CERN-TH.5750/90

Radiative quark masses constrained by the gauge group only

Riccardo Rattazzi¹

CERN, CH-1211 Geneva, Switzerland

We discuss the possibility of obtaining the observed pattern of quark masses and mixings as a consequence of radiative corrections, gauge invariance and particle content of the theory. We do not allow any kind of additional symmetry, such as family and discrete symmetries. A model based on the gauge group $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ is considered. It turns out that the correct values of quark masses can be reasonably reproduced. The typical strength of the flavour changing couplings of the Z^0 -boson is however at least one order of magnitude above the experimental upper bounds.

A comparison is made with a model in which an additional discrete symmetry is present. In this case flavour changing phenomena can be kept under control.

CERN-TH.5750/90

May 1990

¹ On leave from Scuola Normale Superiore, Pisa 56100, Italy

1. Introduction

It is an experimental fact that both in the lepton and in the quark sectors there exists a mass hierarchy [1] with $m_e : m_\mu : m_\tau \simeq 1 : 2 \times 10^2 : 3.5 \times 10^3$, $m_d : m_s : m_b \simeq 1 : 26 : 6.5 \times 10^2$, $m_u : m_c : m_t \simeq 1 : 3.3 \times 10^2$, $m_i/m_c > 60$. Furthermore there exists a mixing angle hierarchy in the quark sector with $V_{us} = 0.23$, $V_{cs} \simeq 0.05$, $V_{cb} < 0.01$. The standard model of the weak interactions, as it stands, does not give a natural explanation for these facts. In this model, after spontaneous symmetry breaking, fermion masses are generated by bare mass terms in the Lagrangian, so that they are completely unpredictable. In particular, in order to obtain the observed hierarchical pattern of masses and mixings, an analogous hierarchy of couplings must be imposed in the Yukawa sector.

In order to have a more predictable fermion mass spectrum we must look for theories in which Yukawa interactions are more constrained than in the Standard Model.

In this paper we shall concentrate on the quark mass spectrum.

As far as the hierarchy problem is concerned, a natural idea is that the masses of the first two generations and possibly the bottom are generated through radiative corrections whereas the mass of the top is already present at the tree level [2-3]. In theories with spontaneous symmetry breaking, the vanishing at the zeroth order of a certain class of mass terms is a consequence of the representation content under the gauge group of the fields appearing in the Lagrangian [2]. In fact a zeroth order pattern of quark masses with only a massive top is obtained by extending the gauge group of the standard model to $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ [4], provided there exist in the theory two Higgs doublets $H_L = (1/2, 0, -1)$, $H_R = (1/2, 0, 1)$ and an electroweak singlet heavy up-quark $g = (0, 0, 2/3)$ [5]. The role of this isosinglet is fundamental in triggering the top quark mass generation.

On the other hand, the fermion mass hierarchy can be obtained by means of radiative corrections as shown in Ref.[6]. In the scheme of that paper, the masses of successive generations come out of successive higher loops, so that they are smaller and smaller. Starting from that idea and using the gauge group

$SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, many authors [7-8-9] have tried to give a qualitative description of the pattern of fermion masses and mixings. The purpose of this work is to ask whether it is possible to get a realistic (not only qualitative) picture of the quark mass spectrum. We start with a theory based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. We then require that the mass and mixing hierarchy arises as a mere consequence of the gauge symmetry and of the particle content of the model itself. We bar any kind of additional symmetry, such as family and discrete symmetries.

The essential features of the models under consideration are:

- i) an extra isosinglet up-type quark with general Yukawa couplings to the standard ones. This explains why only the 3rd generation is heavy.
- ii) the chiral symmetries of the 1st and 2nd generations broken by Yukawa couplings to coloured scalar fields. This accounts for the hierarchical pattern of quark masses.

In sect.2 we consider the masses of the third generation. We give a model in which only the top quark is massive at the tree level, while the mass of the bottom is generated by one-loop graphs involving the gauge and Higgs bosons. This mechanism gives, in a natural way, a mass m_b of the right order of magnitude. Furthermore no dangerous mixing with the lighter generations is introduced. Due to the gauge nature of the interactions generating m_b , the mass eigenstates t and b fall into the same $SU(2)_L$ doublet.

In sect.3, starting from a massive 3rd generation, we consider the problem of giving radiative masses to the lighter quarks. There are two possible ways to do that, depending on whether the scalar field involved in mass generation is a weak singlet or a weak triplet. In the first case [6-7] an isospin flip mechanism is operating in mass generation. The charm mass arises out of a graph with a circulating b , while the strange mass is given by a graph with a top quark. This fact, together with the observation that $m_c > m_s$ and $m_t > m_b$, gives some problems when we try to get the correct values of the masses. In the second case the pattern of mass generation can be made more realistic: the mass of each quark of a given charge arises out of a graph with the heavier quark of the same charge.

