants. Given these constants, a well-defined method exists for making predictions.

7. CHIRAL SYMMETRY IN ACTION

In this section, I want to show you a couple of examples of chiral perturbation theory applied phenomenologically. This is important both to get a feel for the nature of the method and to learn about the limits of validity of the energy expansion.

The simplest process that I can think of to illustrate the method is the pion electromagnetic form factor

$$\begin{aligned} \langle \pi^+(p') | J_\mu^{e.m.} | \pi(p) \rangle &= f(q^2) (p + p')_\mu \\ q_\mu &= (p' - p)_\mu \\ f(q^2) &= 1 + \frac{1}{6} \langle r^2 \rangle_\pi q^2 + \dots \end{aligned} \quad (33)$$

Using the Lagrangian $\mathcal{L}_2$, one obtains the result $f(q^2) = 1$, not a surprising answer. At order q^4 there is more content. One includes the effect of $\mathcal{L}_4$ plus the loop diagram of

Fig. 2. The result after defining a renormalized parameter L_9^r

$$f(q^2) = 1 + q^2 \left[\frac{2L_9^{ren}}{F_\pi^2} - \frac{1}{96\pi^2 F_\pi^2} \left(\ell_n \frac{m_\pi^2}{\mu^2} - \frac{1}{3} \right) \right] + \frac{1}{96\pi^2 F_\pi^2} (q^2 - 4m_\pi^2) H\left(\frac{q^2}{m_\pi^2}\right) \quad (34)$$

with

$$\begin{aligned} H\left(\frac{q^2}{m_\pi^2}\right) &\equiv \left\{ 2 + \sigma \ell_n \left| \frac{\sigma - 1}{\sigma + 1} \right| + i\pi\theta(q^2 - 4m_\pi^2) \right\} \\ \sigma &\equiv \sqrt{1 - \frac{4m_\pi^2}{q^2}} \end{aligned} \quad (35)$$

This latter function contains the imaginary parts required by unitarity (to this order in the energy expansion), and the kinematic effects of low energy rescattering. This process by itself is not predictive. One is required to simply fit the low energy constant L_9^r . Before going on, however, let us use this example to learn more about

Fig. 2: Loop diagram for $\gamma \rightarrow \pi\pi$.

the energy expansion. We know from the success of vector dominance ideas, and from a direct look at the data, that the form factor behaves roughly as

$$\begin{aligned} f(q^2) &= \frac{1}{1 - \frac{q^2}{m_\rho^2}} \\ &= 1 + \frac{q^2}{m_\rho^2} + \frac{q^4}{m_\rho^4} + \dots \end{aligned} \quad (36)$$

(Let us for clarity drop the imaginary parts and loop effects.) The first term in the expansion requires $2L_9^{ren} \approx F_\pi^2/m_\rho^2$. However, it also shows the nature of the expansion, which in this case is in q^2/m_ρ^2 . As $q^2 \rightarrow m_\rho^2$, keeping only the first terms becomes a bad approximation, but for $q^2/m_\rho^2 \ll 1$ the expansion is well behaved.

This example becomes more predictive if we go to other processes [6,7]. There exists a radiative complex of pionic transitions ($\gamma \rightarrow \pi^+\pi^-\pi^+\pi^+ \rightarrow e^+\nu\gamma; \pi^+ \rightarrow$

$e^+\nu e^+e^-; \gamma\pi \rightarrow \gamma\pi$), that describes six form-factors in terms of two low-energy constants (L_9, L_{10}). Besides the charge radius with pion form factor described above, one has three form factors in weak radiative decays

$$\begin{aligned} M_{\mu\nu}(q, p) &= \text{pion pole} + h_v \varepsilon_{\mu\nu\alpha\beta} p p^\alpha q^\beta \\ &+ h_A (p - q)_\mu q_\nu - g_{\mu\nu} (p - q) \cdot q \\ &+ r_A A(g_{\mu\nu} q^2 - q_\mu q_\nu) \end{aligned} \quad (37)$$

and two in $\gamma\pi$ reactions

$$\mathcal{M}(\gamma\pi \rightarrow \gamma\pi) = \vec{\varepsilon}_1 \cdot \vec{\varepsilon}_2 \left[-\frac{\alpha}{m_\pi} \left(1 - \frac{1}{6} \langle r_m^2 \rangle (q_1^2 + q_2^2) \right) + \omega_1 \omega_2 \alpha_E + \vec{\varepsilon}_1 \times \vec{q}_1 \cdot \vec{\varepsilon}_2 \times \vec{q}_2 \right] \beta_M \quad (38)$$

