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JET CONE SIZE DEPENDENCE OF INCLUSIVE CROSS-SECTIONS TO $O(\alpha_s^3)$

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ABSTRACT

We present a detailed description of $O(\alpha_s^3)$ inclusive jet cross-sections for finite cone sizes and give the explicit dependence on the cone opening angle, which is absent at the Born level. Other phenomenological implications are also discussed.

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1. Introduction

In a previous series of papers [1] we have studied the problem of higher-order corrections to inclusive hadron and jet production in hadron collisions. The calculation of $\mathcal{O}(\alpha_s^3)$ terms is of interest both theoretically and phenomenologically for the following reasons. Indeed the $\mathcal{O}(\alpha_s^2)$ QCD-improved parton model results have a twofold limitation due to the absence of any dependence upon the jet cone aperture and the large uncertainty of the resulting cross-sections associated to the choice of the factorization and renormalization mass scales. This fact does not allow a clear test of the theory, giving in addition an ill-defined prediction for the jet production phenomena which must, on the contrary, be clearly understood in order to establish possible evidence for new physics at large transverse momenta.

These theoretical limitations have been partially overcome by adding (α_s^3) corrections, which have been calculated independently by two collaborations [1,2] using different jet algorithms. In Ref. [1] we have given analytical results for all partonic subprocesses, following Furman's definition [3] of a jet as a set of hadrons of momenta $\vec{p}_i$ contained in a given cone C_δ of semi-aperture δ , satisfying $\sum_i \vec{p}_i \equiv \vec{P}_{jet}$. In particular we have assumed $\delta \ll 1$.

Alternatively, the jet definition of Ellis et al. [2] considers a cone C_R in the pseudorapidity-azimuthal angle (η, ϕ) plane, such that $(\eta_i - \eta_{jet}^2 + (\phi_i - \phi_{jet})^2 < R^2$, with $R < \pi/3$ and (η_i, ϕ_i) being the generic co-ordinates of the generic parton i , and, in addition, the transverse energy $E_T \equiv \sum_i E_{Ti}$ is considered instead of the three-momentum. The previous analysis limited to the pure gluon case has been extended [4] to all parton subprocesses, leading in addition to a comparison with CDF data [5].

On the other hand the extension of our results to finite cone sizes and the corresponding successful comparison with UA2 [6] and CDF [5] data has been recently presented [7] in a very concise form. The aim of the present paper is to give a thorough description of our method which extends our δ -range to $\pi/2$, adding further phenomenological implications.

The plan of the paper is the following. In Section 2, we discuss the $\mathcal{O}(\alpha_s^2)$ results for finite cone sizes, giving a detailed description of our numerical algorithm for the jet cross-section. The corresponding phenomenological implications are given in Section 3, where we present the

resulting behaviour with the jet cone size, and discuss the dependence on the jet algorithms. We also give there our conclusions. A discussion of the numerical integration procedure is presented in the Appendix.

2. $\mathcal{O}(\alpha_s^3)$ results for finite cone size

We start from the basic results given in [1], and use the same notation. Then the inclusive cross-section for producing one jet at large transverse momentum within a small cone of semi-aperture δ , in the reaction

$$H_1(k_1) + H_2(k_2) \rightarrow jet(P) + X \quad (1)$$

is written as:

$$E \frac{d\sigma}{d^3P} = \frac{1}{\pi S} \sum_{ij} \int_{VW}^V dv \int_{VW/v}^1 \frac{dw}{1-v} F_p^{H_1}(x_1, M^2) F_p^{H_2}(x_2, M^2) \left[\frac{1}{v} \frac{d\sigma_{ij \rightarrow jet}^0}{dv}(s, v) \delta(1-w) + \frac{\alpha_s(\mu^2)}{2\pi} K_{ij \rightarrow jet}(s, v, w; \mu^2, M^2, \delta) \right] \quad (2)$$

