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INTERMITTENCY STUDIES IN $\bar{p}p$ COLLISIONS AT $\sqrt{s} = 630 \text{ GeV}$

Abstract

A significant intermittency signal is observed in 630 GeV $\bar{p}p$ collisions measured in the UA1 central detector. It occurs with similar magnitude in different variables: pseudorapidity, rapidity and azimuthal angle. The signal increases with decreasing charged particle multiplicity in the event. Its strength in a sample of low $p_{\perp}$ tracks and its multiplicity dependence are not reproduced by commonly used Monte Carlo models of high energy interactions.

1 Introduction

The aim of this work is to study quantitatively charged particle density fluctuations in individual events in small phase space regions at the CERN $\bar{p}p$ collider. Such fluctuations have been reported in cosmic ray experiments [1] and accelerator experiments [2,3].

The strength and the statistical significance of particle density fluctuations are measured by means of the factorial moment formalism developed by A. Bialas and R. Peschanski [4]. This method has been applied to several experiments with the observation that intermittency patterns (i.e. selfsimilar fluctuations or clustering effects down to very small rapidity intervals) emerge in heavy ion reactions [5] and in $\pi^+(K^+)p$ reactions [6] at $\sqrt{s} = 22$ GeV. There are indications that such patterns are more pronounced in e^+e^- reactions [7]. The existence of fluctuations in very small rapidity bins, as small as $\delta y \approx 0.1$, represents a new and unexpected phenomenon¹. It has led to an intensive and lively discussion involving theorists and experimentalists [11].

In this paper we present a measurement of the strength of the intermittency signal at $\sqrt{s} = 630$ GeV and discuss whether we are observing a new phenomenon or whether we can explain our measurements with known effects such as resonance production, cluster production with a correlation length $\delta y \approx 1$, the Bose-Einstein effect and jet production.

The paper is divided as follows:

In section 2 the factorial moment analysis is described and intermittency is defined. In section 3 we present the data sample. The experimental results are given in section 4, and a search for the origin of intermittency with a comparison with Monte Carlo models is done in section 5.

It has been shown [12] that there exists a simple relation between the intermittency parameters and the generalized dimensions in a description in terms of multifractals [13]. These generalized dimensions D_i are calculated from our data and are presented in section 6 together with those of other experiments. The results are summarized in section 7.

2 The Factorial Moment Analysis and the Measurement of Intermittency

Ref. [4] shows how the observed multiplicity distributions in rapidity bins can be obtained from a convolution of the underlying dynamical density distribution with the Bernoulli (or Multipoissonian) distribution, where the latter accounts for

¹In principle the factorial moments can be obtained from integration over the inclusive correlation functions [8,9,10] but information about the latter for very small rapidity distances ($\delta y \lesssim 0.4$) is not yet available.

statistical fluctuations due to finite multiplicities of the observed events. As a consequence the authors show that, by measuring the Factorial Moments (FM) of the particle distribution, one gets the moments of the underlying density distribution.

The FM are calculated as follows: a rapidity region Δy is divided into M bins of size δy . For the m^{th} bin ($m = 1, \dots, M$) the normalized factorial moment of order i is given by

$$\langle F_m^i \rangle = \frac{\langle n_m(n_m - 1) \cdots (n_m - i + 1) \rangle}{\langle n_m \rangle^i} \quad (1)$$

where n_m is the number of charged particles of one event in bin m and $\langle \rangle$ denotes the average over the event sample. For the investigation of particle density fluctuations in the range Δy it has been proposed to average over all M bins

$$\langle F^i \rangle = \frac{1}{M} \sum_{m=1}^M \langle F_m^i \rangle \quad (2)$$

and to study the dependence of $\langle F^i \rangle$ on the binsize $\delta y = \Delta y/M$.