The latter two are the electron and magnetic polarizabilities, describing the polarization of the pion in E and B fields

$$\mathcal{L} \sim \text{Born} + (\alpha_E E^2 + \beta_M B^2) \vec{\pi} \cdot \vec{\pi} \quad (39)$$

In a parameter-free manner one predicts (here I do not display the small effects from loops):

$$\begin{aligned} \alpha_E + \beta_\mu &= 0 \\ h_v &= \frac{m_\pi}{4\sqrt{2}\pi^2 f_\pi^2} = \frac{0.026}{m_\pi} \end{aligned} \quad (40)$$

in agreement with measurements

$$\begin{aligned} (\alpha_E + \beta_\mu)_{\text{expt}} &= (1.4 \pm 3.1) \times 10^{-4} \text{ fm}^2 \\ h_v &= (0.029^{+0.019}_{-0.014}) \end{aligned} \quad (41)$$

The determination of L_9^r from the pion form factor above allows the predictions

$$\frac{r_A}{h_v} = 32\pi^2 L_9 = 2.6 \quad (42)$$

compared to the experimental value

$$\left(\frac{r_A}{h_v}\right)_{\text{expt}} = 2.3 \pm 0.6 \quad (43)$$

A new low-energy constant L_{10} enters in h_A

$$\frac{h_A}{h_v} = 32\pi^2(L_9 + L_{10}) \quad (44)$$

and the experimental value

$$\left(\frac{h_A}{h_v}\right)_{\text{expt}} = 0.46 \pm 0.02 \quad (45)$$

serves to determine this. Finally, this then serves to predict the electric polarizability

$$\alpha_E = \frac{4\pi}{m_\pi F_\pi^2}(L_9 + L_{10}) = 2.8 \cdot 10^{-4} \text{ fm}^2 \quad (46)$$

Interestingly, this result does not seem to work, with the only experimental measurement being $\alpha_E = (6.8 \pm 1.4) \times 10^{-4} \text{ fm}^2$. The experiment is a difficult one, done by a Soviet group [8] scattering pions off heavy Z atoms $\pi Z \rightarrow Z\pi\gamma$ where one uses Coulomb exchange to provide the initial photon. It involves a delicate procedure of subtracting off the Born terms and studying the remainder. Before drawing any conclusion from the apparent disagreement, the parameter should be remeasured.

One can also look at the applications of chiral Lagrangians to $\pi\pi$ scattering [6,9]. Here the amplitudes are expressed in a partial wave expansion as scattering amplitudes T^J_I with $I = \text{isospin}$ and $J = \text{angular momentum}$. At low energies the five possible amplitudes ($T^0_0, T^0_1, T^1_1, T^2_0, T^2_2$) can be expressed with 10 parameters (scattering lengths, slopes, curvatures), which in principle are independent. Chiral symmetry relates them with two low-energy constants (i.e. L_1, L_2). The analysis works remarkably well with sample results shown in Fig. 3.

Fig. 3: (left) Tree-level $\pi\pi$ amplitudes. a) Lowest order, b) order q^4 ; (right) Full theory to one loop, order q^4 .

Here curve a is the lowest order Weinberg prediction [10], i.e. from $\mathcal{L}_2$. It works at low energy but has clear problems as the energy increases, even violating unitarity below 700 MeV. The full theory to order q^4 , curve b, does significantly better at correlating the various channels. Again one can see the nature of the energy expansion, for example by looking at T^1_1 . The bump here is the ρ resonance, and keeping only two terms in the energy expansion can never reproduce the full ρ Breit-Wigner shape. It does, however, match on to the low energy tail of the ρ .

