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Measuring Colour Flows in Hard Processes via Hadronic Correlations¹

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Abstract

We propose a method for revealing the connection between observed hadronic distributions and the colour structure of an underlying hard process. The method does not require any special event selection or jet finding. It involves measuring a ratio of energy-multiplicity correlations which is especially sensitive to colour flows in jet formation. This quantity is infrared stable and can be calculated completely perturbatively. We discuss in detail the case of e^+e^- annihilation.

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The first data on hadronic Z^0 decays from SLC [1] and LEP [2-5] appear to be in very good agreement with Monte Carlo simulations [6-8] based on a QCD parton shower mechanism of multihadron production in hard processes (see e.g. the reviews [9,10] and references therein). In this mechanism, hadron distributions are mainly determined by those of underlying parton cascades, whose properties can be calculated in detail using perturbative QCD. The conversion of partons into hadrons is supposed to occur at a low virtuality scale, independent of the scale of the primary hard process, and to involve only low momentum transfers, leading to a close similarity or “local duality” (LPHD) [11] between parton and hadron distributions. Such local duality follows naturally from the pre-confinement property of QCD [12].

A fundamental feature of the parton shower mechanism is the connection between the *colour flow* in the hard process and the observed flow of hadron multiplicity [9,10,13].

This connection was beautifully illustrated in e^+e^- annihilation at lower energies by the observation of the “string” or “drag” effect in three-jet final states. Here the colour flow gives rise to destructive interference in the “antenna pattern” of parton emission in the angular region between the quark and antiquark jets. The corresponding depletion of hadron flow into this region was confirmed both by comparison of hadron multiplicities between the jets [14] and by comparison of $q\bar{q}g$ and $q\bar{q}\gamma$ final states [15]. It should be possible to provide further evidence for colour interference in three-jet events by the same type of analysis of the LEP data, with greatly increased statistics, higher energy, and the possibility of identifying quark jets through the observation of heavy quark decays.

As a demonstration of the connection between colour and hadronic flows, the “string” analysis of three-jet events suffers from some inherent difficulties and weaknesses that one would prefer to avoid if possible. First of all, the necessity of selecting a three-jet event sample reduces the statistics and may introduce biases into the observed hadron flow. The need to define jet directions introduces a dependence on the jet-finding algorithm. Discrimination between quark and gluon jets on the basis of their relative energies reduces the effect and prevents the use of symmetrical jet configurations.

In this paper we propose a different test for essentially the same colour interference phenomenon as the classic string effect. The advantages of our proposal are that it avoids the selection of a three-jet sample, the determination of jet axes, and the identification of jets. Instead, it focuses on the correlation between hadron flows in different directions transverse to the main direction of energy flow in hadronic final states. Thus it should provide a simple, high-statistics test of the intimate connection between colour flows and hadron production.

In the following, we first give an explanation of the phenomenon to be studied in terms of the antenna patterns of soft gluon emission from hard parton configurations. We then define the ratio of energy-multiplicity correlation functions that can be used to investigate the effect experimentally. Next, we discuss the corrections to the lowest-order antenna pattern that arise from multiparton emission and hard parton contributions. We present results of a Monte Carlo simulation that includes estimates of these corrections, together with those due to hadronization and heavy quark decays.

We start by recalling the antenna pattern for soft gluon emission from a hard massless quark-antiquark-gluon system, which leads to the string effect. Denoting the quark, antiquark

and hard and soft gluon directions by $\mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3$ and $\mathbf{n}_4$, respectively, we have

$$W^{q\bar{q}g}(\mathbf{n}_4) \propto N_c \left(\frac{a_{13}}{a_{14}a_{43}} + \frac{a_{23}}{a_{24}a_{43}} \right) - \frac{1}{N_c} \frac{a_{12}}{a_{14}a_{42}} \quad (1)$$

where $W^{q\bar{q}g}(\mathbf{n}_4)$ represents the angular distribution of the soft gluon and $a_{ij} = 1 - \mathbf{n}_i \cdot \mathbf{n}_j$. This may be compared with the pattern for a quark-antiquark system, with a hard photon replacing the gluon jet 3 to permit the directions $\mathbf{n}_1$ and $\mathbf{n}_2$ to remain the same:

$$W^{q\bar{q}\gamma}(\mathbf{n}_4) = W^{q\bar{q}g}(\mathbf{n}_4) \propto 2C_F \frac{a_{12}}{a_{14}a_{42}} \quad (2)$$

where $C_F = (N_c^2 - 1)/2N_c$. Taking for illustration the threefold symmetric $q\bar{q}g$ or $q\bar{q}\gamma$ configuration, the ratio of emission in the direction opposite to the hard gluon or photon, $\mathbf{n}_4 = -\mathbf{n}_3$, in the two processes is

$$\frac{W^{q\bar{q}g}}{W^{q\bar{q}\gamma}} = \frac{N_c^2 - 2}{2(N_c^2 - 1)} = \frac{7}{16}, \quad (3)$$

showing the destructive interference that reduces emission into this region in the $q\bar{q}g$ case (see e.g. [10]).