Ref. [4] shows that if clustering effects and particle fluctuations occur only at a scale $\delta y \gtrsim 1$ the FM are expected to increase with decreasing δy as long as $\delta y \gtrsim 1$, and approach a constant value for $\delta y < 1$. On the other hand, selfsimilar fluctuations at all scales δy lead to a power law dependence of $\langle F^i \rangle$ on δy :

$$\langle F^i \rangle \propto \left(\frac{1}{\delta y} \right)^{\varphi_i} \quad \text{with} \quad 0 < \varphi_i \leq i - 1 \quad (3)$$

This particular behaviour is called intermittency². In the strict sense intermittency can only be said to occur if the power law behaviour holds for $\delta y \rightarrow 0$. Experimentally one can only test this behaviour in a limited δy -region. In practice we look for a linear rise with slope φ_i of $\log \langle F^i \rangle$ with $\log(1/\delta y)$ in the region $\delta y < 1$ down to the limit of resolution [14]. The magnitudes of the *slope parameters* φ_i characterize the strength of the intermittency signal.

In the following study the FM analysis is performed mainly using the pseudo-rapidity variable η . Results in rapidity y (with all particles assumed to be pions) and azimuthal angle ϕ (angle in the plane perpendicular to the collision axis) are given in some specific cases.

3 The Data Sample

The data sample consists of approximately 160,000 non-singly-diffractive events at $\sqrt{s} = 630$ GeV collected with a minimum bias trigger [15] in the UA1 1985

²The upper limit $\varphi_i = i-1$ is obtained if all particles are emitted at exactly the same rapidity, see also section 6.

run. Only vertex associated charged tracks in the central detector (CD) [16] with transverse momentum $p_{\perp} \geq 0.15 \text{ GeV}/c$, good measurement quality and fitted length $l \geq 30 \text{ cm}$ have been used.

The following possibilities for the introduction of a systematic error into the determination of the $\langle F^i \rangle$ and φ_i have been studied:

- a) acceptance loss in the CD
- b) measurement accuracy of η, y, ϕ
- c) double counting of tracks and loss of nearby tracks of the same charge
- d) γ conversion in the beam pipe wall
- e) limited number of events and limited event multiplicity
- f) the influence of the $p_{\perp}$ -cut.

The following comments can be made:

- a) The overall acceptance for obtaining good quality tracks in the CD has been estimated to be 82% in the central rapidity region ($-1.5 \leq \eta \leq 1.5$). The resulting systematic error for the $\langle F^i \rangle$ and the slope parameters φ_i have been studied using Monte Carlo simulations. An example is shown in fig. 1. When comparing Monte Carlo events (PYTHIA including the Bose-Einstein effect) without and with limited detector acceptances applied, we found a systematic decrease of the FM values - leaving, however, the slope parameters in the fit region unchanged. We conclude that the influence of the acceptance loss on the slopes φ_i is negligible. This is confirmed by the data themselves: the observed φ_i values do not change significantly when measured in different detector regions with varying acceptances.
- b) The measurement errors δx_{err} are small compared to our smallest bin size used for the fits ($\delta x \geq 0.1$). We estimate $\delta x_{err} \lesssim 0.01$ where $x = \eta, y, \phi$.
- c) All tracks recorded in the CD are reconstructed first from the drift time measurements in the xy -plane which includes the beam axis (x) and is normal to the direction of the magnetic field (z). The dip angle λ with respect to this plane is determined from the charge division on the wires which are spanned parallel to the magnetic field of 0.7 Tesla [16]. For 1% of the tracks, the pattern recognition program interprets two segments of the same track as two separate tracks that are close together in phase space ("split tracks"). All reconstructed track pairs of same charge, similar momentum ($\Delta p < 0.1 \text{ GeV}/c$) and with an opening angle $\leq 18^\circ$ have been analysed separately and the second piece of a split track has been eliminated. Moreover, all pairs with

like charge, an opening angle $\alpha_{xy} \lesssim 1.4^\circ$ in the xy plane and $\Delta\lambda < 2.5^\circ$ have been eliminated in order to avoid problems with the two-track resolution. This introduces a systematic error on the φ_i values. Its magnitude has been estimated by studying the effect of successively removing pairs in the neighbouring region $1.4^\circ \leq \alpha_{xy} \leq 2.5^\circ$. Assuming the same particle density in $0 \leq \alpha_{xy} \leq 1.4^\circ$ we find that the value of φ_i is underestimated by at most 10%.