In this way $m_c > m$, is obtained as a consequence of $m_t > m_b$. The FCNC which arise in this case are however too large. At one loop, the electroweak interactions of the scalar triplet give rise to unacceptably large flavour changing couplings of the quarks to the Z^0 boson. These couplings, normalized to the gauge ones, turn out to be of order $10^{-2} \div 10^{-3}$, that is one or two orders of magnitude above their experimental upper bounds [10].

In sect.4 we draw our conclusions.

2 . The third generation masses: a model for calculating the b-mass

In this section we show a model in which the pattern of masses $m_u = m_d = m_c = m_s = 0, 0 \neq m_b$ with the K.M. matrix equal to unity, is obtained as a mere consequence of the gauge invariance and the particle content of the model.

We consider a model based on the gauge group $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{[4]}$, with quarks Q_i and Q_i^* transforming as left- and right-handed doublets with $(B-L)$ charge respectively $1/3$ and $-1/3$ (i is the family index). We add to these fields two $SU(2)_L \times SU(2)_R$ singlet fermions: g ($B-L = 4/3$) and g^c ($B-L = -4/3$). In the Higgs sector we introduce a $SU(2)_L$ doublet H ($B-L = 1$) and a $SU(2)_R$ doublet H_c ($B-L = -1$). The most general Yukawa interactions for this model are given by :

$$\mathcal{L}_Y = \left\{ h_i (H \epsilon Q_i) g_c - h_i^* (H_c \epsilon Q_i^*) g + \tilde{m}_g g g^c \right\} + \text{h.c.} \quad (2.1)$$

where h_i and h_i^* are Yukawa couplings and ϵ is the 2×2 antisymmetric matrix with $\epsilon_{12} = 1$. We can rotate Q_i and Q_i^* so that only the third family is involved in Yukawa interactions. The v.e.v. of the scalars are:

$$\langle H \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix} \quad \langle H_c \rangle = \begin{pmatrix} v_c \\ 0 \end{pmatrix} \quad (2.2)$$

and the resulting tree-level mass terms are given by:

* We use Weyl fermions for notational convenience

$$\mathcal{L}_{mass} = (u_3 \ g) \begin{pmatrix} 0 & h v \\ h_c v_c & \tilde{m}_g \end{pmatrix} \begin{pmatrix} u_3^c \\ g^c \end{pmatrix} + \text{h.c.} \quad (2.3)$$

The mass v_c represents the scale of breaking of $SU(2)_R$; we take $\tilde{m}_g, h_c v_c \gg h v$. In this limit the masses of t and g are:

$$m_t = (1-r)^{1/2} h v \quad r = \frac{\tilde{m}_g^2}{\tilde{m}_g^2 + h_c^2 v_c^2} \quad (2.4a)$$

$$m_g = \frac{\tilde{m}_g}{\sqrt{r}} \quad (2.4b)$$

Since $m_t \gtrsim 80 \text{ GeV}$ [11] and $v = 174 \text{ GeV}$, we see from 2.4a that, as long as h is not very large, r cannot be too close to 1. It follows that $\tilde{m}_g \leq O(h_c v_c)$.

At this stage the b-mass is zero at any order in perturbation theory. Notice that m_b could have been generated in precisely the same way as m_t , by adding a pair of down-type singlet quarks f and f_c , and by introducing Yukawa interactions analogous to eq(2.1) with $H, H_c \rightarrow H^* \epsilon, H_c^* \epsilon$ [6]. This way of generating m_b is, however, unsatisfactory. The relation $m_b \ll m_t$ is not obtained in a natural way. In fact an analogous inverted relation ($\tilde{m}_g \ll \tilde{m}_t$) must be imposed on the isosinglet mass parameters. In addition, a large mixing angle is in general introduced between the 2nd and the 3rd generations. This is due to the fact that the Yukawa coupling vector of the top t and that of the bottom b (where the notation is obvious) are in general not proportional *.

On the other hand, a realistic picture can be obtained if the b-mass is generated by radiative corrections. The generation of m_b requires the introduction of additional scalar fields. These can act as a source of mixing both between W_L and W_R , and between H^+ and $(H^-)^*$. In this way m_b arises out of radiative corrections due to gauge (fig.2a) and Yukawa (fig.2b) interactions. No mixing is introduced with the lighter quarks. For instance a scalar multiplet P , transforming as $(1, 2, 3, +1)$ under the gauge group, can do the job. When the gauge

* In Ref.[7] h_t and h_b are equated by means of a softly broken discrete charge symmetry, under which $g \leftrightarrow f, H H_c \leftrightarrow H^* \epsilon H^{c*} \epsilon$. This solution is however unsatisfactory if we want to obtain the hierarchy of masses and mixings as a consequence of the gauge symmetry alone.

group is broken to $SU(3) \times U(1)_Q$ and P gets a v.e.v, mass mixing terms are in general induced between W_L and W_R . Furthermore P allows for scalar potential terms of the form:

$$L_{\text{scalar}} = \lambda ({}^t H P_j) ({}^t H_c r^j \epsilon H_c) \quad (2.5)$$

which mix H^+ and $(H_c^-)^*$ after spontaneous symmetry breaking †.