where $S = (K_1 + K_2)^2$, $T = (K_1 - P)^2$, $U = (K_2 - P)^2$, $V = 1 + (T/S)$, $W = -U/(S+T)$, $x_1 = VW/vw$, $x_2 = (1-V)/(1-v)$ and $s = x_1 x_2 S$. $F_p^H(x, M^2)$ are the structure functions of a parton p inside the hadron H at the factorization scale M^2 , $d\sigma_{ij \rightarrow jet}^0(s, v)$ are the Born partonic cross-sections - which can be found in [1] - and K_{ij} is the correction term. Furthermore the running coupling constant $\alpha_s(\mu^2)$, evaluated at the renormalization scale μ^2 , is given by:

$$\alpha_s(\mu^2) = \frac{2\pi}{\beta_0 \ln(\mu^2/\Lambda^2)} \left\{ 1 - \frac{\beta_1 \ln[\ln(\mu^2/\Lambda^2)]}{\beta_0^2 \ln(\mu^2/\Lambda^2)} \right\} \quad (3)$$

with $\beta_0 = (\frac{11}{3})N - (\frac{1}{3})N_F$, $\beta_1 = (\frac{17}{6})N^2 - (\frac{5}{6})NN_F - (\frac{1}{6})C_F N_F$, and $N = 3(N_F)$ being the number of colours (flavours) with $C_F = (N^2 - 1)/2N$.

The calculation of the correction terms K_{ij} in Eq. (2) starts from the basic results of Ellis and Sexton [8] for the squared matrix elements of all $(2 \rightarrow 2)$ and $(2 \rightarrow 3)$ real and virtual

subprocesses to $0(\alpha_s^3)$. Then we perform the phase space integration of the real processes in $n = 4 - 2\epsilon$ dimensions, cancel the $(1/\epsilon^2)$ divergences by adding the virtual corrections and absorb the $(1/\epsilon)$ mass singularities from incoming legs into evolved structure functions. The collinear divergences associated to final partons are automatically cancelled by adding the contributions of one and two partons in the cone.

Then the correction factor $K_{ij \rightarrow jet}$ can be written as:

$$K_{ij \rightarrow jet}(s, v, w, \mu^2, M^2, \delta^2) = \frac{\pi \alpha_s^2(\mu^2)}{2C_i C_j s} \left\{ \left[c_1 + \tilde{c}_1 \ln\left(\frac{s}{M^2}\right) + \tilde{c}_1 \ln\left(\frac{s}{E^2 \delta^2}\right) + \tilde{c}_1 \ln\left(\frac{s}{\mu^2}\right) \right] \delta(1-w) + \left[c_2 + \tilde{c}_2 \ln\left(\frac{s}{M^2}\right) + \tilde{c}_2 \ln\left(\frac{s}{E^2 \delta^2}\right) \right] \frac{1}{(1-w)} + c_3 \left[\frac{\ln(1-w)}{(1-w)} \right] \right\} + K'_{ij \rightarrow jet}(s, v, w, \delta) \quad (4)$$

The coefficient C_i is equal to N for quarks and to $N^2 - 1$ for gluons. The explicit expression for the coefficients c_i of the distributions can be obtained upon request. The regular terms $K'_{ij \rightarrow jet}$ are numerically small but analytically very cumbersome. Of course the form of the coefficients $c_i(\tilde{c}_i, \dots)$ depends upon the factorization scheme used in the parametrization of the structure functions to next-to-leading order. As discussed in detail in Ref. [1], we prefer to use a factorization scheme which absorbs the kinematical terms becoming large near the boundary of the phase space into the finite next-to-leading corrections to the structure functions. In this framework we have modified the Diemöz et al. [9] evolution programme consistently, and our parametrization of structure functions will be denoted by ACGG and can also be obtained upon request. We have used this set of structure functions in all our numerical calculations.

Alternatively, the use of different parametrizations of structure functions, obtained in other factorization schemes to next-to-leading order such as, for example, the set of structure functions by Martin et al. [10], requires the appropriate modification of the coefficients c_i . This is explicitly done in our Fortran code for the jet production cross-sections.