- d) The number of γ conversion pairs in the beam pipe wall with a thickness of $0.019 \chi_0$, where both the e^+ and e^- have $p_\perp \geq 0.15$ GeV/c, and their influence on φ_i has been determined by Monte Carlo calculations. The values of φ_i could be overestimated by at most 9% due to this effect.
- e) A systematic uncertainty of the φ_i values is expected from the limited number of events in our sample and from the limited event multiplicities. Both effects lead to an underestimation of the $\langle F^i \rangle$ values in the smallest bins and therefore to an underestimation of the φ_i values. However, an event simulation [17] with a random cascading model (α -model [4]), which incorporates intermittency of known strength, shows that these effects are negligible in our data ($i \leq 5$), provided that the mean particle densities are greater than 0.3 in the smallest $\delta\eta$ binning considered. In models without intermittency these effects could cause a decrease of $\langle F^i \rangle$ with decreasing bin size leading to negative φ_i values.

Special attention has been given to the error determination: fewer and fewer events contribute to the $\langle F^i \rangle$ when the bin size becomes small. To test the error calculation, the total event sample has been subdivided into 9 subsamples with 18,000 events each. The fluctuations of the nine φ_i values around their mean value have been compared to the errors σ_i obtained from the fits. These two methods of error estimation are consistent for $i \lesssim 4$. In the case of φ_5 the fluctuations turned out to be stronger, therefore the corresponding errors σ_5 have been increased.

- f) No change of the $\langle F^i \rangle$ and φ_i values is observed when the $p_\perp$ -cut is varied between 0.1 and 0.2 GeV/c. About 17% of all tracks are lost with a cut $p_\perp \geq 0.15$ GeV/c. The correction is expected to be small but cannot be determined from our measurements.

We summarize the results of these studies as follows: the raw $\langle F^i \rangle$ values are systematically underestimated (by 5% in the case of $\langle F^2 \rangle$, but by 60% in the case of $\langle F^5 \rangle$) due to the acceptance loss in the central detector. The amount of the systematic shifts has been estimated by a Monte Carlo calculation and the resulting corrections have been applied to the $\langle F^i \rangle$ values shown below. However, the sum of all systematic errors as determined from a)–e) on the slope values φ_i is

expected to be small and the different effects partially cancel each other, e.g. in the cases c) and d). Therefore we assume a systematic error of $\pm 10\%$ which represents an upper limit and determine the φ_i -values without applying corrections. All $\langle F^i \rangle$ and φ_i values refer to a sample of charged tracks with $p_{\perp} \geq 0.15$ GeV/c.

4 Experimental Results

Before presenting the slope parameters φ_i which indicate the fine structure of the FM, we show that the FM obtained by UA1 in large bins are compatible with those from other experiments.

Fig. 2 shows FM values in the central region with $\delta\eta = 3$ together with published data at other energies [18]. The UA1 values confirm the observed energy dependence very well.

We concentrate here on the study of the variation of the $\langle F^i \rangle$ with the bin size in terms of the slope parameters φ_i according to equ.(2). No previous measurement exists at collider energies for $\delta\eta < 0.4$. Preliminary UA1 data have been given in ref.[19]. The analysis has been performed (unless stated otherwise) in the central pseudorapidity range $\eta = [-1.5, 1.5]$.

Table 1 gives the fitted φ_i values ($i = 2, \dots, 5$) from the FM analysis in the variables η , y and ϕ . All φ_i values are significantly greater than zero and indicate intermittency behaviour. Figs.3a-c are plots of $\log \langle F^i \rangle$ vs. $\log(1/\delta\eta)$ or $\log(1/\delta\phi)$ to show the quality of linearity. The linear fits in the region $1 \geq \delta\eta(\delta\phi) \geq 0.1$ are indicated as full lines. The extrapolation of these lines outside the fit regions are drawn as dashed lines. In fig.3a we observe a strong rise of the FM with decreasing bin size in the region $\delta\eta > 1$. For $\delta\eta < 1$ the slope is much smaller but still significant. This behavior is in qualitative agreement with the results at lower energy [6]. The dependence of $\log \langle F^i \rangle$ on the binning in the azimuthal angle ϕ in figs.3b and 3c shows for $\delta\phi < \pi$ a similar behaviour to that in $\delta\eta$ bins. The higher $\langle F^i \rangle$ values in the first bin with $\delta\phi = 6 \approx 2\pi$ are caused by correlations induced by $p_{\perp}$ compensation. This effect is reproduced by Monte Carlo Models (e.g. GENCL, PYTHIA).

