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Even Larger Contributions to the Neutron Electric Dipole Moment

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ABSTRACT

Constraints on theories of CP-violation, from limits on the neutron electric dipole moment, and mediated by a CP-odd three-gluon operator, are current best sellers. We introduce novel CP-odd operators involving one photon and three gluons. We find that effects mediated by these operators result on bounds on supersymmetry an order of magnitude more stringent than earlier results: they are the tightest known bounds. For left-right models we derive richer limits than previously found. We also recalculate the anomalous dimensions of the three-gluon operator and find them to be *minus* those originally used; this weakens considerably its strictures on theory, though it still mediates the dominant effect in multi-Higgs models.

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Though perhaps required by the observer's existence [1] CP-violation has only been observed in the neutral kaon system. For decades on end, these measurements have been refined, and the limits on the electric dipole moment (EDM) of the neutron have been painstakingly improved. These parallel lines of research will not soon intersect, should the standard model stubbornly remain unscathed. In its realm, only the beauty analog to the $K_0 \bar{K}_0$ system offers hope for progress, in the form of non-null—but non-novel—results. Rather mysteriously, nature has chosen the number and properties of fundamental fields *just so* that all of the above may be snugly so. Thickening the enigma, a Peccei-Quinn [2] symmetry, and a suspiciously evasive axion [3] must be invoked to state all this with a poker face, lest disaster strike in the form of a gigantic θ -term contribution [4] to the neutron's EDM. It is hard to believe that there is no CP-odd skeleton in the closet.

Weinberg [5] has recently unearthed such a potential skeleton, in the form of an effective CP-odd three-gluon operator that mediates—in non-standard models—a relatively large neutron EDM. We, and others [6], have found that the operator's anomalous dimensions [7], as originally quoted [5], were of the wrong sign. This considerably reduces this operator's numerological impact. Moreover, Bander [8] claims that the very same Peccei-Quinn mechanism that disposes of the θ -term would, in the large N_c limit, completely erase the three-gluon operator from the low-energy effective QCD Lagrangian, and partially reduce its impact¹ for $N_c = 3$. We introduce and study novel one-photon-three-gluon operators that are not subject to Bander's threat and that, in supersymmetric and “left-right” models, mediate the dominant sources of a non-vanishing neutron EDM.

The electric dipole moment d_f of an on-shell spinor f is the value at vanishing momentum transfer of the coefficient of an effective dimension-five interaction of the form:

$$\mathcal{L}_f^j = \frac{1}{2} d_f^j \bar{\psi} i \sigma_{\mu\nu} \gamma_5 \psi F^{\mu\nu}. \quad (1)$$

The electric dipole vertex is the lowest-dimension CP-violating operator describing the coupling of a photon to a spinor. The generalization of an electric dipole moment as in (1) to its chromodynamic counterpart d_f^g (CEDM), is obvious:

$$\mathcal{L}_f^g = \frac{1}{2} d_f^g \bar{\psi} i \sigma^{\mu\nu} \gamma_5 \frac{\lambda_a}{2} \psi G_{\mu\nu}^a, \quad (2)$$

with $G_{\alpha\beta}^a$ the non-Abelian field tensor of QCD.

¹ It is not clear to us that the argument is decisive, if only because the neutron matrix elements of the three-gluon operator are enhanced by N_c : the number of valence quarks.

Habit has established the use of the bastardized units “e-cm” in quantifying electric dipole moments, as opposed to some dimensionless characterization. The “e” is for kicks, but there is no universal way to trade off the “cm” for something less gross, since different models of CP-violation suggest different powers of masses of external and virtual particles in the expression for an EDM or its generalizations. But all sensible theories coincide in introducing these dipole moments at the weak scale or beyond. As a particle-demagogical compromise to get rid of dimensions, and to characterize EDM's in a more natural scale, we introduce a notion of “weak magneton” and define dimensionless quantities, δ_f , as follows:

$$d_f^j \equiv \frac{e}{M_\pi} \delta_f^j, \quad d_f^g \equiv \frac{g_s}{M_\pi} \delta_f^g \quad (3)$$

Notice that we have extracted the coupling strength of the vector bosons in the definition of the different δ_f 's. This departs from “magneton” tradition, but allows for a fairer comparison between the various CP-violating dipole moments.