Suppose now that we consider the emission of two soft gluons by a hard quark-antiquark system. Replacing the constant of proportionality in Eq. (1) by the soft limit of the $q\bar{q}g$ matrix element squared, we obtain the angular distribution of two soft gluons with $E_{cm} \gg E_3 \gg E_4$:

$$W^{q\bar{q}}(\mathbf{n}_3, \mathbf{n}_4) \propto 2C_F N_c \left(\frac{a_{12}}{a_{13}a_{34}a_{42}} + \frac{a_{12}}{a_{23}a_{34}a_{41}} - \frac{1}{N_c^2} \frac{a_{12}^2}{a_{13}a_{32}a_{41}a_{42}} \right). \quad (4)$$

Note that this expression is symmetric under exchange of the two gluons and therefore it is also valid in the other strongly ordered region $E_{cm} \gg E_4 \gg E_3$. Dividing by the product of the single-gluon distributions gives the gluon-gluon correlation function

$$C^{q\bar{q}}(\mathbf{n}_3, \mathbf{n}_4) = \frac{W^{q\bar{q}}(\mathbf{n}_3, \mathbf{n}_4)}{W^{q\bar{q}}(\mathbf{n}_3)W^{q\bar{q}}(\mathbf{n}_4)} = \frac{N_c}{2C_F} \left(\frac{a_{13}a_{24} + a_{14}a_{23}}{a_{12}a_{34}} - \frac{1}{N_c^2} \right). \quad (5)$$

Since the hard quark and antiquark directions will be (anti-)collinear in the soft gluon limit, we may define the gluon pseudorapidities $\eta_{3,4}$ and azimuthal angles $\phi_{3,4}$ relative to the direction $\mathbf{n}_1 \simeq -\mathbf{n}_2$ and write Eq. (5) in the simple form

$$C^{q\bar{q}}(\eta_{34}, \phi_{34}) = 1 + \frac{N_c}{2C_F} \left(\frac{\cos \phi_{34}}{\cosh \eta_{34} - \cos \phi_{34}} \right) \quad (6)$$

where $\eta_{34} = \eta_3 - \eta_4$ and $\phi_{34} = \phi_3 - \phi_4$.

Equation (6) provides an infrared-finite measure of the correlation between colour flows in the directions (η_3, ϕ_3) and (η_4, ϕ_4) . According to the local duality hypothesis, it can be applied directly to hadronic flows. Considering, for example, the flows in orthogonal azimuthal directions at the same pseudorapidity, we have

$$C^{q\bar{q}}(0, \pi/2) = 1, \quad (7)$$

implying that hadronic multiplicities in these directions should be uncorrelated. For back-to-back azimuths, on the other hand,

$$C^{q\bar{q}}(0, \pi) = \frac{N_c^2 - 2}{2(N_c^2 - 1)} = \frac{7}{16}, \quad (8)$$

showing that in this configuration there is destructive interference, of the same magnitude as the string effect in threefold symmetric jets [Eq. (3)]. Thus measurements of hadronic flow correlations in the orthogonal and back-to-back azimuthal directions should demonstrate the same type of colour coherence as the string effect, without requiring the selection of a three-jet event sample.

The lowest-order result (6) has a collinear singularity in the direction $\eta_{34} = \phi_{34} = 0$, which will be regularised by parton showering in higher orders. Thus the expression (6) is not reliable at small values of η_{34} and ϕ_{34} . As the rapidity difference η_{34} increases, the dependence on ϕ_{34} decreases and the hadronic flow should become uncorrelated.

Notice that the ratio $7/16$ in (8) is directly related to the colour flow in the $q\bar{q}gg$ matrix element. For instance, the correlation function C^{gg} , for emission of two soft gluons by a pair of hard gluons gg instead of $q\bar{q}$, is obtained from (5) by replacing C_F by N_c . Then we would still have no correlation in the orthogonal azimuthal direction ($C^{gg} = 1$), while in the back-to-back direction we would have

$$C^{gg}(0, \pi) = \frac{3}{4}. \quad (9)$$

Next we explain how to measure the correlated hadronic flows discussed above without having to define explicitly any jet axes for the hard process. To be specific we consider again e^+e^- annihilation. The basic idea is to use an energy-weighted three-particle correlation to emphasize the direction of energy flow. For each set of three hadrons i, j and k , we define an interval of pseudorapidity for j and k relative to the direction of i , $\eta_{\min} < \eta_j, \eta_k < \eta_{\max}$, where $\eta_j = -\ln(\theta_{ij}/2)$. Then we define the energy-multiplicity-multiplicity correlation (EMMC),