The φ_i values do not vary significantly over the whole rapidity plateau region. This is shown in fig. 4a-d for all FM orders $i = 2-5$.

Fig.5 shows a comparison of the UA1 slope parameters φ_i with those at lower energy [6] and those from heavy ion reactions [5]. The slopes of our total sample are much smaller than the NA22 values, but the slopes of a low multiplicity sample are larger than those observed at lower energy. (In the sample with $n_{ch} \leq 15$ the value of φ_5 has not been measured because the mean number of tracks is too low in the small $\delta\eta$ bins - see e) discussed in section 3.)

5 Search for the Origin of Intermittency, Comparison with Monte Carlo models

In this section we consider whether the intermittency signal can be explained in terms of known physics.

The following effects are discussed:

- a) *Resonance and cluster production.* In principle every resonance or decaying particle (like K_s^0 , Λ) imposes correlations. Special attention should be attributed to resonances with masses near threshold (e.g. $\phi_{1020} \rightarrow KK$) where the decay particles are close to each other in rapidity and azimuthal angle.
- b) *Jet production.* High $p_\perp$ jets consist of many particles emitted in a small angular region giving rise to clusters in rapidity and azimuthal angle [20]. Indeed, we verified in a Monte Carlo study that jets emitted at 90° with respect to the collision axis produce rising FM in the region $\delta y < 1$.
- c) *Bose-Einstein (B.E.) correlations.* An enhancement in the production of pions of same charge and similar momentum occurring in all types of reactions is a natural candidate for producing intermittency both in rapidity and azimuth [21].

The following Monte Carlo models which include some or all of these effects have been considered:

- PYTHIA (Version 4.8) [22], a model based on the LUND-string fragmentation mechanism, including the production and decay of K_s^0 , Λ , vector and pseudoscalar resonances (e.g. ρ , K_{890}^* , ω , η , η' , ϕ), spin 1/2 baryons (" Σ -like", " Λ -like") and spin 3/2 baryons. It also includes hard parton-parton scattering according to perturbative QCD with jet production ($p_{\perp min} \sim 1.6$ GeV/c) and multiple hard interactions in one event. This model reproduces the observed particle multiplicity distribution, as well as low $p_\perp$ jet and multijet production [23]. The jet fragmentation proceeds according to the Lund-shower model [24].
- PYTHIA+B.E. which includes the Bose Einstein correlations [25] in a phenomenological way: after the basic event generation according to the standard PYTHIA model, identical particles nearby in phase space are shifted towards smaller invariant masses by "final state interactions". The shift is chosen so as to reproduce the parametrized³ two particle enhancement close to the kinematical threshold.

³A simple exponential parametrization has been chosen with an effective source radius of 1 fm and the incoherence parameter 1 (i.e. a factor 2 enhancement at vanishing momentum separation).

- GENCL, the UA5 cluster model [26], which includes the production and decay of resonances and clusters. It accounts for the measured particle multiplicity and transverse momentum distributions, short range rapidity correlations and the dependence of the average transverse momentum on the event multiplicity.
- FRITIOF (Version 3.1) [27] which differs from PYTHIA by the treatment of the soft multigluon radiation.

For each model about 100,000 events have been generated.

In the following discussion the measured φ_i values are compared with the model predictions⁴ in table 2 and fig.6. Table 2 shows φ_i for two data sets: (i) using only low $p_\perp$ tracks ($p_\perp < 0.5 \text{ GeV}/c$) and (ii) using only tracks with $p_\perp \geq 0.5 \text{ GeV}/c$. Fig.6 shows the dependence of the φ_i on the total number n_{ch} of observed charged tracks in every event.