The most stringent results on the neutron's EDM are $d_n^j = (-1.4 \pm 0.6) \cdot 10^{-25}$ e cm [9] and $d_n^g = (-0.3 \pm 0.5) \cdot 10^{-25}$ e cm [10]. The tightest of these bounds, to the 95% ($\sim 2\sigma$) confidence level to which we shall subscribe throughout, entails the constraint $|\delta_n^j| < 6 \cdot 10^{-10}$. Use of the naive quark model or SU(4) relation $\delta_n = (4/3)\delta_d - (1/3)\delta_u$ allows one to conclude that, barring cancellations²,

$$|\delta_n^j| < 4.4 \cdot 10^{-10}, \quad |\delta_n^g| < 1.8 \cdot 10^{-9}. \quad (4)$$

“Direct” limits on the chromo-electric dipole moments of u , d and s can be extricated from bounds on the EDM's of the baryons that incorporate these quarks as valence constituents, using a technique [11] that has gained wild acceptance. The aim is to estimate the coefficient of an “external” operator constructed exclusively with “external” fields, such as the neutron's EDM d_n^j in the effective interaction term $d_n^j \bar{n} \gamma_5 \sigma^{\mu\nu} n F_{\mu\nu}$. The operator can be generated by the cooperative effect of various “internal” operators involving “constituent” fields (gluons and “valence” quarks). In the example at hand these internal operators will be the quark's minimal coupling $Q_q e \bar{q} \gamma_\mu q A^\mu$ and the quark's CEDM of Eq. (2). Let an operator of dimension D , $\mathcal{O}_N^D$, be built of a total of $\mathcal{N}$ fields, and let g be its coupling strength in the relevant effective Lagrangian. For the neutron's EDM and the

² Should we capitalize on the allegedly large strange sea within a nucleon, the strange quark would also carry an extra “direct” contribution to the neutron's EDM.

quark's CEDM $\mathcal{N} = 3$, $D = 5$ and $g = d_\pi^q$ or d_π^g , respectively; while for the minimal quark-photon interaction $\mathcal{N} = 3$, $D = 4$ and $g \sim e$. Let $M = 2\pi f_\pi \simeq 1190$ MeV be the scale of chiral symmetry breaking. Define the reduced coupling of $\mathcal{O}_N^D$ as $\bar{g} = (4\pi)^{2-\mathcal{N}} M^{D-4} g$. The rule [11] is that the reduced coupling of the external operator can be estimated as the product of the reduced couplings of the internal operators that suffice to generate it. In the case of a CEDM of a *constituent* quark and its minimal photonic coupling conspiring to generate a baryon's EDM, the rule leads to $d_B^q \sim (e/4\pi)d_q^q$, or, in our weak-magneton parlance, $\delta_B^q = (g_s(M)/4\pi)\delta_q^g$. The size of $g_s(M)$ may be estimated [5] by demanding that the first and second order terms in the perturbative expression for its scale-dependence be of equal magnitude: $g_s(M) \sim 4\pi/\sqrt{6}$. Given the limits on the neutron's EDM, this entails:

$$|\delta_q^g|, |\delta_q^q| < 1.5 \cdot 10^{-9}, \quad (5)$$

Consider a specific theory of CP-violation that generates quark EDM's or CEDM's at a scale Λ close to or above M_π . Focus now on a quark q that is *not* a valence constituent of the neutron. An EDM or a CEDM of one of these medium-weight or heavy quarks can induce a non-zero neutron electric dipole moment via quantum effects, whose contributions are not chirally suppressed. Examples of loop diagrams that may transmit the CP-odd infection are given in fig. 1 and fig. 2, in which the original CP-violating vertices are depicted as heavy dots. Since the (C)EDM is the lowest-dimension CP-odd vertex operator³, it is adequate, in the limit $\Lambda \gg m_q$, to “integrate out” the heavier degrees of freedom and to embody all CP-violating effects in these single operator(s). (For a t -quark the approximation is dubious and complete satisfaction may require a two-loop calculation). If the quark in the loop is much heavier than the inverse of the neutron's radius (dubious, this time, for a s -quark), it can in turn be integrated out to generate new “local” effective interactions involving only the photon and gluon fields that propagate with a longer range. Finally, the neutron matrix elements of these lower-scale local operators are to be evaluated. The