The analytical expressions [2-4] are exact up to terms $0(\delta^2)$. Indeed we observe that the neglected terms in the calculation are of order of $\delta^2/4$, originating from the expansion of

$(1 - \cos \delta)/(1 + \cos \delta)$. It is important to extend the above results without restrictions on the jet cone size, to understand the limit of validity of the analytical formulae and also for phenomenological implications for collider experiments.

To this aim we first consider a cone C_Δ of size Δ - with no limitations on Δ , apart from the obvious one $\Delta < \pi/2$ - and we divide it into a smaller one C_δ centred on the jet axis of size δ ($\delta \ll 1$) and a corresponding crown $C_{\Delta-\delta}$. Then we write

$$E_J \frac{d\sigma}{d^3 P_J}(\Delta) = E_J \frac{d\sigma_{\text{an}}}{d^3 P_J}(\delta) + E_J \frac{d\sigma_{\text{sum}}}{d^3 P_J}(\Delta - \delta) \quad (5)$$

where the first contribution is obtained from Eq. (2), while the second one is computed by performing the numerical integration of the matrix element squared, in $n = 4$ dimensions, in the complementary region. This procedure, however, needs some care in order to avoid double counting and also requires a definition of the kinematical cuts consistent with the jet algorithm. Schematically, one can write

$$\begin{aligned} d\sigma_{\text{sum}}(\Delta - \delta) = & d\sigma(p_1 \text{ and } p_2 \in C_{\Delta-\delta}; \vec{p}_1 + \vec{p}_2 = \vec{P}_J) \\ & + d\sigma(p_1 \in C_\delta, p_2 \in C_{\Delta-\delta}; \vec{p}_1 + \vec{p}_2 = \vec{P}_J) \\ & - d\sigma(p_1 \in C_\delta, p_2 \in C_{\Delta-\delta}; \vec{p}_1 = \vec{P}_J) \\ & + \text{permutations} \end{aligned} \quad (6)$$

where p_1 and p_2 are two generic final partons. The first two terms on the right-hand side of Eq. (6) are obvious; the third one has to be subtracted since it is already included in the definition of the jet cross-section $d\sigma_{\text{an}}(\delta)$ for the cone of size δ . Furthermore the second and the third terms together cancel the soft divergence, corresponding to $E_2 \rightarrow 0$, which is present in each term separately. The finite left-over contribution corresponds to a soft cut given by

$$E_2 \geq E_{2\text{min}} \equiv E_J \int_0^\delta \frac{\sin x}{\sin(x + \Delta)} dx \quad (7)$$

In the numerical analysis this soft cut reproduces the right behaviour of Eq. (5) for Δ small, where we can use the analytic formula to check our numerical algorithm.

Having settled the kinematics, the numerical computation corresponds to an integration on four variables, chosen to be η_3 , η_2 , ϕ_2 and p_{T7} , as discussed in more detail in the Appendix. This concludes our discussion for the case of production of a jet of fixed momentum in a given cone C_Δ , which corresponds to the UA2 jet algorithm. On the other hand, when the jet algorithm implies the measurement of the transverse energy only within a finite region of radius R in the (η, ϕ) space, as for example in the CDF algorithm - where one has $(\eta - \eta_J)^2 + (\phi - \phi_J)^2 < R^2$ - one could modify the numerical contribution in Eq. (5) by taking into account the conditions

$$\begin{aligned} E_{TJ} &= \sum_i E_{Ti} \\ E_{TJ}\eta_J &= \sum_i \eta_i E_{Ti} \\ E_{TJ}\phi_J &= \sum_i \phi_i E_{Ti} \end{aligned} \quad (8)$$

For $\eta_J = 0$ the cone of radius R in the (η, ϕ) space is transformed into an approximate cone in the (θ, ϕ) space, for $R \lesssim 1$. On the other hand, for $\eta_J \neq 0$, there is an additional deformation of the cone due to the Lorentz boost.

In the following section we will discuss the phenomenological implications of Eq. (5) for current $p\bar{p}$ collider experiments.

