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QCD Phase Transitions in an Effective Field Theory

Bruce A. Campbell
Physics Department, University of Alberta
Edmonton, Canada

John Ellis
CERN, Geneva

Keith A. Olive
School of Physics and Astronomy, University of Minnesota
Minneapolis, Minnesota 55455 U.S.A.

Abstract

We analyze the transitions between the hadronic and quark-gluon phases of QCD using an effective low-energy lagrangian for chiral and scale invariance, which embodies quark and gluon condensation. We argue that the quark condensation and deconfinement transitions take place at the same temperature, T_q , and that gluon condensation and confinement occur at a common temperature T_g . The requirement that scale invariance be spontaneously broken before quarks condense imposes $T_q \leq T_g$. The gluon transition is in general first order, whereas the quark transition may be second order if $T_q < T_g$, or first order if $T_q = T_g$, which is favoured for large N_c . We argue on general grounds and demonstrate with several model calculations that massive hadrons play a key role in driving the gluon transition, and suggest that T_g is below the Hagedorn temperature. We also estimate the surface tension for hadronic bubbles forming in a quark-gluon plasma, finding it to be somewhat too small for inhomogeneities to affect the standard calculations of cosmological nucleosynthesis.

1. Introduction

Transitions between hadronic matter and the quark-gluon plasma play important roles in particle physics, astrophysics and cosmology. Present experiments with relativistic heavy ions at CERN are already reaching in extended regions energy densities that should be sufficient for quark-gluon plasma to be formed¹, and other experiments with heavier nuclei and higher in energies are planned at CERN and elsewhere. In the light of the recently-observed supernova 1987a and the apparent observation of a very rapid pulsar in its remnant², there has been renewed speculation about the dense hadronic equation of state, and the possibility that neutron star cores might contain some quark-gluon plasma. Finally, there has recently been much speculation about the possible generation of inhomogeneities during the quark-hadron transition in the very early Universe, which might modify the successful standard picture of cosmological nucleosynthesis³.

Several theoretical tools are available for analyzing the nature of the quark-hadron transition or transitions. One of these is perturbative QCD, which should be applicable at high enough temperatures and or pressures⁴. Unfortunately, QCD perturbation theory exhibits infrared singularities which complicate quantitative analysis close to the transition temperature. Another tool is lattice QCD⁵, which can in principle be applied close to the transition temperature, but requires a large amount of computer time when dynamical fermions are included and especially when one wants to incorporate a non-zero baryon chemical potential. Then there are effective lagrangians for the light hadrons in QCD which can be used to approach the QCD phase transitions from low temperatures, and may give some insight into their nature^{6,7}, but are not applicable above the QCD phase transitions. In the past, the QCD effective

lagrangian approach has not been explored as extensively as the perturbative and lattice QCD approaches, and the purpose of this paper is to deepen and extend the steps that were taken in this direction in Ref. 8.

The QCD lagrangian contains quark and gluon degrees of freedom that are confined in the real low-temperature world. Perturbative and lattice QCD studies prove that quarks and gluons are indeed unconfined at high temperatures, giving rise to discussion of quark and/or gluon deconfinement phase transitions at a finite temperature or temperatures. However, the technical definition of these phase transitions has been hampered by the absence of a local order parameter linked to confinement in the effective low-energy theory. This has no explicit quark and gluon degrees of freedom, but does realize the approximate chiral and scale symmetries of the underlying QCD lagrangian. As well as being explicitly broken by quark masses and, in the case of scale symmetry, by the renormalization prescription, these chiral and scale symmetries are spontaneously broken by quark and gluon condensates in the vacuum. The finite-temperature behavior of these condensates can be studied by applying finite temperature field-theoretical techniques to the effective low-energy theory^{6,7}. We argue⁸ that the quark and gluon condensates are in fact order parameters for the phase with confined quarks and gluons, and hence that these studies of their temperature dependences give us insight into the quark and gluon deconfinement transitions.

In the framework of the multi-flavour chiral symmetry present if there are 2 or 3 light quarks, baryons appear as topological solitons of the quark condensate field⁹. When the value of this condensate falls to zero, the baryon masses also vanish and their radii become infinite, signalling quark deconfinement⁸. In the low-energy theory of broken scale invariance¹⁰, a scalar glueball appears as a quantum fluctuation of the gluon condensate field.

When the value of this condensate falls to zero, all composite particle masses vanish and the glueball interpretation of quantum fluctuations becomes invalid, signalling gluon deconfinement.

It is well-known that the chiral phase transition would be second-order if the dynamics of other hadrons could be neglected^{6,7}. In fact, scale invariance tells us that quarks can condense only if gluons also condense⁸. This leaves us with two possibilities, either the chiral quark condensate vanishes and quarks deconfine at a temperature lower than the corresponding temperature for gluons, or both quark and gluon phase transitions occur simultaneously and are driven by gluon decondensation. Lattice studies¹¹ indicate that the latter occurs in $N_c = 3$ QCD with 3 light flavours. We argue from the form of the effective gluon condensate potential that this gluon decondensation transition is likely to be first order, in agreement with lattice results¹¹. We further argue that the gluon (and hence the quark) transition is driven by the infinite set of hadronic degrees of freedom present in the confined phase: $n(m) \sim \exp(\beta m)$. These drive the order parameters to zero at a temperature somewhat below the Hagedorn¹² temperature $T_0 = 1/\beta$. We exhibit several approximate phenomenological calculations demonstrating that they do indeed drive a first-order quark/gluon decondensation and deconfinement phase transition.