³ The “axial convection” operator $\bar{d}_f^i \bar{\psi}_f (\vec{D}_\mu - \vec{D}_\mu) \gamma_5 \psi_f A^\mu$ coincides with the dipole moment (1) on the fermion's mass-shell, but is an independent entity off-shell. Since we shall use these objects and their QCD analogs off-shell, we ought to keep track of how specific models generate one or the other of the two off-shell dipole types: they may be expected to contribute to the final answers in similar amounts. Indeed, they do: the two types of operator *equally* contribute to each of the individual quantities of interest: K_w , K_G and K_F , defined below. It thus suffices to compute the (C)EDM expectations on-shell.

end result is an estimate of the neutron EDM in terms of the original quark (C)EDM's or the physics that underlies them. The details of the plot are thickened by QCD scale-dependent effects. The quark CP-odd dipole moments and the quark mass itself are, by convention, defined at a scale m_q . The effective operator resulting from the integration of the quark loop must be QCD-evolved from m_q down to the quark-confining scale, prior to its insertion into the neutron. Moreover, the (C)EDM's themselves must be evolved up from m_q to the scale Λ , to make contact with the theory that generates them.

The result of integrating out the quark in the loop of fig. 1 is an effective Lagrangian [5] involving a CP-odd three-gluon operator:

$$\mathcal{L}_w = K_w(\mu) g_s^3(\mu) \mathcal{O}_w(\mu) \quad (6)$$

$$\mathcal{O}_w = -\frac{1}{3!} f_{abc} \tilde{G}_\beta^{ac} G_\gamma^{b\beta} G_\alpha^{c\gamma}, \quad (7)$$

with μ the renormalization scale, $\tilde{G}_{\mu\nu} = (1/2)\epsilon_{\mu\nu\rho\sigma} G^{\rho\sigma}$ the dual to the gluon field G , and $\epsilon_{023} = 1$.

The explicit value of the coefficient in (6) is:

$$K_w(m_q) = \frac{1}{16\pi^2 m_q} \frac{\delta_q^g(m_q)}{M_\pi}. \quad (8)$$

We have computed in background gauge the anomalous dimension of the operator (7) and found that the scale-dependence of (6) is governed by

$$\frac{K_w(\mu')}{K_w(\mu)} = \left[\frac{g_s(\mu')}{g_s(\mu)} \right]^{-\frac{10}{33-2N_f}}. \quad (9)$$

Our exponent in (9) is equal in magnitude but of opposite sign to that [7] quoted by S. Weinberg [5] in his influential paper, and agrees with a more recent calculation [6]. The correct sign considerably weakens the potential constraints on heavy quark CEDM's and on the theories that may generate them.

These results for the diagram of fig. 1, and the rules [11] to estimate matrix elements that we have reviewed, yield an expected EDM for the neutron, of magnitude:

$$\begin{aligned} \delta_n^q &= \frac{1}{32\pi^3} \frac{M}{m_q} g_s^3(M) \left[\frac{g_s(M)}{g_s(m_q)} \right]^{-\frac{10}{33-2N_f}} \delta_q^g(m_q) \\ &\sim \frac{1}{3\sqrt{6}} \frac{M}{m_q} \left[\frac{33-2N_f}{18} \ln \left(\frac{m_q^2}{\Lambda^2} \right) \right]^{-\frac{5}{33-2N_f}} \delta_q^g(m_q), \end{aligned} \quad (10)$$

where we have used the conventional expression for the QCD running coupling constant for N_f active quark flavours, with $\lambda \sim 150$ MeV. The limits obtained from the inversion of (10), for s, c, b, t quarks of mass 0.45, 1.3, 4.9, 100 GeV, are:

$$|\delta_1^g| < 1.7 \cdot 10^{-8}, |\delta_2^g| < 2.0 \cdot 10^{-7}, |\delta_3^g| < 2.5 \cdot 10^{-6}, |\delta_4^g| < 2.7 \cdot 10^{-4}. \quad (11)$$

We re-emphasize that the limits for $q = s, c, b, t$ are not as defensible as the others, thus the “caveat” dot over the inequality sign, a notation that we shall again use when needed.