3. Phenomenological applications

We first discuss the Δ dependence of our cross-section (5) using the jet algorithm where the full momentum is measured. We have considered the subprocesses $q_k \bar{q}_k, g g, g g \rightarrow jet(P, \Delta) + X$, and compared the resulting cross-sections to the corresponding analytical results of [1] [Section 2, Eqs. (2) - (4)]. This is shown in Figs. 1, 2, 3. The numerical values have been obtained putting $\delta = 0.1$ in Eq. (5) and we have checked they are quite stable for other choices of δ . Although the analytical results are expected to be valid for $\Delta \ll 1$, surprisingly enough we observe the excellent validity of the approximation up to $\Delta \lesssim 0.7 \div 0.8$, particularly for $x_T \simeq 0.3 \div 0.5$. For $x_T = 0.1$ the two results coincide up to $\Delta \simeq 0.4$, differ by $\lesssim 5\%$ for $\Delta \simeq 0.7$ and by about 10% for $\Delta \simeq 1.1$.

This behaviour might be understood by observing that the neglected terms in the analytical calculation are of order $(\delta^2/4)$, since they all come from the expansion of $(1 - \cos \delta)/(1 + \cos \delta)$. This interesting feature can be of help in future applications, since the Fortran codes relative to the analytical results are much faster than those relative to the numerical ones.

In order to test the sensitivity to the jet measurement, we have considered two extreme detector configurations, defined as follows. The first one (denoted I) is given by a detector of cone size $\Delta = 0.7$, measuring a jet of transverse momentum p_T centred at 90° . The second one (II) corresponds to a configuration of seven smaller cones, each of size $\Delta/\sqrt{7}$, centred around 90° and covering the same total area, each cone measuring jets of the same transverse momentum p_T . In the latter configuration, the θ -variation around 90° is clearly taken into account. The resulting cross-sections relative to the two-detector configuration, are shown in the Table, for various x_T , at $\sqrt{S} = 1.8 TeV$, for gluon - gluon scattering.

From inspection of the Table it follows that an inclusive jet measurement with a poor angular definition of the jet tends to overestimate the cross-section.

Next, we study the dependence of the jet cross-section upon the cone size R . Since the Born cross-section is independent of R , such an effect is expected to test the next-to-leading order in a non-trivial way. Similar studies have been performed in e^+e^- annihilation [11,12], and are currently being examined at hadronic colliders. In Fig. 4, we plot the R dependence of the jet cross-section, for fixed $p_T = 100 GeV$ and $\sqrt{S} = 1.8 TeV$, at $\eta_J = 0$ and $(9/16)p_T^2 \leq \mu^2 = M^2 \leq 4p_T^2$. The resulting R -dependence seems to be weaker than the preliminary one observed by the CDF collaboration [13], but in good agreement with the results of Ellis et al. [4] evaluated at a scale $\mu^2 = p_T^2/4$, using the set of structure functions of Ref. [10] (MRSB). The theoretical uncertainty due to the scale dependence - of order 25% at $\delta = 1$ and smaller at $\delta = 0.4$ - seems unable to match the experimental result at $\delta = 0.4$. If confirmed by inclusion of systematic errors, this discrepancy at low δ would indicate the necessity of higher-order corrections and/or the importance of fragmentation effects in this kinematical domain. The latter seems more appropriate to play a rôle at small δ . It should be noticed, however, that the CDF data have been averaged in the range $0.1 < |\eta| < 0.7$. Nevertheless, we expect the variation with η to be of order (10%), in the above η -range.

To conclude, we have presented a detailed description of $O(\alpha_s^2)$ inclusive jet cross-sections for finite cone sizes, by extending our previous analytical work for $\delta \ll 1$. Our results show that this approximation holds with good accuracy up to $\delta \lesssim 0.7 \div 0.8$. The dependence on the cone size - which is absent at the Born level - is then presented as a non-trivial test of the next-to-leading calculations. Different definitions of jet algorithms have also been discussed.