The structure of this paper is as follows. In Section 2 we set up the effective low-energy lagrangian for broken chiral and scale invariance¹⁰, discussing within this framework the interpretation of baryons and glueballs, and links between chiral and scale symmetry breaking. We also review calculations of finite-temperature corrections calculated^{6,7} within this effective theory, and argue that although they might be adequate for describing the behaviour of the quark condensate at finite temperature, a treatment of

the full spectrum of hadrons is needed to discuss the disappearance of the gluon condensate. In Section 3 we show how several different approximate treatments of hadrons lead to the same conclusion of a first order phase transition. These include the use of the known spectrum of hadrons with masses ≤ 2.5 GeV, a perfect Hagedorn gas, a Hagedorn spectrum with a hard-core potential, and an excluded-volume treatment of a Hagedorn spectrum. Finally, in Section 4 we discuss our results and present some conclusions. In particular, we estimate the surface tension of hadronic bubbles, and argue that these are unlikely to modify the successful standard picture of cosmological nucleosynthesis¹³.

2. Low-Energy Effective Lagrangian Considerations

In this section we first introduce the low-energy effective lagrangian for approximate chiral and scale symmetry¹⁰ that we use. Then we discuss the interpretation of baryons and glueballs in this effective lagrangian framework, arguing that these hadrons cease to exist when the quark and gluon condensates vanish. Finally, we discuss finite-temperature corrections in the low-energy effective theory.

2.1 Effective lagrangian for chiral and scale symmetry

The low-energy effective lagrangian for spontaneously broken chiral symmetry¹⁴ is constructed out of the dimensionless meson field

$$U(x) = \exp \left(i \sum_{i=0}^8 \frac{\lambda_i \phi_i(x)}{f_\pi} \right) \quad (2.1)$$

where the ϕ_i ($i=0,1,\dots,8$) are fields for the nine (pseudo-)Goldstone pseudoscalar mesons and $f_\pi \approx 93$ MeV is the pion decay constant. Expanding in powers of field derivatives and quark masses the lowest-order lagrangian terms are

$$L = \frac{f_\pi^2}{16} \text{Tr}(\partial_\mu U \partial^\mu U^\dagger) - c \text{Tr}(m_q(U+U^\dagger)) - \frac{m_0^2}{2} \phi_0^2 + \dots \quad (2.2)$$

where $m_q = \text{diag}(m_u, m_d, m_s)$ is the light quark mass matrix, c is a constant with mass dimension 3, and $m_0^2 = 0(1/N_c)$ is an extra $U(1)$ -breaking mass term for the ninth pseudoscalar meson ϕ_0 . Terms of higher order in ∂_μ , m_q and $1/N_c$ are

not essential for our discussion. The role of the quark condensate is played by a vacuum expectation value for $U(x)$:

$$\langle 0 | \bar{q}q | 0 \rangle = c \langle 0 | (U + U^\dagger) | 0 \rangle \quad (2.3)$$

as can be seen from the effective lagrangian (2.2).

Although all the terms in the lagrangian were constructed to have the mass dimension 4 appropriate to give a dimensionless action $\int d^4x L(x)$, they do not reflect correctly the operator scaling properties of QCD¹⁵. At the tree level, quantum field theories admit scale transformations generated by a dilation charge $D(t) = \int d^3x S_0(x)$, where $S_\mu(x)$ is the scale current, and

$$[D, \Phi(x)] = i(x_\mu \partial^\mu + d_\Phi) \Phi(x) \quad (2.4)$$

for a generic quantum field $\Phi(x)$ with scale dimension d_Φ . Conventionally, spin -1 gauge fields $A_\mu(x)$ are assigned scale dimension $d_A = 1$ and spin - $\frac{1}{2}$ fermion fields $\psi(x)$ have $d_\psi = 3/2$. Thus the operators $F_{\mu\nu}(x)F^{\mu\nu}(x)$ and $\bar{\psi}(x)\psi(x)$ appearing in the QCD lagrangian have scale dimension 4 so that $[D, L_{\text{QCD}}(x)]$ is a total derivative and hence massless QCD is scale-invariant at the tree level, while quark mass terms have scale dimension 3 and hence break scale invariance. Comparing the forms of $U(x)$ (2.1) and the general scale transformation law (2.2), it is apparent that the only consistent choice is $d_{\phi_i} = d_U = 0$, which means that the three terms in the effective lagrangian (2.2) have scale dimensions 2, 0 and 0 respectively.

In order to be able to write down effective lagrangian terms with the correct scale dimensions, the minimal possibility¹⁰ is to introduce a flavour-singlet field χ with scale dimension $d_\chi = 1$ and a vacuum expectation value

$$\chi_0 = \langle 0 | \chi | 0 \rangle \quad (2.5)$$

that breaks scale invariance spontaneously. We can then restore the correct tree-level QCD scaling properties if we multiply each of the terms in (2.2) by the appropriate power of (χ/χ_0) and introduce a kinetic term for the χ field:

$$\begin{aligned} L = \frac{f^2}{16} \left(\frac{\chi}{\chi_0}\right)^2 \text{Tr} \partial_\mu U \partial^\mu U^\dagger + \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - c \left(\frac{\chi}{\chi_0}\right)^3 \text{Tr} (m_q (U + U^\dagger)) \\ - \frac{m_0^2}{2} \phi_0^2 \left(\frac{\chi}{\chi_0}\right)^4 + \dots \end{aligned} \quad (2.6)$$

We could also introduce a scale-invariant effective potential $\propto \chi^4$, but this would not stabilize the χ field in such a way that $\chi_0 \neq 0$.