Limits on quark EDM’s and CEDM’s can also be extracted from the diagrams of fig. 2, whose analysis parallels that of fig. 1. The result of integrating out the quark loops is an effective Lagrangian involving two CP-odd, one-photon-three-gluon operators:

$$\mathcal{L}_4 = e g_s^3(\mu) [K_F(\mu) \mathcal{O}_F(\mu) + K_G(\mu) \mathcal{O}_G(\mu)] \quad (12)$$

$$\mathcal{O}_F = \frac{1}{3!} d_{abc} \tilde{F}_{\mu\nu} G^{\alpha\mu\nu} G_{\rho\sigma}^b G^{\rho\sigma a} \quad (13)$$

$$\mathcal{O}_G = \frac{1}{3!} d_{abc} \tilde{G}_{\mu\nu}^a G^{b\mu\nu} G_{\rho\sigma}^c F^{\rho\sigma} \quad (14)$$

The explicit values of the coefficients in (12) in terms of the EDM of fig. 2a and the CEDM of fig. 2b are:

$$K_G = \frac{1}{64 \pi^2 m_q^3 M_z} (Q_q \delta_q^g + \delta_q^7) \quad (15)$$

$$K_F = \frac{1}{64 \pi^2 m_q^3 M_z} (2 Q_q \delta_q^g) \quad (16)$$

The eigenstates of the renormalization group are not $\mathcal{O}_G$ and $\mathcal{O}_F$, but the combinations

$$\mathcal{O}^+ = \mathcal{O}_F + \frac{\sqrt{6}-2}{2} \mathcal{O}_G \quad (17)$$

$$\mathcal{O}^- = \mathcal{O}_G - (\sqrt{6}-2) \mathcal{O}_F, \quad (18)$$

so that it behooves us to re-express (12) in terms of $\mathcal{O}^\pm$, the scale-evolution of whose coefficients we have computed in background gauge to be:

$$R^\pm \equiv \frac{K^\pm(\mu')}{K^\pm(\mu)} = \left[\frac{g_s(\mu')}{g_s(\mu)} \right]^{-\frac{11 \pm 3\sqrt{6}}{32-2N_f}} \quad (19)$$

Even for the longest-distance run we undertake (that from m_t to the confining scale) the ratio R^+/R^- is only ~ 2 , certainly within the uncertainty of the inescapable final estimate

of neutron matrix elements. It would therefore be pedantic to maintain the difference between R^+ and R^- in our expressions. We simply evolve K_G and K_F with the average of the anomalous dimensions in (19). This, and the rules [11] to estimate matrix elements, yield an expected EDM for the neutron, of magnitude:

$$\delta_n^7 \sim \frac{1}{12\sqrt{6}} \left(\frac{M}{m_q} \right)^3 \left[\frac{33-2N_f}{18} \ln \left(\frac{m_q^2}{\lambda^2} \right) \right]^{-\frac{7}{2(33-2N_f)}} [3 Q_q \delta_q^g(m_q) + \delta_q^7(m_q)]. \quad (20)$$

Assuming no accidental cancellation between EDM’s and CEDM’s, the limits obtained from the inversion of (20) are, for s, c, b, t quarks of mass 0.45, 1.3, 4.9, 100 GeV:

$$|\delta_n^g| < 4.8 \cdot 10^{-9}; |\delta_n^c| < 1.5 \cdot 10^{-7}; |\delta_n^b| < 3.6 \cdot 10^{-5}; |\delta_n^t| < 5.0 \cdot 10^{-1}, \quad (21)$$

$$|\delta_n^7| < 4.8 \cdot 10^{-9}; |\delta_n^c| < 3.0 \cdot 10^{-7}; |\delta_n^b| < 3.6 \cdot 10^{-5}; |\delta_n^t| < 1.0.$$

These are tighter than (11) on the light-quark side, and much looser on the heavy side, as could be expected from the diverse dimensionalities of the effective operators in (6) and (12), and the different resulting m_q -dependences of (8) versus (15) or (16).