It is interesting to see what are the general features of m_b as calculated from the graphs of fig.2. We indicate by m_b^W the contribution of graph 2.a and by m_b^H that of graph 2.b. In the R_t -gauge m_b^H contains the contribution of both physical and unphysical Higgs bosons. The total contribution $m_b = m_b^W + m_b^H$ is gauge independent, while the single addenda are not. In what follows we report the calculation in the Feynman gauge.

The contribution of fig.2a is given by:

$$m_b^W = \frac{g_L g_R}{16\pi^2} \sin 2\theta \cos \beta \cos \beta_c m_t (F(M_{W_R}^2, m_t^2, m_b^2) - F(M_{W_L}^2, m_t^2, m_b^2)) \quad (2.6)$$

$$F(M^2, m_1^2, m_2^2) = M^2 \left(\frac{1}{M^2 - m_1^2} \log \frac{M^2}{m_1^2} - \frac{1}{M^2 - m_2^2} \log \frac{M^2}{m_2^2} \right)$$

where θ is the mixing angle between W_L and W_R ; β is the mixing angle between t and g ; β_c is the mixing angle of t_c and g_c . We have $\cos \beta \cos \beta_c = \sqrt{r}$. g_L and g_R are the gauge couplings of $SU(2)_L$ and $SU(2)_R$; we suppose $g_L \sim g_R$. When $M_{W_R}, m_g \gg M_{W_L}, m_t$ eq.(2.6) becomes

$$m_b^W = \frac{g_L g_R}{16\pi^2} \sin 2\theta \cos \beta \cos \beta_c m_t \left(\ln \frac{M_{W_R}^2}{m_t^2} - \frac{M_{W_R}^2}{M_{W_R}^2 - m_g^2} \ln \frac{M_{W_R}^2}{m_g^2} + \frac{M_{W_L}^2}{M_{W_L}^2 - m_t^2} \ln \frac{M_{W_L}^2}{m_t^2} \right) \quad (2.7)$$

† Notice that a scalar multiplet $D=(2,2,0)$ would also allow the mixing $W_L - W_R, H - H_c^*$. On the other hand, according to gauge invariance, a scalar D would also allow interactions of the form $Q_i D Q_j^*$. These terms would give rise to tree-level masses for the quarks, so that we would be back to the standard model situation.

The contribution of the diagram 1.b is determined by all charge 1 scalar fields in the theory. The boson line represents the mixing propagator $G(H^+ H_c^-)$. We indicate with H_a^+ ($a = 1, \dots, n$; $H_1^+ = H^+$; $H_2^+ = (H_c^-)^*$) the charge 1 fields appearing in the various multiplets, and with ϕ_b^+ ($b = 1, \dots, n$) the mass eigenstates. These fields are related through a unitary matrix U_{ab} :

$$U_{ab} \phi_b^+ = H_a^+ \quad (2.8)$$

The propagator $G(H^+ H_c^-)$ is given by:

$$G(H^+ H_c^-) = \sum_{b=1}^n U_{1b} U_{2b}^* \frac{1}{p^2 - M_b^2 + i\epsilon} \quad (2.9)$$

Exploiting the unitarity condition $\sum_{b=1}^n U_{1b} U_{2b}^* = 0$, eq.(2.9) can be rewritten as:

$$G(H^+ H_c^-) = \sum_{b=1}^n U_{1b} U_{2b}^* \frac{M_b^2}{p^2 - M_b^2 + i\epsilon} \quad (2.10)$$

This is a sort of GIM-mechanism. The $(p^2)^{-2}$ behaviour of the propagator at high p makes the diagram 1.b convergent; its contribution is given by:

$$m_b^H = \frac{\sqrt{r} m_g h_c}{16\pi^2} \left(\sum_{b=1}^n A_b M_b^2 G(M_b^2, m_t^2, m_b^2) \right) \quad (2.11a)$$

$$G(M^2, m_1^2, m_2^2) = \frac{1}{M^2 - m_1^2} \left[\ln \frac{M^2}{m_1^2} - m_2^2 \left(\frac{1}{m_2^2 - m_1^2} \ln \frac{m_2^2}{m_1^2} + \frac{1}{m_2^2 - M^2} \ln \frac{M^2}{M^2} \right) \right] \quad (2.11b)$$

where $A_b = U_{1b} U_{2b}^*$. Expression (2.11) contains the contribution of both physical and unphysical Higgses. Indicating the unphysical ones by ϕ_1^+, ϕ_2^+ , we have

$$m_b^{\text{unphys}} = \frac{\sqrt{r} m_g h_c}{16\pi^2} \left(\sum_{b=1}^2 A_b M_b^2 G(M_b^2, m_t^2, m_b^2) \right) = -\frac{m_b^W}{4} \quad (2.12)$$

which corresponds to subtracting the contribution of the longitudinal part of the W -propagator. Hence the total 1-loop contribution to the b -mass is