Beyond the tree level, even massless QCD is no longer scale-invariant because of the scale anomaly¹⁶

$$\partial^\mu S_\mu = \frac{\beta}{2g} F_{\mu\nu} F^{\mu\nu} \quad (2.7)$$

Within the effective lagrangian framework, this is modelled¹⁷ by adding to L (2.6) a potential term

$$V_\chi = B \left[\frac{\chi_0^4}{4} + \chi^4 \ln(\chi/e^{1/4}\chi_0) \right] \quad (2.8)$$

where the constants inside the square parentheses are chosen by: $V_\chi = \partial V / \partial \chi = 0$ at $\chi = \chi_0$, and the overall normalization factor B is related to a conventional bag constant B_B :

$$B_B = B \chi_0^4 / 4 \quad (2.9)$$

Applying (2.4) to the effective potential (2.8) it is easy to see that

$$\partial^\mu S_\mu = -B \chi^4 \quad (2.10)$$

We therefore relate χ_0 to the gluon condensate:

$$\langle 0 | \frac{\beta}{2g} F_{\mu\nu} F^{\mu\nu} | 0 \rangle = -B \chi_0^4 \quad (2.11)$$

and interpret (2.6, 2.8) as the effective low-energy theory of QCD in the hadronic phase with quark and gluon condensates. In terms of the gluon condensate $G_0 = \langle 0 | F_{\mu\nu} F^{\mu\nu} | 0 \rangle$, we have $B_B = (9/32) G_0$. We note that Equation (2.3) is replaced by

$$\langle 0 | \bar{q}q | 0 \rangle = c(\chi/\chi_0)^3 \langle 0 | (U+U^\dagger) | 0 \rangle \quad (2.12)$$

when scale invariance is taken into account as in the effective lagrangian (2.6).

2.2 Interpretation of mesons, baryons and glueballs

From the effective lagrangian point of view, hadrons can be divided into 6 classes: (a) flavour-octet pseudoscalar mesons ϕ_i , (b) the flavour-singlet $\bar{q}q$ pseudoscalar meson ϕ_0 , (c) other $\bar{q}q$ mesons, (d) baryons, (e) the lowest-lying scalar glueball, (f) other glueballs. Classes (a) and (b) appear explicitly in the lagrangian (2.2) or (2.6) as quantum fluctuations in the quark condensate, analogous to spin waves in condensed matter physics. Other $\bar{q}q$ mesons (c) can be introduced into the effective lagrangian framework but at low energies their effects are adequately represented by integrating them out, a procedure which gives higher-order terms in the effective lagrangian, e.g. $(\partial_\mu U)^4$ terms due to vector meson exchange.

Baryons (d) could in principle also be introduced into the effective lagrangian as external fields, but this would be double-counting since they already exist as topological solitons in the effective low-energy theory. As was first pointed out by Skyrme⁹, the non-trivial topology $\Pi_3(SU(N_f)) = \mathbb{Z}$ for $N_f > 1$ permits one to construct classical configurations of the U field which are fermions with half-integer isospin and baryon number 1. These Skyrmons are believed to give exact descriptions of the static properties of baryons in the double limit where the number of colours $N_c \rightarrow \infty$ and the quark masses vanish, and give a phenomenologically accurate description of zero-momentum-transfer axial current matrix elements in the real world¹⁸ with $N_c = 3$ and $m_q \neq 0$. They exist if and only if the quark condensate $\langle 0 | \bar{q}q | 0 \rangle \neq 0$, and if $m_q = 0$ their masses vanish as $\langle 0 | \bar{q}q | 0 \rangle \rightarrow 0$. Moreover, in this limit their classical radii become infinite. These observations are very suggestive that quark (de)condensation is equivalent to quark (de)confinement. Therefore we

regard $\langle 0 | \bar{q}q | 0 \rangle$ as a local order parameter associated with quark confinement, and identify the quark condensation temperature with the quark confinement temperature.

The quantum fluctuations in χ are to be interpreted as the lightest scalar glueball, which has a mass m_χ :

$$m_\chi^2 = \left. \frac{\partial^2 V}{\partial \chi^2} \right|_{\chi=\chi_0} = 4 B \chi_0^2 \quad (2.13)$$

The parameters B and χ_0 are not precisely known, but we might expect $\chi_0 = O(f_\pi)$. Taking $\chi_0 = 100$ MeV and varying B_B between $(140 \text{ MeV})^4$ and $(200 \text{ MeV})^4$, we find $m_\chi \approx 800$ to 1600 MeV, which is a plausible range for the lightest scalar glueball mass. If the effective potential for χ is modified by finite-temperature or -density corrections in such a way that the thermodynamically preferred configuration still has $\langle 0 | \chi | 0 \rangle \neq 0$, this glueball particle interpretation of the quantum fluctuations can be maintained, but this is not necessarily the case if corrections drive $\langle 0 | \chi | 0 \rangle$ to zero. Such corrections generally behave as χ^n : $1 \leq n < 4$ at small χ . If $n > 2$, then $m_\chi^2 = \partial^2 V / \partial \chi^2|_{\chi=0} = 0$ and there is no mass gap in the scalar glueball sector, whereas if $n < 2$ $m_\chi^2 = \partial^2 V / \partial \chi^2|_{\chi=0}$ becomes infinite, and no particle interpretation is possible for fluctuations around the origin $\chi = 0$. This is the range of n we will find generically in the models of Section 3. We infer that a $\langle 0 | \chi | 0 \rangle = 0$ ground state cannot in general be interpreted as a conventional hadronic phase, and interpret it as a phase with free gluons. Thus we regard $\langle 0 | F_{\mu\nu} F^{\mu\nu} | 0 \rangle \propto \langle 0 | \chi | 0 \rangle^4$ as local order parameter associated with gluon confinement, and identify the gluon condensation temperature with the gluon confinement temperature.

2.3 Finite-temperature corrections

The one-loop finite-temperature corrections calculated using the effective chiral lagrangian (2.2) are well-known^{6,7}. They imply a finite-temperature correction to the zero-temperature quark condensate of the form

$$\langle 0 | \bar{q}q | 0 \rangle_T = \langle 0 | \bar{q}q | 0 \rangle_0 \left[1 - \frac{N_f^2 - 1}{2N_f} \frac{T^2}{f_\pi^2} + \dots \right] \quad (2.14)$$

which suggests that decondensation and deconfinement would occur in a second-order phase transition at a temperature

$$T_q \sim f_\pi / \sqrt{N_f} \sim \sqrt{\frac{N_c}{N_f}} \quad (2.15)$$

This conclusion is supported by the 2-loop finite-temperature corrections calculated⁷ using (2.2), which give

$$\delta \langle 0 | \bar{q}q | 0 \rangle_T = \langle 0 | \bar{q}q | 0 \rangle \times O\left(\frac{T^4}{f_\pi^4}\right) \quad (2.16)$$

However, it is not clear that finite-temperature corrections due to higher-mass states can be neglected, since the low-energy theory suggests (2.15) that $T_q \propto \sqrt{N_c}$ whereas generic meson masses are $O(N_c^0)$.