Integration of the heavy degrees of freedom of our yet-unspecified theory of CP violation yields predictions for $\delta_q^7(\Lambda)$ and $\delta_q^g(\Lambda)$. To transcribe the theoretical predictions from a scale Λ to the quark-mass scale we need to know the QCD evolution of the various dipole moments. The relevant anomalous dimensions are known [12] and yield the result:

$$\frac{\delta_q^7(\mu')}{\delta_q^7(\mu)} = \left[\frac{g_s(\mu')}{g_s(\mu)} \right]^{-\frac{5}{33-2N_f}}, \quad \frac{\delta_q^g(\mu')}{\delta_q^g(\mu)} = \left[\frac{g_s(\mu')}{g_s(\mu)} \right]^{-\frac{4}{33-2N_f}}. \quad (22)$$

The exponents in the above expressions are very small in comparison with those describing the evolution of multi-gluon operators, as in (9) and (19). Even for the large gap between $\mu' = \Lambda \sim M_z$ and the confining scale $g(M) = 4\pi/\sqrt{6}$, the numerical values of (22) are $\sim 1.6, 1.3$, indistinguishable from unity above the “noise” in the estimates of hadronic matrix elements. Operator mixing is of even lesser numerical relevance⁴. Thus, we neglect all

⁴ Matching of the coefficients of the various effective Lagrangians (obtained as successive fields are integrated out) mimicks a mixing between operators of different field dimensionality. The numerical incidence of this effect on the evolution of dipole-moment-like operators in the $\Delta S=1$ weak Hamiltonian [12] is minor. Similar “mixing” effects between CEDM’s and the operator (7) have been recently found to be numerically small [6] in Higgs-mediated [13] and supersymmetric [14] models. We expect no surprises in the forthcoming extension of these results to include the operators (13) and (14).

QCD-evolution effects but the large ones pertinent to the evolution of multi-gluon vertices, and proceed to study limits on various models of the origin of CP violation.

In theories in which CP violation percolates down from a rich Higgs sector, quantum effects involving heavy quarks are likely to play a crucial role, since the scalar-fermion couplings scale with the fermion's mass. We reanalyse Weinberg's constraints [5] on Weinberg's theory of CP-violation [16] in terms of chromo-electric dipole moments, to gauge the extent to which we can reproduce them. Let ϕ_2^0 be the neutral member⁵ of the only [16] scalar doublet assumed to couple to $Q_4 = 2/3$ quarks. Following [5], let the vacuum expectation value of ϕ_2^0 , λ_2 , be the smallest of all, so that the forthcoming bounds are pessimised, and $\sqrt{2}G_F \sim |\lambda_2|^{-2}$. The relevant Yukawa couplings can be cast in the form:

$$\mathcal{L}_q = \frac{\tilde{\phi}_2}{|\lambda_2|} m_q [\cos\theta_s \bar{\psi}_q \psi_q + i \sin\theta_s \bar{\psi}_q \gamma_5 \psi_q], \quad (23)$$

with $\tilde{\phi}_2$ a real field and θ_s an unremovable CP-violation parameter. A quark CEDM arises from the diagram of fig. 3a:

$$\delta_q^g = \frac{1}{16\pi^2} \sqrt{2} G_F M_z \frac{m_q^3}{M_z^2} \sin(2\theta_s) \left[\ln \frac{M_z^2}{m_q^2} - \frac{1}{2} \right] \quad (24)$$

When successively inserted into the loop of fig. 1 and into the neutron, this result reproduces Weinberg's [5], for $M_z \gg m_q$. Should this condition not be met, a complete two-loop calculation [17] must be faced. In the particular case of a Higgs model, our novel three-gluon, one-photon operators in (12) do not lead to constraints on the theory competitive to those extracted from the top-quark contribution to the three-gluon operator (7): their higher dimensionality entails extra negative powers of m_q that compensate for the heavy-quark's Yukawa coupling (compare (8) with (16) and (15)). Thus, the tightest constraint [5] on the theory [15] originates from the comparison of (24) with (11) for $q = t$. For $m_t = 100$ GeV and the square bracket in (24) set to unity, we obtain the bound:

$$\left[\frac{M_z}{M_2} \right]^2 \sin(2\theta_s) < 0.24. \quad (25)$$

The frailty of this bound reflects the sign of the anomalous dimensions in (9).

⁵ The effects of charged scalars in the same theory have also been recently analysed in terms of CEDM's [13].