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APPENDIX

We give here the details of the numerical integration over the three-parton final state, as required in Eq. (5). We have to compute, up to constant factors, expressions of the type

$$d\sigma \sim \int dx_1 dx_2 \frac{d^2 p_1}{E_1} \frac{d^2 p_2}{E_2} \frac{d^2 p_3}{E_3} |M(ab \rightarrow 123)|^2 \quad (9)$$

$$\delta^4(ab \rightarrow 123) \delta^3(\vec{P}_J - \vec{p}_1 - \vec{p}_2) F_6^{H_1}(x_1, M^2) F_6^{H_2}(x_2, M^2)$$

where the jet has been assumed to be obtained from the two partons p_1 and p_2 . Defining, as usual,

$$p_i = p_{Ti}(\cosh \eta_i, \cos \phi_i, \sin \phi_i, \sinh \eta_i) \quad (10)$$

we first get rid of the integration on x_1 and x_2 from the zeroth and third components of $\delta^4(ab \rightarrow 123)$. After we use the left-over components to fix p_{3T} and ϕ_3 variables.

Then the condition $\delta^3(\vec{P}_J - \vec{p}_1 - \vec{p}_2)$, which defines our jet algorithm, fixes the $\vec{p}_1$ momentum. Notice that a different jet definition would lead to a simple change of this conservation law. For example, Eq. (8) can be used in the case of the CDF algorithm. We are finally left with four variables η_3, η_2, ϕ_2 and p_{2T} which are integrated numerically by random generation with VEGAS. This procedure allows us to generate the integration variables directly within the kinematical region defined by our δ and Δ cones and maximize the statistics of the generation.

Our numerical programme has been tested by comparing it with the analytical results for small Δ . In particular we have been able to reproduce with very good accuracy the previous results for, say, $\Delta = 0.2 \div 0.3$ by adding to the analytic term relative to $C_{f=0.1}$ the contribution of the corresponding crown $C(\Delta, \delta = 0.1)$, according to Eq. (5).

X_T	I		II	
	$p_T^A \frac{E}{\sqrt{s}} [\text{nb GeV}^2]$	$p_T^A \frac{E}{\sqrt{s}} [\text{nb GeV}^2]$	$p_T^A \frac{E}{\sqrt{s}} [\text{nb GeV}^2]$	$p_T^A \frac{E}{\sqrt{s}} [\text{nb GeV}^2]$
0.1	5.37	E + 04	5.39	E + 04
0.2	1.64	E + 03	1.51	E + 03
0.3	7.50	E + 01	6.62	E + 01
0.4	4.27	E + 00	3.31	E + 00
0.5	2.10	E - 01	1.60	E - 01

TABLE 1

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FIGURE CAPTIONS

- 1) Size cone dependence of the scaled cross-section $p_T^4 E \frac{d\sigma}{p_T^4 dy}$ at $\eta = 0$, $\sqrt{S} = 1.8\text{TeV}$ for gluon-gluon subprocess with $\mu = M = p_T$. Small dashed curve: analytical calculation only. Dashed curve: full calculation of Eq. (5). Fig. 1a: $x_T \equiv \frac{2p_T}{\sqrt{S}} = 0.1$; Fig. 1b: $x_T = 0.3$; Fig. 1c: $x_T = 0.5$.
- 2) Same caption as for Fig. 1 for quark-gluon subprocess.
- 3) Same caption as for Fig. 1 for quark-antiquark of same flavour subprocess.
- 4) Size cone dependence of $\frac{d\sigma}{p_T^4 dy}$ (a) and ratio to 0.7 versus cone size (b) at $\eta = 0$ compared with CDF data [13] and calculation of Ref. [4] at $\mu = M = \frac{p_T}{2}$ (open circles). The dashed bars correspond to our prediction for scales between $\mu = M = \frac{3p_T}{4}$ and $\mu = M = 2p_T$.

- Figure 1c -

- Figure 2a -

- Figure 1a -

- Figure 1b -

- Figure 3a -

- Figure 3b -

- Figure 2b -

- Figure 2c -

$X_p=0.5$ SCALE PT 30=4

- Figure 3c -

- Figure 4 -