Let me just mention, without formulae, an analysis [11] which is coming out soon. The process $K^+ \rightarrow \pi^+\pi^-e^+\nu$ (K_{e4} is determined by the same low-energy

constants which are relevant for $\pi\pi$ scattering, with our addition. However, the large N_c limit of QCD predicts that one combination of the three constants must vanish. (This is similar to a Zweig rule argument.) By the phenomenological study of K_{L4} , one can then test this large N_c selection rule. The result seems to work at about the 1σ level, although higher experimental precision could tighten this considerably. The chiral constraints between $\pi\pi$ scattering and K_{L4} are especially stringent, and the low-energy constants can become well known.

There are many other applications of those ideas in other systems. Space and energy (*i.e.* my energy) limitations prevent me from continuing this survey. Hopefully, these examples have been sufficient for you to see both the beauty and the limitations of the theory.

8. THEORETICAL DIRECTIONS

There is a wide range of motivations of theorists using chiral symmetry. Even among those who apply chiral perturbation theory to phenomenology, there are many underlying interests. Most of us do not have as our primary interest phenomenology for its own sake. Rather the phenomenology serves a larger purpose. For many, the main motivation is to "tame" low energy QCD. Here we have a tool that is well defined, and the phenomenology lets us use it to uncover the ingredients important to QCD in this region. Others want to use chiral symmetry to solve other interesting problems. For example, the accurate prediction of CP violating amplitudes is an important goal for the testing of this unproved aspect of the standard model, or of theories beyond the standard model. For kaons, chiral symmetry is a main tool and we need to master it in order to test CP theories. In order to understand what phenomenology is to be desired, it helps to know the nature of questions which theorists are presently asking. The following is probably an incomplete list, but hopefully will give an indication of the range of questions.

- i) What is the origin of chiral Lagrangians in QCD? We have seen above at that order q^2 , the form of the chiral Lagrangian is universal, given F_π and

m_π . One cannot tell QCD from the linear σ model at this level. However at order q^4 the situation is different. The low-energy constants L_i differ from one theory to the next, and by determining them from experiment we are obtaining the low energy imprint of QCD. There are half a dozen or so different suggestions in the literature as to what the important physics is for the effective Lagrangians. These amount to attempts to obtain the L_i from QCD. When compared to the data, there is a clear winner [12]. This is a generalized form of vector dominance. The low-lying spectrum determines the low energy constants, to a remarkable accuracy. Further work is needed to make the connection to QCD more precise.

- ii) How does the energy expansion break down? There is often much information contained in the pattern of breakdown of the procedure. The symmetry ideas work best for pions at low energy, and have larger corrections for kaons or for pions at $q^2 = m_K^2$. The η brings in yet other corrections. If we regard the order q^2 results as fixed (which we should) it is the corrections at order q^4 and higher which are the interesting feature.

- iii) What is the connections with dispersion relations and other techniques? Chiral perturbation theory satisfies all of the general properties of field theory, such as analyticity, crossing and unitarity, order by order in the energy expansion. It can therefore match on nicely to other techniques, such as dispersion relations, which make use of these properties [13]. The combination is more powerful than either one by itself. One hopes that these ideas will allow the results of chiral perturbation theory to be extended to higher energies. Correspondingly, dispersion theory by itself is filled with problems, such as the values of subtraction constants at low energy, which chiral symmetry can resolve. However, this marriage of ideas needs a long and careful courtship in order to determine the suitability of the union. One cannot just do anything, as for example, there are many ways to build exact unitarity into the theory, each yielding somewhat different results. One is risking giving up the rigour of the procedure. However, it seems likely

that chiral perturbation theory will gradually absorb ideas from dispersion theory, to its ultimate improvement.

iv) What is the physics of the chiral anomaly? The axial anomaly is a fascinating subject which merges nicely with chiral perturbation theory [14]. The phenomenology of this has not been explored much beyond $\pi^0 \rightarrow \gamma\gamma, \eta \rightarrow \gamma\gamma$, but this is starting to change.

v) What is going on with the non-leptonic weak interactions? The weak non-leptonic decays are theoretically obscure. While their symmetry structure is known and well verified, the dynamics are not understood. Several applications come to mind. The higher order weak Lagrangians are now known [15]. If we can predict their relative coefficients, as in i) above, we can have widespread application. On the other hand, if we determine them from experiment, these coefficients contain information on the physics which drives the $\Delta I = 1/2$ enhancement. Finally, there is some hope of using chiral ideas, plus other physics, as an attempt to calculate the non-leptonic amplitudes.