Next, we turn our attention to CP-violation in softly broken supersymmetry [18], wherein the neutron's EDM is expected [19] to be dominated by virtual-gluino effects⁶. The leading diagrams generating a quark CEDM or EDM are those of fig. 4. Following a beaten path, we may gather all non-trivial phases into a complex mass for the gluino, $m_{\tilde{g}} \cos\Phi + i\gamma_5 m_{\tilde{g}} \sin\Phi$, and neglect the mixing of quarks and squarks of different generations in rotating fields by an angle ρ to express the left-right squark mass matrix in terms of its eigenvalues $m_{\tilde{q},1}$ and $m_{\tilde{q},2}$. This allows one to trade squark-mass differences for a universal combination, $\tilde{M}$, of the other abundant parameters, times the mass of the concerned quark:

$$\tilde{M} \equiv [A m_{3/2} + (v_1/v_2)\mu], \quad (26)$$

$$[m_{\tilde{q},1}^2 - m_{\tilde{q},2}^2] \sin\rho \cos\rho = \tilde{M} m_q, \quad (27)$$

with $A = O(1)$ and $(v_1/v_2)\mu \sim m_{3/2}$, the gravitino mass. Denote the average q -type squark mass by $m_{\tilde{q}} \equiv m_{\tilde{q},1}/m_{\tilde{q},2} \sim O(1)$. Armed with this notational arsenal, we may express the results of the integrations of the loops in fig. 4 as:

$$\delta_q^g(m_{\tilde{g}}) = \frac{m_q(m_{\tilde{g}}) M_z \tilde{M}}{m_{\tilde{g}}^3} \frac{g_s^2(m_{\tilde{g}})}{64\pi^2} \sin(2\Phi) \left\{ \frac{8}{3} F'(z_q) + 3 G'(z_q) \right\}, \quad (28)$$

$$\delta_q^{\gamma}(m_{\tilde{g}}) = Q_q \frac{m_q(m_{\tilde{g}}) M_z \tilde{M}}{m_{\tilde{g}}^3} \frac{g_s^2(m_{\tilde{g}})}{64\pi^2} \sin(2\Phi) \left\{ \frac{8}{3} F''(z_q) \right\}, \quad (29)$$

with

$$F(z) \equiv \frac{1 - z^2 + 2z \ln z}{2(1 - z)^3}; \quad G(z) \equiv \frac{z - 1 - z \ln z}{(1 - z)^2}. \quad (30)$$

To compare constraints on the supersymmetric parameters stemming from our various bounds on δ_q^{γ} and δ_q^g , we use once more the scale M_z to construct a dimensionless object:

$$B \equiv \frac{M_z^2 \tilde{M}}{m_{\tilde{g}}^3} \sin(2\Phi), \quad (31)$$

and we set, for the sake of definiteness, $z_q = 1$ in the expressions within curly brackets in (28) and (29) (which at this point take the values $+5/18$ and $-2/9$, respectively). We use $\alpha_s(m_{\tilde{g}}) = 0.126$, corresponding to the lowest order renormalization-group-improved

⁶ "Super Kobayashi-Maskawa" contributions are small [20], Wino-Higgsino mixing effects are larger [21], though not competitive with gluino contributions for equally-sized phases.

expression with $\lambda=150$ MeV, $m_j \sim 100$ GeV and five active flavours. “Current” quark masses are to be used for the light quarks in (28) and (29); we choose 5, 7, and 150 MeV, for u, d, s respectively⁷. The direct limits (4) for the lightest quarks imply:

$$B(\Leftarrow \delta_j^7) < 1.6 \cdot 10^{-2}, \quad B(\Leftarrow \delta_u^7) < 4.4 \cdot 10^{-2}, \quad (32)$$

where “ $\Leftarrow$ ” is a reminder of the origin of the bound. The limits (5) from the light quark’s CEDM are a little tighter:

$$B(\Leftarrow \delta_j^g) < 1.4 \cdot 10^{-2}, \quad B(\Leftarrow \delta_u^g) < 1.9 \cdot 10^{-2}. \quad (33)$$

The strange quark limits in (21), stemming from our one-photon three-gluon operators in (12), imply:

$$B(\Leftarrow \delta_s^g) < 2.1 \cdot 10^{-3}, \quad B(\Leftarrow \delta_d^7) < 3.1 \cdot 10^{-2}. \quad (34)$$