More specifically, scale invariance as embodied in the lagrangian (2.6) tells us in Equation (2.12) that $\langle 0 | \bar{q}q | 0 \rangle \neq 0$ is possible only if $\langle 0 | \chi | 0 \rangle = \langle 0 | F_{\mu\nu} F^{\mu\nu} | 0 \rangle^{1/4} \neq 0$, i.e. if scale invariance is spontaneously broken. This implies that $T_q \leq T_g$. On the other hand, we expect $T_g = O(N_c^0)$, since

glueballs have masses $O(N_c^0)$, starting with the lightest scalar glueball described by the quantum fluctuations of χ itself. One-loop finite temperature corrections for the χ sector of the scale-invariant lagrangian (2.6) alone give a contribution to the pressure $P = -V - \delta V_g(T)$, where

$$\delta V_g(T) \approx -\frac{\pi^2}{90} T^4 + \frac{1}{24} \frac{\partial^2 V}{\partial \chi^2} T^2 + \dots \quad (2.17)$$

where the dots represent terms that are negligible when $T \gg \chi$. In this limit, the minimum of $V_g + \delta V_g(T)$ is at

$$\chi = \chi_0 e^{-5/6} \quad (2.18)$$

The energy at this value of χ is to be compared with that at $\chi = 0$, where we argued above that the theory is in the quark-gluon phase, and where we have a pressure $P = -V_g^0$:

$$V_g^0 = \frac{B\chi_0^4}{4} - \frac{\pi^2}{90} (2N_g + \frac{7}{8} \cdot 3 \cdot 4 \cdot N_f) T^4 : N_g = 8, N_f = 3 \quad (2.19)$$

Clearly this phase is energetically favoured when T is large enough, so this naive calculation suggests that there is a first-order phase transition from a confined and condensed gluon phase with χ given approximately by Equation (2.18) to the unconfined phase.

There is no clear justification for neglecting other mesons with masses $O(N_c^0)$, but we believe⁸ that this qualitative conclusion of a first-order gluon transition is likely to be correct. As seen in Fig. 1, any correction $\delta V_g(T) \sim \chi^n$: $n < 4$ for small χ will give a first-order phase transition when combined

with the zero-temperature potential V_g (2.8), simply because the latter is very flat a' la Coleman-Weinberg, although this transition could be very weakly first-order. For example in the (unjustified) approximation of the previous paragraph the first-order jump in $\langle 0 | F_{\mu\nu} F^{\mu\nu} | 0 \rangle$ is $\sim e^{-10/3}$ of the zero-temperature value. In the next section we will present various more complete models for $\delta V_g(T)$ incorporating massive states. Fig. 1 also suggests the existence of a temperature T_x at which the local minimum at $\langle 0 | \chi | 0 \rangle \neq 0$ ceases to exist, having coalesced with the local maximum separating it from $\chi = 0$, forming a point of inflection. As we discuss in more detail in Section 3, this corresponds to a critical point in the hadron phase, which cannot exist even as a metastable state when $T > T_x$.

Assuming the gluon transition is indeed first-order, and recalling that $T_q \leq T_g$, we are left⁸ with the two possible scenarios shown in Fig. 2. Either (a) $T_q < T_g$: the quark transition is second-order and the gluon transition is first-order, or (b) $T_q = T_g = T_c$ and both transitions are first-order. In view of equation (2.15), the former would be more likely if N_c were small or N_f were large. In the case of QCD with $N_c = 3$ and $N_f = 3$ one might expect scenario (b) to be relevant, and this suggestion is supported by lattice QCD studies¹¹ indicating a common transition temperature T_c for several measureable quantities. It would in principle be possible to check the relevance of scenario (a) or (b) in lattice gauge theories with different values of N_c and N_f .

The coincidence at some temperature T_c of the transition in the quark condensate, in the Polyakov loop and in the topological susceptibility was discussed at length in the last paper of Ref. 11. There the hypothesis was advanced that the behavior of the topological susceptibility drove those of the other quantities. Instead, in our view this driving role is played by the

gluon condensate, whose transition to a non-zero value breaks scale invariance dynamically and allows the other quantities to be non-zero. We have already commented (2.12) on its role in the quark condensate. We note also in (2.6) that $\langle 0|\chi|0\rangle \neq 0$ is a necessary condition for the topological susceptibility contribution to the singlet pseudoscalar meson mass to be non-zero.

3. Hadronic Models

As we discussed earlier, there are two critical temperatures which we are interested in: 1) T_c , the temperature at which the pressure in the quark-gluon phase is equal to the pressure in the hadron phase, which determines the deconfinement transition temperature; 2) T_x , the temperature above which the vacuum with $\chi \neq 0$ no longer exists, which represents a critical point in the hadron phase. Clearly any reasonable model will have only a single value for T_c (i.e. the quark-gluon and hadron pressures cross once and only once), and $T_c \leq T_x$. Neither of these two conditions can be taken for granted and we will show some examples where in fact they are not satisfied. Let us assume that the hadron spectrum contains at least the known states. In the most naive treatment where hadrons and quarks are all absolutely free, though there is only one value of T_c , one would predict quarks and gluons at low temperatures and hadrons at high temperatures as the thermodynamically preferred state, simply because there are more hadrons. If we subtract from the quark-gluon pressure a bag constant as in Eq. (2.19), then in most cases one could expect two values of T_c : hadrons being preferred at both very low and very high temperatures with only an intermediate temperature range for quarks and gluons. Previously, the absence of a double crossing or a second value for T_c could be obtained only by including hard-core repulsive interactions¹⁹ or by using an excluded volume approach²⁰. In many cases, even the singularity associated with the Hagedorn spectrum could be avoided. These models, however, are largely empirical and give very little insight into the identity of an order parameter. Here, we will combine the effective Lagrangian approach outlined above with several of the hadronic models.