$$C_{EMM}(\eta_{\min}, \eta_{\max}, \phi) = \frac{1}{\sigma} \int E_i dE_i dE_j dE_k \int_{\eta_{\min}}^{\eta_{\max}} d\eta_j d\eta_k \int_0^{2\pi} d\phi_j d\phi_k \delta(\phi - \phi_j + \phi_k) \frac{d\sigma}{dE_i dE_j dE_k d\eta_j d\eta_k d\phi_j d\phi_k}, \quad (10)$$

where ϕ_j and ϕ_k are azimuthal angles around the direction of hadron i . To obtain a quantity related to the colour flow correlation in Eq. (6), we should normalize to the corresponding two-particle energy-multiplicity correlation squared, divided by the total energy flow, to obtain

$$C(\phi) = \frac{C_{EMM}(\eta_{\min}, \eta_{\max}, \phi) C_E}{[C_{EM}(\eta_{\min}, \eta_{\max})]^2} \quad (11)$$

where

$$C_E = \frac{1}{\sigma} \int E_i dE_i \frac{d\sigma}{dE_i} \quad (12)$$

and

$$C_{EM}(\eta_{\min}, \eta_{\max}) = \frac{1}{\sigma} \int E_i dE_i dE_j \int_{\eta_{\min}}^{\eta_{\max}} d\eta_j \int_0^{2\pi} d\phi_j \frac{d\sigma}{dE_i dE_j d\eta_j d\phi_j}. \quad (13)$$

Because of the energy weighting factor E_i in (10), the hadron i is preferentially associated (via local duality) with the hard quark or antiquark initiating the QCD emission. For a suitable choice of the pseudorapidity interval $[\eta_{\min}, \eta_{\max}]$, the other two hadrons j and k will be associated with soft gluon emission. Thus to leading perturbative order, the integrand of (10) is given by $W^{q\bar{q}}(\mathbf{n}_3, \mathbf{n}_4)$ where we associate $\mathbf{n}_1$ with the direction of hadron i and $\mathbf{n}_3$ and $\mathbf{n}_4$ with those of hadrons j and k . Similarly the integrand of (13) is given to leading order by $W^{q\bar{q}}(\mathbf{n}_3)$.

The correlation functions C_{EMM} and C_{EM} separately are not infrared finite quantities and therefore they receive leading contributions from all perturbative orders. These leading terms can be summed by replacing the soft gluons 3 and 4 by their associated parton cascades. Thus the leading contributions to the radiation pattern $W^{q\bar{q}}$ in Eq. (4) can be taken into account, away from the collinear directions $\mathbf{n}_{1,2}$, by multiplying the right-hand side by a cascading factor $N'_g(\ln E_{cm})$ for each gluon 3 and 4, where $N_g(\ln E)$ is the mean multiplicity in a gluon jet at scale E and the prime denotes differentiation. Since

$$N_g(\ln E) \sim \exp \sqrt{(16N_c/\beta) \ln E}, \quad (14)$$

where $\beta = 11N_c - 2N_f$, we have

$$N'_g = \sqrt{2N_c \alpha_S(E)/\pi} N_g [1 + O(\sqrt{\alpha_S})]. \quad (15)$$

Thus the general energy-(multiplicity) ^{n} correlation of the form (10) or (13) will be of the order of $(\sqrt{\alpha_S} N_g)^n E_{cm}$. Since the leading effects of cascading factorise, they cancel in quantities such as $C(\phi)$. As a result, at high energies $C(\phi)$ is determined entirely by soft gluon radiation and is asymptotically given by Eq. (6):

$$C(\phi) \sim C^{q\bar{q}}(0, \phi) \quad (16)$$

for a sufficiently narrow rapidity interval $[\eta_{\min}, \eta_{\max}]$.

We now discuss the most important finite energy corrections to this asymptotic expression.

1) Three jet events. The factor $\sqrt{\alpha_S} N_g$ from each gluon cascade implies that the contribution we have calculated, from $q\bar{q}$ plus two soft gluons in lowest order, is of the same order as a hard three-jet contribution to the same quantity, when one of the jets defines the energy flow and the other two can both be recorded in the pseudorapidity interval $[\eta_{\min}, \eta_{\max}]$. Such a three-jet contamination, which would contribute mainly around $\phi \simeq \pi$ and for negative rapidities, can be eliminated by making it kinematically impossible for two jets in a three-jet event to enter the interval, for example by choosing $\eta_{\min}$ and $\eta_{\max}$ both positive.