vi) Can we apply what we have learned in new systems? I have already mentioned CP violation as an important application of chiral Lagrangians. Another one, in a far different area, is the study of $W_L W_L$ scattering [16]. There is an equivalence theorem which says that under certain conditions (a range of energies and a strongly interacting symmetry breaking sector), the scattering of longitudinal gauge bosons mirrors the Goldstone bosons of the symmetry breaking. These couplings are described by symmetry, and one obtains universal predictions at order q^2 and non-universal behaviour at order q^4 , with the latter in principle being able to distinguish between the different possible underlying origins of symmetry breaking.

9. OPEN EXPERIMENTAL QUESTIONS

My survey here will be even less complete than the previous theory survey. If I try to list the reactions for which chiral symmetry has something to say, I come up with Table I. Each of these reactions has its own notation, jargon, phenomenology and literature. The underlying feature is that they describe the interactions of pions, kaons and etas with themselves or with other fields, in the limits where the π, K, η energies are small. For some of these reactions, especially those with heavy fields, this involves only a portion of the available phase space. However, given the right conditions chiral ideas can be applied. Here, I describe a subset of possible reactions for which I know that interest exists.

i) Pions - Here a remeasurement of the $\gamma\pi \rightarrow \gamma\pi$ amplitude is needed in order to clear up the disagreement mentioned in Section 6. The two photon reactions $\gamma\gamma \rightarrow \pi^+\pi^-, \pi^0\pi^0$ are related, but one is interested in the threshold region. Some effort tying together these reactions would be worthwhile. Data on $\gamma\gamma \rightarrow 3\pi$ would prove interesting in connection with the axial anomaly. These are related to $\gamma\gamma \rightarrow \pi$, but because of the three-body final state, contain more kinematic information.

ii) Kaons - The reaction $K \rightarrow \pi\pi e\nu$ contains within its form factors a test of the large N_c selection rule, as described briefly above. The last experiment was performed 15 years ago [17], but kaon physics has developed immensely since then. The last experiment was analyzed in a way which does not match well with the chiral prediction and the sensitivity hints at a small violation of the large N_c prediction, but is only at 1σ accuracy. This reaction can also yield the most model independent results on $\pi\pi$ scattering. A higher precision experiment would help these issues.

There exists a kaon radiative complex of decays ($K \rightarrow e\nu\gamma, K \rightarrow e\nu^+e^-, K \rightarrow \pi e\nu, K \rightarrow \pi e\nu\gamma, \gamma K \rightarrow \gamma K$) analogous to the pionic reactions described above. These are all described without any new parameters, to order q^4 . Predictions then exist for all reactions, most of which have not

been tested. In $K \rightarrow 3\pi$, one can uncover the effects of higher order weak chiral Lagrangians by studying the quadratic dependence of the Dalitz plot distribution [15,18]. The full theory has recently been worked out. There are connections of this to other interesting issues in the weak interactions, such as the B parameter, the origin of the $\Delta I = 1/2$ rule and the ability of lattice methods to predict it, and rare K decays.

The reaction $K_L \rightarrow \pi^0 \gamma \gamma$ is at the centre of a controversy right now [19,20]. There is a calculation of the amplitude and energy distribution using chiral loops. There are no tree level contributions at order q^4 , so the result is purely loop dominated. However, from another study [20], it was suggested that there is a q^6 contribution from the ρ which is comparable. This is important to sort out because it influences strongly the search for CP violation in $K_L \rightarrow \pi^0 e^+ e^-$ (see below). It is also interesting in its own right as to the relative roles of resonances and loops in the structure of chiral Lagrangians. The reaction $K_L \rightarrow \pi^0 e^+ e^-$ is one of the prime modes for the observation of direct CP violation. The best signal involves an interference of the CP-conserving channel with a two-photon exchange (hence the relevance of $K_L \rightarrow \pi^0 \gamma \gamma$) and the CP-violating one-photon mode. Although considerable theoretical work has been done, I feel that more can be accomplished. The observation of $K_s \rightarrow \pi^0 e^+ e^-$ would be a major step as it would provide an absolute normalization of the effect of the ε parameter, allowing one to clearly see a non- ε effect.