$$m_b = \frac{3}{4} m_b^W + m_b^{\text{phys}} \quad (2.13)$$

3 . The masses of the lighter generations

In this section we start from a massive 3rd generation and add flavour mixing Yukawa interactions in order to generate the lighter quark masses through radiative corrections. The only Yukawa interactions which do not give rise to direct mass terms after the breaking of the gauge symmetry are given by:

$$L = \lambda_{ij} \bar{Q}_i Q_j \Omega + \lambda'_{ij} \bar{Q}'_i Q'_j \Omega' + \text{h.c.} \quad (3.1)$$

where Ω and Ω' are scalar fields with suitable transformation properties under the gauge group. λ_{ij} and λ'_{ij} are Yukawa couplings which mix the generations. There are the following possibilities for Ω and Ω' :

- 1) Ω and Ω' are $SU(2)_L \times SU(2)_R$ singlets. In this case we can take $\Omega'' = \Omega = (3 \text{ or } \bar{6}, 1, 1, -2/3)$.
- 2) Ω is a $SU(2)_L$ triplet $\Omega = (3 \text{ or } \bar{6}, 3, 1, -2/3)$ and Ω' is a $SU(2)_R$ triplet $\Omega' = (\bar{3} \text{ or } 6, 1, 3, 2/3)$.

The fields Ω and Ω' , being coloured, do not receive any vacuum expectation value. Radiative corrections due to interactions of the type (3.1) modify the fermion mass spectrum and give rise in a natural way to the fermion mass hierarchy. One-loop corrections are given by the diagrams of fig.3.1a-b †. These graphs give finite results since no gauge invariant counterterm for them can be formed. Their effect is to add a small correction to the mass matrix. This correction is proportional to $\lambda_c P_3 \lambda$, where P_3 is the projector on the third family. The rank of $\lambda_c P_3 \lambda$ is 1. The tree-level mass matrix is proportional to P_3 , so that the rank of the mass matrix is in general increased by 1 unit when the 1-loop correction is summed. This means that there is one more non-zero eigenvalue. Furthermore, this eigenvalue is small since it vanishes with the loop correction. The graphs in fig.3.1a-b give a mass to the c and s quarks. As we go to two-loop order the rank of the mass matrix is increased by another unit so that all generations become massive[6].

† In case 2) a mixing must be present between Ω and Ω' after $SU(2)_L \times SU(2)_R$ breaking

To get the physical b -mass $m_b(m_b)$ versus $m_t(m_t)$ eq.(2.13) should be scaled by the usual QCD factor $(\alpha_s(m_b)/\alpha_s(m_t))^{12/23}$. Let us analyze the two addenda in (2.13). The first cannot account for the experimental value of m_b . Given the present upper limits on m_t [12] and on the mixing angle θ [13], this term can give at most $m_b \sim 0.1 \text{ GeV}$. The smallness of the vector boson contribution is related to the fact that it vanishes when M_{W_R} and m_g go to infinity. In this limit the expression in brackets in (2.6) grows like $\ln(M_{W_R}/M_{W_L})^2$ while $\sin 2\theta$ behaves like $O(M_{W_L}/M_{W_R})^2$. On the other hand the physical Higgses' contribution does not vanish, in general, as we push the new physics scale (M_{W_R}, m_g) to a high energy. This fact is a consequence of the different mass insertions of graph 1.b with respect to those of graph 1.a:

- i) in 1.b the mass insertion on the fermion line is m_g , while in 1.a it is m_t .
- ii) in 1.b the mass insertion on the boson line is in general of order m_g . This is a consequence of the presence of interaction terms like (2.5). On the other hand the mass insertion on the vector boson line is of order v^2 , since the v.e.v.s of fields which are not $SU(2)_L$ singlets are constrained to be of the order of the Fermi scale.

These two facts neutralize in m_b^{phys} the $(M_{W_L}/M_{W_R})^2$ behaviour of eq.(2.7). As an example we give the expression for m_b^{phys} in the case of $n = 4$ and $M_3 \simeq M_4 \simeq M_{W_R}$:

$$m_b = \frac{\sqrt{r} m_t}{32\pi^2} \left\{ g_L g_R \cos \theta \sin \theta \frac{m_g^2 M_3^2}{M_{W_L}^2 (M_3^2 - m_g^2)} \log \frac{M_3^2}{m_g^2} \right\} \quad (2.14)$$

Eq.(2.14), after rescaling m_b at low energies, may well give $m_b \simeq 1 \text{ GeV}$, which is not far from the correct value. However the crucial feature of (2.14) is not the fact that it gives $m_b \simeq 1 \text{ GeV}$ instead of $m_b \simeq 5 \text{ GeV}$: the correct value may well be obtained by increasing the number of scalar fields in the theory. The central point is the essential scale independence of eq (2.14). If we change M_3 , M_{W_R} , m_g to λM_3 , λM_{W_R} , λm_g expression (2.14) remains unchanged **.