To begin with let us discuss the quark-gluon equation of state as it will be common to all of our subsequent analyses. The pressure in the quark-gluon phase can be approximated by

$$P_Q = N \frac{\pi^2}{90} T^4 - B_B \quad (3.1)$$

where N counts the number of effectively massless degrees of freedom at a temperature T . Perturbative QCD and lattice calculations give deviations from the ideal quark-hadron gas formula (3.1), but these do not alter the qualitative features of our analysis. We will throughout include the u , d and s quarks (and antiquarks) as well as eight gluons giving $N = 47.5$. The bag constant B_B will take values between $50 - 450 \text{ MeV/fm}^3$ or $B_B^{1/4} = 140-240 \text{ MeV}$. In most cases, it is the value of B_B which largely determines the transition temperature T_c .

In the hadron phase, the total pressure can be divided into several contributions

$$P_H = P_o + P_i - V \quad (3.2)$$

where P_o is the pressure due to the (massless) octet of pseudo-scalar mesons

$$P_o = 8 \frac{\pi^2}{90} T^4 \quad (3.3)$$

and P_i contains the pressure due to the massive states in the theory which will differ in each of the models below. Finally, the effective potential V has been described in Section 2

$$V = \frac{9}{8} G_0 (x/x_0)^4 \left(\ln x/x_0 - \frac{1}{4} \right) + B_B \quad (3.4)$$

where $B_B = \frac{9}{32} G_0$ so that $V(x_0) = 0$. As discussed above, $m_\chi = 1-2$ GeV and $B_B^{1/4} = 140 - 240$ MeV constrain $G_0 \approx 1.4 \times 10^{-3} - 1.2 \times 10^{-2} \text{ GeV}^4$ and $x_0 \approx 40 - 230$ MeV, though G_0 and x_0 may not be chosen independently.

In what follows, we choose different hadron models, leading to various expressions for P_i . Then T_c is determined by $P_H = P_Q$ and T_x is determined by the absence of a maximum in $\partial P / \partial \chi$ for $\chi \neq 0$. Figure 1 indicates how the global minimum in the effective χ potential is in general shifted by finite-temperature effects and ceases to be the global minimum of the potential (i.e. maximum of the pressure) at a critical temperature T_c . At some higher temperature T_x , the local maximum of the potential that separates the $\chi = 0$ state from the local minimum of the potential eventually merges with the latter to give a point of inflection. Beyond T_x there is no maximum in the pressure for $\chi \neq 0$, and hence not even a metastable hadronic phase.

3.1 Known particles

As known particles, we include all hadronic states (both baryons and mesons) in the particle data book²¹ which are ranked by three or four stars. In addition, we include all one and two star states which are required to complete a multiplet and furthermore, when masses for three out of four members of a multiplet are known, we use standard mass formulas to complete the multiplet. This brings our total to 151 hadronic mass states (excluding the pseudo-scalar nonet) and a grand total of 2844 spin states.

To compute P_i , we make a non-relativistic Boltzmann approximation

$$P_i = \sum_j \frac{1}{3} \int \frac{g_j p_j^4 dp_j}{2\pi^2 E_j} e^{-E_j/T} \quad (3.5)$$

with $E_j = m_j + p_j^2/2m_j$ and g_j being the number of degrees of freedom. Upon integration one finds,

$$P_i = \frac{T^{5/2}}{(2\pi)^{3/2}} \sum_j g_j m_j^{3/2} e^{-m_j/T} \quad (3.6)$$

By defining the parameter

$$\hat{m}_j = (m_j/\chi_0)|_{\chi=\chi_0} \quad \text{and} \quad m_j = \hat{m}_j \chi \quad (3.7)$$

as the scaled hadron mass ($\hat{m}_j \chi = m_j$ when χ takes its vacuum value $\chi = \chi_0$), we can write

$$P_i = \frac{T^{5/2}}{(2\pi)^{3/2}} \chi^{3/2} \sum_j g_j \hat{m}_j^{3/2} e^{-\hat{m}_j \chi/T} \quad (3.8)$$

Note that $P_i \sim \chi^{3/2}$ for small χ , inconsistent with a glueball particle interpretation in the neighborhood of $\chi = 0$.

In Figure 3, we show P/T^4 versus T for two choices of parameters. In one case (solid) we take $\chi_0 = 0.04$ GeV, $G_0 = .0014$ GeV⁴ so that $B_B^{1/4} = 140$ MeV and $m_\sigma \approx 2$ GeV. In this case $T_c \approx 100$ MeV and $T_x = 137$ MeV. Despite the low values of both T_c and T_x , this is a good example of the scenario we have been advocating. In the second case (dashed), we take the other extreme with $\chi_0 = 0.23$ GeV, $G_0 = 0.012$ GeV⁴ and $B_B^{1/4} = 240$ MeV with $m_\sigma \approx 1$ GeV. Now $T_c \approx 170$

MeV and $T_x = 173$ MeV. This is also a perfectly good example with higher values of T_c and T_x . As one can see from the dotted curve which is a continuation of the hadronic pressure assuming that $V(\chi) = 0$ and $\chi = \chi_0$, there is a double crossing which was alluded to earlier.

Thus in perhaps the simplest case, using only known particles, our conjecture is realized over a wide set of the allowed parameters.