iii) Eta - Eta physics has not yet been as thoroughly explored as that of π, K . More theoretical work will be forthcoming if the experiments are to be done. Here the rare modes $\eta \rightarrow \pi \gamma \gamma$ and $\eta \rightarrow \pi e^+ e^-$ share many of the same issues as $K \rightarrow \pi \gamma \gamma$ in relation to kaon CP violation. However, the analysis is in some ways clearer in η decay, as the weak interactions are not present. The study of both $K \rightarrow \pi \gamma \gamma$ and $\eta \rightarrow \pi \gamma \gamma$ would allow one to separate the weak and strong dynamics.

Decays such as $\eta \rightarrow \gamma e^+ e^-$ or $\gamma \gamma^* \rightarrow \eta$ test the q^2 dependence of the

anomaly analysis. The formalism has been worked out but needs better data to be applied [17]. The ultimate issue is again the understanding of the physics underlying these processes.

iv) Nucleons - There are several possible applications here, but let me just mention the one with the most outside interest at present [21]. The sigma term in $\pi N \rightarrow \pi N$ scattering measures the presence of explicit chiral symmetry breaking, i.e. quark masses. Combined with the Gell-Mann-Oakes-Renner analysis of baryon masses, it appears to indicate a sizeable strange quark content of the proton. There are a host of experimental and theoretical issues which have to be resolved before this question is settled definitively. However, it is important enough to justify further study.

v) Direct $p\bar{p}$ - Since this talk is being given at a $p\bar{p}$ school, I thought it worthwhile to display how one can generate chiral predictions even in systems like this. One requires the pions to have low energy, which most often is not the case in $p\bar{p}$ annihilation. However, if one studies $p\bar{p} \rightarrow M + \pi$'s, with M being a heavy resonance, the mass of M can use up most of the $p\bar{p}$ energy, leaving little for the π s. In this case chiral relations enter. The NN can be in $I = 0, 1$ state and $J = 0, 1, \dots$. Let me for simplicity, treat just one case here. The general formalism will be given elsewhere. I will look at $(NN)(I = 0, J = 0) \rightarrow M(I = 0, J = 0) + \pi$'s such as $p\bar{p} \rightarrow \eta(1440) + \pi\pi$. The NN is treated as a heavy source, Φ . The M mass is also heavy. One expands an effective Lagrangian in terms of the energy of the pions. There is no term with zero derivatives, and two possible terms with two derivatives on the pions;

$$\mathcal{L} = a_1 \Phi M \quad \text{Tr}(\partial_\mu U \partial^\mu U^\dagger) = a_2 \partial_\mu \Phi \partial_\mu M \quad \text{Tr}(\partial^\mu U \partial^\nu U^\dagger) \quad (47)$$

The extra derivatives on the heavy fields in the second term do not count in the energy expansion, as they yield energies which are not small. A

possibility such as

$$\mathcal{L}' = b_1 \partial_\lambda \Phi \partial^\lambda M \quad T r(\partial_\mu U \partial^\mu U^\dagger) \quad (48)$$

yields the same results as the a_1 form given above, to leading order in the pion momenta since $p_\Phi \cdot p_M = p_\Phi \cdot (p_\Phi - p_\pi) = \mathcal{M}_\Phi^2 - \theta(p_\pi)$. I have not included a quark mass term, as this is generally small for pions. The type of predictions that follows is that

$$\mathcal{M}(\Phi \rightarrow M \pi^+ \pi^-) = \bar{a}_1 p_+ \cdot p_- + \bar{a}_2 E_+ E_- + \dots \quad (49)$$

which says that the Dalitz plot distribution has a special kinematic suppression as E_π gets small. This is an "Adler zero" and forces the amplitude to grow with $\pi\pi$ energy in a way clearly distinguished from the phase space distribution. One also predicts $p\bar{p} \rightarrow M + 4\pi$ in terms of $p\bar{p} \rightarrow M + 2\pi$. At the moment, I do not see detailed uses for these types of predictions. Right now they are more of a curiosity as an application of chiral symmetry. However, perhaps with some study they can be put to more use, perhaps as flavour tags for the properties of the meson states. They also may be useful as a probe of how well chiral predictions do with heavy sources.