** Notice that $\theta \sim (M_{W_L}/M_{W_R})^2$

In what follows we consider in turn case 1) and case 2) for the field Ω . We discuss the possibility of obtaining a realistic mass spectrum.

a) The case of a singlet scalar.

In this case the interaction (3.1) flips isospin: $Q_i Q_j \Omega = (u_i d_j - d_i u_j) \Omega$. This means that m_c is given by a graph with a propagating b -quark (fig.3.1b), while m_s is given by a graph with a top quark. Experimentally m_c and m_b are of the same order of magnitude. It might be troublesome for perturbation theory to have m_c as a radiative effect of m_b .

Let us, however, consider this possibility in more detail. To be definite we take $\Omega = \Omega_c^+ = S$, and S a colour triplet. In this case λ and λ_c are symmetric matrices. We can further assume, for simplicity, L-R-symmetry: $\lambda_{ij} = \lambda_{ji}^*$. We indicate in general by Λ_t and Λ_b the energy scales at which the effective masses m_t and m_b vanish. For instance, in the model of the previous section, Λ_t is given by the mass m_g of the heavy isosinglet quark, while Λ_b is of the order of the typical scale of the graph contributing to eq.2.14: $\Lambda_b \simeq M_{W_R} \simeq m_g$. We choose a flavour basis in which $\lambda_{13} = 0$. We give the second-generation masses in the two cases $\Lambda_t, \Lambda_b < M_S$ and $\Lambda_t, \Lambda_b > M_S$ (M_S is the mass of the coloured scalar).

i) $\Lambda_t, \Lambda_b < M_S$. Apart from logarithmic factors the rough behaviour is

$$m_s = \sqrt{\tau} \frac{|\lambda_{23}|^2}{8\pi^2} \left(\frac{\Lambda_t}{M_S} \right)^2 m_t \quad (3.2a)$$

$$m_c = \frac{|\lambda_{23}|^2}{8\pi^2} \left(\frac{\Lambda_b}{M_S} \right)^2 m_b \quad (3.2b)$$

ii) $\Lambda_t, \Lambda_b > M_S$. The graphs diverge logarithmically with the cut-off

$$m_s = \sqrt{\tau} \frac{|\lambda_{23}|^2}{8\pi^2} \ln \left(\frac{\Lambda_t}{M_S} \right)^2 m_t \quad (3.3a)$$

$$m_c = \frac{|\lambda_{23}|^2}{8\pi^2} \ln \left(\frac{\Lambda_b}{M_S} \right)^2 m_b \quad (3.3b)$$

Notice that expression (3.2b) gives too small a value for m_c , as long as λ_{23} remains in the perturbative domain *. On the other hand eq.(3.3b) can give the correct value, provided the logarithm is very large. For $|\lambda_{23}| = 1$, $\ln(\Lambda_b/M_S)^2$ must be

* This fact has been overlooked in ref. [7]. In that case $\Lambda_t = m_g$, $\Lambda_b = m_f$

$\simeq 24$ †. The strange quark mass given in eq.(3.3a) is proportional to m_t . In the model of the previous section $\Lambda_b \sim \Lambda_t$, so that also $\ln(\Lambda_b/M_S)^2$ is large. In order to account for the smallness of m_s/m_t it must be $\sqrt{\tau} < 10^{-2}$. This restriction on $\sqrt{\tau}$ renders the b -mass calculated in the previous section too small. Thus, the case of a radiatively generated m_b and of singlet scalar field S is excluded.

Observe, however, that when m_b is generated at the tree level via the mixing with an isosinglet down type quark [6-7], the restriction $\sqrt{\tau} \simeq 0$ is harmless. Although we consider this way of generating m_b unsatisfactory, it is worthwhile saying a few words on this possibility. If $\sqrt{\tau}$ were exactly zero, mass generation would proceed along the "chain":

$$m_d \leftarrow m_u \leftarrow m_s \leftarrow m_c \leftarrow m_b$$

where each arrow represents the step from n - to the $(n+1)$ -loop order. Thus, m_u and m_d would arise respectively at three- and four-loop order: this would be in contrast with the experimental observation that $m_d > m_u$. The contribution to m_s from the top quark graph (eq.3.3a) must be relevant. In this case mass generation follows roughly the chain:

$$m_u \leftarrow m_s \leftarrow m_t$$

$$m_d \leftarrow m_c \leftarrow m_b$$

Notice that if all the $|\lambda_{ij}|$ were nearly equal we would have $m_d/m_c \sim O(m_c/2m_b)^{**}$. This relation implies $m_d \sim 100$ MeV, which is too large. In fact, and $m_g/m_f \simeq m_b/m_t$. When $M_g > m_g$, the mass ratios turn out fair $m_c/m_s \simeq m_f/m_g \simeq m_t/m_b$. The applicability of perturbation theory, however, implies too small a value for m_c .