After this precaution has been taken to eliminate three-jet contributions, all remaining corrections to Eq. (16) are of relative order $\sqrt{\alpha_S}$:

2) Incomplete parton shower factorization and cancellation. There are corrections due to incomplete cancellation of parton showering in Eq. (5), since the arguments of the multiplicity factors in the numerator and denominator are not exactly the same. More precisely, each singularity α_{ij}^{-1} has an associated factor of $N'_{g,1/2} \ln(E_i E_j \alpha_{ij})$. From Eq. (15) we see the resulting corrections will be of relative order $\sqrt{\alpha_S}$.

3) Non-soft contributions. Further corrections of order $\sqrt{\alpha_S}$ arise when the pseudorapidity band $[\eta_{\min}, \eta_{\max}]$ contains a hard jet and a soft jet, or two soft jets with energies of the same order.

To estimate the effects of finite energy corrections and hadronization, we used the Monte Carlo program HERWIG [8], which contains the necessary features of perturbative QCD together with a simple cluster hadronization model that obeys the local duality hypothesis. Figure 1 shows the results of a simulation of e^+e^- annihilation at $E_{\text{cm}} = 91$ GeV. The pseudorapidity interval used was from $\eta_{\min} = 1$ to $\eta_{\max} = 2$, to avoid three-jet contamination as discussed above. The results shown are based on an unbiased sample of 10^5 events, without any event or particle selection. The leading-order formula (6) is shown by the curves for various values of η_{34} .

Destructive interference, that is, $C(\phi) < 1$ for $\pi/2 < \phi < \pi$, is clearly seen in the Monte Carlo results at both the parton and hadron levels. According to the cluster model used in the program, the effects of hadronization are small at these energies.

The difference between the Monte Carlo points and the leading-order curves for $\eta_{34} \leq 1$ gives an estimate of the finite-energy corrections, which are seen to be significant. As discussed above, they are of relative order $\sqrt{\alpha_S}$, where for some contributions the scale of α_S is associated with the relatively soft emitted gluons 3 and 4. Thus it is not surprising that such corrections are large. $O(\sqrt{\alpha_S})$ corrections to some other multiparticle observables have been computed analytically and are also found to be large [10,16]. It would be interesting to compute the corrections to $C(\phi)$ analytically if possible. Meanwhile we have to rely on the Monte Carlo estimate. The approximation scheme used in HERWIG is known to underestimate destructive interference effects in certain regions of phase space, but is expected to be reliable for $C(\phi)$.

In order to explore the energy dependence of $C(\phi)$, we also performed a simulation at $E_{\text{cm}} = 2$ TeV. We obtained similar results, with some evidence of convergence to the asymptotic curves at $\phi < \pi/2$ but little change in the region $\pi/2 < \phi < \pi$. Note that even at 2 TeV $\sqrt{\alpha_S}$ has only decreased by about 20% relative to its value at 91 GeV.

An interesting area for further study is the colour flow in heavy flavour jets. In this case the eikonal distributions that build up the antenna patterns in Eqs. (1-4) are modified by the presence of the heavy quark mass (see e.g. [17,18]). These modifications are included in the HERWIG program. In Fig. 2 we show the predicted hadronic flow correlation in $b\bar{b}$ compared with light quark ($d\bar{d}$) final states, again at $E_{\text{cm}} = 91$ GeV. The value of $C(\phi)$ for $\pi/2 < \phi < \pi$ is slightly larger than that for light quarks if b -flavoured hadrons are treated as stable particles, but slightly smaller if their decay products are included. The decays give rise to secondary jets, which tend to increase $C(\phi)$ at small ϕ and decrease it at large ϕ .

An analysis of colour flow along the above lines may be performed for other hard processes. For instance, in deep inelastic scattering the jet structure in the Breit frame is similar to that of e^+e^- annihilation in the centre of mass. The incoming and outgoing struck parton plays the role of the produced quark-antiquark pair and therefore the correlated hadronic flow should again be approximately described by Eq. (6). A similar effect should be seen in electroweak boson production via the Drell-Yan process in hadron-hadron collisions, although

its magnitude will be diluted by the presence of soft spectator interactions. Analogous correlations should also be present in jet production at hadron colliders, but the corresponding antenna patterns are more complicated than Eq. (6), owing to the presence of additional hard partons and alternative colour flows [10,13].

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Figure Captions

Fig. 1. Hadronic flow correlation defined by Eq. (11), as a function of the azimuthal angle ϕ , for a rapidity interval $1 < \eta_{1,2} < 2$. The points are Monte Carlo predictions from the program HERWIG [8] at the parton and hadron levels, for e^+e^- annihilation at $E_{cm} = 91$ GeV. The curves show the leading-order prediction (6) for various rapidity differences η_{34} .

Fig. 2. The same as Fig. 1, comparing $b\bar{b}$ and $d\bar{d}$ final states and showing the effect of weak b -flavoured hadron decays.

Fig. 1

Fig. 2