Several conclusions can be drawn:

In general, the models which include jet production (PYTHIA, FRITIOF) overestimate the φ_i values ($i = 3,4,5$) in the high $p_\perp$ sample but underestimate them in the low $p_\perp$ sample. The cluster model GENCL can neither reproduce the φ_i values of the high $p_\perp$ sample nor of the low $p_\perp$ sample in table 2.

Now we turn to the effects a)-c) quoted above:

- a) Resonance production is included in all models. However, apart from their failure to reproduce the signal in the low $p_\perp$ sample⁵ (table 2) they also underestimate the φ_i values at low multiplicity (fig.6). We again draw attention to GENCL which also includes cluster production and reproduces well the known short-range correlations in rapidity [28].
- b) By comparing PYTHIA with the data in table 2 and fig.6, we conclude that the intermittency signal from jet production is expected to show up mainly in the high $p_\perp$ sample but is far too weak to explain the low $p_\perp$ and low multiplicity data. Moreover, fig.7 shows φ_i values for a jet-enriched sample (events where the UA1 jet-algorithm [29] found at least one jet with transverse energy $E_\perp \geq 5 \text{ GeV}$) and are compared with φ_i values in a sample with no such jets. No significant difference can be found.
- c) A comparison between PYTHIA and PYTHIA + B.E. shows that the intermittency signal is indeed enhanced by the inclusion of Bose-Einstein correlations and the experimental data at high multiplicity (fig.6) are almost

⁴The model calculations have been performed without taking into account the influence of the detector. However, as shown in section 3 the influence is small. The resulting additional uncertainty is included in the error bars of the model calculations.

⁵The negative φ_i values obtained in some models indicate even decreasing FM values with decreasing bin size, see also e) of section 3.

reproduced. However, the low multiplicity and low $p_{\perp}$ samples are still left unexplained. It is possible that the implementation of the B.E. effect in the Monte Carlo can be improved: there is a striking similarity in the multiplicity dependence of the strength of the B.E. effect [30] and the strength of the intermittency signal which leads to the suspicion that there may be a close connection between those effects. Here, we investigate whether the B.E. effect is the main source of intermittency. It has been pointed out in ref. [21], and is intuitively evident, that in this case we should expect a much stronger signal (twice as large) if we measure φ_i from positively or negatively charged particles separately. In table 3 the φ_i values for each charge separately are compared to those for all charges. We do not see a significant difference and conclude that the B.E. effect is too weak to be the main source of intermittency.

To illustrate the failure of the models to reproduce the low multiplicity data in more detail, fig.8 displays a comparison of the dependence of $\langle F^3 \rangle$ on the bin size $\delta\eta$.

Fig.8 shows that the GENCL model reproduces the data very well for $\delta\eta \gtrsim 0.3$ and that the discrepancy arises only in the region of very small $\delta\eta$. However, GENCL is only an "ad hoc model aiming to reproduce as many features of particle production as possible" [26]. On the contrary, PYTHIA expresses our knowledge about the dynamics of particle interactions [22]. Fig.8 shows that we do not really understand the physical origin of short range correlations yet - not only at very small $\delta\eta$, but also in the region $\delta\eta \approx 1$.

We summarize the situation: known mechanisms like resonance production, jet production and the B.E. effect produce rising factorial moments in the region $1 \geq \delta\eta \geq 0.1$, but the current Monte Carlo models which include them are inadequate to reproduce the intermittency signal observed in the data at low $p_{\perp}$ and at low multiplicity as shown in table 2 and figs. 6,8.

6 Description of Data in Terms of Multifractals

A relation between the slopes φ_i and generalized dimensions D_i [31] has been established in ref. [12]: $D_i = 1 - \varphi_i/(i - 1)$. These dimensions are commonly used for the description of fractals and multifractals in other domains of physics. Moreover, it has been suggested that QCD-branching might exhibit fractal behaviour [32,33].

The intuitive meaning of the generalized dimensions becomes clearer in the two limiting cases:

- i) if no intermittency is present, i.e. all $\varphi_i = 0$, all dimensions D_i are equal to 1, meaning that the particles are equally distributed in the considered rapidity interval.

- ii) at maximum slopes, i.e. $\varphi_i = (i - 1)$, the dimensions are equal to 0. This (unlikely) situation occurs if *all* particles lie *always* within one rapidity bin — no matter how small the bin size δy is.