† The leading contribution of n -loop graphs is of order $1/n! (\lambda^2/8\pi^2 \ln \Lambda^2/M_S^2)^n \times m_{t,b} \sim 1/n! (m_c/m_b)^n m_{t,b}$. Summing up the series of the leading terms is the same thing as solving the 1-loop renormalization group equations, with suitable boundary conditions at the cut-off scale $\Lambda \sim \Lambda_t, \Lambda_b$. Notice that the logarithmic enhancement of eq.(3.3b) cannot be obtained in a supersymmetric version of the model [9]. In that case $\Omega \neq \Omega_c^+$. There must be a mass insertion on the scalar line. Thus, the graph is no longer log-divergent.

** The factor $1/2$ comes from the loop expansion

by analysing the two loop corrected mass matrices, it turns out that the correct values of m_u , m_d and of the Cabibbo angle ($\sin \theta_c = 0.23$) are obtained, provided that λ_{12} is somewhat smaller than the others. It must be $|\lambda_{12}|^2 \simeq 0.1|\lambda_{33}|^2$.

b) The case of a weak triplet scalar.

In this case the interactions of eq.3.1 have the form:

$$\mathcal{L}_Y = \frac{\lambda_{ij}}{2} \left\{ u_i u_j T_{-1/3} + \frac{(u_i d_j + d_i u_j)}{\sqrt{2}} T_{-1/3} + d_i d_j T_{2/3} \right\} + \frac{\lambda'_{ij}}{2} \left\{ u_i^c u_j^c T_{1/3}^c + \frac{(u_i^c d_j^c + d_i^c u_j^c)}{\sqrt{2}} T_{1/3}^c + d_i^c d_j^c T_{-2/3}^c \right\} + \text{h.c.} \quad (3.4)$$

where T_Q represents the triplet component with charge Q . After the breaking of $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, the fields T and T^c get mixed. Observe that the mixing between $T_{-1/3}$ and $T_{1/3}^c$ would give, as in the case of a singlet scalar, a large value to m_s . This mixing can however be avoided in lowest order. This is done by choosing suitable scalar fields to trigger the $T T^c$ -mixing. We introduce four multiplets of scalars N_+ , N_- , N_+^c , N_-^c , with $N_\pm = (1, 3, 1, \pm 1)$ and $N_\pm^c = (1, 1, 3, \pm 1)$. Their interactions with T , T^c are given by:

$$\mathcal{L}_{TT^c} = \lambda_1 (T N_+ N_-^c T^c) + \lambda_2 (T N_- N_+^c T^c) + \text{h.c.} \quad (3.5)$$

After the breaking of $SU(2)_L \times SU(2)_R \times U(1)$ to $U(1)_Q$, the potential terms (3.5) provide mixing mass terms between $T_{1/3}$ and $T_{-4/3}^c$, and between $T_{2/3}$ and $T_{-2/3}^c$. $T_{1/3}$ and $T_{-4/3}^c$ remain unmixed at tree level. Thus, mass generation follows the chain:

$$m_u \leftarrow m_c \leftarrow m_t$$

$$m_d \leftarrow m_s \leftarrow m_b$$

which looks promising, as far as the relation $m_s < m_c$ is concerned. The second generation masses out of the 1-loop graphs of fig.3.1 are

$$m_c = \frac{3}{16\pi^2} |\lambda_{33}|^2 \left\{ \sqrt{r} \cos \eta \sin \eta \ln \frac{M_{1u}^2}{M_{1u}^2} \right\} m_t \quad (3.6a)$$

$$m_s = \frac{3}{16\pi^2} |\lambda_{33}|^2 \left\{ \cos \xi \sin \xi \ln \frac{M_{2d}^2}{M_{1d}^2} \right\} m_b \quad (3.6b)$$

where (η, M_{1u}, M_{2u}) are the mixing angle and masses out of the mass matrix for $T_{-4/3}$ and $T_{1/3}^c$. (ξ, M_{1d}, M_{2d}) are the same thing for $T_{2/3}$ and $T_{-2/3}^c$. We have taken, for simplicity, T and T^c to be colour sextets. In this case λ is symmetric and we can take $\lambda_{13} = 0$. The above expressions are valid when the cut-off scales of m_t and m_b are larger than the masses of the T -fields. This limit has been chosen for calculational convenience. However, Λ_t and Λ_b cannot be decreased too much below the T -masses, otherwise the calculated m_c and m_s become suppressed by a factor $(\Lambda/M_T)^2$ (see eq.3.2). The first generation masses out of two-loop graphs are given by

$$m_u = \frac{3}{16\pi^2} |\lambda_{12}|^2 \left\{ \cos \eta \sin \eta \ln r_{4/3} \left[\frac{1}{2} + \frac{H(r_{4/3})}{\ln^2 r_{4/3}} \right] \right\} m_c \quad (3.7a)$$

$$m_d = \frac{3}{16\pi^2} |\lambda_{12}|^2 \left\{ \cos \xi \sin \xi \ln r_{2/3} \left[\frac{1}{2} + \frac{H(r_{2/3})}{\ln^2 r_{2/3}} \right] \right\} m_s, \quad (3.7b)$$