3.2 A pure Hagedorn spectrum

In these models, we will assume a Hagedorn¹² density of states,

$$n(m) = c m^{-a} e^{m/T_0} \quad (3.9)$$

where c and a are constants and T_0 is the Hagedorn temperature. In this subsection we will neglect hadron interactions, which is formally justified to leading order in $1/N_c$, but subsequent subsections will relax this assumption. The number of massive degrees of freedom given by (3.9) is

$$N = \int_{m_0}^{\infty} n(m) dm \quad (3.10)$$

where $m_0 = O(1)$ GeV is a low mass cutoff. Typically, one expects T_0 to correspond to the point where $P_Q = 0$ or $T_0 \approx 100 - 160$ MeV. This can be seen by noting that when $P = 0$, the entropy $S = E/T_0$ where E is the internal energy of the system. The number of states available is just $n(E) \sim e^S \sim e^{m/T_0}$ in the non-relativistic approximation. Hence we expect the temperature where $P_Q = 0$ to be close to the Hagedorn temperature. For $a = 3$, $c = .01 - .1$ corresponds to $N \approx 800 - 8000$ for the number of spin states between 1 and 2.5 GeV when

$T_0 = 160$ MeV. (95% of the 2844 states included in the previous section have $m < 2.5$ GeV.) When $a = 1.5$, $c = .01 - .1$ corresponds to $N \approx 2700 - 27000$ spin states in the same mass range.

In these models, the pressure P_i is also straightforward to calculate. Making the same non-relativistic Boltzmann approximation, we now write

$$P_i = \frac{1}{3} \int_0^\infty dp \int_{m_0}^\infty dm \frac{n(m)p^4 e^{-E/T}}{2\pi^2 E} \quad (3.11)$$

where again $E = m + p^2/2m$. Integrating over p one finds,

$$P_i = \hat{c} T^{5/2} \int_{m_0}^\infty m^{(3/2-a)} e^{m(T-T_0)/TT_0} dm \quad (3.12)$$

where $\hat{c} = c/(2\pi)^{3/2}$. The integral over m can also be done analytically. The result is

$$P_i = \hat{c} T^{5/2} \left(\frac{T T_0}{T_0 - T} \right)^{5/2-a} \Gamma \left(\frac{5}{2}-a, \frac{m_0(T_0-T)}{T T_0} \right) \quad (3.13)$$

For the special case where $a = 1.5$, this reduces to a simpler form

$$P_i = \hat{c} T^{5/2} \left(\frac{T T_0}{T_0 - T} \right) e^{-m_0(T_0-T)/TT_0} \quad (3.14)$$

In this case, the dependence on χ comes from the m_0 dependence in (3.13). By replacing m_0 with $m_0 \chi/\chi_0$, we get the appropriate scaling for the density of states (when $\chi \rightarrow 0$, all the hadrons become massless and the exponential density of states should be integrated from $m = 0$ upwards).

For a pure Hagedorn spectrum, the pressure P_i diverges for $a < 5/2$ at $T = T_0$. This is particularly bad if T_0 is chosen such that $P_Q(T_0) = 0$. There is never a transition, i.e. there is no value of T such that $P_Q = P_H$. However, to test the applicability of the Hagedorn spectrum to the problem at hand we simply choose T_0 to be larger, say $T_0 = 0.25$ GeV to mimic, not fit, the hadronic spectrum. First let us consider $a = 1.5$. In Figure 4, we again test our extreme values for $B_B^{1/4} = 140, 240$ MeV. For $T_0 = 250$ MeV, $c = 0.3 - 3$ corresponds to $N = 500 - 5000$ in the mass range $1 - 2.5$ GeV, and are shown by the two hadron curves as labelled. Unless $c \leq 0.5$, T_x always occurs (if at all) when $P_H > P_Q$, namely we have a double crossing. For $c > 0.5$, it becomes possible for $P_H(T_x) < P_Q(T_x)$ as we desire. For the cases shown, where $c = 3$, $T_x = 180$ MeV when $B_B^{1/4} = 140$ MeV but again occurs too late when $B_B^{1/4} = 240$ MeV. For $a = 3$, $c = 1 - 10$ corresponds roughly to $N \approx 500 - 5000$, for $m = 1 - 2.5$ GeV. Figure 5 shows the results in this case. For $c = 10$, $P_H(T_x) < P_Q(T_x)$ for all values of $B_B^{1/4} = 140 - 240$ MeV. However when $c = 1$, and $B_B^{1/4} = 240$ MeV, T_x does not occur before T_0 . Note that $P(T_0)$ is finite when $a = 3$.

3.3 A Hagedorn spectrum with hard-core repulsive potentials

The inclusion of hard-core repulsive potentials was motivated by the thermodynamic implications shown in Figure 3 when comparing the dotted (free) hadron curve with a quark curve. The problem of thermodynamically preferred hadrons at high temperatures is due to the large number of unsuppressed hadronic states. However at high temperatures, the density of hadrons (or pions for that matter) is significantly larger than nuclear densities and the number of hadrons per fm^3 is $\gg 1$. One possible resolution to this problem is to consider interactions between particles as mean fields. To simplify matters greatly, a common baryon potential, (based on the Reid potential) and a

common meson potential (based on the effective chiral π - π lagrangian²³) are used to obtain a potential energy per particle¹⁹.