In fig.9 a compilation of generalized dimensions D_i is given for the UA1 results and recent data of other reactions. As has been pointed out in [12], the signals observed in particle reactions are generally rather weak as compared, for example, to measurements in fluid mechanics, but consistent with the picture of independent superposition of more elementary sources, each of which exhibits a much stronger fractality (i.e. lower dimensions).

7 Summary

- (1) A power law dependence of $\langle F^i \rangle$ on the bin size (intermittency) is observed in the data when analysed in pseudorapidity, rapidity or azimuthal angle bins in the region $1 \geq \delta\eta(\delta y, \delta\phi) \geq 0.1$. This rise is expressed by the slope parameters φ_i which are significantly greater than zero and range from 0.01 to 0.15 for the orders $i = 2$ to 5, respectively.
- (2) The φ_i values increase with decreasing event multiplicity.
- (3) The φ_i values at $\sqrt{s} = 630$ GeV are smaller than those at lower energy ($\sqrt{s} = 22$ GeV). However, by selecting a low multiplicity sample, φ_i values comparable to or even larger than those at low energy are obtained. Therefore the observed energy dependence reflects a multiplicity dependence.
- (4) The φ_i values obtained from samples of positively or negatively charged particles only are equal to those from the total sample, suggesting that the Bose-Einstein effect is not the main source of intermittency.
- (5) There are known mechanisms such as resonance and cluster production, jet production and the Bose-Einstein effect which can produce rising factorial moments in the region $1 \geq \delta\eta \geq 0.1$. Monte Carlo models which include these effects (PYTHIA, GENCL, FRITIOF) have been compared to the data: none of the models can, however, reproduce the observed intermittency signal in a sample of low $p_\perp$ tracks or at low multiplicity. Further studies are necessary to find the origin of the effect.
- (6) Adopting the assumption of ref.[4] that one can extract with the FM analysis the underlying dynamical fluctuations from the observed fluctuations of particle numbers per bin, the generalized dimensions D_i of a multifractal description have been calculated. The data in the (1-dimensional) rapidity space show a very weak fractality with D_i values only slightly below 1.

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Table 1: φ_i values from FM analysis in the variables η, y, ϕ . The φ_i were determined by fits to $\langle F^i \rangle = (C_i/\delta y)\varphi_i$ with two free parameters C_i and φ_i using the MINUIT program. The fit region is $1 \geq \delta\eta(\delta y, \delta\phi) \geq 0.1$. The first error is statistical and the second error is systematic.

Variable	φ_2		φ_3		φ_4		φ_5	
η	0.011	± 0.001	0.025	± 0.003	0.050	± 0.005	0.077	± 0.020
		± 0.001		± 0.0025		± 0.005		± 0.008
y	0.012	± 0.001	0.028	± 0.002	0.049	± 0.005	0.062	± 0.020
		± 0.001		± 0.003		± 0.005		± 0.006
ϕ	0.010	± 0.001	0.027	± 0.002	0.077	± 0.006	0.152	± 0.030
		± 0.001		± 0.003		± 0.008		± 0.015

Table 2: φ_i values of the whole event sample (i) where only low $p_\perp$ tracks were selected ($0.15 \leq p_\perp \leq 0.5$ GeV/c) and (ii) where only high $p_\perp$ tracks were selected ($p_\perp \geq 0.5$ GeV/c). The same cuts were applied to the Monte Carlo generated events.