These limits may have a caveat dot, but represent an improvement over their classical counterpart (32) by one order of magnitude. If $\tilde{M}$ and m_j in (31) are $O(M_x)$, $\Phi \lesssim 5 \cdot 10^{-4}$, which appears to be unseaworthy. If $\sin(2\Phi) \sim O(1)$ and $\tilde{M} \sim m_j \sim \tilde{m}$, then $\tilde{m} \gtrsim 4$ TeV, which is not encouraging. To cast it all in a positive light, if supersymmetry at the weak scale is a choice that nature has made, then experiments on the neutron’s EDM should be tantalizingly close to finding indirect evidence for it: a well-deserved gift after lustra of brilliant effort. The charmed quark limits in (21), also from our one-photon three-gluon operator in (12):

$$B(\Leftarrow \delta_c^g) < 7.5 \cdot 10^{-3}, \quad B(\Leftarrow \delta_c^7) < 2.9 \cdot 10^{-2}, \quad (35)$$

carry no warning dot, and are also an improvement, though marginal⁸, over (32). Limits on supersymmetric parameters via heavier quarks are not so stringent. The tightest, stemming from Weinberg’s three-gluon operator in (6), is:

$$B(\Leftarrow \delta_g^g) < 3.6 \cdot 10^{-2}. \quad (36)$$

⁷ We use a “constituent” mass $m_s = 450$ MeV in (10) and (20), and a “current” mass $m_s = 150$ MeV in (28) and (29). This contradiction is only apparent: the strange-quark mass refers to two different scales in the two cases. Moreover, the choices combine to give the most conservative limits.

⁸ Our limit on B via the c -quark contribution to Weinberg’s operator is comparable to (32) and not some 20 times less strict, as in [14]. A factor 4 can be traced to their definition $\mathcal{O}_\gamma = (1/4)\bar{q}\sigma_{\mu\nu}q\tilde{F}^{\mu\nu}$, that should, in our opinion, imply an extra $(1/4)$ in the right hand side(s) of their Eq. (11). The rest is in the passage of their Eqs. (10) to (12), where they use $g_s/4\pi = .1$, at a scale at which the rules [11] recommend $g_s = 4\pi/\sqrt{6}$. They apply these rules to half the operator in (2), their inalienable right.

As a last example we consider CP violation in left-right symmetric models. The neutron’s EDM is expected [22] to be dominantly due to virtual- W effects, involving the non-trivial phase of the complex W_L - W_R mass matrix. Let ξ and α be the angle and phase in the diagonalization of that matrix. Let W refer to the lighter, dominantly left-handed eigenstate, and neglect the virtual effects of the heavier one. Let $U = u, c, t$ and $D = d, b, s$, and let $\mathcal{L}$ and $\mathcal{R}$ be the left- and right-handed Cabbibo-Kobayashi-Maskawa matrices, with entries $L_{UD} e^{i\lambda_{UD}}$ and $R_{UD} e^{i\theta_{UD}}$. The leading diagrams generating a quark EDM or CEDM are those of fig. 5. They result [22] in the prediction:

$$\delta_{D,u}^{\gamma,g} = \frac{e^2 \sin(2\xi) M_x}{64 \pi^2 M_W^2 \sin^2 \theta_W} \text{Im} \left\{ e^{i\alpha} \mathcal{R}^\dagger \mathcal{U}^{\gamma,g} \mathcal{L} \right\}_{D,D}, \quad (37)$$

and an analogous one with $\mathcal{L} \leftrightarrow \mathcal{R}$, and its ups and downs, also interchanged. Here $\mathcal{U}^{\gamma,g}$ and $\mathcal{D}^{\gamma,g}$ are diagonal matrices that are functions of $x_U \equiv m_U^2/M_W^2$ and $x_D \equiv m_D^2/M_W^2$ and whose entries are:

$$\mathcal{U}_{UU}^\gamma = m_U \left\{ I_1(x_U, x_D) + \frac{2}{3} I_2(x_U, x_D) \right\} \quad (38)$$

$$\mathcal{D}_{DD}^\gamma = m_D \left\{ I_1(x_D, x_U) + \frac{1}{3} I_2(x_D, x_U) \right\} \quad (39)$$

$$\mathcal{U}_{UD}^g = m_U I_2(x_U, x_D), \quad \mathcal{D}_{DD}^g = m_D I_2(x_D, x_U). \quad (40)$$