The Reid potential,

$$U(r) = (-10.46e^{-m_\pi r} - 1650.6e^{-4m_\pi r} + 6484.3e^{-7m_\pi r})/m_\pi r \quad (3.15)$$

containing π , σ , and ω exchanges, leads to an energy per particle when integrated over all space

$$U_B = 18.7 \left(\frac{4\pi}{m_\pi}\right)^3 n_B = 680 \text{ MeV } (n_B/\text{fm}^{-3}) \quad (3.16)$$

where n_B is the baryon density. Likewise the π - π interaction leads to

$$U_M = \frac{3}{4f_\pi^2} n_\pi = 663 \text{ MeV } (n_\pi/\text{fm}^{-3}) \quad (3.17)$$

The similarity in these coefficients leads us to further simplify matters and consider a universal hadron potential

$$U_H = 660 \text{ MeV } (n_H/\text{fm}^{-3}) \quad (3.18)$$

When combined with a finite number of hadron species, (3.18) alone is enough to flatten the hadronic pressure and lead to a well-defined phase transition¹⁹. When combined with a asymptotic mass spectrum, the simple form (3.18) is not sufficient, i.e. the pressure continues to diverge at $T = T_0$ ²⁴. However the potential energy per particle is proportional to the hadronic volume which in the bag model is proportional to mass. Furthermore, the bag

model predicts that the interaction strength should also be proportional to mass leading to²⁴

$$U_H(n, m) = Kn (m/m_N)^2 \quad (3.19)$$

for $m > m_N$ and $K = 660 \text{ MeV fm}^3$, where m_N is the nucleon mass. The mass dependence of U_H was studied in greater detail in Ref. 24.

The pressure for the hadronic system with a hard-core potential was studied in Ref. 19. In the non-relativistic limit, we can write

$$P = nT + Kn^2/2 \quad (3.20)$$

where the second term is directly due to the interaction potentials. (Writing the energy density as $\rho \propto \int E_i e^{-E_i/T} d^3 p_i + f(x)$ determines the form of $f(x)$ by equating $E_i = \partial \rho / \partial n_{p_i}$, $n_{p_i} = e^{-E_i/T}$). From Ref. 19, we can write the corresponding expression for P_i with an asymptotic spectrum and hard-core potentials,

$$n = \int_0^\infty dp \int_{m_0}^\infty dm n(m) \frac{p^2}{2\pi^2} e^{-E/T} \quad (3.21)$$

remembering that $E = m + p^2/2m + U(x)$. Note that (3.21) must be solved self-consistently in n . Integrating over p gives

$$P_i = \hat{c} T^{5/2} \int_{m_0}^\infty e^{-\left[\frac{T_0 - T}{T_0} + \frac{m^2 Kn}{2 T}\right]} dm + \frac{1}{2} Kn^2 \quad (3.22)$$

For the case where $a = 1.5$, this can be done analytically

$$P_i = \hat{c} T^{5/2} \sqrt{\pi\beta} e^{\gamma^2\beta} \operatorname{erfc}\left(\gamma\sqrt{\beta} + \frac{m_o}{2\sqrt{\beta}}\right) + \frac{1}{2}Kn^2 \quad (3.23)$$

where

$$\beta = \frac{m_N^2 T}{4Kn} \quad (3.24)$$

$$\gamma = \frac{T_o - T}{T T_o} \quad (3.25)$$

and $\operatorname{erfc}(x) = 1 - \operatorname{erf}(x)$.

In Figure 6, we show the $P - T$ diagram for the asymptotic spectrum with a hard-core potential. Clearly the repulsive interactions do not allow for a second crossing. Here we used $T_o = 160$ MeV and $a = 1.5$, $c = 0.1$ and $.01$. In all cases shown we find T_x in an acceptable place. Note that for larger values of c , i.e. $c \geq .6$, $T_x < T_c$ i.e. there are no crossings of P_Q and P_H .

3.4 Excluded volumes

The excluded volume approach²⁰ is based on a Van der Waals treatment of deformable hadron bags. The pressure in this approximation is found by replacing the volume V in the partition function with $V - \sum_{i=1}^N E_i/4B_B$, for hadron i with energy E_i . We refer the reader to Refs. 20, 24 for a complete derivation of P_i . Here we only quote the result

$$P_i = P_i^{(Pt)}(T^*) / (1 - (P_i^{(Pt)}(T^*)/4B_B)) \quad (3.26)$$

where $P_i^{(Pt)}$ is the pressure given in Eq. (3.11) for point particles and

$$T = T^* / (1 - (P_i^{(Pt)}(T^*) / 4B_B)) \quad (3.27)$$

where T^* is simply a parameter used to determine P_i and T .

In Figure 7, we show the $P - T$ diagram for the excluded volume approach for $a = 1.5$, $T_0 = 0.16$ GeV, and $c = .1$ only. As one can see, T_x occurs but at significantly higher temperatures.

4. Discussion and Conclusions

We have given in this paper an analysis of the approach from low temperatures towards the quark/hadron phase transitions, using an effective low-energy lagrangian¹⁰ embodying approximate chiral and scale invariance. We have argued⁸ on the basis of the Skyrme model⁹ that quark condensation and confinement occur at the same temperature T_q , and on the basis of the effective glueball potential that gluon condensation and confinement are also coincident at some temperature T_g . Previous authors^{6,7} have argued that the quark transition would be second order if other dynamics would be neglected and we have argued on the basis of the glueball effective potential that the gluon transition is first order. We have argued on the basis of spontaneous scale symmetry breaking that $T_q \leq T_g$, with $T_q = T_g = T_c$ for large N_c . In this latter case, the quark transition would be driven by the gluon transition and both would be first order. Lattice simulations¹¹ suggest that this situation is obtained when $N_c = 3$ and there are 3 light flavours.

We have also argued that the gluon transition cannot be treated using the light effective degrees of freedom alone, but that states with masses $O(\Lambda_{QCD} \times N_c^0)$ should be taken into account. We have modelled their effects in 4 different ways: using the known hadronic states with masses ≤ 2.5 GeV, using a perfect Hagedorn¹² gas, using a Hagedorn gas with a hard-core potential¹⁹, and applying an excluded-volume approach to a Hagedorn gas^{20,24}. In each case, for generic parameter choices we find a well-behaved first-order transition between the hadronic and quark-gluon phases at a critical temperature of order 200 MeV, with the hadronic phase ceasing to exist even in a metastable state at a somewhat higher temperature.