10. FINAL COMMENTS

The low energy experimental frontier is devoted to rare processes or high statistics. The field is active, with several low energy hadron facilities under way or being planned. While there are many motivations for such facilities, one component of the program should be chiral symmetry studies. These issues reflect QCD, there are open questions, and a community of theorists exists. While there are extremely compelling reasons for doing experiments to find processes which are forbidden by the standard model, any given experiment may not find such events. When one gets tired of measuring zero to higher precision, the same experiment may be sensitive to some of the rare process relevant for chiral studies. As a by-product of rare meson experiments, one can then contribute to the phenomenology described above. The mix of high states, high risk searches with safer and more conventional reactions provides a richer program for low energies.

I have tried to impress on you that the chiral framework is well defined and solid within its known limits. It needs to be more widely familiar as it is one of the foundations of phenomenology of the standard model. Theorists have returned to it in the hopes of taming QCD. I hope that at least some of the many experimenters who want to do something to study QCD will learn more about the field and keep it in mind when planning/doing experiments. Perhaps the 90's will see a renewal of experimental input to the field. If so, we will get closer to the goal of understanding low energy QCD.

$\pi \rightarrow e\nu$	$K \rightarrow \mu\nu$	$\eta \rightarrow 3\pi$
$\pi \rightarrow e\nu\gamma$	$K \rightarrow \pi\pi$	$\eta \rightarrow \gamma\gamma$
$\pi \rightarrow \pi^0 e\nu$	$K \rightarrow 3\pi$	$\eta \rightarrow 2\pi\gamma$
$\pi \rightarrow e\nu e^+ e^-$	$K \rightarrow \pi e\nu$	$\eta \rightarrow \pi^0 \mu^+ \mu^-$
$\pi \rightarrow \gamma\gamma$	$K \rightarrow \pi e\nu$	$\eta \rightarrow \pi\gamma\gamma$
$\pi \rightarrow \gamma e^+ e^-$	$K \rightarrow \pi\gamma\gamma$	$\eta \rightarrow \gamma\mu^+ \mu^-$
$\gamma\gamma \rightarrow \pi^+ \pi^-$	$K \rightarrow e\nu e^+ e^-$	$\gamma\gamma^* \rightarrow \eta$
$\gamma\gamma \rightarrow \pi^0 \pi^0$	$K \rightarrow e\nu\gamma$	$\eta \rightarrow \pi\pi\gamma$
$\gamma \rightarrow \pi^+ \pi^- \pi^0$	$K \rightarrow \pi\pi\gamma$	$B_u \rightarrow \tau\nu$
		$B \rightarrow \pi e\nu$
		$B \rightarrow \pi\pi e\nu$
$\gamma\gamma^* \rightarrow \pi^0$	$K \rightarrow \pi e^+ e^-$	$D \rightarrow K e\nu$
$\tau \rightarrow 3\nu\tau$	$\gamma K \rightarrow \gamma K$	$D \rightarrow K\pi e\nu$
$\tau \rightarrow 2\nu\tau$	$K \rightarrow 3\pi\gamma$	$D_s \rightarrow \mu\nu$
$\tau \rightarrow 4\nu\tau$	$K_s \rightarrow \gamma\gamma$	$(N\bar{N}) \rightarrow M + \pi's$
$\pi\pi \rightarrow \pi\pi$	$K_L \rightarrow \gamma\gamma$	$(N\bar{N}) \rightarrow (N\bar{N}) + \pi's$
		$\psi' \rightarrow \psi + \pi\pi$
$\gamma p \rightarrow \pi p$	$\Sigma \rightarrow p\pi$	$\Upsilon \rightarrow \Upsilon + \pi\pi$
$\pi p \rightarrow \pi p$	$\Xi \rightarrow \Lambda\pi$	$\Upsilon' \rightarrow \Upsilon + \pi^0 \eta^0$
$\pi p \rightarrow \pi\pi p$		

Table 1

Chiral Reactions

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