We shall only need $I_{1,2}$ in the approximation wherein their second argument vanishes:

$$I_1(x, 0) = \frac{2}{(1-x)^2} \left(1 - \frac{11}{4}x + \frac{1}{4}x^2 - \frac{3x^2 \ln x}{2(1-x)} \right) \quad (41)$$

$$I_2(x, 0) = \frac{2}{(1-x)^2} \left(1 + \frac{1}{4}x + \frac{1}{4}x^2 + \frac{3x \ln x}{2(1-x)} \right) \quad (42)$$

We have checked that our results exactly coincide with those of Chang *et al* [23], who have performed a complete two-loop calculation of the contribution of the diagram involving a top-bottom pair to the three-gluon operator of (7). For this case, use of the limit on δ_g^g in (11) and the expressions (37), (40) results in the bound:

$$|R_{46} L_{46} \sin(2\xi) \sin(\alpha - \rho_{46} + \lambda_{46})| < 3.2 \cdot 10^{-3}, \quad (43)$$

where we have assumed the angles to be such that the top quark, with $m_t = 100$ GeV, dominates. The limit emanating from the “direct” bound on δ_d^7 in (4) is:

$$|R_{4c} \sin \theta_c \sin(2\xi) \sin(\alpha - \rho_{4c} + \lambda_{4c})| < 1.2 \cdot 10^{-5}, \quad (44)$$

where we have assumed the charmed quark to dominate⁹, and used (37) and (38). As a last example, we use the bound on δ_7^7 in (21), assume once again c -dominance and use (37) and (38), to get:

$$|R_{sc} \cos \theta_c \sin(2\xi) \sin(\alpha - \rho_{sc} + \lambda_{sc})| < 1.3 \cdot 10^{-4}. \quad (45)$$

If $R_{4c} \sim \sin \theta_c$ and $R_{sc} \sim 1$, this bound, based on our three-gluon, one-photon operators, constrains the fundamental phases a bit more tightly than the consuetudinary limit (44), and a lot more tightly than the three-gluon-operator mediated result (43), with $R_{4s} L_{4s} \sim 1$.

To summarize, the top quark plays the leading role in models where CP violation is mediated by neutral scalars; given their couplings to mass, a quark's (C)EDM is proportional to m_q^2 , and only one of these powers of mass is cancelled in the coefficient of Weinberg's operator, which wins the context for relevance. Supersymmetry, which emerges severely constrained, is an example of the extreme opposite behaviour: $\delta_{q,7}^{q,7} \sim m_q$, the coefficients of our four-field operators are $\sim m_q^{-3}$; they and the s and c quarks win the competition for the largest contribution to the neutron's EDM. It is difficult to imagine a non-standard model of CP-violation whose behaviour lies not between the scalar and supersymmetric extremes. Left-right models are indeed an intermediate case, in which the (C)EDM of a $Q = -1/3$ quark is linear in the masses of $Q = 2/3$ quarks, and viceversa. Given that these models are particularly rich in CP-odd phases, all quarks and multigluon operators are relevant.

We have seen that heavy quark contributions are suppressed by the large anomalous dimensions associated with the two kinds of three-gluon operators that we have considered: with and without an extra photon. It would be interesting to know whether or not the anomalous dimensions of all CP-odd operators constructible with more than three gluon fields also have the same sign, to gauge their potential impact on the neutron's EDM¹⁰.

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⁹ W -exchange between two valence quarks is a competitive contribution [22], of similar magnitude to (44), once QCD renormalization effects are taken into account [24].

¹⁰ We are indebted to R. Kallosh for bringing this point to our attention.

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Figure Captions

Fig. 1: A CEDM (•) generates a neutron EDM via the three-gluon operator (7).

Fig. 2: [a] A CEDM generates a neutron EDM via the operators (13) and (14);

[b] A quark EDM (also a •) does the same thing.

Fig. 3: [a] A neutral scalar generates a CEDM via its complex couplings (•);

[b] The same for an EDM.

Fig. 4: [a],[b] Superloops generate a CEDM via the complex gluino mass (•);

[c] Same for an EDM.

Fig. 5: [a],[b] The lighter W of a left-right model generates an EDM;

[b] Same for a CEDM, also via the W 's complex mass (•) and couplings.

Fig. 1

Fig 2 a

Fig. 2 b

Fig. 3 a

Fig. 3 b

Fig. 4 a

Fig. 4 b

Fig. 4 c

Fig. 5 a

Fig. 5 b

Fig. 5 c