where $r_{4/3} = M_{1u}^2/M_{2u}^2$ and $r_{2/3} = M_{1d}^2/M_{2d}^2$; the function $H(x)$ is reported in the appendix. Formulae (3.6) and (3.7) may well account for the observed spectrum, provided the mixing between T and T^c is substantial. By taking, for instance, $|\lambda_{33}| \simeq 1$, we have that the expression in curly brackets in eq.(3.6b) must be $1 \div 2$. It is curious to observe that, by using the relations $\sqrt{r} < 1$ and $H(x)/\ln^2 x < 0.145$ (see the Appendix), we get

$$m_t \geq \frac{1}{1.3} m_b(m_t) \frac{m_d m_s^2}{m_u m_t^2} \simeq 140 \text{ GeV} \quad (3.8)$$

where the values of quark masses given in ref[1] have been used. In eq.(3.8) the largeness of m_t is related to the mass inversion in the first generation: $m_d/m_u \simeq 1.8$.

Unfortunately the $T - T^c$ -mixing gives rise to large flavour changing couplings of the Z^0 -boson. The graphs in fig.3.2 sum up to give the following effective Lagrangian

$$\mathcal{L}^{FCNC} = \frac{g}{\cos \theta_w} Z^\mu \left\{ \bar{u}_L^i \gamma_\mu u_L^j f_{ij}(u) - \bar{d}_L^i \gamma_\mu d_L^j f_{ij}(d) \right\}$$

$$f_{ij}(u) = \frac{3}{16\pi^2} (\lambda^\dagger \lambda)_{ij} \sin^2 \eta \cos^2 \eta \left\{ 1 - \frac{M_{1u}^2 + M_{2u}^2}{2(M_{1u}^2 - M_{2u}^2)} \ln \frac{M_{1u}^2}{M_{2u}^2} \right\} \quad (3.9)$$

$f_{ij}(d)$ is the analogue of $f_{ij}(u)$ with $(\eta, M_u) \rightarrow (\xi, M_d)$. Formulae (3.9) result from graphs 3.2 in the limit of heavy T-scalars, i.e. when M_T 's are much larger than the external momenta. If we assume that the λ_{ij} are all of the same order of magnitude, and, moreover, that they are in the perturbative domain $|\lambda_{ij}| < O(1)$, we have that eqs.(3.6) and (3.7), together with the observed values of quark masses, give the generic lower bound: $|f_{ij}| \gtrsim O(10^{-3})$ †. This lower bound is in conflict with experimental observations: $|f_{ds}| < 6 \times 10^{-6}$, $|f_{sd}| < 4 \times 10^{-4}$, $|f_{cu}| < 5 \times 10^{-4}$ [10]. Notice that in the case of the singlet S the analogues of graphs 3.2 do not give rise to FCNC. The simplest way to understand this fact is to consider the effective action before spontaneous symmetry breaking. Interactions like (3.9) arise from generalized kinetic terms, i.e. those that are made up of two fermions, one covariant derivative plus any number of scalar fields. The lagrangian (3.9) is determined in lowest order by two mass insertions on the scalar lines. The associate term in the effective action has the form

$$\mathcal{L}_{eff} \propto (\lambda^\dagger \lambda)_{ij} \bar{Q}_i (N_+ N_-)^{-1} \bar{P} (N_+ N_-) Q_j \quad (3.10)$$

Observe that when S.S.B. occurs the covariant derivative cannot be commuted with $< N_+ N_- >$. In particular we have $[T_L^3, < N_+ N_- >] \neq 0$: this fact implies that the flavour mixing matrices in the kinetic term and in the T_L^3 -current term cannot be reduced simultaneously to unity. In the case of a singlet the terms in the effective action do not contain such scalar fields: kinetic and current terms are reduced simultaneously to the canonical diagonal form.

A last comment on eq. (3.9) is in order. Observe that the f_{ij} 's are unchanged by the rescaling of the scalar masses $M_{1,2} \rightarrow \lambda M_{1,2}$. Thus the interactions of eq. (3.9) do not vanish when $\lambda \rightarrow \infty$, provided the mixing angles η and ξ remain finite. This fact contradicts the usual expectation that flavour violating phenomena vanish as a power of v/Λ_F , where v stays for the Fermi scale and Λ_F for the typical scale of flavour physics. Eq.(3.10) should be scaled by a dimensional factor Λ_F^{-4} : in our case Λ_F is given by the gauge invariant mass

† We have barred relevant cancellations in such entries as $(\lambda^\dagger \lambda)_{12} = \lambda_{11}^\dagger \lambda_{12} + \lambda_{12}^\dagger \lambda_{22}$.

parameters of the T-fields. Flavour violating operators like eq. (3.10) are scaled by a power of $(< N N^c > / \Lambda_F^2)$. The limit considered above corresponds to the special case in which both Λ_F and $< N^c >$ go to infinity, while $< N N^c > / \Lambda^2$ stays finite. In this case the largeness of $< N^c >$ enhances the effect of the scalar fields insertions in operators like (3.10). The decoupling of flavour physics is recovered when $\Lambda_F^2 \gg < N N^c >$: in this case the angles η and ξ become vanishingly small.