Our physical motivation for this study was three-fold: the quark/hadron phase transition in the early Universe, the possible quark/hadron phase transition in heavy-ion collisions and the possible presence of a quark-gluon core inside the SN1987a neutron star. Application of our approach to the latter two problems requires some extension of the methods used here, to include bulk fluid motion in the heavy-ion case, and to non-zero chemical potential for baryon number in the neutron star case. This latter application will be discussed in Ref. 25. We conclude this paper with some discussion of the application of our results to the quark/hadron phase transition in the early Universe.

The question has been raised³ whether baryon inhomogeneities could originate during a first-order quark/hadron phase transition that would alter the successful conventional calculations of cosmological nucleosynthesis¹³. An essential ingredient in these calculations is the surface tension for bubbles of the new hadronic phase. Our methods are not powerful enough to calculate this surface tension precisely, but they do give a useful order-of-magnitude estimate. We remind the reader that in the thin-wall approximation - which is hard to justify in this application - the surface tension for a bubble where a scalar field ϕ has the value ϕ_1 , in a medium where $\phi = \phi_2$, has the value²⁶

$$\sigma = \int_{\phi_1}^{\phi_2} \sqrt{2V(\phi)} \, d\phi \quad (4.1)$$

In our case, we need to evaluate such an integral for the gluon condensate order parameter χ between 0 and the scale-breaking local minimum χ_c at the critical temperature T_c :

$$\sigma = \int_0^{\chi_c} \sqrt{2V_{\text{eff}}(T_c, \chi)} \, d\chi \quad (4.2)$$

where we use the full effective potential including finite-temperature corrections. Unfortunately, we cannot model the effective potential accurately for small χ when the quark-gluon phase is approached. However, we believe that we can model the effective potential adequately for χ between the local minimum at χ_c and the local maximum at $\chi_{\max}$ that separates the confined and unconfined states. We then hypothesize that

$$\sigma \approx 2 \int_{\chi_{\max}}^{\chi_c} \sqrt{2V_{\text{eff}}(T_c, \chi)} d\chi \quad (4.3)$$

and calculate this integral in several of the models introduced in the previous section. On the basis of these calculations, we estimate that

$$\sigma^{1/3} \approx 70 \text{ MeV} \quad (4.4)$$

but clearly with large uncertainties. Note that this is in quite good agreement with the estimates made in Ref. 27.

Is this value of σ large enough to generate a significant effect on cosmological nucleosynthesis calculations? In Ref. 3 it was argued that the length scale associated with the inhomogeneities is given by

$$\ell \approx (3.4 \times 10^4 \text{ m}) (\sigma/\text{MeV}^3)^{3/2} (T_c/\text{MeV})^{-13/2} \quad (4.5)$$

(Eq. 4.5 takes into account the correction given in Ref. 28.) Because of the strong dependence of T_c , ℓ will be largest for small T_c . When $B_B^{1/4} \approx 140 \text{ MeV}$ and $T_c \approx 100 \text{ MeV}$, the case in Figure 3 of real particles $\sigma^{1/3} \approx 70 \text{ MeV}$ gives $\ell \approx 1 \text{ m}$, while the case in Figure 6, using an asymptotic spectrum with hard-core repulsion gives $\sigma^{1/3} \approx 85 \text{ MeV}$ and $\ell \approx 2 \text{ m}$. Significant effects on

nucleosynthesis require $\ell > 10\text{m}$ and probably $\ell > 100\text{m}$. The strong dependence on T_c makes models with a larger bag constant even less interesting in this context. For $B_B^{1/4} = 240\text{ MeV}$ for real particles we find $\sigma^{1/3} \approx 67\text{ MeV}$ and $T_c = 165\text{ MeV}$ so that $\ell \approx 2\text{cm}$, making it hardly of interest for nucleosynthesis.

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Figure Captions

- Figure 1: Sketch of the likely behavior of the effective potential for the gluon condensate parameter χ as the temperature T is increased (the behavior for increasing chemical potential may be similar). At T_c there are two competing vacua, one with $\chi = 0$ and one with $\chi \neq 0$. At T_x , there ceases even to be a metastable state with $\chi \neq 0$.
- Figure 2: Sketches of the possible behaviours of the gluon order parameter $\langle 0|\chi|0\rangle$ and of the quark-antiquark condensate: In case (a), $T_q < T_g$ and the quark transition is second order. In case (b), $T_q = T_g$ and the quark transition is (weakly) first order. The dotted line indicates how $\langle 0|\bar{q}q|0\rangle$ would have behaved in the absence of other degrees of freedom.
- Figure 3: The pressure P/T^4 vs. T for hadrons (H) using the known particles with $m \leq 2.5$ GeV and for quark and gluons (Q) for $B_B = 0.14$ GeV (solid), $B_B = 0.24$ GeV (dashed). T_x is represented by an X on the hadron curve at the end of the solid portion when $B_B = 0.14$ GeV and at the end of the dashed portion when $B_B = 0.24$ GeV. The dotted portion assumes $V_g = 0$.
- Figure 4: As in Figure 3, but for the hadron spectrum taken as a pure Hagedorn spectrum with $a=1.5$ and $T_0=0.25$ GeV. The two hadron curves correspond to $c=3$ and $c=.3$ as marked.
- Figure 5: As in Figure 4 but for $a=3$, and $c=1$ and $c=10$ as marked. Again the dotted portion of the hadron curve assumes $V_g = 0$.

Figure 6: As in Figure 5, but for a hadron spectrum using the Hagedorn spectrum and hard-core potentials. In this case $a=1.5$, $T_0=0.16$ GeV and $c=0.1$ and 0.01 as marked.

Figure 7: As in Figure 6 but for a hadron spectrum using the Hagedorn spectrum and an excluded volume approach. In this case $a=1.5$, $T_0=0.16$ GeV and $c=0.1$. Notice the break in the temperature scale.

Figure 8: The potential V_g for the case of real particles, when $B_B^{1/4}=0.14$ GeV.

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- Fig. 1 -

a)

b)

- Fig. 2 -

Figure 3.

Figure 5.

Figure 7