SELECTION		DATA	GENCL	PYTHIA	PYTHIA+B.E.	FRITIOF
$\langle F^2 \rangle$	(i)	0.015 ± 0.001 ± 0.0015	0.005 ± 0.002	0.0002 ± 0.002	0.007 ± 0.002	0.007 ± 0.002
	(ii)	0.015 ± 0.002 ± 0.0015	0.009 ± 0.002	0.013 ± 0.003	0.022 ± 0.003	0.034 ± 0.003
$\langle F^3 \rangle$	(i)	0.037 ± 0.003 ± 0.004	0.005 ± 0.004	-0.001 ± 0.006	0.010 ± 0.005	0.016 ± 0.004
	(ii)	0.045 ± 0.005 ± 0.005	0.021 ± 0.006	0.061 ± 0.009	0.080 ± 0.010	0.143 ± 0.009
$\langle F^4 \rangle$	(i)	0.070 ± 0.009 ± 0.007	0.003 ± 0.010	-0.009 ± 0.011	-0.002 ± 0.011	0.015 ± 0.010
	(ii)	0.103 ± 0.013 ± 0.010	0.015 ± 0.013	0.118 ± 0.026	0.190 ± 0.035	0.347 ± 0.021
$\langle F^5 \rangle$	(i)	0.096 ± 0.024 ± 0.010	0.005 ± 0.023	-0.035 ± 0.024	-0.077 ± 0.023	0.006 ± 0.023
	(ii)	0.115 ± 0.032 ± 0.012	-0.115 ± 0.030	0.130 ± 0.062	0.228 ± 0.093	1.105 ± 0.013

Table 3: Comparison of φ_i values of positively charged particles and negatively charged particles with the slopes φ_i of all charged particles. Analysis in η .

sample	φ_2		φ_3		φ_4	
positively charged particles	0.012	± 0.002	0.030	± 0.004	0.052	± 0.011
		± 0.001		± 0.003		± 0.005
negatively charged particles	0.011	± 0.002	0.025	± 0.004	0.045	± 0.011
		± 0.001		± 0.003		± 0.005
all charged particles	0.011	± 0.001	0.025	± 0.003	0.050	± 0.005
		± 0.001		± 0.003		± 0.005

Figure Captions

- Fig.1: Monte Carlo simulation of the influence of acceptance loss in the UA1 central detector on the FM. Full circles: $_{10}\log \langle F^4 \rangle$ from the original event sample generated by PYTHIA. Open circles: $_{10}\log \langle F^4 \rangle$ after application of the detector acceptance.
- Fig.2: Energy dependence of $\langle F^2 \rangle - \langle F^5 \rangle$. Open squares: the FM values of this experiment after correction for acceptance loss. Full circles: values from UA5 [18]. The values at $\sqrt{s} = 540$ GeV are obtained from the k-parameter of a negative binomial fit.
- Fig.3a),b) Dependence of $_{10}\log \langle F^i \rangle$ on the bin size $\delta\eta$, $\delta\phi$, respectively.
c) Dependence of $_{10}\log \langle F^2 \rangle$ on the bin size $\delta\phi$ at a magnified scale.
- Fig.4: Dependence of the slope parameters φ_i on pseudorapidity for each order i separately. The full circles are displayed in the centre of each analysed pseudorapidity interval with an original width of $\Delta\eta = 2$. The crosses are obtained in the region $1.6 \leq |\eta| \leq 2.4$ where the detector acceptance is almost 1.
- Fig.5: Comparison of φ_i values from this experiment to other measurements in hadron-hadron and heavy ion interactions for each order i separately.
- Fig.6: The slope parameters φ_i for three distinct multiplicity samples as indicated. The distances on the abscissa are arbitrary. For easy comparison, the three experimental values and also the corresponding three values of each Monte Carlo model are connected by lines.
- Fig.7: The slope parameters φ_i of the minimum bias sample are compared to a jet-enriched sample and a sample of events in which no jets were found.
- Fig.8: A comparison of $\langle F^3 \rangle$ vs $\delta\eta$ at low multiplicity (total observed charged multiplicity $n_{ch} \leq 15$) with Monte Carlo predictions.
- Fig.9: A comparison of the generalized dimensions $D_i = 1 - \varphi_i/(i - 1)$ in various reaction types. The dimensions from the NA22, KLM and the UA1 samples correspond to the φ_i values shown in Fig.5. Estimated dimensions from two e^+e^- -samples (all and a twojet (low sphericity) selection) of the HRS experiment [7] are added. For clarity only some representative error bars are drawn.

Fig. 1

Fig. 2

Fig. 3 a

Fig. 3 b

Fig. 3 c

Fig. 4

Fig. 5.

Fig. 6

Fig. 7

Fig. 8

Fig. 9