4 . Conclusions

We have studied the possibility of the quark mass spectrum being a consequence of radiative corrections. We have considered a theory based on the gauge group $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, in which no additional symmetry (family symmetries, discrete symmetries etc.) is allowed. We have followed the idea that only the heaviest fermions are massive at the tree level. The lighter fermions get a radiative mass through flavour violating interactions.

The experimental observation that the off-diagonal entries in the K.M. matrix are small implies that only the heaviest quark, namely the top, can be massive at the tree level. Giving a tree-level mass to the b-quark too would in general introduce a large mixing between the second and the third generations. By generating the b-mass through radiative corrections due to gauge interactions the relation $V_{KM} \simeq 1$ is preserved in a natural way.

The experimental relations $m_s < m_c$ and $m_b < m_t$ imply that the coloured scalar field involved in mass generation must be a weak triplet. In this way the quark mass spectrum can be reasonably reproduced. The weak interactions of the triplet scalar, however, give rise to unacceptable FCNC. Thus, the observed pattern of quark masses and mixings seems difficult to obtain as a mere consequence of gauge invariance, without tunings of parameters.

Allowing for additional symmetries, a model can be built [7] in which mass generation proceeds via a weak singlet scalar. In this way large FCNC are avoided. On the other hand, in order to get the right value for m_c without the breakdown of perturbation theory, large mass ratios must be present: $(m_t/m_s) = O(10^5)$.

Our discussion is not peculiar to the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. Analogous considerations can be made for other groups, the critical feature being the $SU(2)_L$ transformation property of the scalar field involved in mass generation.

Acknowledgements

I would like to thank Riccardo Barbieri for bringing to my attention the problems discussed above and for stimulating discussions during the work. I would also like to thank Guido Altarelli for helpful comments. I am indebted with the Theory Division at CERN for its hospitality.

Appendix

We report the expression of the function $H(r)$.

We define:

$$f(r) = \int_0^\infty \frac{dz}{z(z+r)} \ln(z+1) \quad A.1$$

H is given by:

$$H(r) = f(r) + f(1/r) - \frac{\pi^2}{3} \quad A.2$$

$H(r)/\ln^2 r$ is a decreasing positive function of r ; its maximum value, at $r = 1$, is $\pi^2/6 - 3/2 \simeq 0.145$.

References

- [1] H.Gasser and H.Leutwyler, Phys. Rep. 87 (1982) 77
- [2] S.Weinberg, Phys. Rev. Lett. 29 (1972) 388
- [3] H.Georgi and S.L.Glashow, Phys. Rev. D6 (1972) 2977; D7 (1973) 2457
R.N.Mohapatra, Phys. Rev. D9 (1974) 3461
- S.M.Barr and A.Zee, Phys. Rev. D15 (1977) 2652; D17 (1978) 1854
- R.Barbieri and D.V.Nanopoulos, Phys. Lett. 91B (1980) 369; 95B (1980) 43
- R.Barbieri, D.V.Nanopoulos and A.Masiero, Phys. Lett. 104B (1981) 194
- [4] J.C.Pati and A.Salam, Phys. Rev. D10 (1974) 275
- R.N.Mohapatra and J.C.Pati, Phys. Rev. D12 (1975) 2558
- G.Senjanovic and R.N.Mohapatra, Phys. Rev. D12 (1975) 1502
- [5] R.Barbieri and R.Rattazzi, IFUP-Th 20/89. To appear in the proceedings of the La Thuile Meeting On Particle Physics (March 1989)
- [6] B.S.Balakrishna, Phys. Rev. Lett. 60 (1988) 1602
- [7] B.S.Balakrishna, A.L.Kagan and R.N.Mohapatra, Phys. Lett. 205B (1988) 345
- [8] A.L.Kagan, University of Maryland Preprint, UMDEPP 89-109 (1989) to appear in Phys. Rev. D
- B.S.Balakrishna, Phys. Lett. 214B (1988) 267
- E.Ma, Phys. Rev. Lett. 63 (1989) 1042
- [9] K.S.Babu, B.S.Balakrishna and R.N.Mohapatra, University of Maryland Preprint, MDdP-PP-90-103 (1990)
- [10] E.W.N.Glover and J.J.van der Bij, CERN Yellow Report, CERN 89-08 Volume 2 (1989)
- [11] Abe et al., CDF Collab., Fermilab-Pub-89/212-E (1989)
- [12] U.Amaldi et al., Phys. Rev. D36 (1987) 1385

Figure captions

2.1a-b One-loop graphs contributing to the b-mass.

3.1a-b One-loop graphs contributing to the lighter generations masses.

3.2 Graphs generating the flavour changing couplings of the W_3^I vector boson to quarks.

fig. 2.1 a

fig. 2.1 b

fig. 3.1 a

fig. 3.1 b

fig. 3.2